Title: Coordinate Charts on Character Varieties via Non-abelianization

Speakers: Benedict Morrisey

Series: Mathematical Physics

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Abstract:

Classical work by Thurston in the theory of surfaces gives symplectic co-ordinate charts on TeichmÃ¼ller space, associated to quadratic differentials. Motivated by wall crossing in 4d field theories Gaiotto, Moore and Neitzke defined a generalization of these; giving maps from the moduli of one dimensional local systems on a spectral curve to the moduli space of n-dimensional local systems on a non-compact Riemann surface. I will describe joint work with M. Ionita which extends this construction to arbitrary reductive algebraic groups G. Time permitting I will describe the interpretation of this construction in terms of the wild Riemann--Hilbert Correspondence in the closely related setting of Frobenius manifolds.

Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita.

Local systems on X.

1. A locally constant sheaf of vector spaces on X

Example: (holomorphic) VB w/ flat connection.
$\rightsquigarrow$ Sheaf of flat sections/solutions.

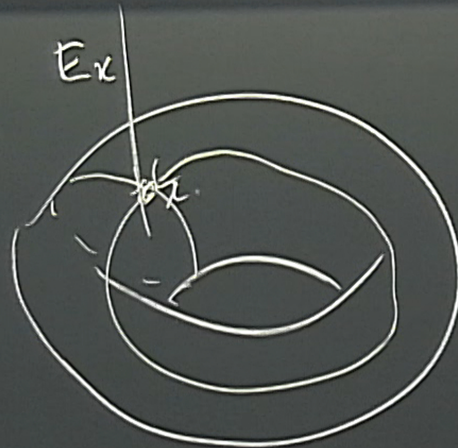
abelianization.

spaces

ection.

/ solutions.

2.



$$\pi_1(X,x) \longrightarrow \mathrm{Aut}(E_x).$$

$$\left\{\pi_1(X) \longrightarrow GL(n)\right\} / \text{conj}$$

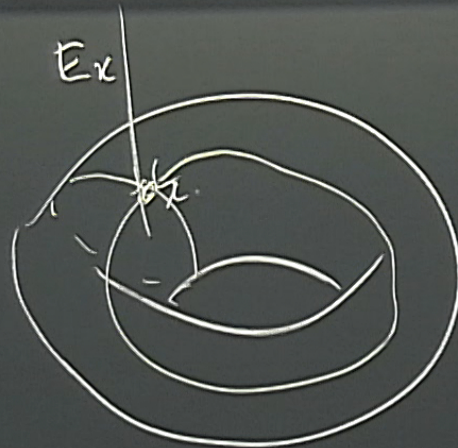
abelianization.

spaces

ection.

/ solutions.

2.



$$\pi_1(X,x) \longrightarrow \mathrm{Aut}(E_x).$$

$$\{\pi_1(X) \longrightarrow GL(n)\} / \text{conj}$$

Moduli spaces of these.

1. 1 dimensional. $(\mathbb{C}^\times)^{2n} / \mathbb{C}^\times$

A mo

(E_x)).

$\Big)\Big\}\Big/\mathrm{Conj}$

A more interesting example:

If we have a top surface X^{top} $\quad (g \geq 2)$

then X with $\mathrm{top}(X) \cong X^{top}$

$$\pi_1(X) \subset \begin{matrix}\text{universal} \\ \text{covering} \\ \text{space}\end{matrix} \cong \mathbb{H}$$

opposite orientation

$$\mathrm{Isom}(\mathbb{H}) \cong PSL_2(\mathbb{R})$$

$$\mathcal{M}_B(X, PSL_2(\mathbb{R})) \cong \mathcal{T}(X^{top}) \sqcup \mathcal{T}(\overline{X}^{top})$$

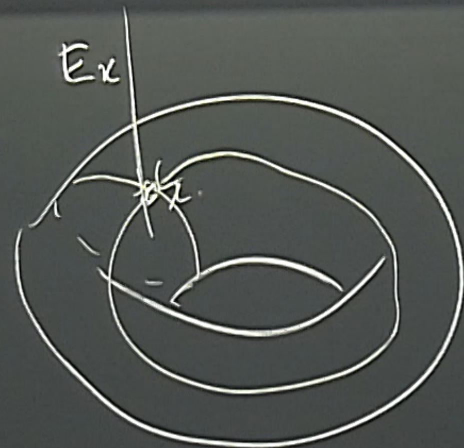
non-abelianization.

~~vector spaces~~

t connection.

sections/solutions.

2.



$\pi_1(X,x) \longrightarrow \mathrm{Aut}(E_x).$

$\{\pi_1(X) \longrightarrow G\}/\mathrm{conj}$

Moduli spaces of these.

1. 1 dimensional. $(\mathbb{C}^\times)^{2n}/\mathbb{C}^\times$

In GMN (non-abelianization)

$\left\{ \begin{array}{l} \text{moduli space} \\ \text{of } \mathbb{C}^\times\text{-connections} \\ \text{on } \overline{X} \setminus \pi^{-1}(\mathcal{P}) \end{array} \right\}$

$$\begin{array}{c} \overline{X} \supset \pi^{-1}(\mathcal{P}) \\ n{:}1 \ \Big\downarrow \pi \\ X \supset \mathcal{P} \end{array}$$

branch pts.

(X non-compact
$X = X^c \setminus \mathcal{P}$)

In GMN (non-abelianization).

$$\left\{\begin{array}{l}\text{moduli space}\\ \text{of } \mathbb{C}^\times\text{-connections}\\ \text{on } \overline{X}\setminus \pi^{-1}(p)\end{array}\right\}\ \text{(\& a monodromy cdn around } \pi^{-1}(p)\text{)} \longrightarrow \left\{\begin{array}{l}GL(n)\\ \text{Local}\\ \text{Systems}\\ \text{on } X\setminus p\end{array}\right\}$$

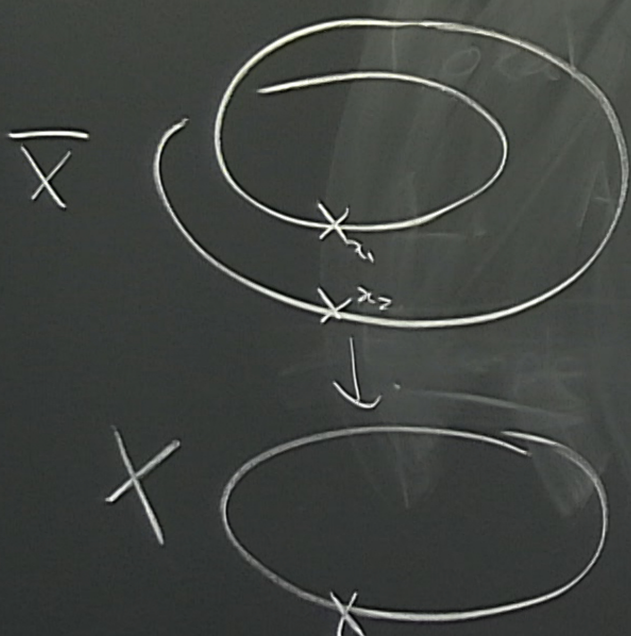
$$\mathcal{L} \longmapsto \pi_*(\mathcal{L})$$

$$\begin{array}{c}\overline{X} \supset \pi^{-1}(p)\\ {\scriptstyle n:1}\ \big\downarrow \pi\\ X \supset p \quad \text{branch locus}\end{array}$$

(X non-compact
$X = X^c \setminus D$)

Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita

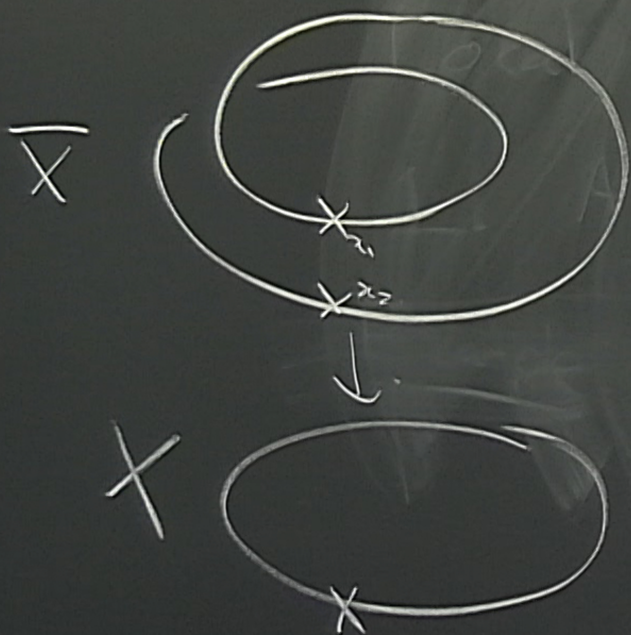


$$\mathbb{Z}|_{x_1} \oplus \mathbb{Z}|_{x_2} \xrightarrow[f]{\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}} \mathbb{Z}|_{x_1} \oplus \mathbb{Z}|_{x_2}$$

β

Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita



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p

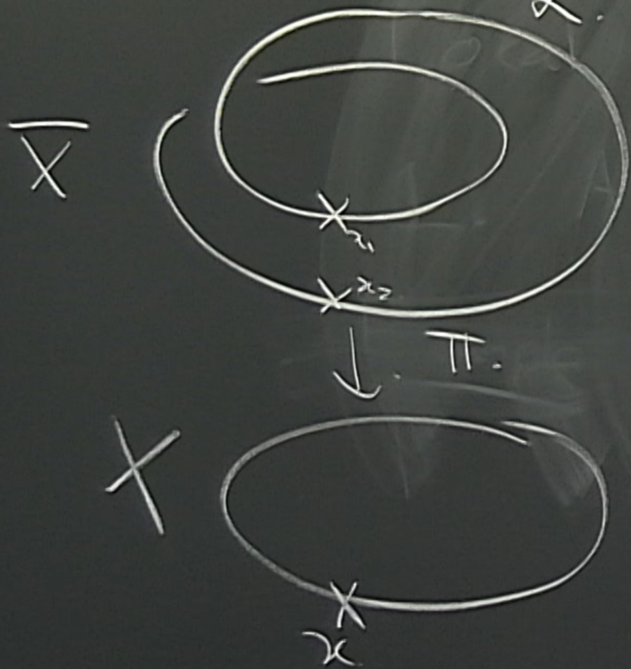
β

Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita

$\mathbb{Z}$.

$$\mathbb{Z}|_{x_1} \oplus \mathbb{Z}|_{x_2} \xrightarrow[f]{\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}} \mathbb{Z}|_{x_1} \oplus \mathbb{Z}|_{x_2}.$$



ρ

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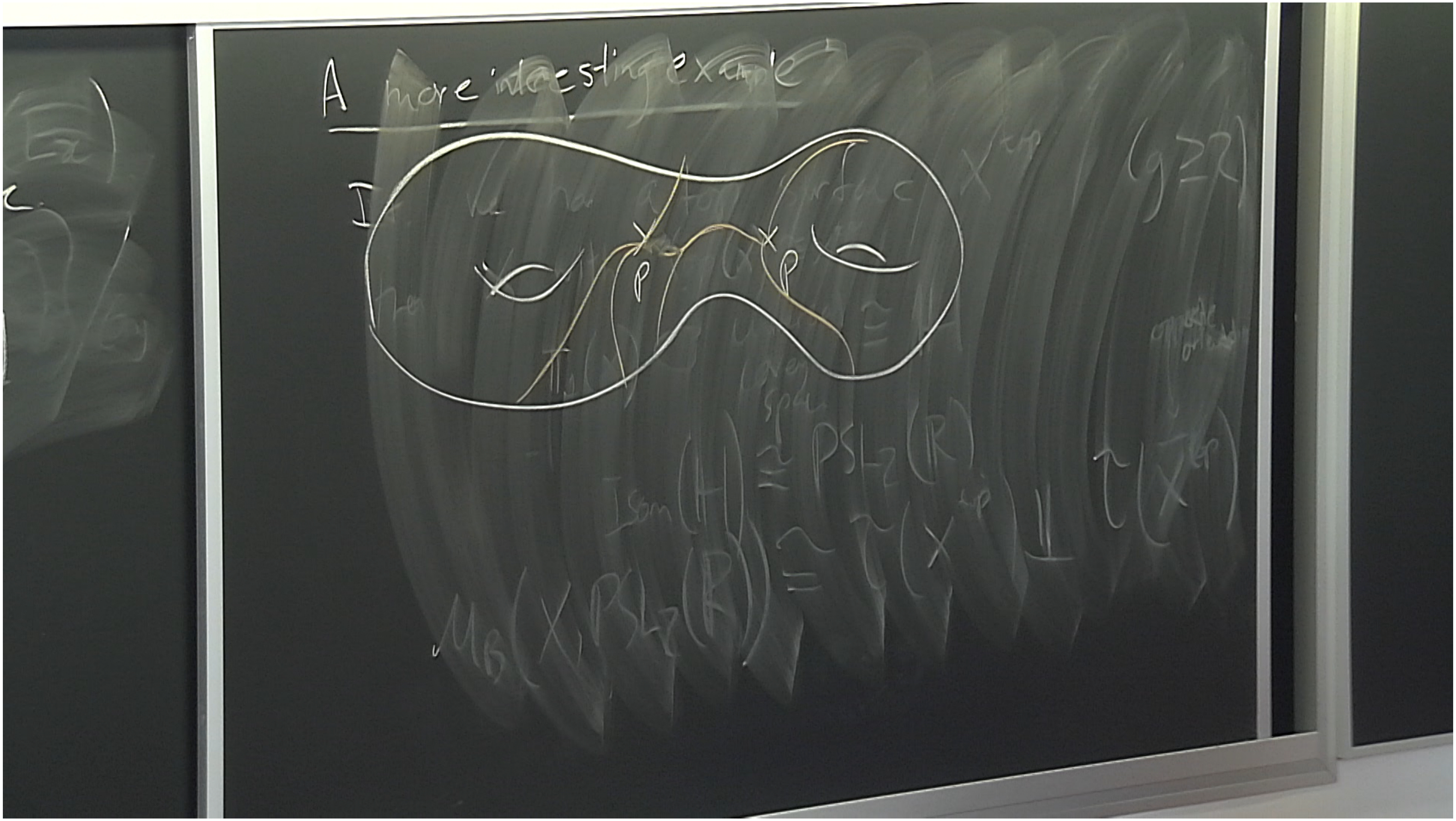
non-abelianization.

$\mathcal{L}|_{x_1}$
$\oplus$
$\mathcal{L}|_{x_2}$.

$p^* \pi_* \mathcal{L}|_x \xrightarrow{g} p^* \pi_* \mathcal{L}|_x$

new local syst w/ monodromy

$g \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$


A more interesting example
(g ≥ 2)
Isom(H) ≅ PSL2(R)
MB(X, PSL2(R))

In GMN (non-abelianization)

$$\left\{\begin{array}{l}\text{moduli space} \\ \text{of } \mathbb{C}^\times\text{-connections} \\ \text{on } \overline{X} \setminus \pi^{-1}(P)\end{array}\right\}$$

(& a monodromy cdn around $\pi^{-1}(P)$)

$$\overline{X} \supset \pi^{-1}(P) \qquad \mathcal{L}$$

$$n:1 \Big\downarrow \pi$$

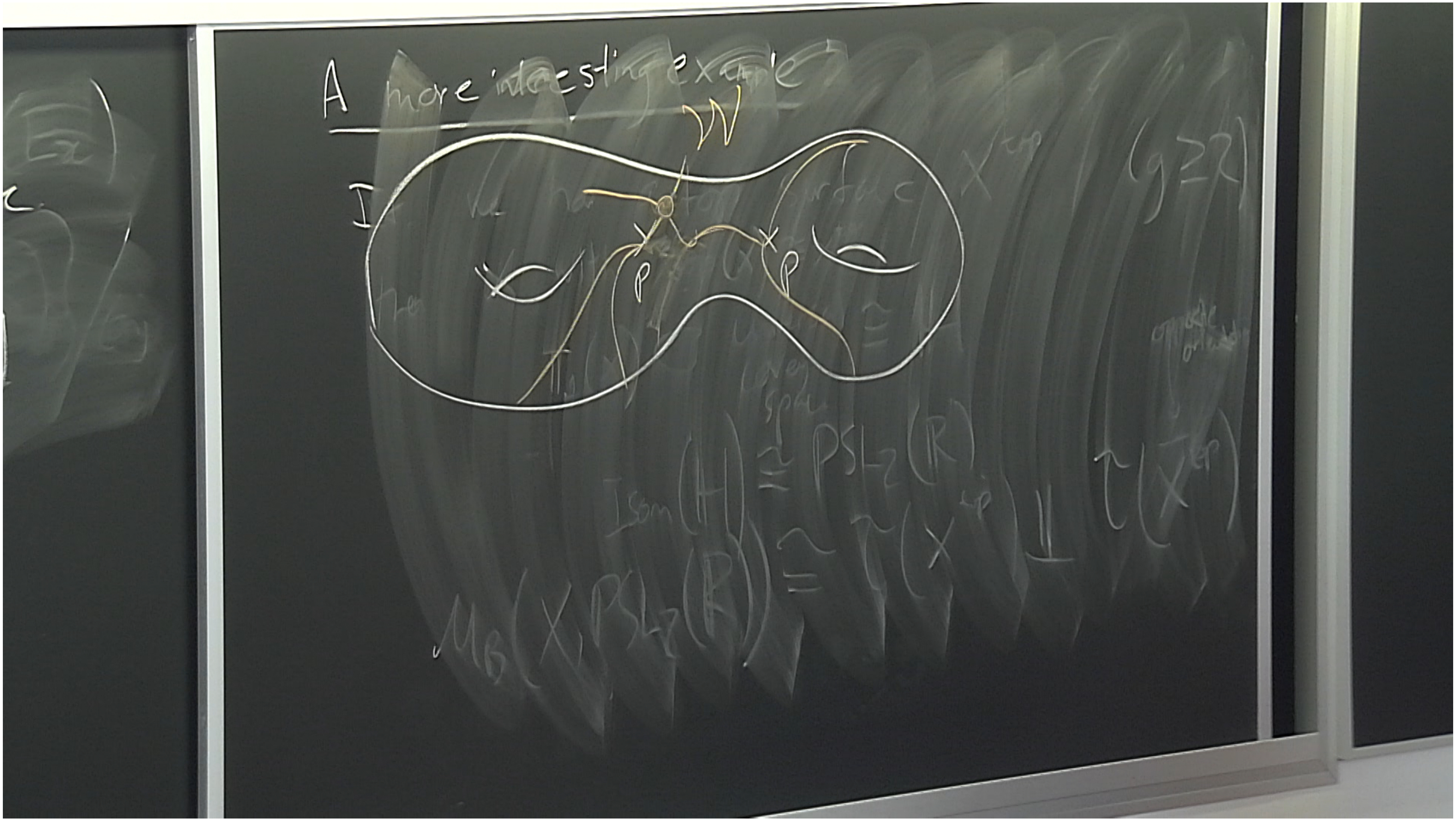
$$X \supset P \quad \text{branch locus}$$

$(X$ non-compact $X = X^c \setminus D)$

$$\mathcal{L} \longmapsto \pi_*(\mathcal{L})$$

$$\left\{\begin{array}{l}GL(n) \\ \text{Local} \\ \text{systems} \\ \text{on } X \setminus P\end{array}\right\}$$

$$\left\{\begin{array}{l}GL(n) \text{ local system } \mathcal{E} \text{ on} \\ X \setminus P \\ \text{together} \\ \text{w/ locally constant} \\ \text{sections of } \mathrm{Aut}(\mathcal{E})\big|_{\text{each connected compt of } \mathcal{W} \subset X \setminus P}\end{array}\right\}$$


A more interesting example
W
ρ
(g ≥ 2)
Isom(H) ≅ PSL2(R)
MB(X, PSL2(R)) ≅

In GMN (non-abelianization)

$$\left\{\begin{array}{l}\text{moduli space}\\ \text{of } \mathbb{C}^\times\text{-connections}\\ \text{on } \bar{X}\setminus\pi^{-1}(P)\end{array}\right\} \text{ (\& a monodromy cdn at } \pi^{-1}(P)) \xrightarrow{\text{nonab}} \left\{\begin{array}{l}GL(n) \text{ local system on}\\ X\setminus P\end{array}\right\}$$

together w/ locally constnt sections of $Aut(E)$ | each conn'd cpt of $\mathcal{W}\subset X\setminus P$

forget $\downarrow$

$$\left\{\begin{array}{l}GL(n)\\ \text{Local}\\ \text{Systems}\\ \text{on } X\setminus P\end{array}\right\}$$

$\bar{X} \supset \pi^{-1}(P)$ $\quad \mathcal{Z}$

$n:1 \downarrow \pi$

$X \supset P$ branch pts

(X non-compact, $X = X^c \setminus D$)

$\pi_*(\mathcal{Z})$

$$\left\{\begin{array}{l}GL(n) \text{ local}\\ \text{system on}\\ X\end{array}\right\}$$

$$\left\{\begin{array}{l}GL(n) \text{ local}\\ \text{system on}\\ X\setminus(P\cup J)\end{array}\right\}$$

intersection of lines in $\mathcal{W}$

In GMN (non-abelianization)

$\left\{ \begin{array}{l} \text{(\& a monodromy cdn around } \pi^{-1}(p)) \\ \text{moduli space of } \mathbb{C}^\times\text{-connections} \\ \text{on } \overline{X} \setminus \pi^{-1}(p) \end{array} \right\} \xrightarrow{\text{nonab}} \left\{ \begin{array}{l} \cancel{GL(n)} \ G \text{ local systems on } X \setminus P \\ \text{together w/ locally constant} \\ \text{sections of } Aut(\mathcal{E}) \end{array} \right| \begin{array}{l} \text{each connd cpt} \\ \text{of } \gamma \subset X \setminus P \end{array} \Big\}$

forget ↓

$\left\{ \begin{array}{l} \cancel{GL(n)} \ G \\ \text{Local systems} \\ \text{on } X \setminus P \end{array} \right\}$

$\overline{X} \supset \pi^{-1}(P) \quad Z \longrightarrow \pi_*(Z)$

$n:1 \Big\downarrow \pi$

$X \supset P$ branch pts

(X non-compact, $X = \overline{X}^c \setminus D$)

$\left\{ \begin{array}{l} \cancel{GL(n)} \ G \text{ local} \\ \text{systems on} \\ X \end{array} \right\}$

$\left\{ \begin{array}{l} \cancel{GL(n)} \ G \text{ local} \\ \text{system on} \\ X \setminus (P \cup J) \end{array} \right\}$

intersection of lines in ∇D

1. Analogue of

$$\mathrm{Loc}_{\mathbb{C}^\times}\left(\overline{X}\setminus\pi^{-1}(P)\right)$$

& analogue of the monodromy condition.

2. What is $\mathcal{W}$?

3. Motivation for cutting & gluing procedure.

In GMN (

no

(& a monodromy cdn around $\pi^{-1}(P)$)

$\left\{ \text{moduli space of } \mathbb{C}^\times\text{-connections on } \overline{X}\setminus\pi^{-1}(P) \right\}$

$$\begin{array}{l} \overline{X} \supset \pi^{-1}(P) \quad Z \\ n{:}1 \big\downarrow \pi \\ X \supset P \quad \text{branch locus} \end{array}$$

(X non-compact

$X = X^c \setminus D$)

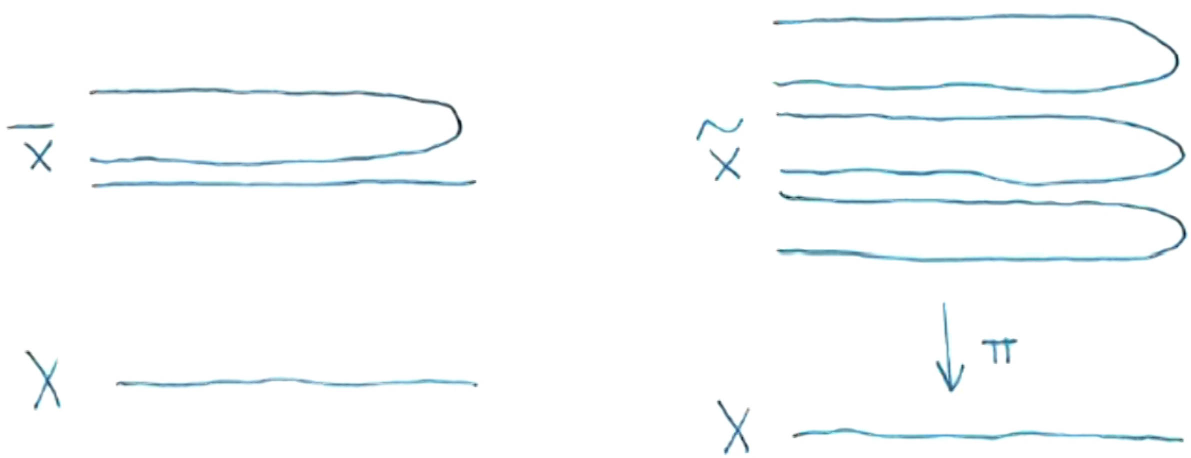
Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita

$GL(n)$	G
$E \in$ Locally constant. vector bdles	Locally constant ppl G-bundle $\mathrm{Hom}^{iso}_{x}(E, \underline{\mathbb{C}^n})$ $GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$
spectral covers $\tilde{X} \xrightarrow{n:1} X$	Cameral cover (Donagi, Faltings, Scognamillo) $\mathrm{Hom}^{iso}_{X}(\tilde{X}, \underline{\{1,2,\dots,n\}}_X)$

$SL(3,\mathbb{C})$ example: spectral cover VS cameral cover

- Take $G = SL(3,\mathbb{C})$, generically 3 distinct eigenvalues.
- Ramification of order 2: two eigenvalues become equal.



Co-ordinate charts, character varieties & non-abelianization.

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$GL(n)$	G
$E \in$ Locally constant. vector bdles	Locally constant ppl G-bundle
spectral covers $\tilde{X} \xrightarrow{n:1} X$	Cameral cover (Donagi, Faltings, Scognamillo) $\mathrm{Hom}_X^{iso}(\tilde{X}, \underline{\{1,2,\dots,n\}}_X)$
$\tilde{X}^c \hookrightarrow K_{X^c}(D)$ $\lambda^n + a_1\lambda^{n-1} + \dots + a_n$. λ is taut section	

$\mathrm{Hom}_{\mathbb{C}}^{iso}(E, \underline{\mathbb{C}}^n)$

$GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$

$\mathcal{E}(n)$ local system on X

non-abelianization.

$GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$

$(E, \underline{\mathbb{C}}^n)$

ttings, Scognamillo)

$a_i \in \Gamma(X^c, K_{X^c}(1)^{\otimes i})$

non-abelianization.

$(E, \underline{\mathbb{C}}^n)$

$GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$

(Hollands, Scognamillo)

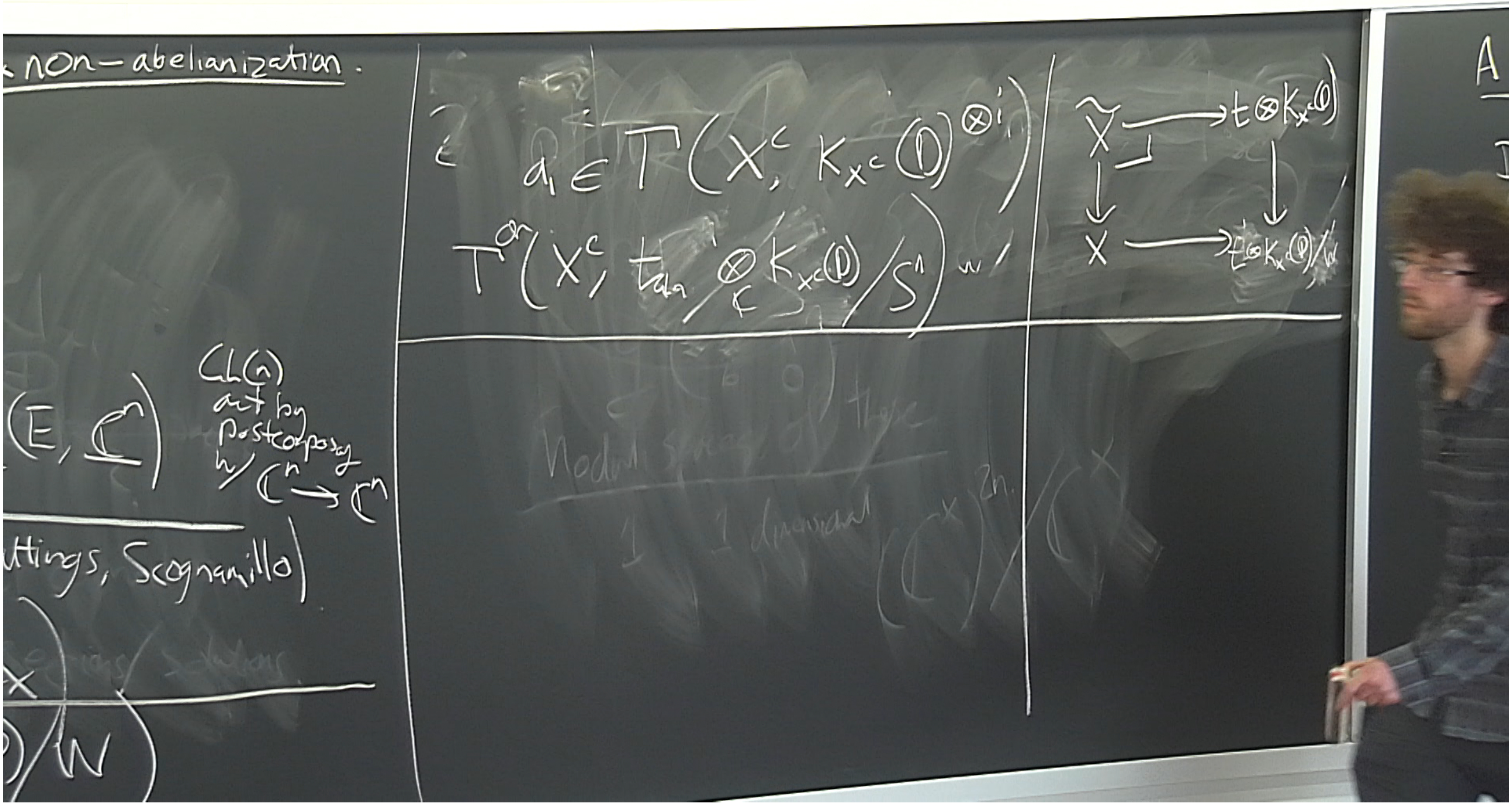
$a_1 \in \Gamma(X^c, K_{X^c}(D)^{\otimes i})$

$\Gamma^{or}(X^c, t_{\mathrm{dia}} \otimes_K K_{X^c}(D) / S^1)$ w/

Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita

$GL(n)$	G
$E \in$ Locally constant. vector bdles	Locally constant ppl G-bundle, $\mathrm{Hom}^{\mathrm{iso}}_{\mathcal{L}}(E, \underline{\mathbb{C}}^n)$ — $GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$
Spectral covers $\begin{array}{c}\overline{X} \\ n:1 \downarrow \\ X\end{array}$	Cameral cover (Donagi, Faltings, Scognamillo) $\mathrm{Hom}^{\mathrm{iso}}_X(\overline{X}, \underline{\{1,2,\dots,n\}}_X)$
$\overline{X}^c \hookrightarrow K_{X^c}(D)$ $\lambda^n + a_1\lambda^{n-1} + \dots + a_n$. λ is taut section	$\Gamma(\overline{X}^c, \mathfrak{t} \otimes K_{X^c}(D)/W)$


non-abelianization.
GL(n) act by postcomposing w/ C^n → C^n
ttings, Scognamillo)
a_i ∈ Γ(X^c, K_{X^c}(D)^{⊗i})
X̃ → t ⊗ K_{X^c}(D)
X → t ⊗ K_X(D)/W

non-abelianization.

$(E, \underline{\mathbb{C}}^n)$ $GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$

(...ttings, Scognamillo)

$\cdot)/N$

$a_i \in \Gamma(X^c, K_{X^c}(P)^{\otimes i})$

$\Gamma^{or}(X^c, t_{diag} \otimes_{\mathbb{C}} K_{X^c}(P) / S^n)$ w/

$$\begin{tikzcd} \tilde{X} \arrow[r] \arrow[d] & t \otimes K_X(P) \arrow[d] \\ X \arrow[r] & t \otimes K_X(P)/W \end{tikzcd}$$

G_m-Local systems on $\tilde{X} \setminus \pi^{-1}(P)$

N-local systems $\mathcal{E}_N$ w/ $\mathcal{E}_N / T \cong \tilde{X} \setminus \pi^{-1}(P)$

(analogue of Donagi-Gaitsgory)

Co-ordinate charts, character varieties & non-abelianization.

jt w/ Matei Ionita

$GL(n)$	G
$E \in$ Locally constant. vector bdles	Locally constant ppl G-bundle, $\mathrm{Hom}^{\mathrm{iso.}}_{\mathbb{C}}(E, \mathbb{C}^n)$
spectral covers $\overline{X} \xrightarrow{n:1} X$	Cameral cover (Donagi, Faltings, Scognamillo) $\tilde{X} := \mathrm{Hom}^{\mathrm{iso}}_X(\overline{X}, \underline{\{1, 2, \ldots, n\}}_X)$
$\overline{X}^c \hookrightarrow K_{X^c}(D)$ $\lambda^n + a_1\lambda^{n-1} + \ldots + a_n$. λ is taut section	$\Gamma(X^c, \mathfrak{t} \otimes K_{X^c}(D) / W)$

$GL(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$

$\cong \left\{ \begin{array}{l} \mathcal{E}_N / \cancel{T} \\ \downarrow \\ \mathcal{E}_N / T \cong \tilde{X} \setminus \pi^{-1}(p) \end{array} \right.$ is a T-bundle on cameral cover which is "N-shifted weakly W-equivariant" $\}$ $(g \geq 2)$

(analog of DG)

$$1 \to T \to N^{\mathcal{E}} \to W \to 1$$

$$\begin{array}{ccccccccc} 1 & \to & T & \to & N & \to & W & \to & 1 \\ \downarrow & & \downarrow & & \searrow & & & & \| \\ 1 & \to & \mathrm{Hom}_c(\tilde{X}, T) & \to & \mathrm{Aut}_X\left(\begin{array}{c}\mathcal{E}_N \\ \downarrow \\ \mathcal{E}_N/T\end{array}\right) & \to & W & \to & 1 \end{array}$$

$\otimes K_X \cdot \mathcal{O})$

$\otimes K_X(\mathcal{P})/W$

ems $\mathcal{E}_N$

$\cong \tilde{X} \setminus \pi^{-1}(p)$

Gaitsgory)

belianization.

$\mathbb{C}h(n)$ act by postcomposing w/ $\mathbb{C}^n \to \mathbb{C}^n$

cognamillo)

$a_i \in \Gamma(X^c, K_{X^c}(\mathbb{P})^{\otimes i})$

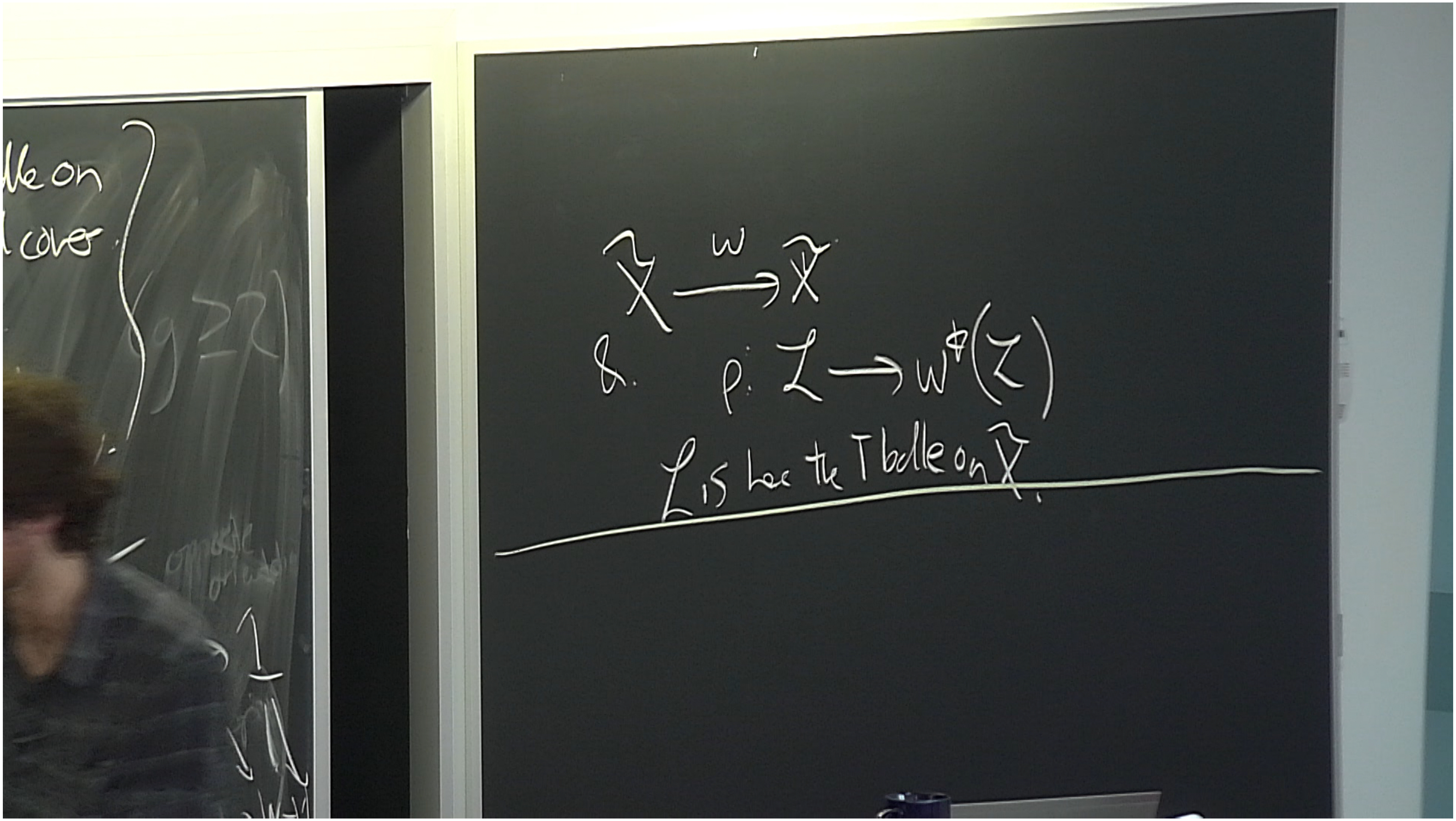
$\Gamma^{or}(X^c, t_{\mathbb{C}h(n)} \otimes_{\mathbb{C}} K_{X^c}(\mathbb{P}) / S^n)$ w/

$$\begin{tikzcd} \tilde{X} \arrow[r] \arrow[d] & t \otimes K_{X^c}(\mathbb{P}) \arrow[d] \\ X \arrow[r] & t \otimes K_X(\mathbb{P}) / W \end{tikzcd}$$

G_m-Local systems on $\tilde{X} \setminus \pi^{-1}(\mathbb{P})$

N-local systems $\mathcal{E}_N^{\to X}$ w/ $\mathcal{E}_N / T \cong \tilde{X} \setminus \pi^{-1}(\mathbb{P})$

(analogue of Donagi-Gaitsgory)



$\tilde{Y} \xrightarrow{w} \tilde{X}$
8. $\rho: \mathcal{L} \longrightarrow w^*(\mathcal{L})$
$\mathcal{L}$ is here the T bdle on $\tilde{Y}$.

Co-ordinate charts, character varieties & non-abelianization.

For $GL(n)$:

$N \cong T \rtimes W$.

$1 \to T \to N \rightleftarrows W \to 1$.

$$\left\{\begin{matrix}\text{N-shifted} \\ \text{weakly} \\ \text{W-equiv} \\ \text{T-local system}\end{matrix}\right\} \cong \left\{\begin{matrix}\text{W-equivariant} \\ \text{T-local systems}\end{matrix}\right\}$$

λ ... ta... secti...

Co-ordinate charts, character varieties & non-abelianization.

For $GL(n)$:

$N \cong T \rtimes W$.

$1 \to T \to N \to W \to 1$

$$\left\{\begin{matrix}N\text{-shifted} \\ \text{weakly} \\ W\text{-equivt} \\ T\text{-local system}\end{matrix}\right\} \cong \left\{\begin{matrix}W\text{-equivariant} \\ T\text{-local systems}\end{matrix}\right\}$$

For $GL(n)$:

$$\begin{matrix} \mathcal{L} & & \tilde{X} \setminus \pi^{-1}(p) \\ \downarrow & \swarrow \pi_i & \downarrow \\ \tilde{X} \setminus \pi^{-1}(p) & \tilde{X} \setminus \pi^{-1}(p) \to & X \setminus p \end{matrix}$$

$S_{n-1} \hookrightarrow S_n$

$\widehat{\bigoplus_{i=1}} \pi_i^* (\mathcal{L})$

$T^{or}($

λ ... taut. section

1. ~~Analogue of~~

~~$\mathrm{Loc}_{\mathbb{C}^\times}(\overline{X} \setminus \pi^{-1}(p))$~~

& analogue of the monodromy condition.

2. What is $\mathfrak{M}$?

3. Motivation for cutting & gluing procedure.

In GMN (no

nonab

(& a monodromy cond. around $\pi^{-1}(p)$)

$\left\{ \text{moduli space of } \mathbb{C}^\times\text{-connections on } \overline{X} \setminus \pi^{-1}(p) \right\}$

$\overline{X} \supset \pi^{-1}(p) \quad Z$

$n:1 \downarrow \pi$

$X \supset D$ branch locus

(X non-compact $X = X^c \setminus D$)

In GMN (non-abelianization)

$\overline{X} \supset \pi^{-1}(P) \quad \mathcal{L}$

$n:1 \big\downarrow \pi$

$X \supset P$ branch locus

(X non-compact
$X = X^c \setminus D$)

{moduli space of $\mathbb{C}^\times$-connections on $\overline{X} \setminus \pi^{-1}(P)$ (& a monodromy of -1 around $\pi^{-1}(P)$)}

nonab

{~~GL(n)~~ local systems on $X \setminus P$ together w/ locally constant sections of Aut($\mathcal{E}$)}

forget

{~~GL(n)~~ $\mathcal{E}$ Local systems on $X \setminus P$}

each connected compt of $\mathcal{W} \subset X \setminus P$

{$\mathcal{E}$ ~~GL(n)~~ local system on $X \setminus (P \cup J)$}

intersection of lines in $\mathcal{W}\mathcal{W}$

{$\mathcal{E}$ ~~GL(n)~~ local system on X}

induce an N-band system to a G local system

$\pi_*(\mathcal{L})$

...dle on
...l cover
...ed
...uivariant

$\to N \to W \to 1$

$T) \to \mathrm{Aut}_X\left(\begin{smallmatrix} \mathcal{E}_N \\ \downarrow \\ \mathcal{E}_r/T \end{smallmatrix}\right) \to W \to 1$

$$\begin{tikzcd} \tilde{X} \arrow[r] \arrow[rr, bend right, "\chi_\alpha"'] & t_{K_c(D)} \arrow[r, "\alpha"] & \mathrm{Tot}(K_{X^c}(D)) \arrow[d] \\ & & \mathrm{Tot}(K_{\tilde{X}^c}(D)) \end{tikzcd}$$

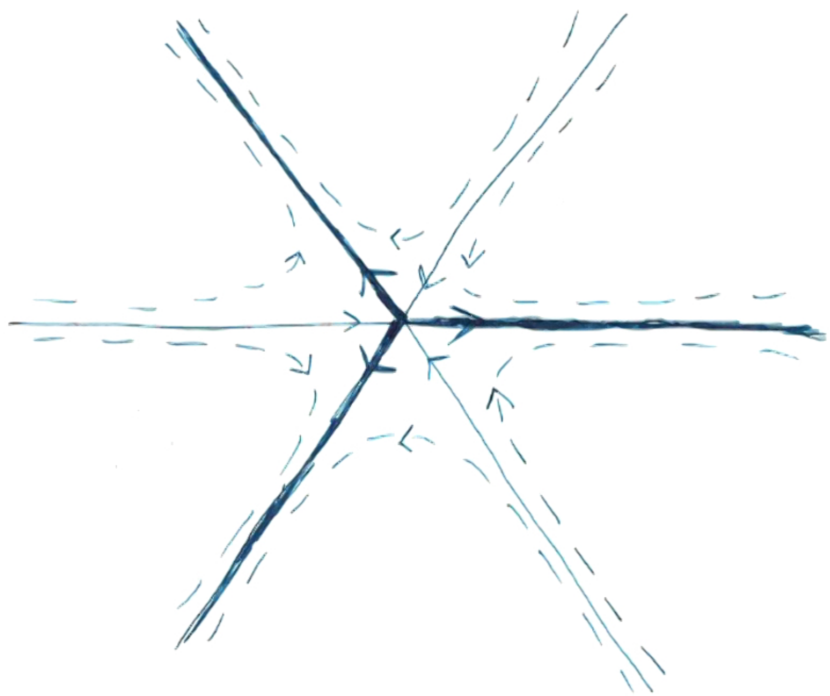
χ_α which is non-vanishing away from ramification locus

real foliation on $\tilde{X} \setminus R$

$\chi_\alpha(V_\alpha) \in \mathbb{R}^+$ real valued projection vector field

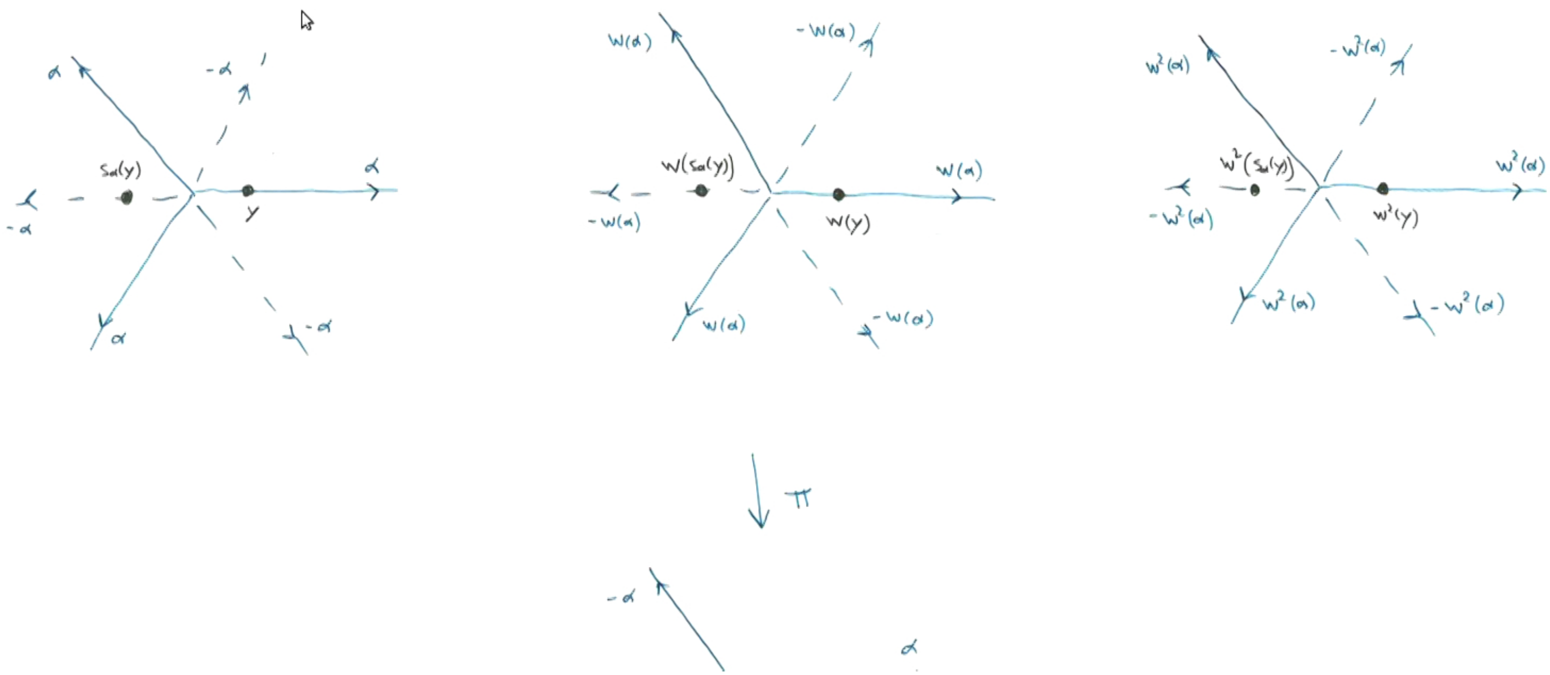
vector field

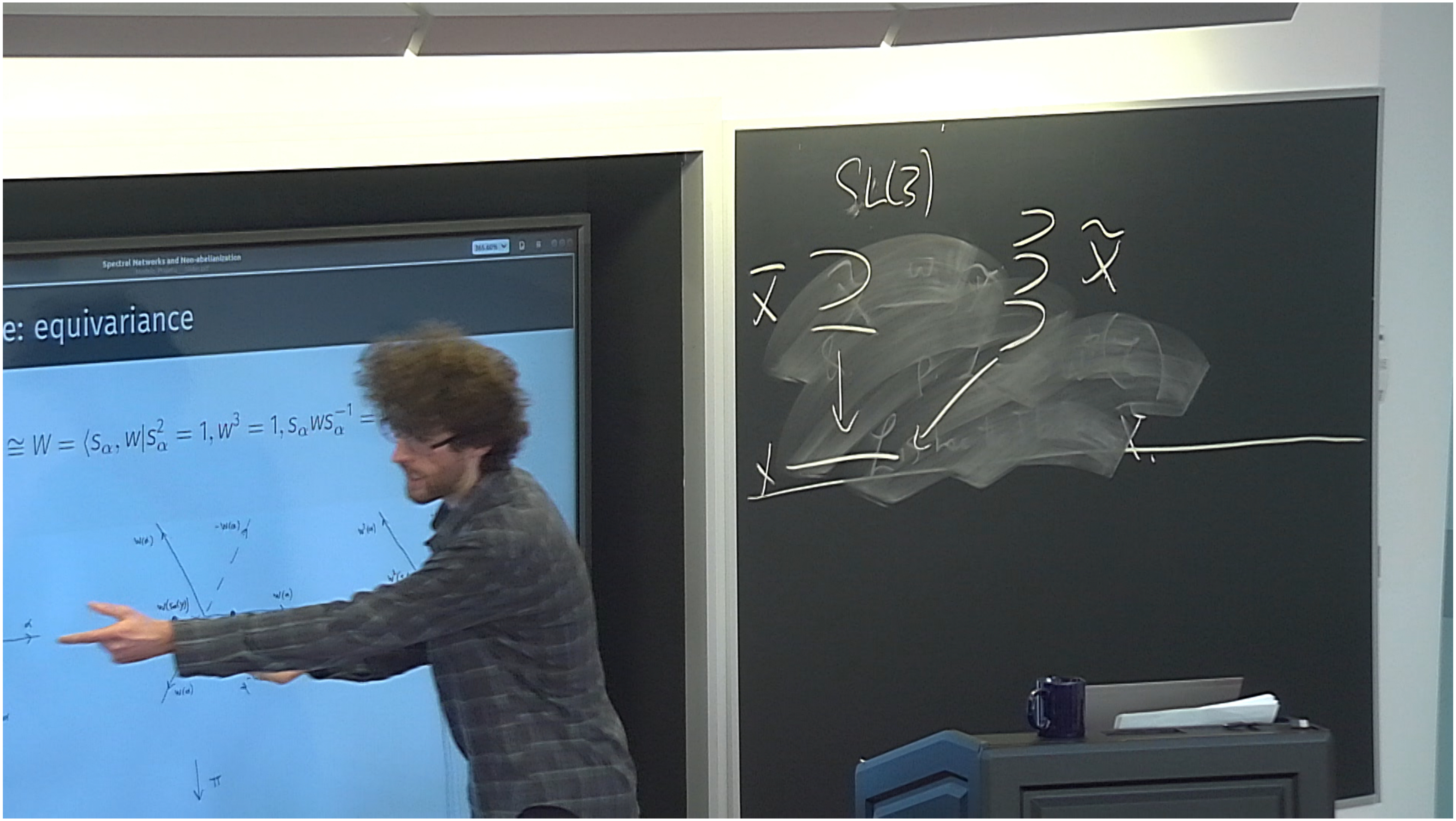
Projective vector field around ramification point



$SL(3,\mathbb{C})$ example: equivariance

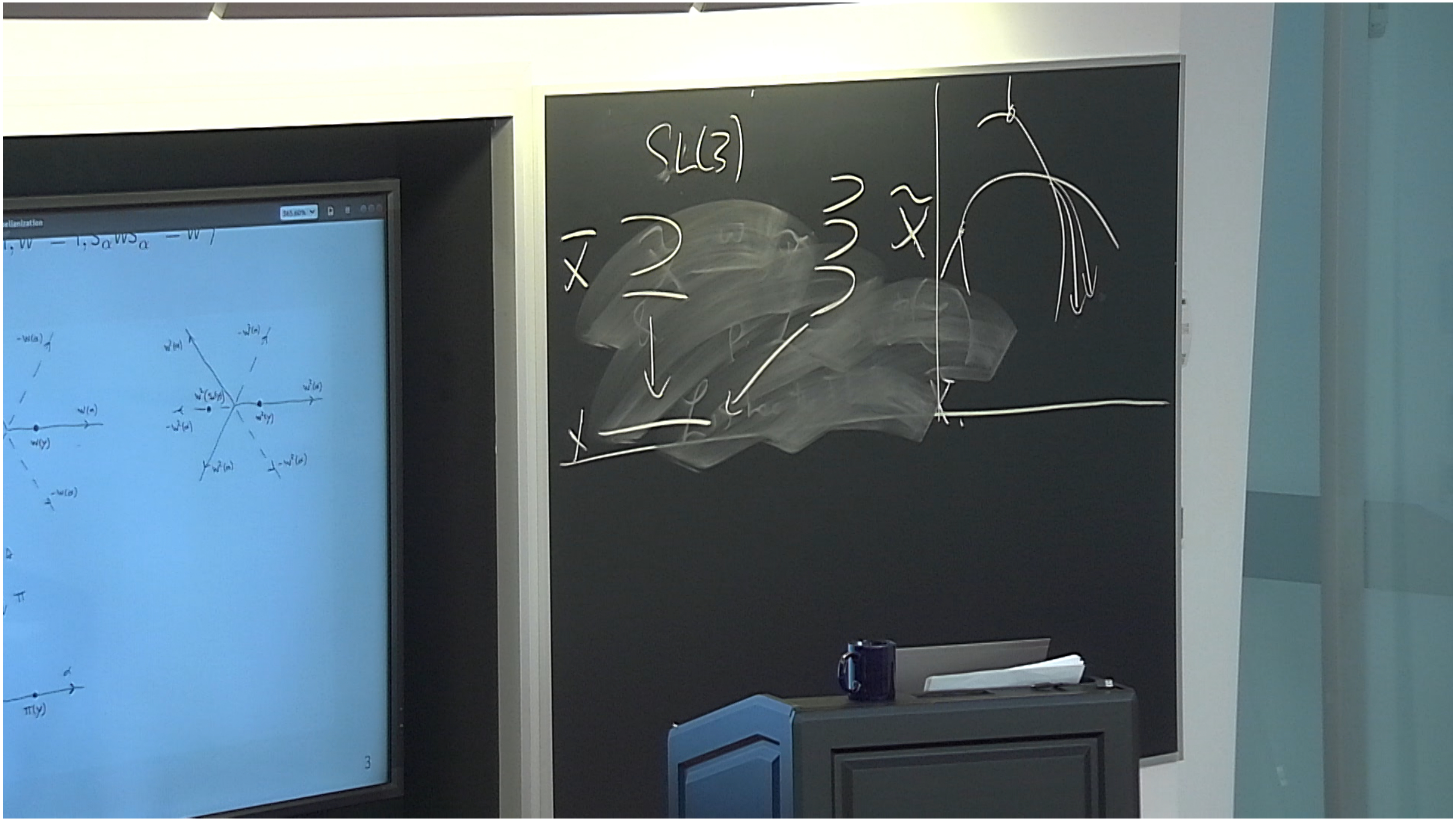
$$S_3 \cong W = \langle s_\alpha, w | s_\alpha^2 = 1, w^3 = 1, s_\alpha w s_\alpha^{-1} = w^2 \rangle$$




Spectral Networks and Non-abelianization
e: equivariance
$\cong W = \langle s_\alpha, w | s_\alpha^2 = 1, w^3 = 1, s_\alpha w s_\alpha^{-1} =$
$\downarrow \pi$
SL(3)
$\bar{X}$
$\tilde{X}$
X

Take the lines starting at $r \in R$.
along the projection vector field (for each α)

Image of these lines in X.


SL(3)
π(y)
3

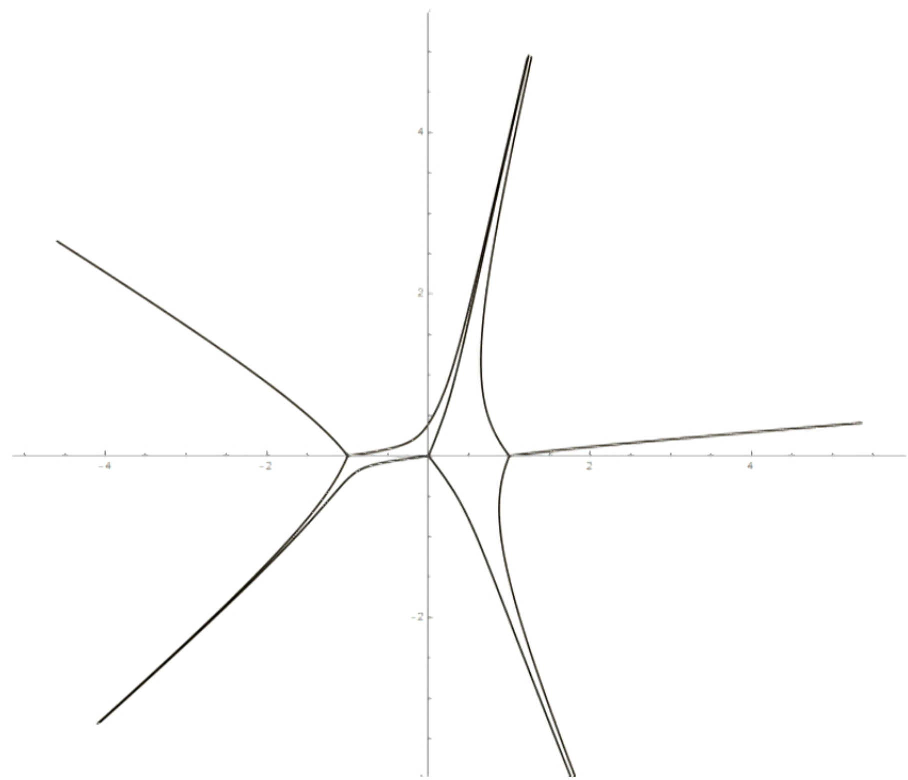
Take the lines starting at $r \in R$.
along the projection vector field (for each α)

At intersections add new lines for
positive natural number combos of roots

Then pushforward to X.

A spectral network for $SL(2,\mathbb{C})$

On $\mathbb{P}^1$, characteristic polynomial $\lambda^2 - z(z-1)(z+1)$.



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Spectral Networks and Non-abelianization

Modelo_Projeto___Slides.pdf

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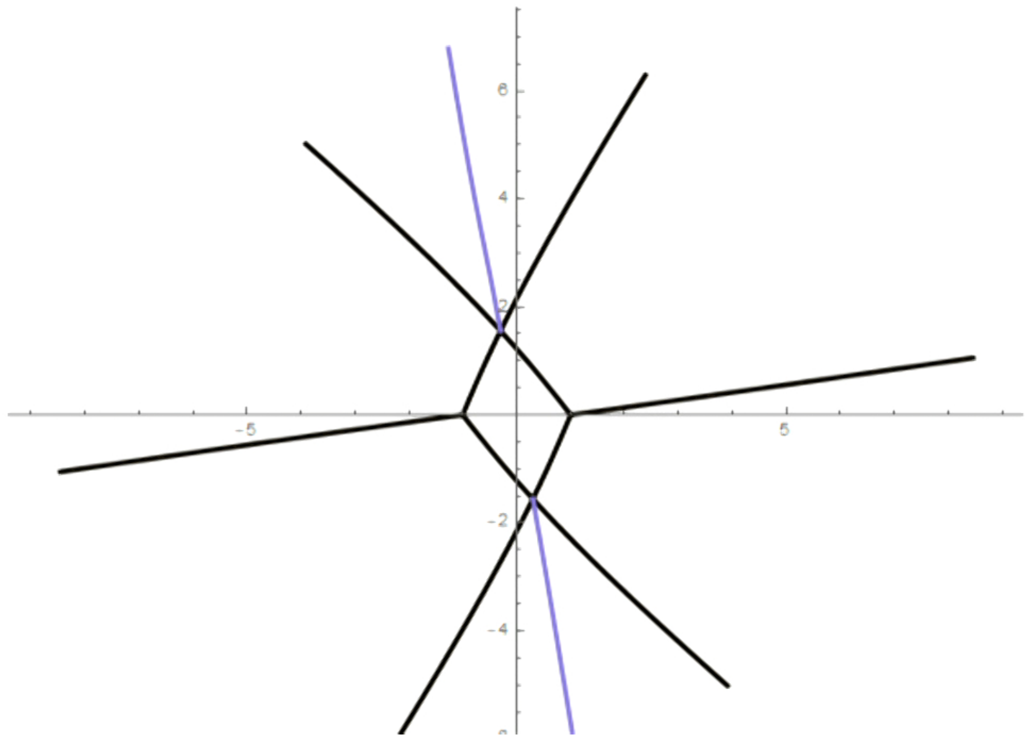
A spectral network for $SL(3, \mathbb{C})$

On $\mathbb{P}^1$, characteristic polynomial $\lambda^3 + 3\lambda + 2iz$.



Co-ordinate charts, character varieties & non-abelianization.

At branch pts (smooth)

$\rightsquigarrow$ trivialize then cand cover near branch pt, so that monodromy of $\bar{X}$ is S_α.

Then: our result on monodromy is

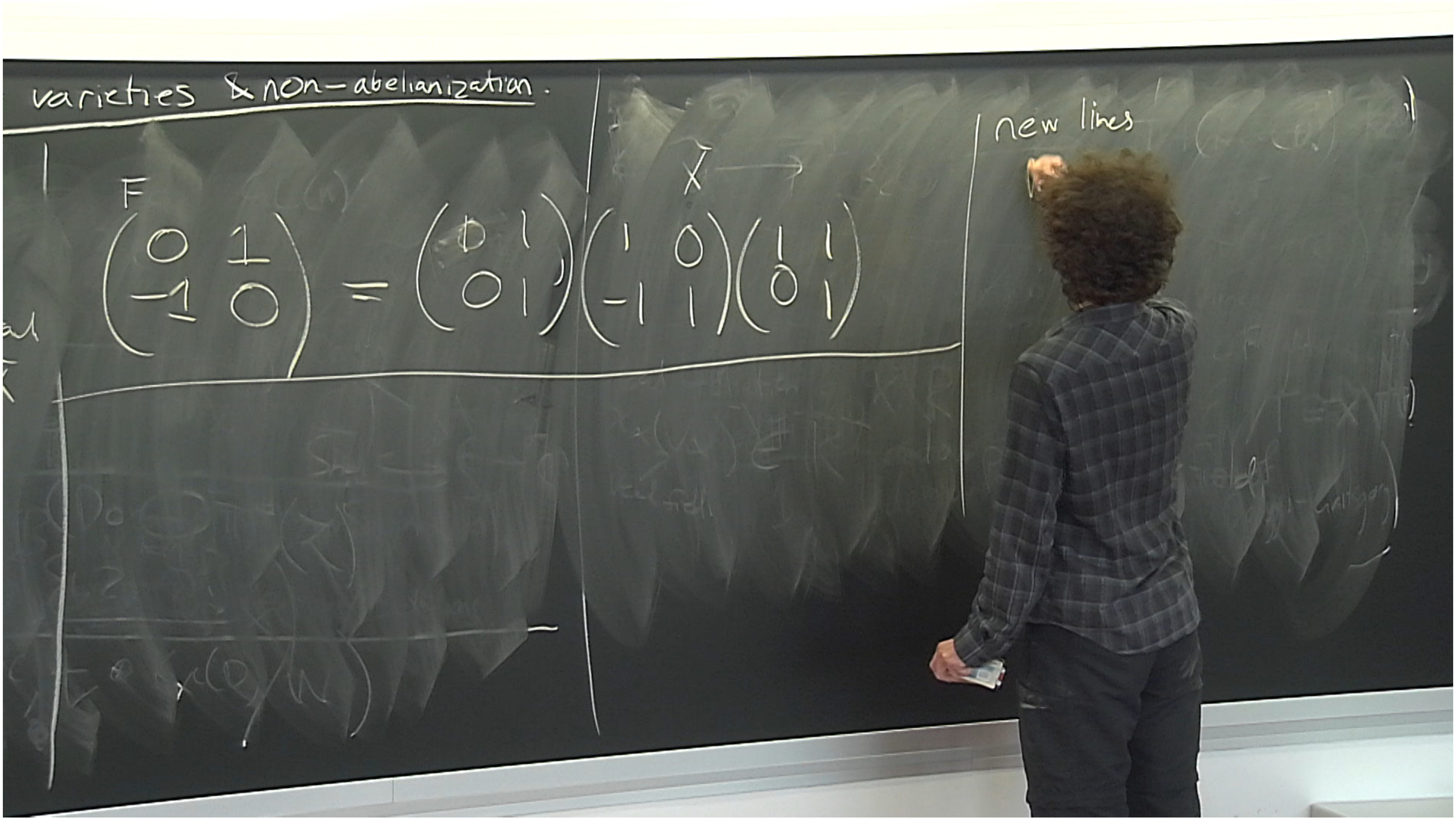
N-monodromy is in $S_\alpha T_\alpha$

$T_\alpha = \mathrm{image}(T_{SL_2}) \subset T_G$

$I_\alpha : SL_2 \to G$

F

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =$$


varieties & non-abelianization.
F
$\begin{pmatrix}0&1\\-1&0\end{pmatrix} = \begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}$
$\tilde{X} \longrightarrow$
new lines

varieties & non-abelianization.

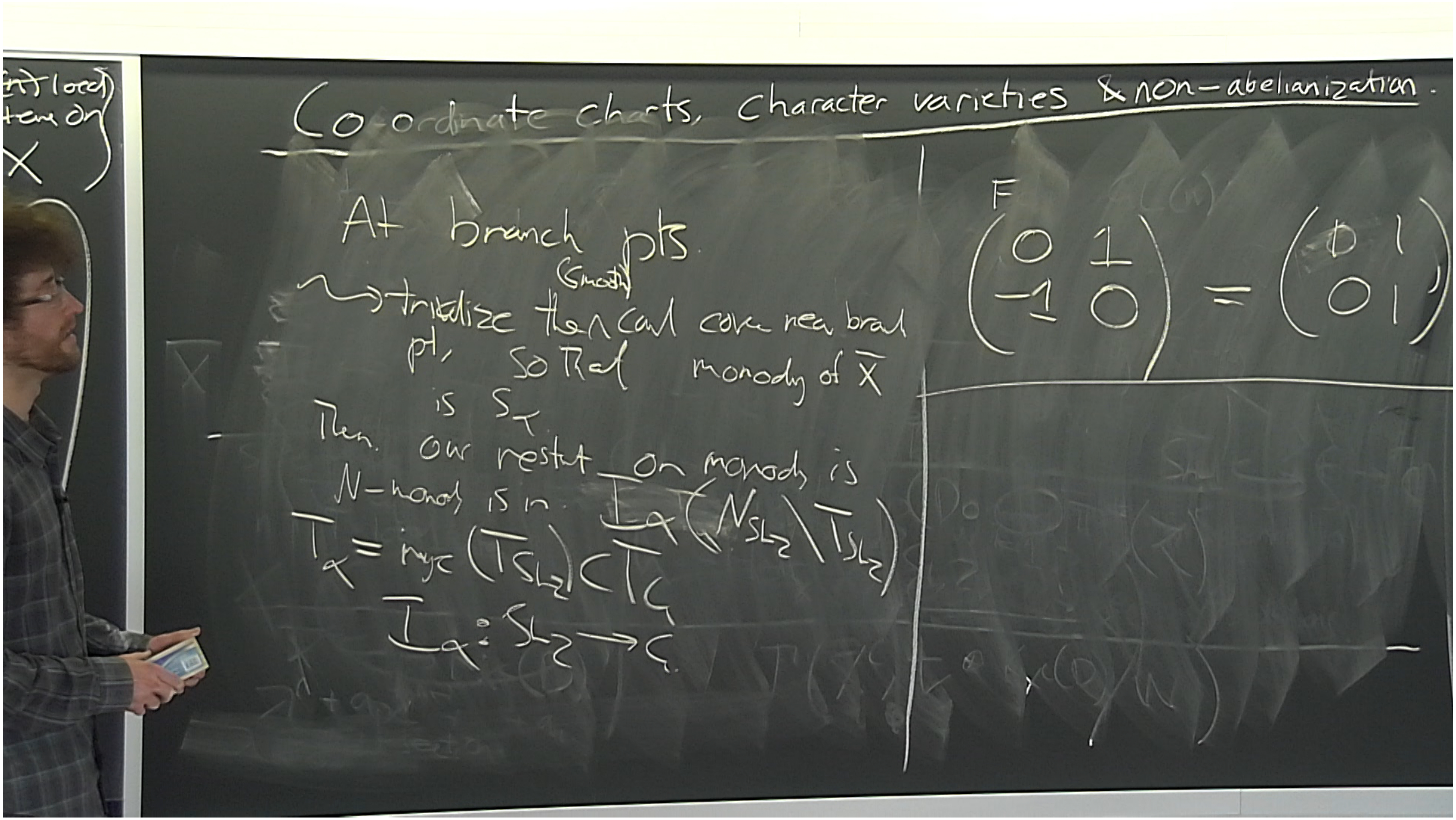
F

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$\tilde{X} \longrightarrow$

new lines (trivial)

Prop: As long as all the root integers are plac on t $\exists$ elem of U_α for each outgoing line s.t. the monodromy we get is 0



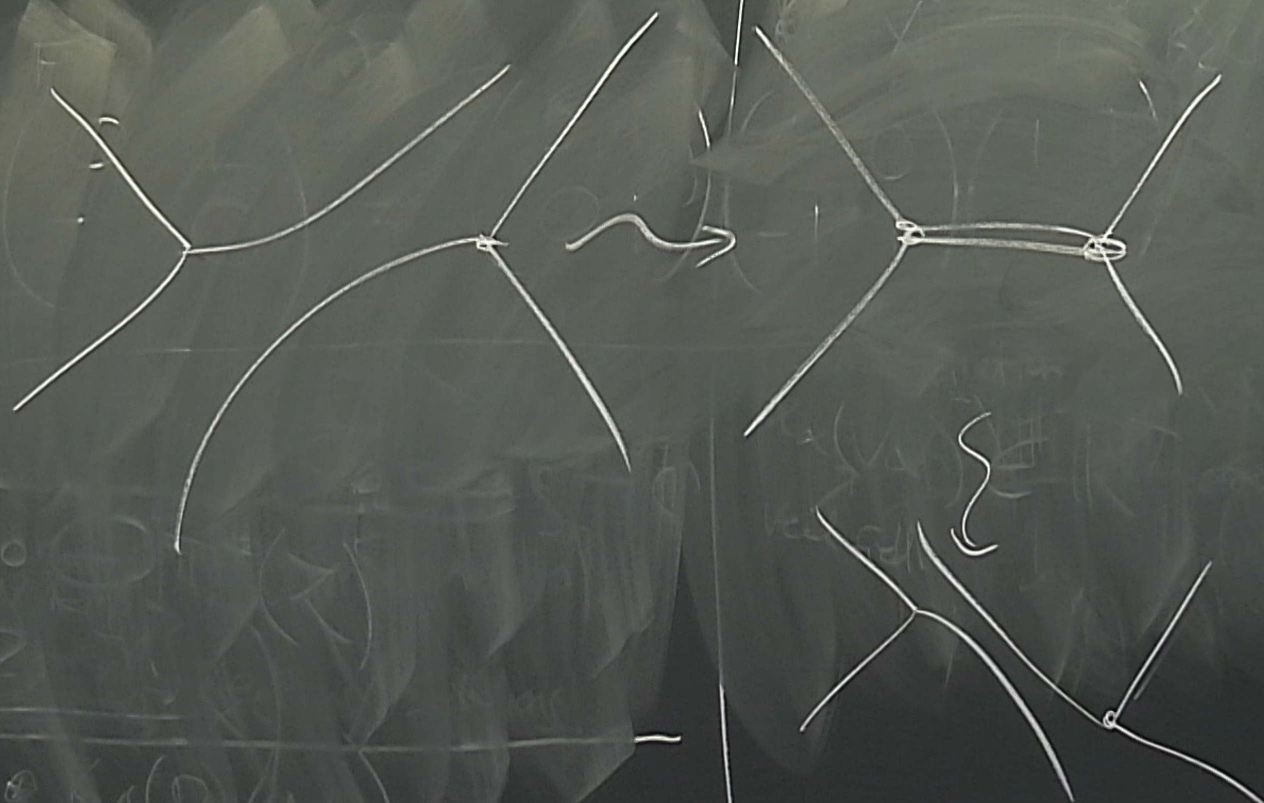
Co-ordinate charts, character varieties & non-abelianization.
At branch pts (smooth)
⇝ trivialize the cover near branch pt, so that monodromy of $\bar{X}$ is S_τ.
Then our restriction on monodromy is
N-monodromy is in $\mathcal{I}_\alpha(N_{SL_2} \setminus T_{SL_2})$
$T_\alpha = \mathrm{image}(T_{SL_2}) \subset T_G$
$\mathcal{I}_\alpha : SL_2 \to G$
$F \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$

-ts, character varieties & non-abelianization.

pts.

cal cover near brad
monodry of $\overline{X}$

on monody is
$(N_{SL_2} \backslash T_{SL_2})$
T_C
$\to C.$



new
P
a
$\exists$
St.