Title: 3d A-twist and analytic continuation of path integrals II

Speakers: Davide Gaiotto

Series: Mathematical Physics

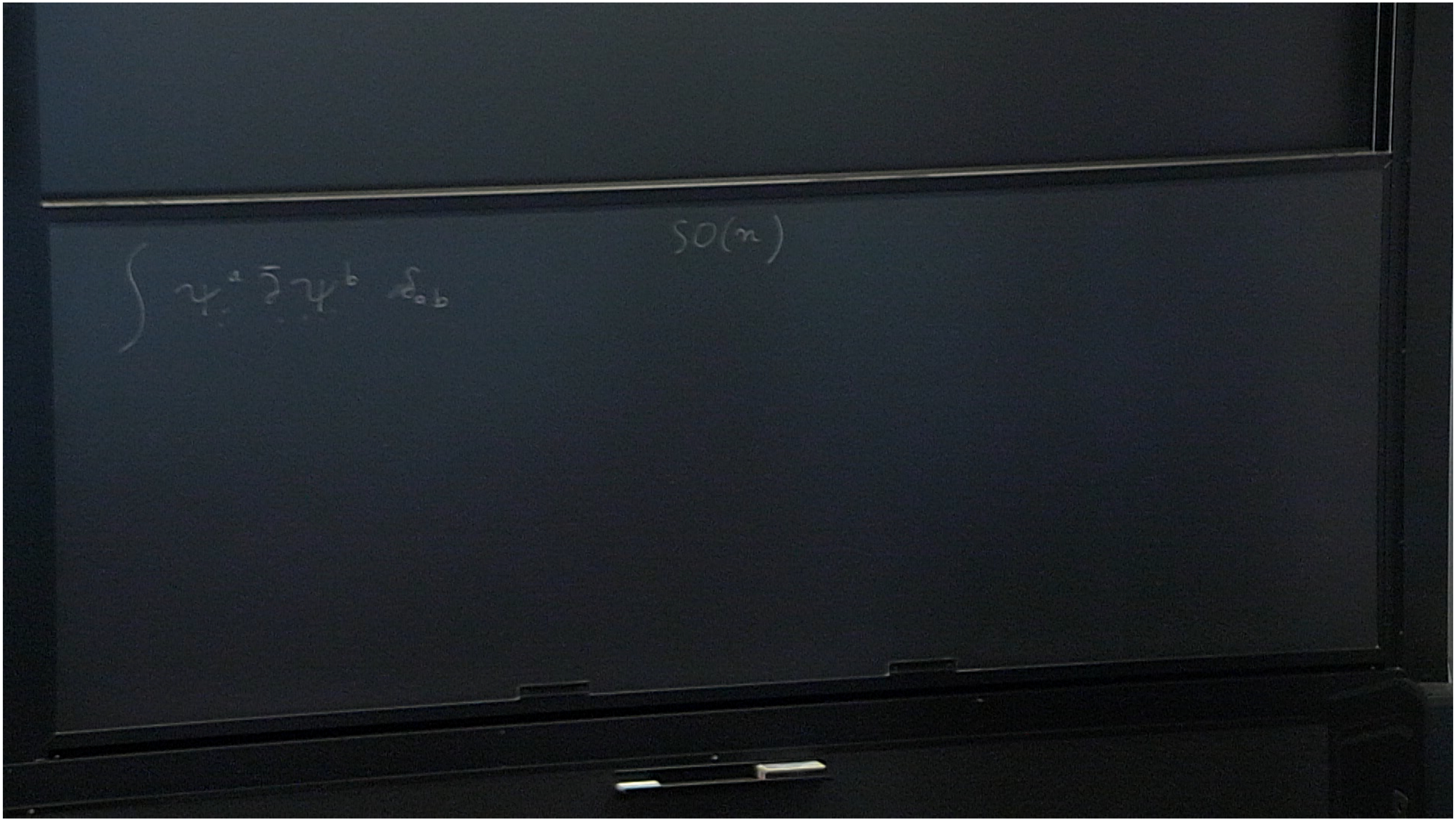
Date: February 27, 2020 - 11:00 AM

URL: http://pirsa.org/20020084

$(\mathbb{C}^{2n}, \omega = dp_I \wedge dq^I)$ $\quad Sp(2n)$

$Z^\alpha \quad \omega_{\alpha\beta} \quad \omega^{\alpha\beta}$

$$\int_\Sigma Z^\alpha \bar\partial Z^\beta \, \omega_{\alpha\beta}$$


$SO(n)$
$\int \psi^a \bar{\partial} \psi^b \delta_{ab}$

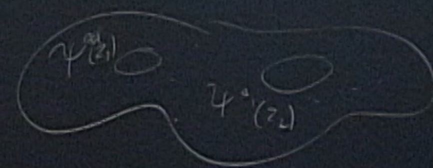
$SO(n)$

$S[\psi] = \int \psi^a \, \bar\partial \psi^b \, \delta_{ab} \, dz \, d\bar z$

DEFINED ON

Σ w/t SPIN STRUCTURE

$\int D\psi \, e^{S[\psi]} = \mathrm{Pfaff}\,\bar\partial$

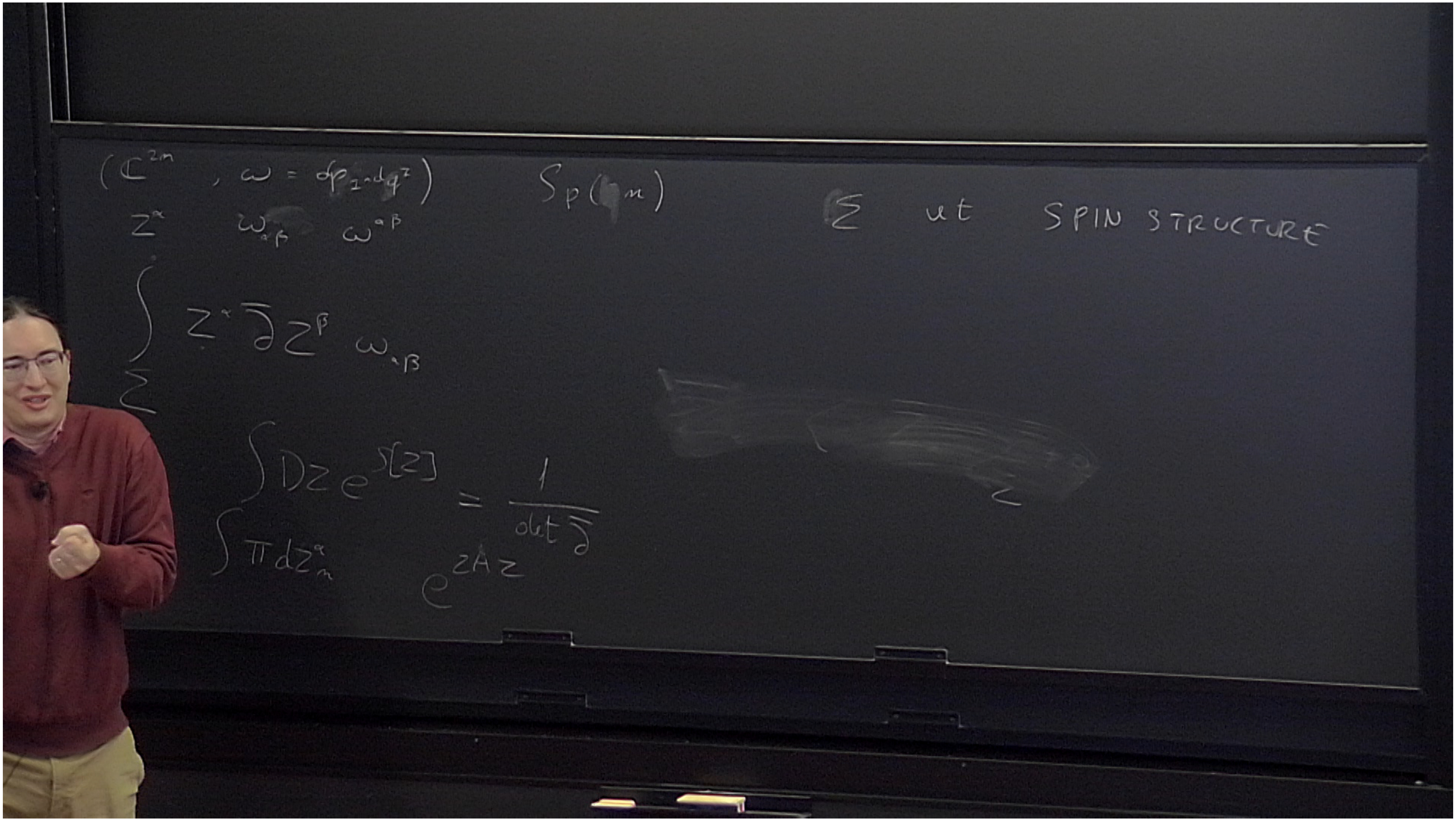


$(\mathbb{C}^{2n}, \omega = dp_1 \wedge dq^1)$ $\quad Sp(n)$ $\quad \Sigma$ $\quad$ w/ $\quad$ SPIN STRUCTURE

$Z^\alpha \quad \omega_{\alpha\beta} \quad \omega^{\alpha\beta}$

$$\int_\Sigma Z^\alpha \bar{\partial} Z^\beta \omega_{\alpha\beta}$$

$$\int DZ e^{S[Z]} = \frac{1}{\det \bar{\partial}}$$


$(\mathbb{C}^{2n}, \omega = dp_i \wedge dq^i)$
$Sp(n)$
Z^α $\omega_{\alpha\beta}$ $\omega^{\alpha\beta}$
Σ ut SPIN STRUCTURE
$\int_\Sigma Z^\alpha \bar\partial Z^\beta \omega_{\alpha\beta}$
$\int DZ\, e^{S[Z]} = \frac{1}{\det \bar\partial}$
$\int \Pi\, dZ^\alpha_n$ e^{ZAZ}

$(\mathbb{C}^{2m}, \omega = dp_I \wedge dq^I)$ $\quad Sp(m)$

$Z^\alpha \quad \omega_{\alpha\beta} \quad \omega^{\alpha\beta}$

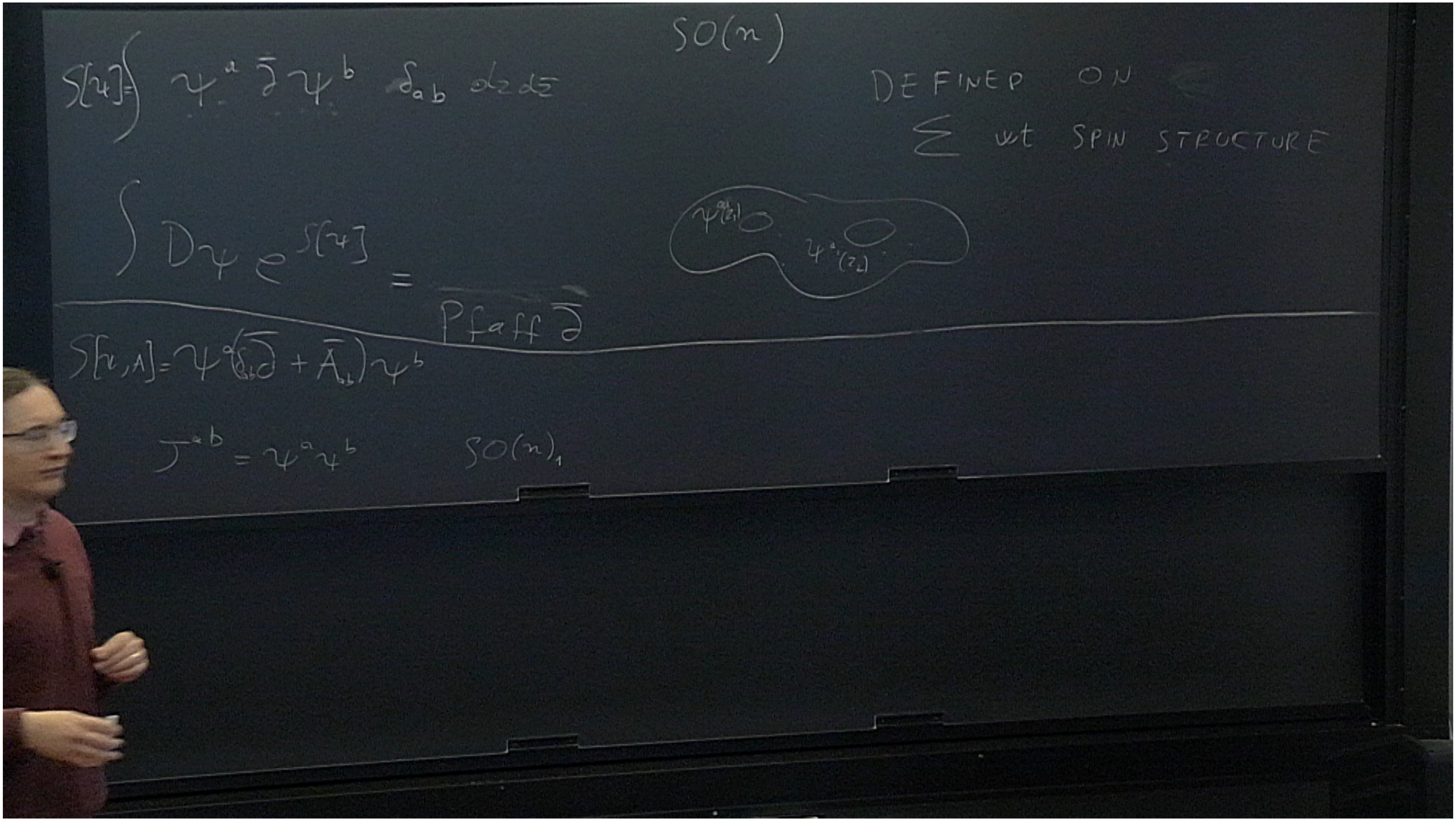
$$\int_\Sigma Z^\alpha \bar{\partial} Z^\beta \omega_{\alpha\beta}$$

$$\int DZ\, e^{S[Z]} = \frac{1}{\det \bar{\partial}}$$

$$\int \prod dZ^\alpha_n \quad e^{ZAZ}$$

Σ ut SPIN STRUCTURE

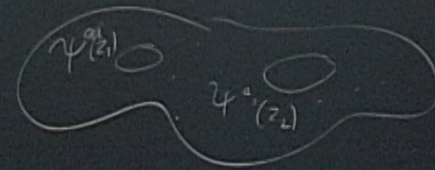
Σ


SO(n)
S[ψ] = ∫ ψ^a ∂̄ ψ^b δ_ab dz dz̄
DEFINED ON
Σ wt SPIN STRUCTURE
∫ Dψ e^S[ψ] = Pfaff ∂̄
ψ^a(z1)
ψ^a(z2)
S[ψ,A] = ψ^a(δ_ab ∂̄ + Ā_ab) ψ^b
J^ab = ψ^a ψ^b
SO(n)_1

$$S[\psi] = \int \psi^a \bar\partial \psi^b \, \delta_{ab} \, dz \, d\bar z$$

DEFINED ON Σ w/t SPIN STRUCTURE

$$\int D\psi \, e^{S[\psi]} = \mathrm{Pfaff}\,\bar\partial$$



$$S[\psi, A] = \psi^a(\delta_{ab}\bar\partial + \bar A_{ab})\psi^b$$

$$\int D\psi \, e^S = \mathrm{Pfaff}(\bar\partial + \bar A) \langle \cdots \rangle_{\bar A + \delta \bar A} = \langle J \delta A \cdots \rangle$$

SECTION OF $\mathcal{L}$ ON $BUN_{SO(n)}$

$J^{ab} = \psi^a \psi^b$ $\quad SO(n)_1$ CURRENTS

$(\mathbb{C}^{2n}, \ \omega = dp_I \wedge dq^I)$ $\qquad Sp(n)$

$Z^\alpha \qquad \omega_{\alpha\beta} \qquad \omega^{\alpha\beta}$

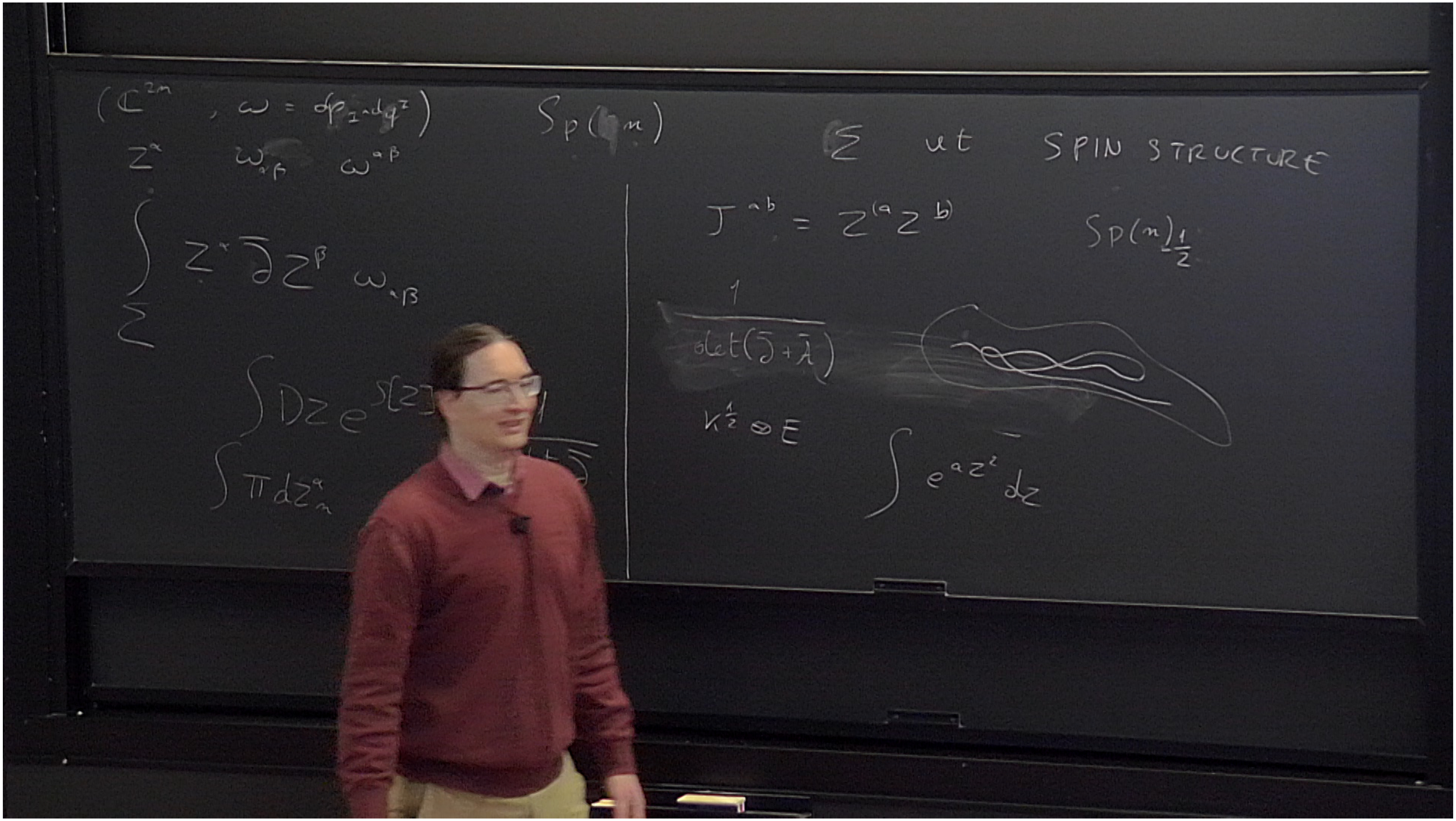
$$\int_\Sigma Z^\alpha \bar\partial Z^\beta \, \omega_{\alpha\beta}$$

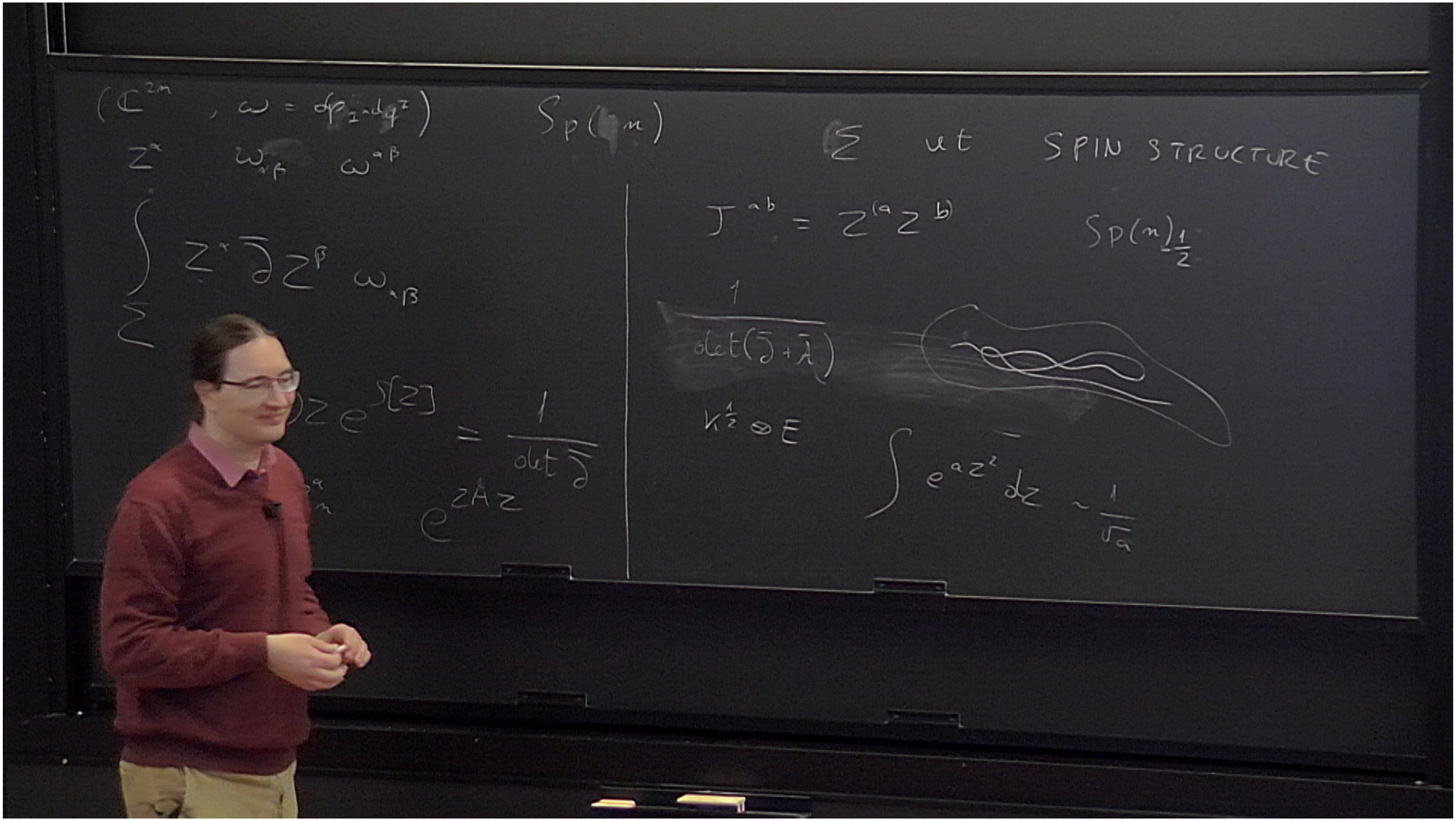
$$\int DZ \, e^{S[Z]} = \frac{1}{\det \bar\partial}$$

$$\int \prod dZ^a_n \quad e^{ZAZ}$$

Σ wt SPIN STRUCTURE

$J^{ab} = Z^{(a} Z^{b)}$ $\qquad Sp(n)_{-\frac{1}{2}}$


SPIN STRUCTURE


(C^{2n}, ω = dp_I ∧ dq^I)
Sp(2n)
Σ
SPIN STRUCTURE
J^{ab} = Z^{(a}Z^{b)}
Sp(n)_{-1/2}
1/det(∂̄ + Ā)
K^{1/2} ⊗ E
= 1/det ∂̄
e^{zAz}
∫ e^{az²} dz ~ 1/√a

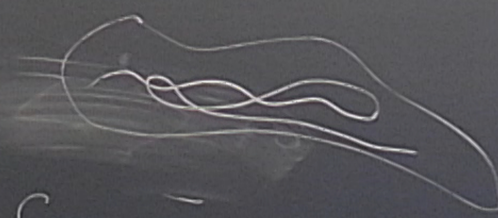
$(\mathbb{C}^{2n}, \omega = dp_I \wedge dq^I)$ $\qquad Sp(n)$ $\qquad \Sigma$ wt SPIN STRUCTURE

$Z^\alpha \qquad \omega_{\alpha\beta} \qquad \omega^{\alpha\beta}$

$$J^{ab} = Z^{(a}Z^{b)} \qquad Sp(n)_{-\frac{1}{2}}$$

$$\int_\Sigma Z^\alpha \bar\partial Z^\beta \, \omega_{\alpha\beta}$$

$$\frac{1}{\det(\bar\partial + \bar A)}$$

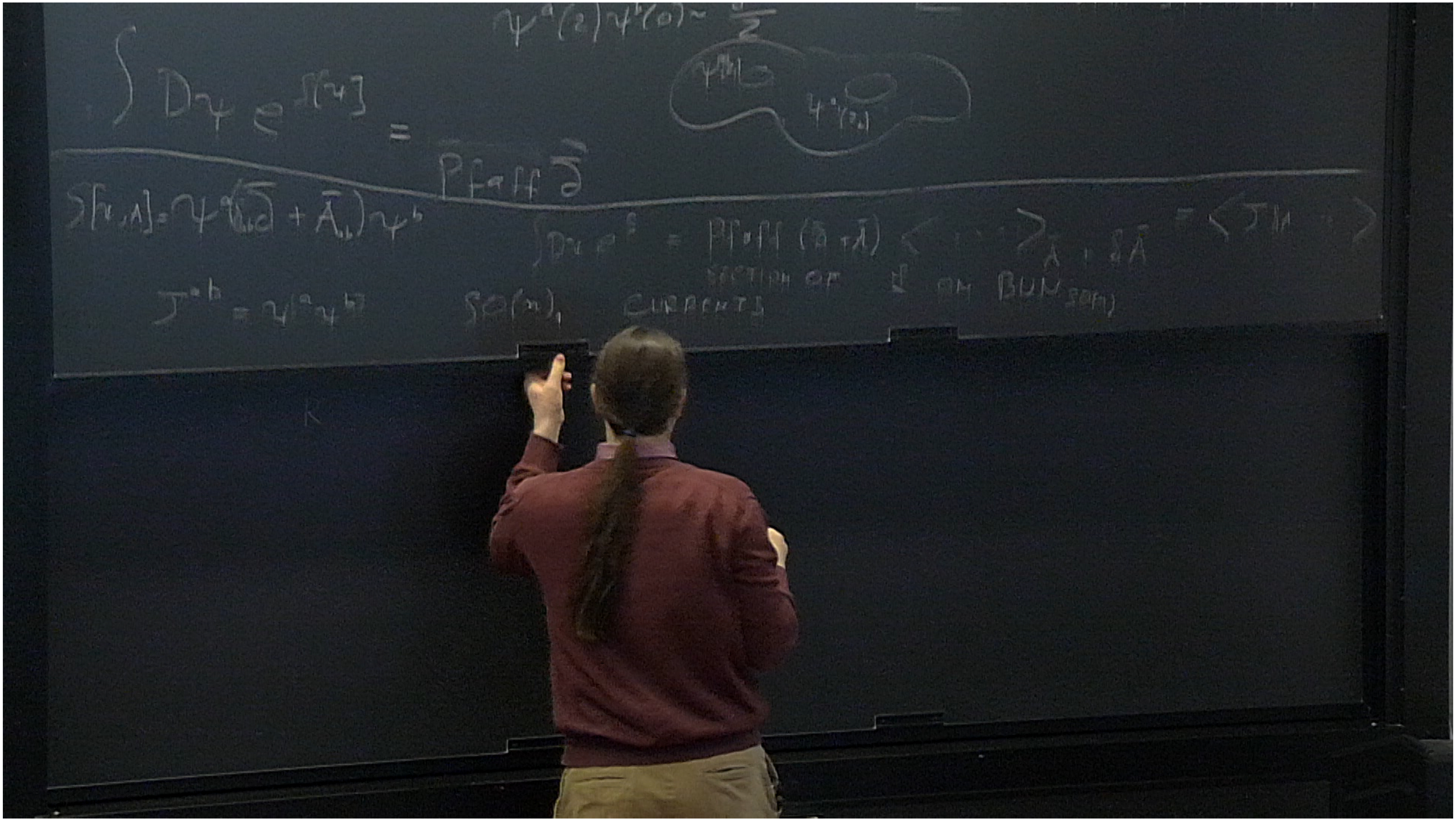


$$K^{\frac{1}{2}} \otimes E$$

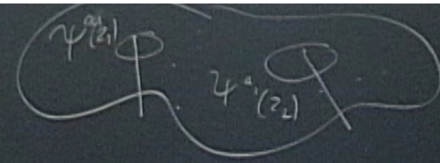
$$\int DZ \, e^{S[Z]} = \frac{1}{\det \bar\partial}$$

$$\int \prod dZ^a_n \quad e^{ZAZ}$$

$$\int e^{az^2} dz \sim \frac{1}{\sqrt{a}}$$


Pfaff
SECTION OF
CURRENTS

$$\int D\psi\, e^{S[\psi]} = \overline{\mathrm{Pfaff}\,\bar\partial}$$



$$S[\psi, A] = \psi^a(\delta_{ab}\bar\partial + \bar A_{ab})\psi^b$$

$$\int D\psi\, e^{S} = \mathrm{Pfaff}\,(\bar\partial + \bar A)\,\langle \cdots \rangle_{\bar A + \delta \bar A} = \langle J\,\delta A \cdots \rangle$$

SECTION OF $\mathcal{L}$ ON $\mathrm{BUN}_{SO(n)}$

$J^{ab} = \psi^{[a}\psi^{b]}$ $\quad SO(n)_1$ CURRENTS

NS $\quad \psi = \sum_{n\in\mathbb{Z}} \frac{\cdots}{z^n}$ $\qquad \{\psi^a_n, \psi^b_m\} = \delta^{ab}\,\delta_{n+m,0}$

$\{\psi^a_{\frac12}, \psi^b_{-\frac12}\} = \delta^{ab} \cdots$

$\psi_n|0\rangle = 0 \qquad n > 0$

$\psi_{-\frac12}|0\rangle,\ \psi_{-\frac32}|0\rangle,\ \psi_{-\frac12}\psi_{-\frac32}|0\rangle \cdots$

R $\qquad \psi = \sum_{n\in\mathbb{Z}} \frac{\psi_n}{z^{n+\frac12}} \qquad \{\psi^a_0, \psi^b_0\} = \delta^{ab} \qquad \psi^a_0|\alpha\rangle = (\Gamma^a)^{\alpha}_{\beta}|\beta\rangle$

$\det(\bar\partial + A)$

$$\int Dz\, e^{S[z]} = \frac{1}{\det \bar\partial}$$

$$\int \prod dz^a_n \quad e^{zAz}$$

$K^{\frac{1}{2}} \otimes E$

$$\int e^{az^2} dz \sim \frac{1}{\sqrt{a}}$$

NS $\quad |0\rangle \quad z^a_{-\frac{1}{2}} |0\rangle \; \ldots$

R $\quad [z^a_0, z^b_0] = \delta^{ab}$

$\det(\bar\partial + A)$

$$\int Dz\, e^{S[z]} = \frac{1}{\det \bar\partial}$$

$$\int \prod dz^a_n \quad e^{zAz}$$

$K^{\frac{1}{2}} \otimes E$

$$\int e^{az^2} dz \sim \frac{1}{\sqrt{a}}$$

NS $\quad |0\rangle \quad z^a_{-\frac{1}{2}} |0\rangle \; \cdots$

R $\quad [z^a_0, z^b_0] = \delta^{ab} \quad W^{(n)}_{\epsilon_n}\text{-MOD} \;\simeq \text{BAA-BRANES IN } \mathbb{C}^{2n}$

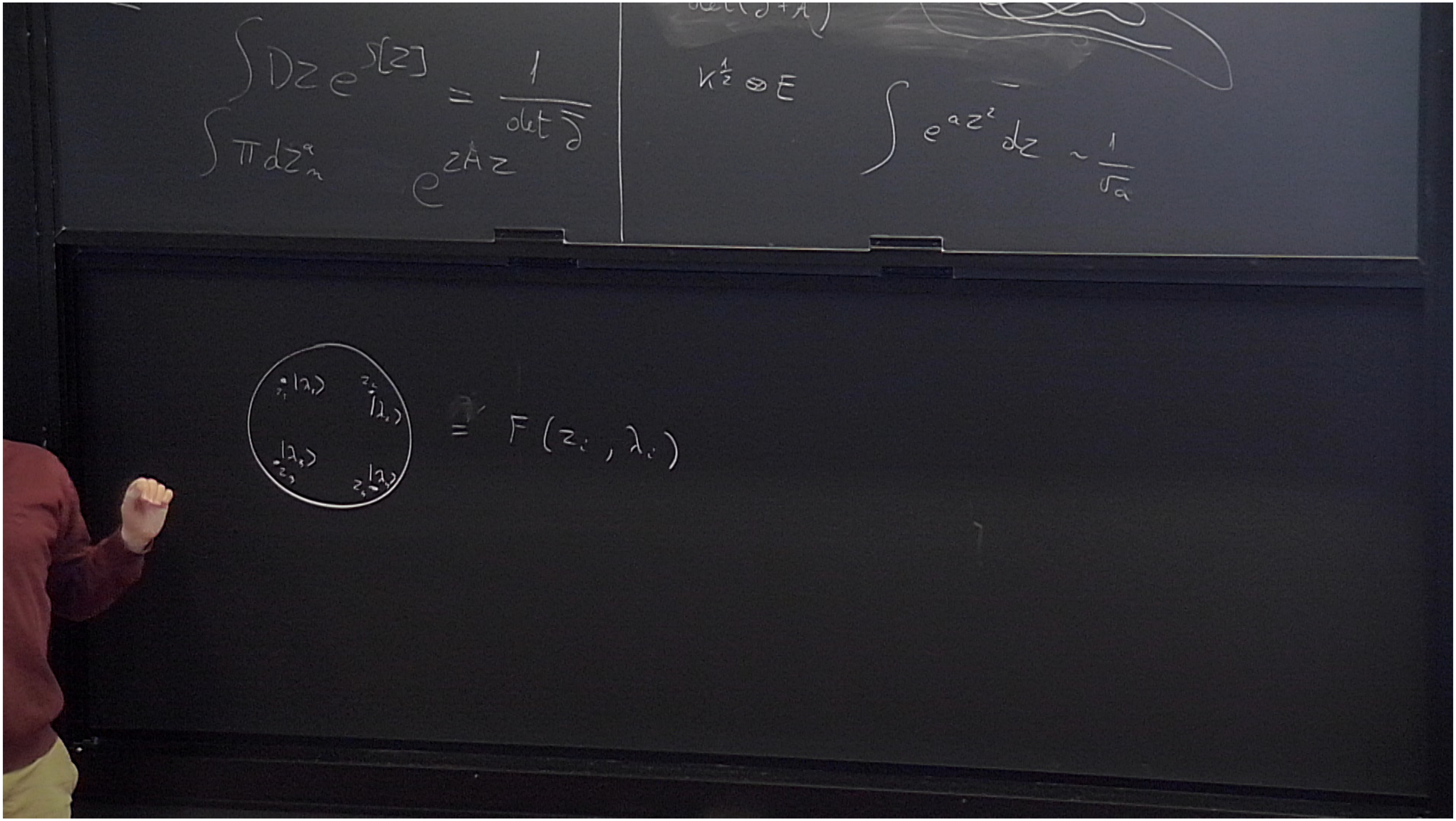
$z^1, z^2 \quad z^1_0 |+\rangle = 0 \quad (z^2_0)^n |+\rangle$

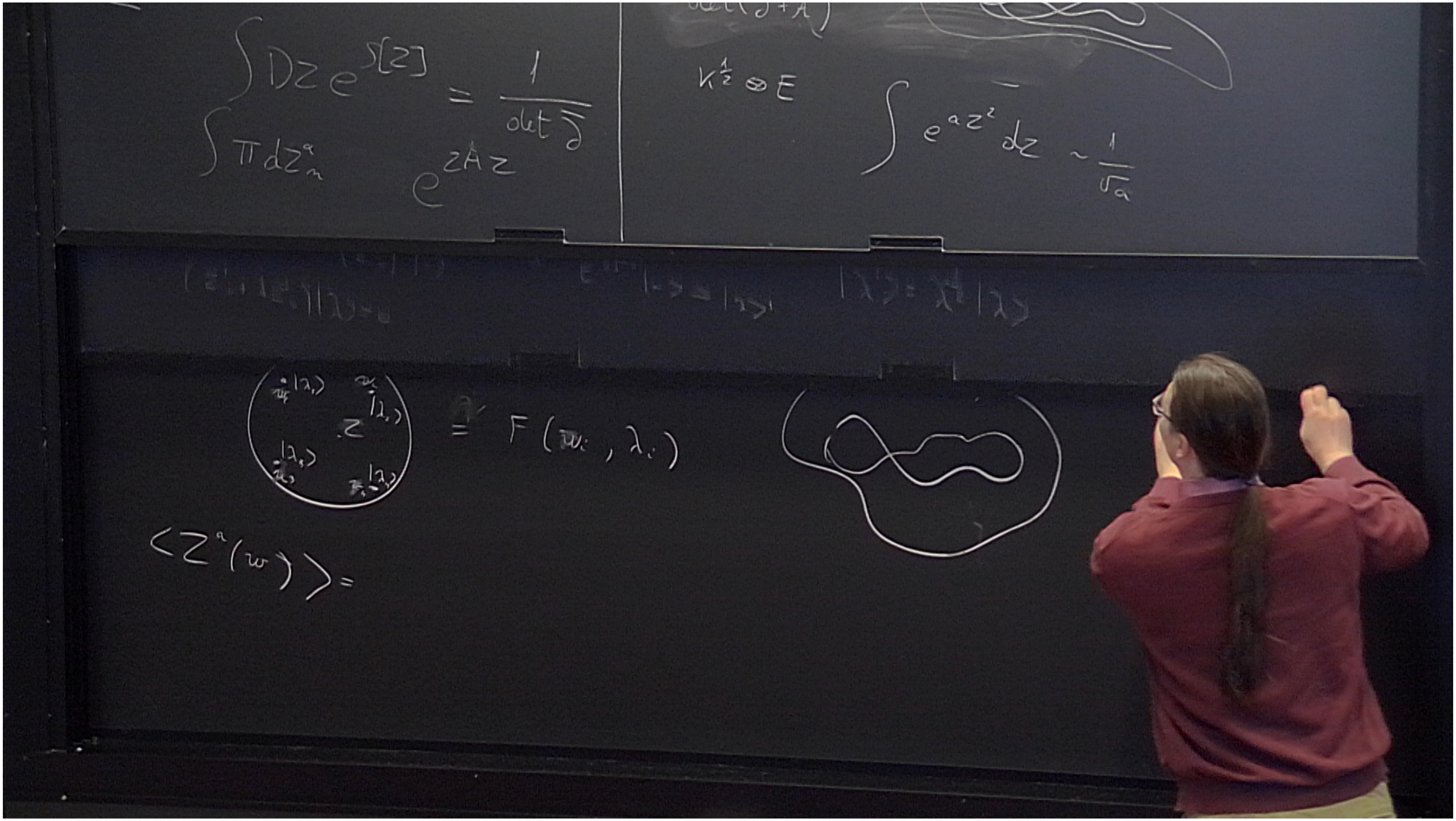
$z^2_0 |-\rangle = 0 \quad (z^1_0)^n |-\rangle$

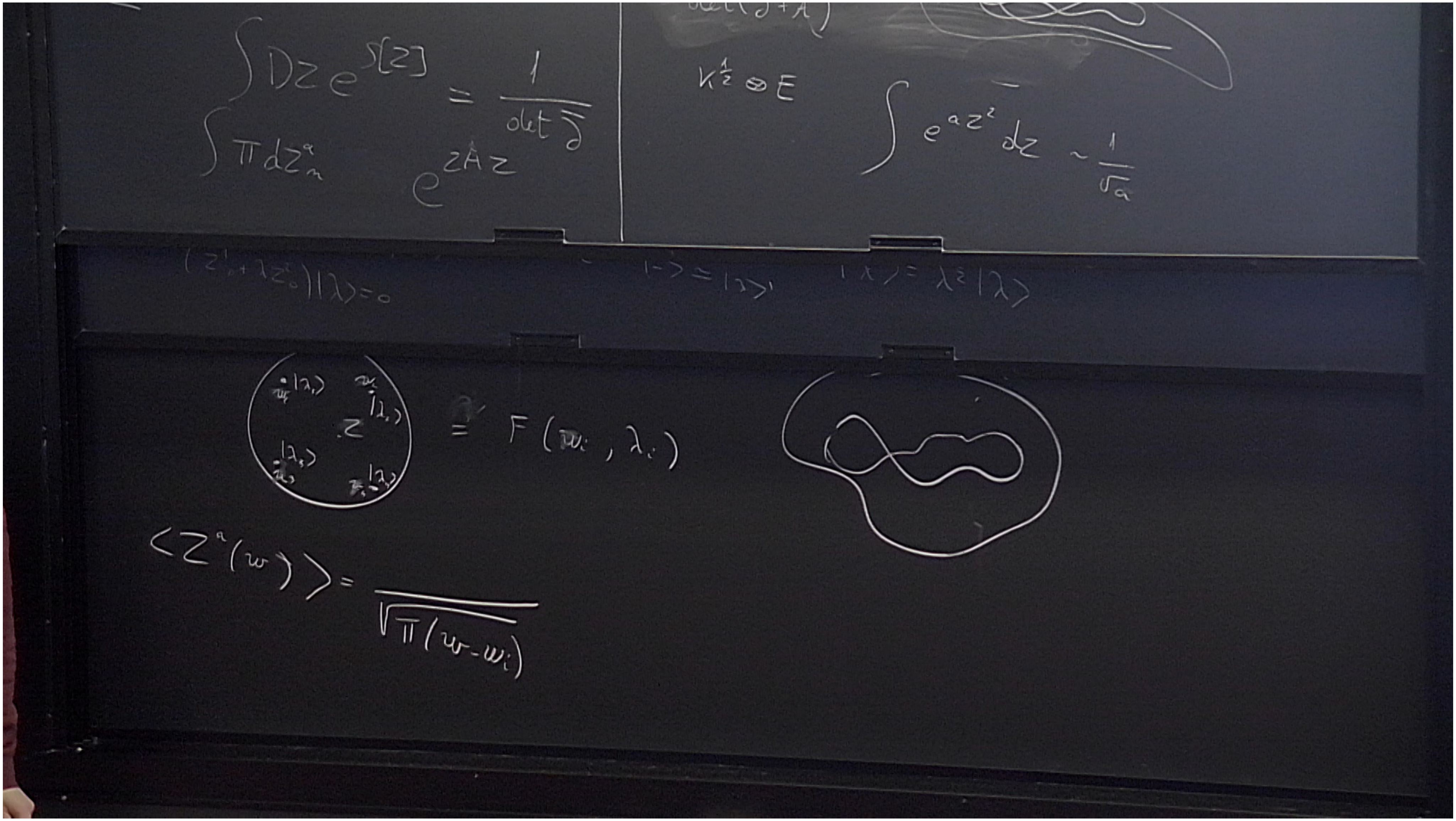
$(z^1_0 + \lambda z^2_0)|\lambda\rangle = 0$

$M_\lambda \quad e^{\frac{1}{2}\lambda (z^2_0)^2} |+\rangle = |\lambda\rangle$

$e^{\frac{1}{2\lambda}(z^1_0)^2} |-\rangle = |\lambda\rangle'$


$\int Dz\, e^{S[z]} = \frac{1}{\det \bar\partial}$
$\int \prod dz^a_n \quad e^{zAz}$
$K^{\frac{1}{2}} \otimes E$
$\int e^{az^2} dz \sim \frac{1}{\sqrt{a}}$
$|\lambda_1\rangle$ z_1
z_2 $|\lambda_2\rangle$
$|\lambda_3\rangle$ z_3
z_4 $|\lambda_4\rangle$
$= F(z_i, \lambda_i)$


∫ Dz e^S[z] = 1/det ∂̄
∫ Π dz^a_n e^zAz
K^1/2 ⊗ E
∫ e^az² dz ~ 1/√a
F(w_i, λ_i)
⟨Z^a(w)⟩ =


$\int Dz\, e^{S[z]} = \frac{1}{\det \bar\partial}$
$\int \prod dz^a_n \quad e^{zAz}$
$K^{\frac{1}{2}} \otimes E$
$\int e^{az^2} dz \sim \frac{1}{\sqrt{a}}$
$(z^1_0 + \lambda z^2_0)|\lambda\rangle = 0$
$|-\rangle = |\lambda\rangle$
$|\lambda\rangle = \lambda^2 |\lambda\rangle$
$= F(w_i, \lambda_i)$
$\langle Z^a(w) \rangle = \frac{\quad}{\sqrt{\prod (w - w_i)}}$

$$\int Dz\, e^{S[z]} = \frac{1}{\det \bar\partial}$$

$$\int \prod dz^a_n \quad e^{zAz}$$

$K^{\frac{1}{2}} \otimes E$

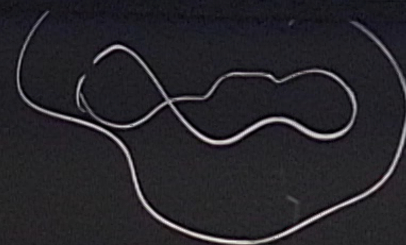
$$\int e^{az^2}\, dz \sim \frac{1}{\sqrt{a}}$$

$(z^1_0 + \lambda z^2_0)|\lambda\rangle = 0$

$e^{\ldots}|-\rangle = |\lambda\rangle$

$|\lambda'\rangle = \lambda^{-\frac{1}{2}}|\lambda\rangle$

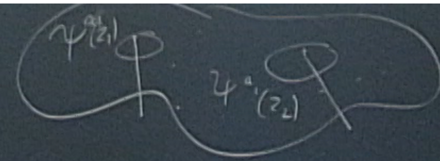
$w_1\,|\lambda_1\rangle$, $w_2\,|\lambda_2\rangle$, $w_3\,|\lambda_3\rangle$, $w_4\,|\lambda_4\rangle$, z $= F(w_i, \lambda_i)$



$$\langle Z^a(w)\rangle = \frac{U^a z + V^a}{\sqrt{\prod(w - w_i)}}$$

$M\mathcal{L}_1$, $M\mathcal{L}_2$, $M\mathcal{L}_3$, $M\mathcal{L}_4$ $= \mathrm{HOM}($BAA BRANE IN $(\mathbb{C}^{2n})^4$, $\mathcal{L}_1 \otimes \mathcal{L}_2 \otimes \mathcal{L}_3 \otimes \mathcal{L}_4$

$$\int D\psi\, e^{S[\psi]} = \mathrm{Pfaff}\,\bar\partial$$

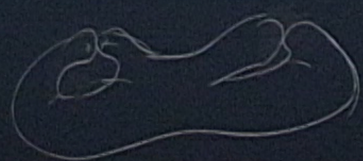


$$S[\psi, A] = \psi^a(\delta_{ab}\bar\partial + \bar A_{ab})\psi^b$$

$$\int D\psi\, e^S = \mathrm{Pfaff}\,(\bar\partial + \bar A) \langle \cdots \rangle_{\bar A + \delta \bar A} = \langle J \delta_A \cdots \rangle$$

SECTION OF $\mathcal{L}$ ON $BUN_{SO(n)}$

$J^{ab} = \psi^{[a}\psi^{b]}$ $SO(n)_1$ CURRENTS



$$\int Dz\, e^{S[z]} = \frac{1}{\det \bar\partial}$$

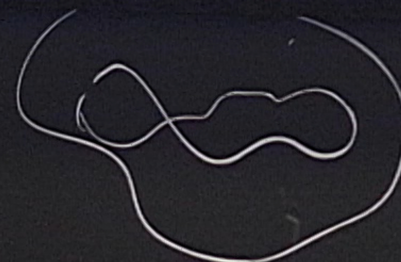
$$\int \prod dz^a_n \quad e^{zAz}$$

$K^{\frac{1}{2}} \otimes E$

$$\int e^{az^2}\, dz \sim \frac{1}{\sqrt{a}}$$

$(z^1_0 + \lambda z^2_0)|\lambda\rangle = 0$ $\quad |-\rangle = |\lambda\rangle'$ $\quad |\lambda\rangle = \lambda^{\bar z}|\lambda\rangle$

$w_1 |\lambda_1\rangle$, $w_2 |\lambda_2\rangle$, z, $w_3 |\lambda_3\rangle$, $w_4 |\lambda_4\rangle$ $= F(w_i, \lambda_i)$



$$\langle Z^a(w)\rangle = \frac{U^a z + V^a}{\sqrt{\prod (w - w_i)}} \sim \frac{\lambda^a_i}{\sqrt{w - w_i}}$$

$M^{\mathcal{L}_1}$, $M^{\mathcal{L}_2}$, $M^{\mathcal{L}_3}$, $M^{\mathcal{L}_4}$ $=$ HOM(BAA BRANE IN $(\mathbb{C}^{2n})^4$, $\mathcal{L}_1 \otimes \mathcal{L}_2 \otimes \mathcal{L}_3 \otimes \mathcal{L}_4$

$(\mathbb{C}^{2n}, \omega = dp_I \wedge dq^I)$ $\quad Sp(n)_{-\frac{1}{2}}$ $\quad \Sigma$ wt SPIN STRUCTURE

$Z^\alpha \quad \omega_{\alpha\beta} \quad \omega^{\alpha\beta}$ $\quad \cup$

G_{κ}

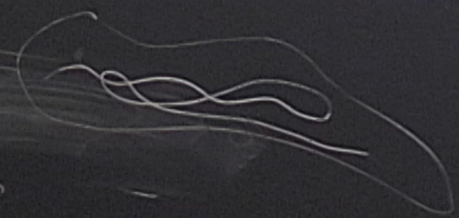
$$\int_\Sigma Z^\alpha \bar\partial Z^\beta \, \omega_{\alpha\beta}$$

$$\int Dz \, e^{S[Z]} = \frac{1}{\det \bar\partial}$$

$$\int \Pi \, dZ^\alpha_n \quad e^{zAz}$$

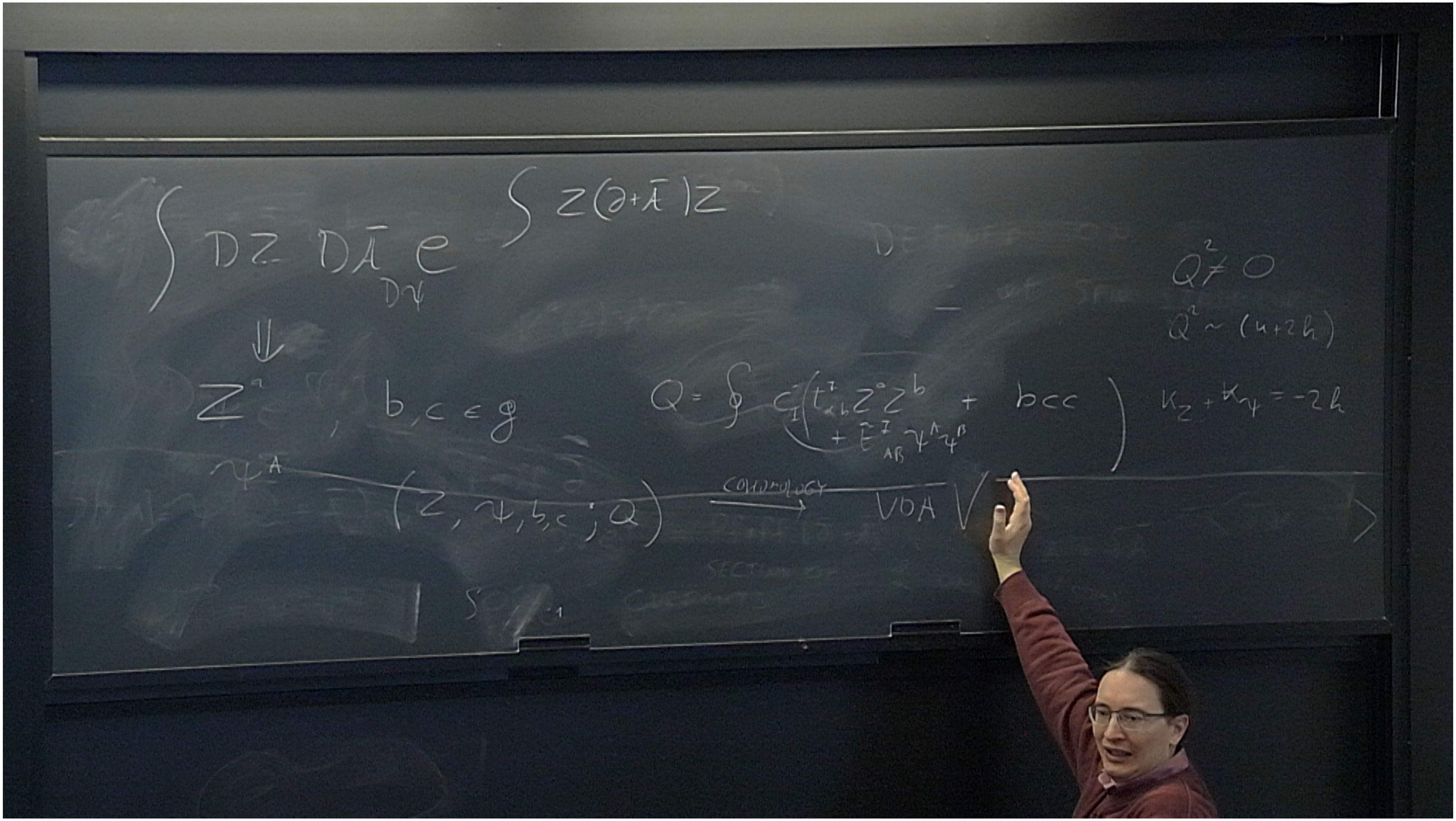
$J^{ab} = Z^{(a} Z^{b)}$ $\quad Sp(n)_{-\frac{1}{2}}$

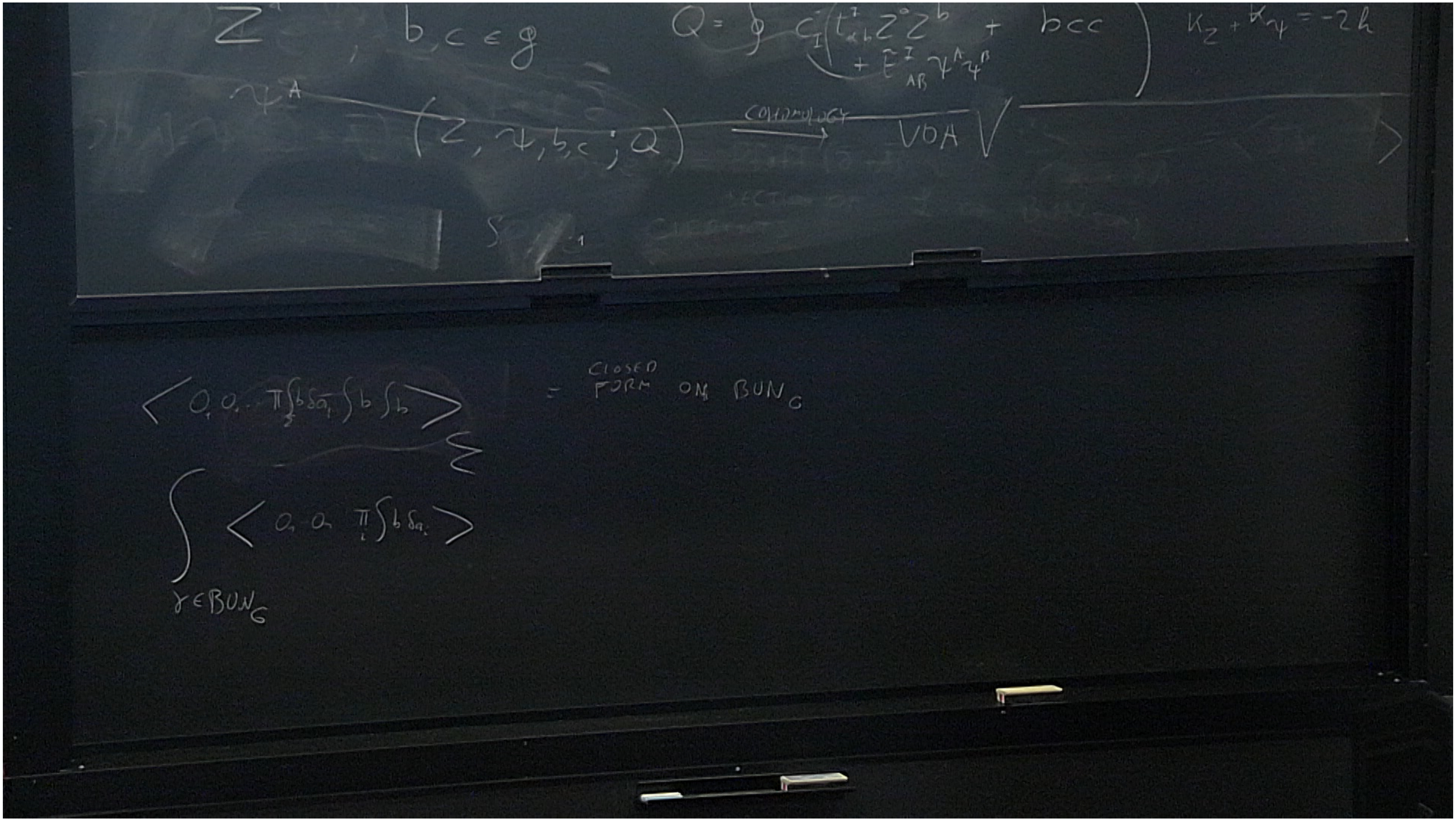
$$\frac{1}{\det(\bar\partial + \bar{A})}$$

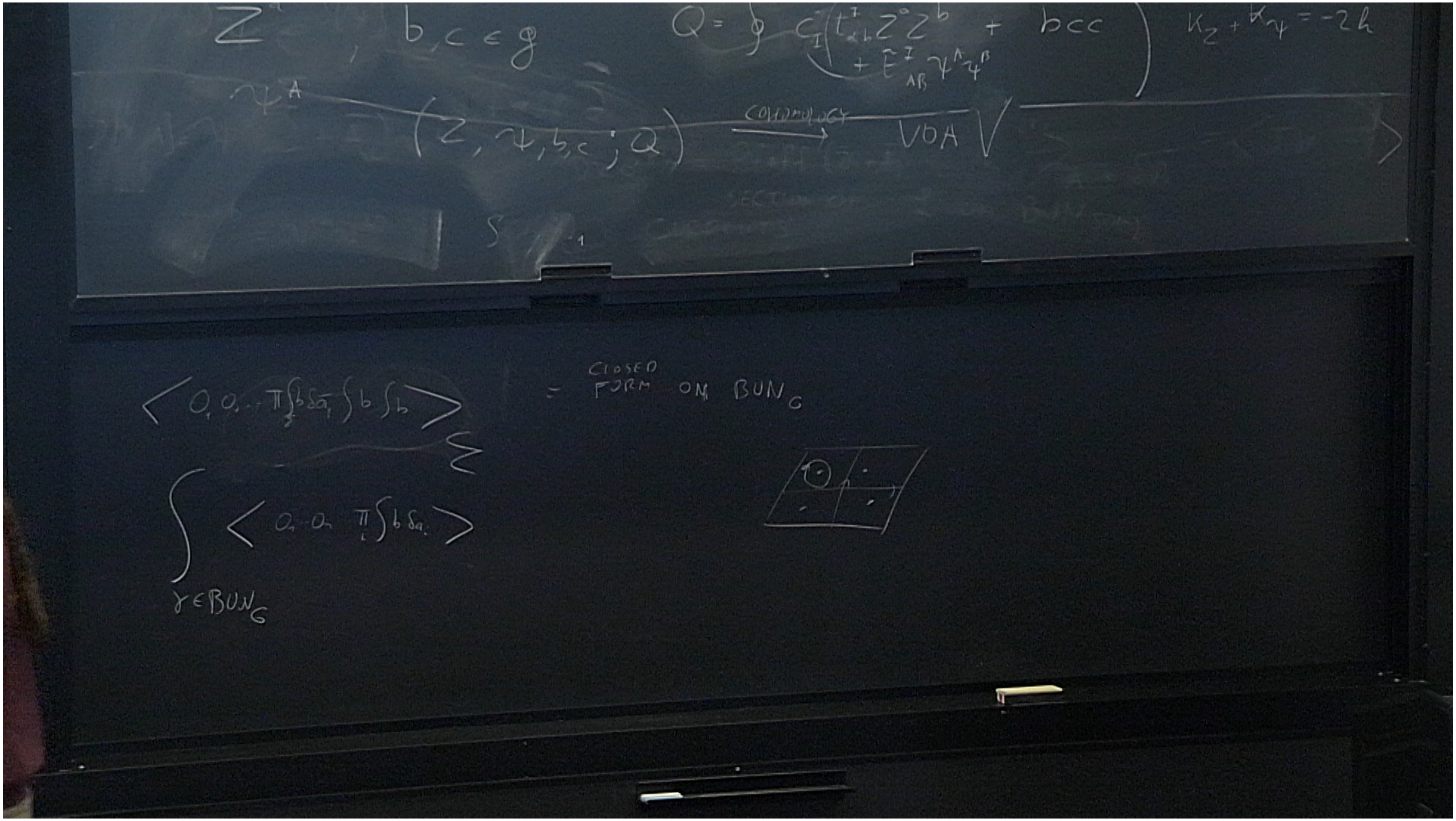


$K^{\frac{1}{2}} \otimes E$

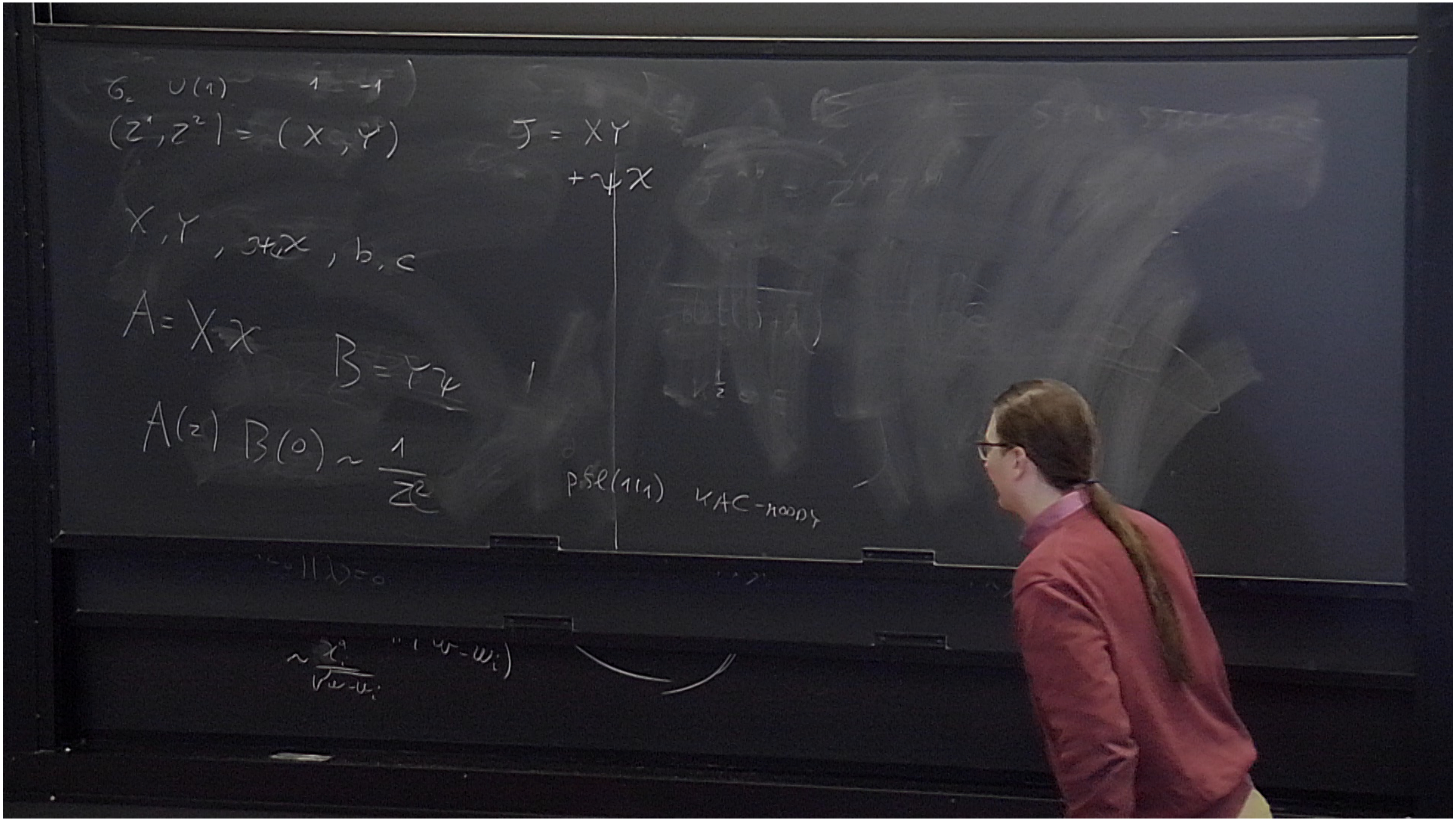
$$\int e^{az^2} dz \sim \frac{1}{\sqrt{a}}$$

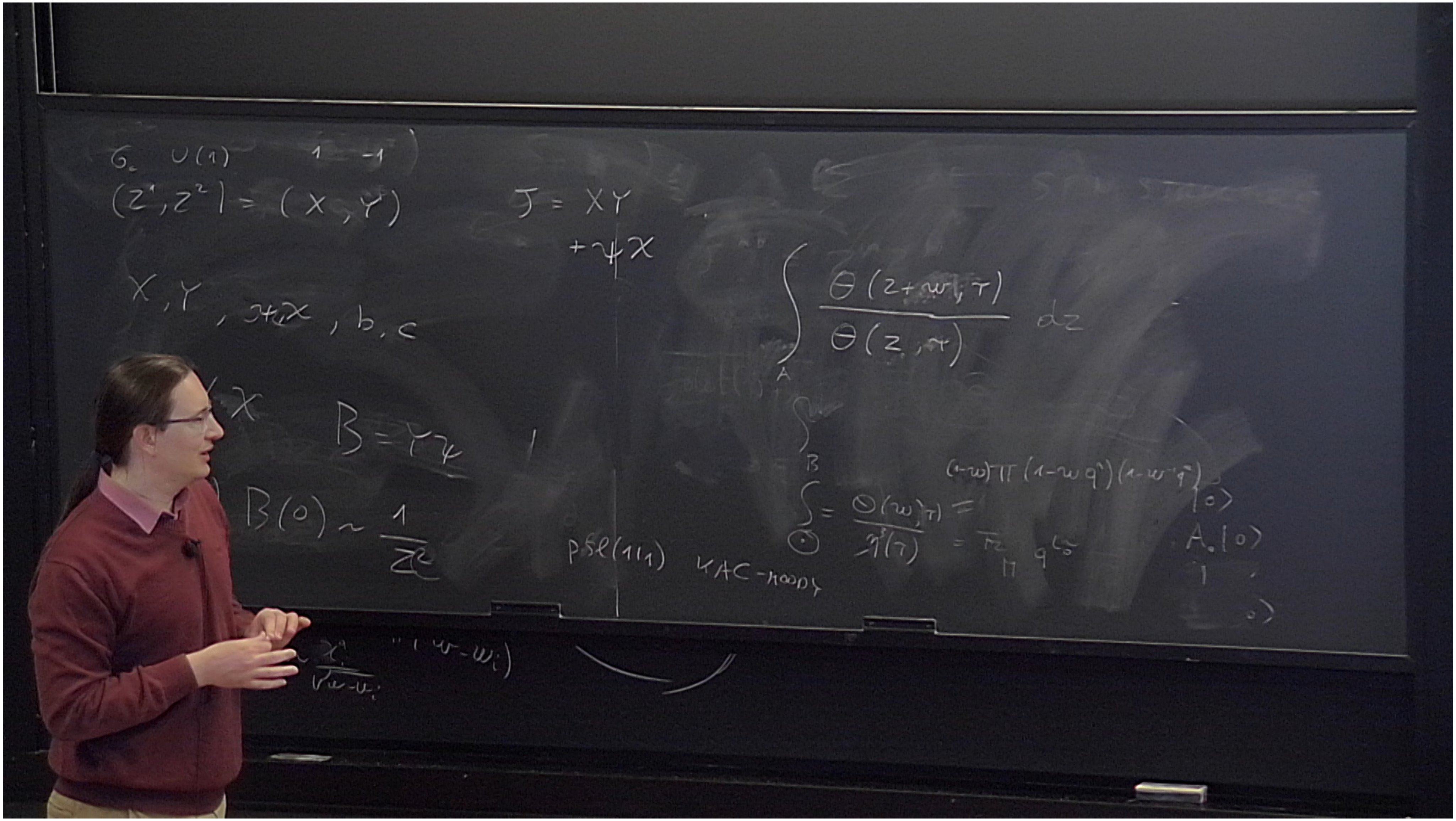

$\int DZ\, D\bar{A}\, D\psi\, e^{\int Z(\partial+\bar{A})Z}$
$\Downarrow$
Z^a, ψ^A, $b, c \in \mathfrak{g}$
$Q = \oint c_I (t^I_{ab} Z^a Z^b + \epsilon^I_{AB} \psi^A \psi^B + bcc)$
$(Z, \psi, b, c \,;\, Q) \xrightarrow{\text{COHOMOLOGY}}$ VOA V
$Q^2 \neq 0$
$Q^2 \sim (k+2h)$
$k_Z + k_\psi = -2h$

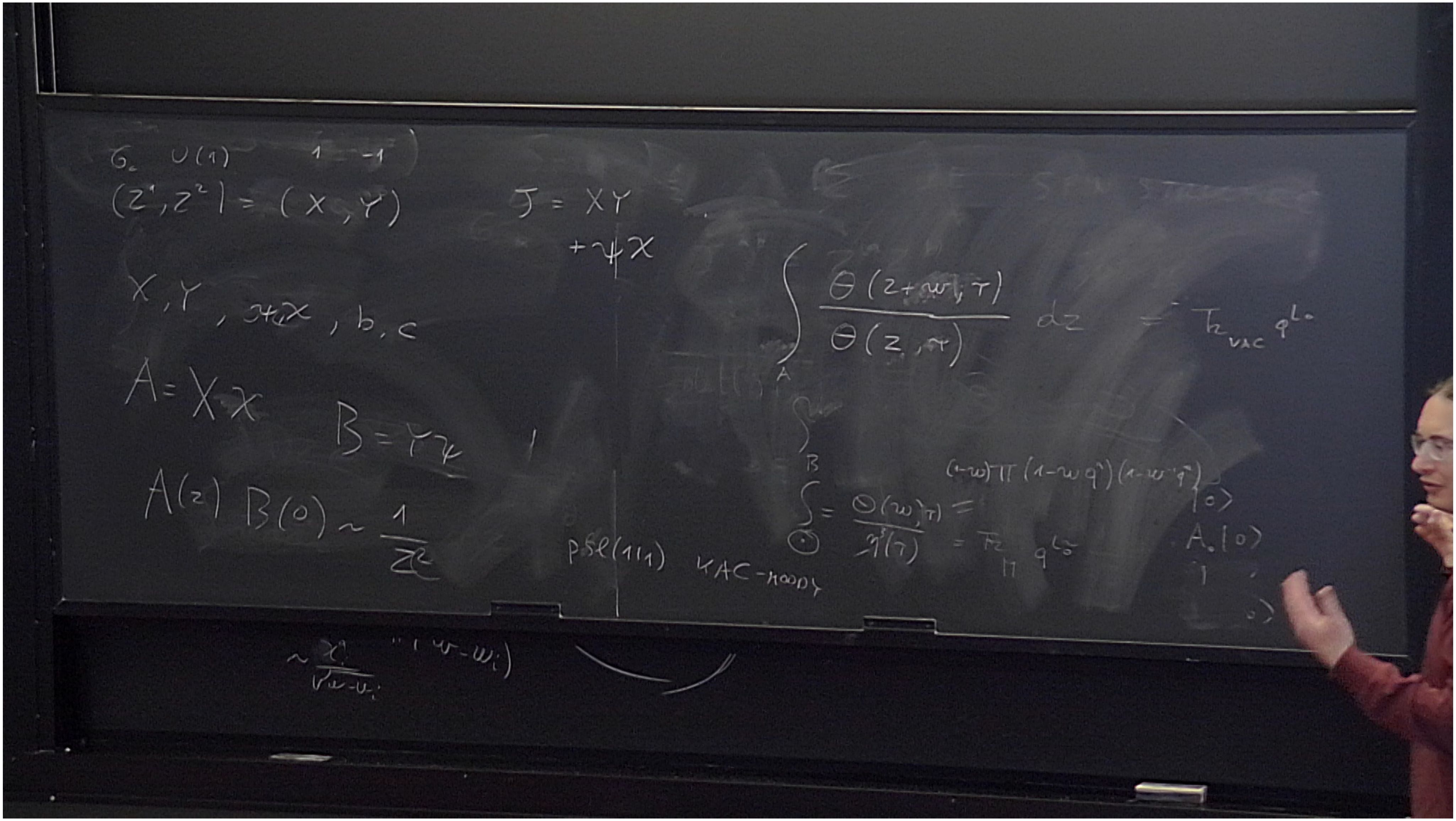

Z ; b, c ∈ g
Q = ∮ c_I (t^I_ab Z^a Z^b + bcc + t̃^I_AB ψ^A ψ^B)
K_Z + K_ψ = -2h
ψ^A
(Z, ψ, b, c ; Q)
COHOMOLOGY
VOA
⟨ O_1 O_2 … ∏_i ∫ b δa_i ∫ b ∫ b ⟩_Σ
= CLOSED FORM ON BUN_G
∫_{γ ∈ BUN_G} ⟨ O_1 O_2 ∏_i ∫ b δa_i ⟩



b, c ∈ g
Q = ∮ c_I (t^I_ab Z^a Z^b + bcc + t̃^I_AB ψ^A ψ^B)
κ_Z + κ_ψ = -2h
(Z, ψ, b, c ; Q) COHOMOLOGY → VOA
⟨O_1 O_2 ... Π ∫ b δa_i ∫ b ∫ b⟩_Σ
= CLOSED FORM ON BUN_G
∫ ⟨O_1 ... O_n Π_i ∫ b δa_i⟩
γ ∈ BUN_G


U(1)
(z¹, z²) = (X, Y)
J = XY
+ψχ
X, Y, ψ, χ, b, c
A = Xχ
B = Yψ
A(z) B(0) ~ 1/z²
psl(1|1) KAC-MOODY


U(1)
J = XY
+ψχ
X, Y, ψ, χ, b, c
B = Yψ
psl(1|1)
KAC-MOODY
|0⟩
A₀|0⟩


$G_c \quad U(1) \quad 1 \quad -1$
$(Z^1, Z^2) = (X, Y)$
$J = XY$
$+ \psi\chi$
X, Y, ψ, χ, b, c
$A = X\chi$
$B = Y\psi$
$A(z)\, B(0) \sim \frac{1}{z^2}$
$\int_A \frac{\theta(z+w, \tau)}{\theta(z, \tau)}\, dz = \mathrm{Tr}_{VAC}\, q^{L_0}$
$\int_B$
$(1-w)\prod(1-wq^n)(1-w^{-1}q^n)$
$\int_{\odot} = \frac{\Theta(w, \tau)}{\eta^3(\tau)} = \mathrm{Tr}_{\Pi}\, q^{L_0}$
$|0\rangle$
$A_0|0\rangle$
$psl(1|1)$ KAC-MOODY
$\sim \frac{\chi_i}{\sqrt{w-w_i}}$
$(w-w_i)$

$$\int DZ\, D\bar{A}\, D\psi\; e^{\int Z(\partial+\bar{A})Z}$$

$$\Downarrow$$

Z^a , $b, c \in \mathfrak{g}$

ψ^A

$\bar{D} Z = 0$

$\mu[Z] = 0$

$$Q = \oint c_I \left(t^I_{ab} Z^a Z^b + \tilde{t}^I_{AB} \psi^A \psi^B + bcc \right)$$

$Q^2 \neq 0$

$Q^2 \sim (u+2h)$

$k_Z + k_\psi = -2h$

$$(Z, \psi, b, c \,;\, Q) \xrightarrow{\text{COHOMOLOGY}} \text{VOA } V$$

$\gamma \in \mathrm{BUN}_G$

G $U(1)$ 1 -1

$(Z^1, Z^2) = (X, Y)$

$J = XY + \psi\chi$

X, Y, ψ, χ, b, c

$A = X\chi$

$B = Y\psi$

$A(z)\,B(0) \sim \frac{1}{z^2}$

psl(1|1) KAC-MOODY

$$\int_A \frac{\theta(z+w;\tau)}{\theta(z;\tau)}\,dz = \mathrm{Tr}_{VAC}\, q^{L_0}$$

$\int_B$

$(1-w)\prod(1-wq^n)(1-w^{-1}q^n)$

$$\oint = \frac{\Theta(w;\tau)}{\eta^3(\tau)} = \mathrm{Tr}_{\Pi}\, q^{L_0}$$

$|0\rangle$

$A_0|0\rangle$

$\sim \frac{\chi_i^a}{\sqrt{w-w_i}}$ " $(w-w_i)$