Title: Echo-Diversity in Binary BH inspiral

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Collection: Echoes in Southern Ontario

Date: February 24, 2020 - 10:15 AM

URL: http://pirsa.org/20020086

Orbital Effects in Perturbations around Exotic Compact Objects

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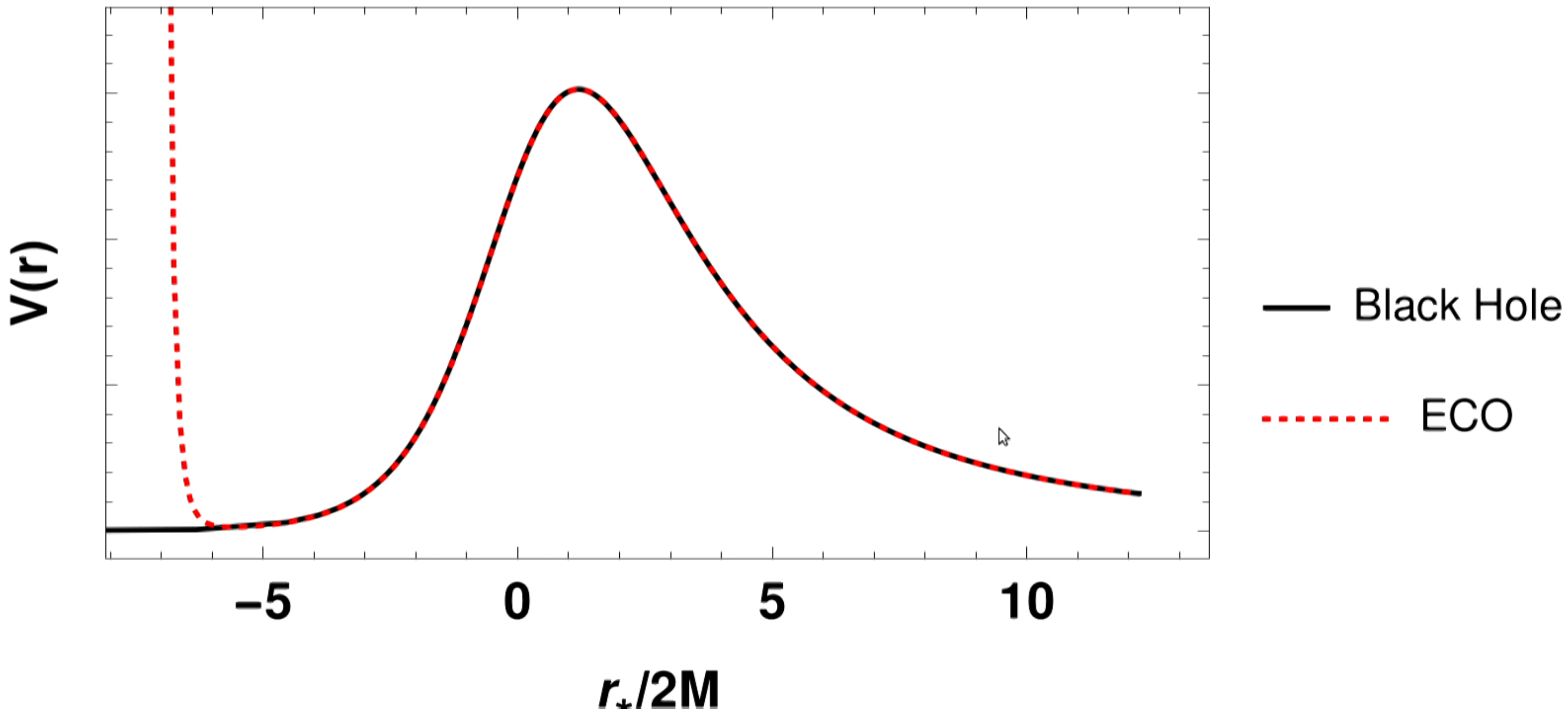
24/02/2020

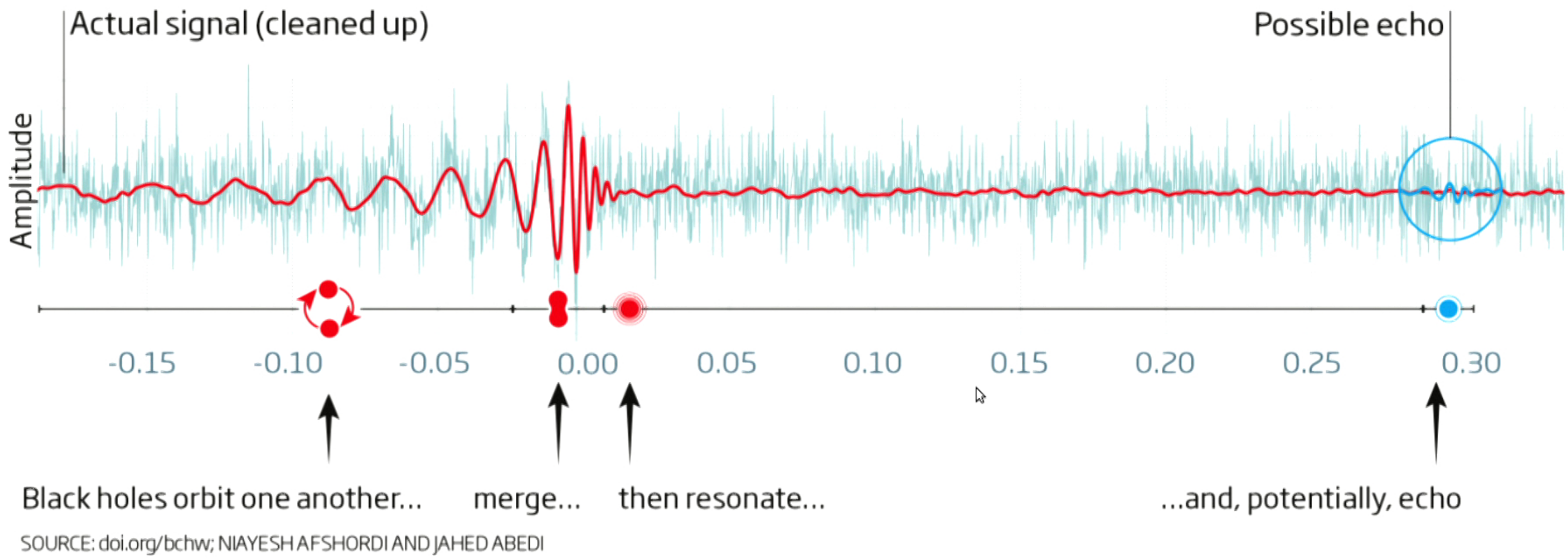
Introduction

- Black Holes or Exotic Compact Objects (ECOs)?
- ECOs well-approximated by Kerr metric outside horizon
- Partial reflection near (would-be) horizon
- Study the perturbative response for different orbital motions
- Why should we care?
  - Test for quantum nature of black holes
  - Black Hole Information (a.k.a. Firewall) paradox
  - Probe the existence of event horizons

Introduction



Data: Tentative Evidence?



Data: Tentative Evidence?

- Do observations support such claims?
  - First tentative evidence: Abedi, Dykaar & Afshordi 2017
  - Several groups found evidence at varying degrees of confidence:
  - p-values ranging from 1.6×10^{-5} to 0.9 (see arXiv:2001.00821)
  - Smaller p-values were encountered for more extreme mass ratios
  - Do we expect any dependence of echoes on the mass ratio of the binary?

Previous Work

- Most works used a gaussian initial conditions
- Only two papers account for orbital motion :
  - arXiv:1912.05419 (L. Micchi and C. Chirenti'19)
  - Phys. Rev. D, 96, 8, 084002 (Z. Mark et all'17)
- They only account for scalar waves and geodesic motion
- Goal: account for possible differences between different mass ratios such as :
  - Which case leads to a large echo amplitude?
  - Resonances due to the orbital motion?

Teukolsky equation

- During this work we looked for solutions of the radial Teukolsky equation:

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d\,{}_sR_{lm\omega}(r)}{dr}\right)+ \tag{1}$$
$$+\left(\frac{K^2-2is(r-M)K}{\Delta}+4is\omega r-{}_s\lambda_{lmc}\right){}_sR_{lm\omega}(r)=S_{lm\omega}(r),$$

- where $\Delta = r^2 - 2Mr + a^2$, $c \equiv a\omega$, $K \equiv (r^2 + a^2)\omega - am$
- ${}_s\lambda_{lmc}$ is a constant coming from the separation of variables

Formulation: Homogeneous Solutions of the Teukolsky equation

- The homogeneous solutions (for s=-2) are defined by their asymptotic behaviour as:

$$R^{\mathrm{in}}_{lm\omega} \sim \begin{cases} B^{\mathrm{trans}}_{lm\omega}\Delta^2 e^{-ikr_*}, & \text{for} \quad r \to r_+ \\ B^{\mathrm{ref}}_{lm\omega} e^{i\omega r_*} r^3 + B^{\mathrm{inc}}_{lm\omega} \dfrac{e^{-i\omega r_*}}{r}, & \text{for} \quad r \to \infty \end{cases} \tag{2}$$

$$R^{\mathrm{up}}_{lm\omega} \sim \begin{cases} C^{\mathrm{ref}}_{lm\omega}\Delta^2 e^{-ikr_*} + C^{\mathrm{inc}}_{lm\omega} e^{ikr_*}, & \text{for} \quad r \to r_+ \\ C^{\mathrm{trans}}_{lm\omega} e^{i\omega r_*} r^3, & \text{for} \quad r \to \infty \end{cases} \tag{3}$$

Formulation: Green's Function

- For the black hole case the GF reads :

$$G^{BH}_{lm\omega}(r|r') = \frac{R^{up}_{lm\omega}(r)R^{in}_{lm\omega}(r')}{W_{lm\omega}}\Theta(r-r') + \frac{R^{up}_{lm\omega}(r')R^{in}_{lm\omega}(r)}{W_{lm\omega}}\Theta(r'-r)$$

- Where $R^{in}_{lm\omega}$ and $R^{up}_{lm\omega}$ are homogeneous solutions of the Teukolsky equation

Formulation: Usual BH response

- Assume a point mass stress energy-tensor
- Integrate the source term against the GF
- Find that there will be two terms, the usual black hole response at infinity

$$R^{\infty}_{BH\,lm\omega} \propto \frac{r^3 e^{i\omega r_*}}{2i\omega B^{inc}_{lm\omega}} \int_{r_+}^{\infty} dr R^{in}_{lm\omega}(r_p(\tau))_0 T_{lm\omega} \Delta^{-2} = Z^{\infty}_{BH\,lm\omega} r^3 e^{i\omega r_*},$$

- and the usual black hole response at the horizon

$$\Delta^2 e^{-ikr_*} R^{trans} \quad \int^{\infty}$$

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$$R^{H}_{BH\,lm\omega} \propto \frac{\Delta^2 e^{-ikr_*} B^{trans}_{lm\omega}}{2i\omega B^{inc}_{lm\omega} C^{trans}_{lm\omega}} \int_{r_+}^{\infty} dr R^{up}_{lm\omega}(r_p(\tau))_0 T_{lm\omega} \Delta^{-2} = Z^{H}_{BH\,lm\omega} \Delta^2 e^{-ikr_*}$$

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$$R^{\rm up}_{lm\omega} \sim \begin{cases} C^{\rm ref}_{lm\omega}\Delta^2 e^{-ikr_*} + C^{\rm inc}_{lm\omega} e^{ikr_*}, & \text{for} \quad r \to r_+ \\ C^{\rm trans}_{lm\omega} e^{i\omega r_*} r^3, & \text{for} \quad r \to \infty \end{cases} \tag{3}$$

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Transforming to Sasaki-Nakamura formalism

- The homogeneous solutions for SN equation are defined by their asymptotic behaviour as:

$$ {}_{SN}R^{\rm in}_{lm\omega} \sim \begin{cases} {}_{SN}B^{\rm trans}_{lm\omega} e^{-ikr_*}, & \text{for} \quad r \to r_+ \\ {}_{SN}B^{\rm ref}_{lm\omega} e^{i\omega r_*} + {}_{SN}B^{\rm inc}_{lm\omega} e^{-i\omega r_*}, & \text{for} \quad r \to \infty \end{cases} \tag{4} $$

$$ {}_{SN}R^{\rm up}_{lm\omega} \sim \begin{cases} {}_{SN}C^{\rm ref}_{lm\omega} e^{-ikr_*} + {}_{SN}C^{\rm inc}_{lm\omega} e^{ikr_*}, & \text{for} \quad r \to r_+ \\ {}_{SN}C^{\rm trans}_{lm\omega} e^{i\omega r_*}, & \text{for} \quad r \to \infty \end{cases} \tag{5} $$

Transforming to Sasaki-Nakamura formalism

- For the Sasaki-Nakamura equation we construct the Green function in a similar way as in the Teukolsky but choosing a different choice of homogenous solution
- First we normalize the homogeneous solutions as :

$$ {}_{SN}\bar{R}^{\mathrm{up}}_{lm\omega} = \frac{{}_{SN}R^{\mathrm{up}}_{lm\omega}}{{}_{SN}C^{\mathrm{trans}}_{lm\omega}} \tag{6} $$

$$ {}_{SN}\bar{R}^{\mathrm{in}}_{lm\omega} = \frac{{}_{SN}R^{\mathrm{in}}_{lm\omega}}{{}_{SN}B^{\mathrm{trans}}_{lm\omega}} \tag{7} $$

- and construct

$$ {}_{SN}\bar{R}^{\mathrm{ECO}}_{lm\omega} = {}_{SN}\bar{R}^{\mathrm{in}}_{lm\omega} + K_{lm\omega}\,{}_{SN}\bar{R}^{\mathrm{up}}_{lm\omega} \tag{8} $$

- Our Green function is constructed by using the pair $\bar{R}^{\mathrm{up}}_{lm\omega}$ and $\bar{R}^{\mathrm{ECO}}_{lm\omega}$

Echoes in SN : Transfer function

- The function $K_{lm\omega}$ is computed in order to impose that the out-going flux of the $\bar{R}^{\rm ECO}_{lm\omega}$ solution at r_0 is proportional to its in-going flux also at r_0
- It can be shown to be

$$K_{lm\omega} = \frac{|b_0|_{SN} C^{trans}_{lm\omega}}{|C|_{SN} C^{inc}_{lm\omega}} \frac{Re^{-2ikr^0_*}}{1 - (|b_0|_{SN} C^{ref}_{lm\omega} / |C|_{SN} C^{inc}_{lm\omega}) Re^{-2ikr^0_*}}, \quad (9)$$

- which in terms of the Teukolsky quantities reads

$$K_{lm\omega} = \frac{-|b_0|c_0 C^{trans}_{lm\omega}}{4|C|\omega^2 g C^{inc}_{lm\omega}} \frac{Re^{-2ikr^0_*}}{1 - (|b_0| d C^{ref}_{lm\omega} / |C| g C^{inc}_{lm\omega}) Re^{-2ikr^0_*}} \quad (10)$$

- R is the proportionality constant that accounts for the reflectivity
- b_0,c_0,d,g and C are frequency dependent quantities that appear in the SN-Teulkosky transformations

Echoes Response in Sasaki-Nakamura

- The final ECO response will then be given by :

$$X = (X^{\infty}_{BH\,lm\omega} + K_{lm\omega} X^{H}_{BH\,lm\omega}) e^{i\omega r_*} \tag{11}$$

- In terms of the Teukoslky quantities we can find that

$$X = -(1 + \frac{c_0}{4d\omega^2} K_{lm\omega}) \frac{c_0}{4\omega^2} Z^{\infty}_{BH\,lm\omega} \tag{12}$$

- where we used the Sasaki-Nakamura transformation for obtaining $X^{\infty}_{BH\,lm\omega}$
- where we used and approximation to write $X^{H}_{BH\,lm\omega}$ in terms of $X^{\infty}_{BH\,lm\omega}$
- Such an approximation discussed in arxiv:1712.06517 should be valid near the QNM frequency

Chosen Orbits: Adiabatic Inspiral + Isco Plunge

- In order to account for the orbital effects we chose two different plunges, varying the mass of the secondary body

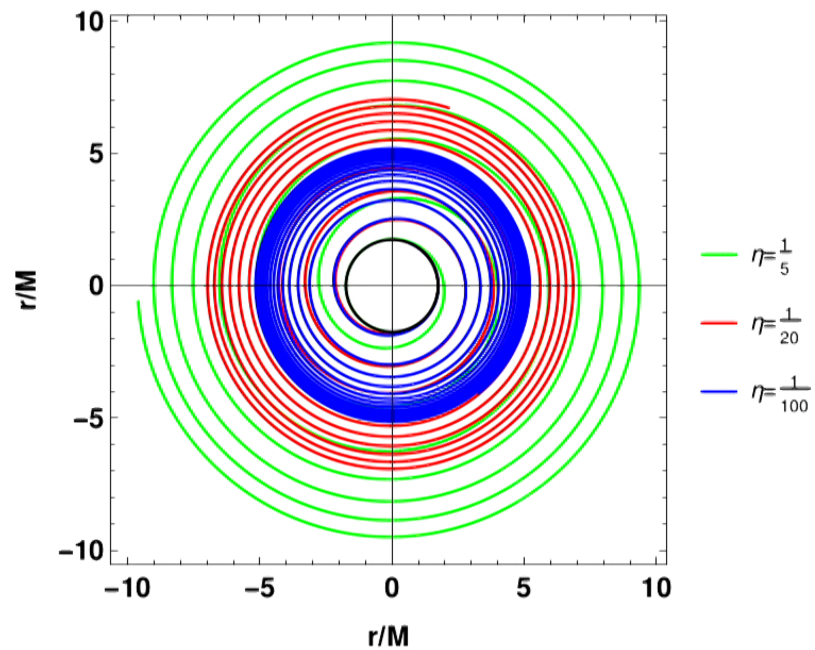

Figure: Example of orbits for three different mass ratios for spinning central object of $a = 0.67M$.

Results: $K_{lm\omega}$ transfer function

- arXiv:1905.00464 described quantum black holes as having a reflectivity of $R = exp(-|k|/T_H)$
- This reflectivity gives a natural cut-off for frequencies much higher than the QNM frequency

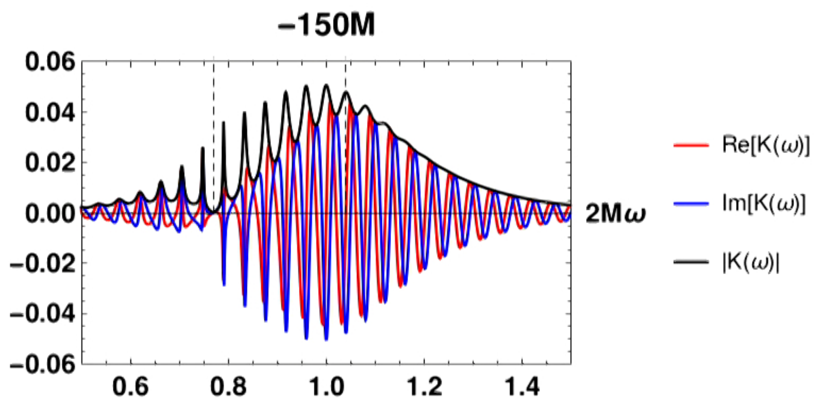

Figure: Transfer function for $a = 0.67M$, $l = m = 2$ and $r_* = -150M$. Vertical lines mark (from left to right) the superradiant bound frequency and the fundamental QNM frequency.

Results: $K_{lm\omega}$ transfer function

- Another proposal found in 2001.11642 shows that we may have multiples of T_H in the reflectivity
- For this reason we also investigate the case of $R = exp(-|k|/2T_H)$

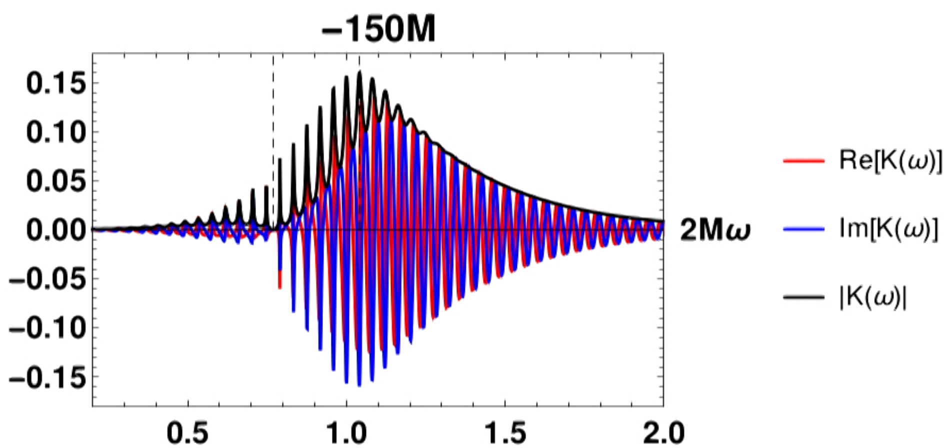

Figure: Transfer function for the same choice of parameters as before but replacing $T_H \to 2T_H$ in the reflectivity.

Results: Standard Waveforms

- First we evolve the standard wave from the orbital motion

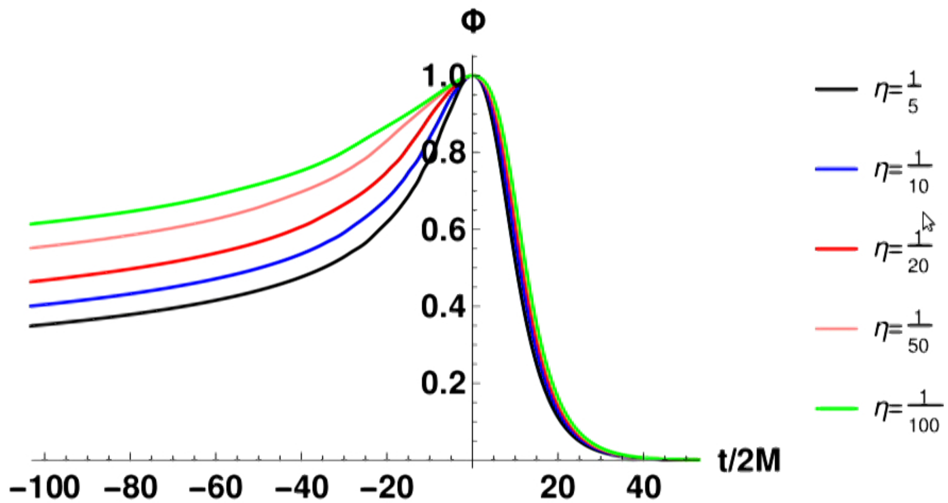

Figure: Amplitude evolution of waveforms with different mass ratios for $a = 0.67M$. All the curves are normalized by the peak of the wave.

Results: Echoes for $R = exp(-|k|/T_H)$

- First we evolve the standard wave from the orbital motion

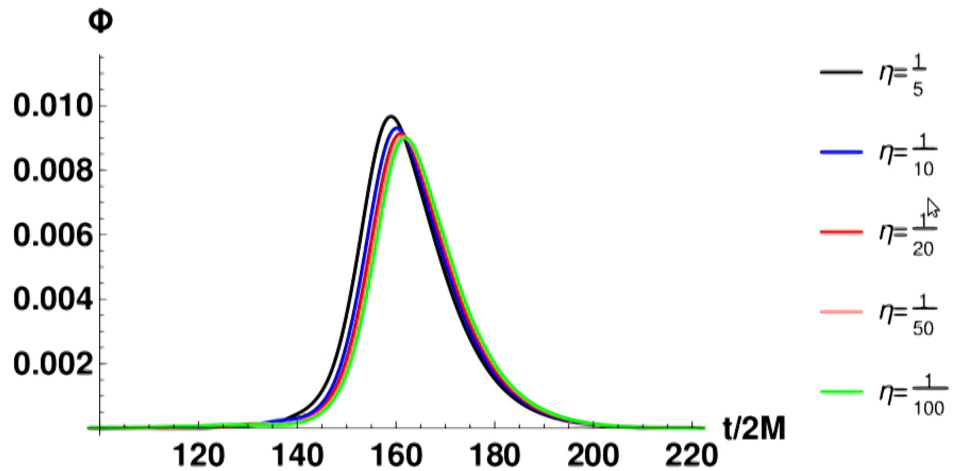

Figure: Amplitude of first echo for different mass ratios η ($a = 0.67M$). All the curves are normalized by the peak of the ringdown wave-form.

Results: Echo amplitude dependence with *eta*

- We notice a dependence on the amplitude of the first echo with the mass ratio

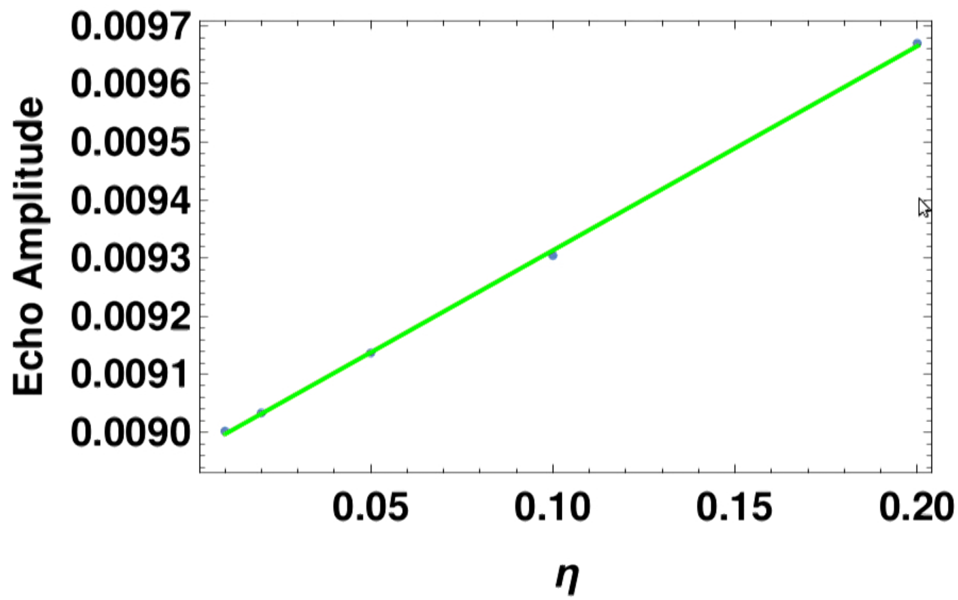

Figure: Amplitude of first echo dependence of *eta* for $R = exp(-|k|/T_H)$.

Results: Echoes for $R = exp(-|k|/2T_H)$

- First we evolve the standard wave from the orbital motion

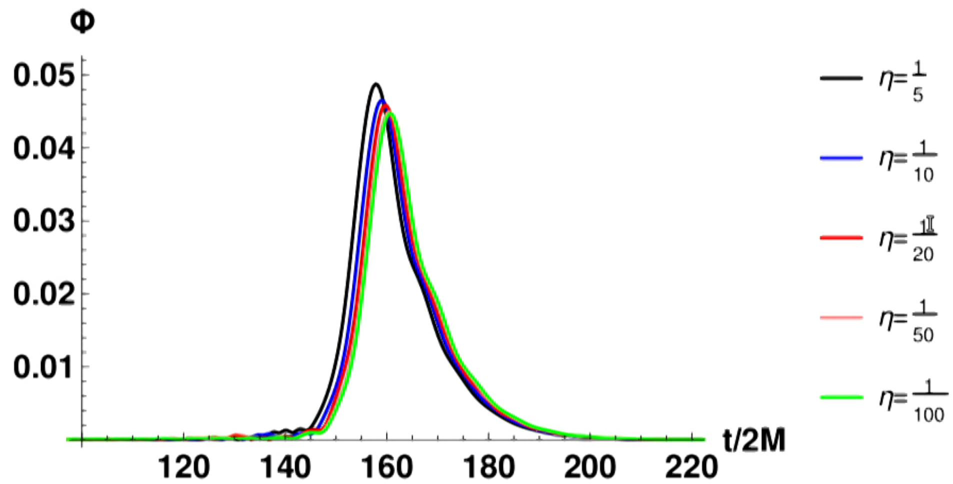

Figure: Amplitude of first echo for different mass ratios η ($a = 0.67M$). All the curves are normalized by the peak of the ringdown wave-form. Now for $R = exp(-|k|/2T_H)$.

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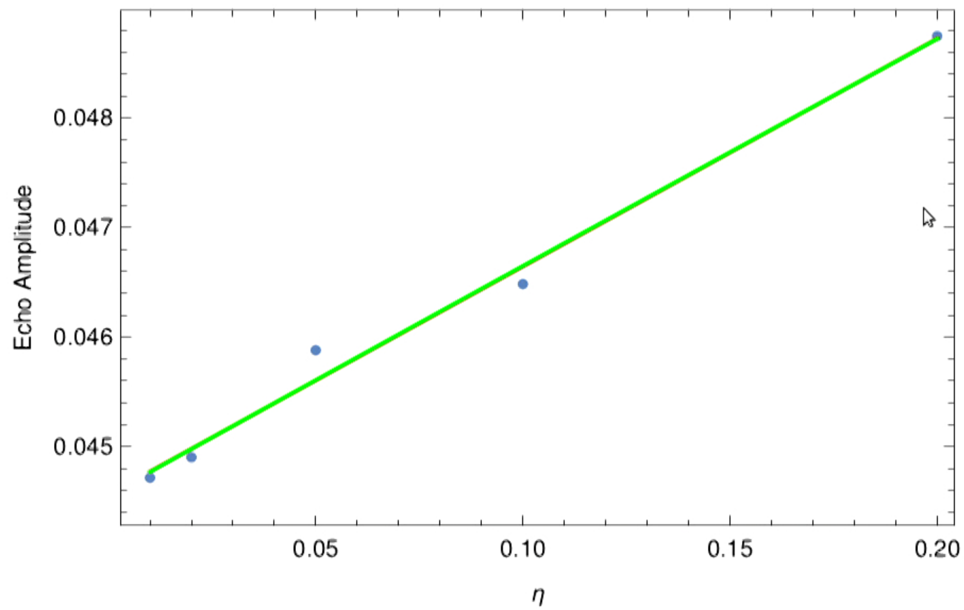

Figure: Amplitude of first echo dependence of *eta* for $R = exp(-|k|/2T_H)$.

Conclusion and Work in Progress

- Smaller mass ratio lead to a smaller relative amplitude of echoes
- Smaller mass ratio lead to a delay in the appearance of echoes
- It seems that larger mass ratios lead to different decay rates of subsequent echoes - under investigation
- Resonances of the orbital phase seems to be negligible, agreement with analytical work in: Phys. Rev. D,100,8,084046 by Cardoso,Río and Kimura