Title: PSI 2019/2020 - Standard Model and Beyond part 2 - Lecture 3

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Collection: PSI 2019/2020 - Standard Model and Beyond - Part 2

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Lecture 16 $\quad SM + \underline{B}$

Grand Unification

SU(5), Pati-Salam

Spin(10)

	SU(3)	SU(2)	U(1)	
q_L	3	2	1/6	(1)
u_R	3	1	2/3	(4)
d_R	3	1	$-1/3$	(-2)
l_L	1	2	$-1/2$	(-3)
ν_R	1	1	0	(0)
e_R	1	1	-1	(-6)
h	1	2	1/2	

$q_L = \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes$

$u_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes$

$d_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes$

$l_L = \mathbb{C} \otimes \mathbb{C}^2 \otimes$

$\nu_R = \mathbb{C} \otimes \mathbb{C} \otimes$

$e_R = \mathbb{C} \otimes \mathbb{C} \otimes$

$U(1)$

$1/6 \quad (1)$

$2/3 \quad (4)$

$-1/3 \quad (-2)$

$-1/2 \quad (-3)$

$0 \quad (0)$

$-1 \quad (-6)$

$1/2$

$$q_L = \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1 \qquad q_L^* = \mathbb{C}^{3*} \otimes \mathbb{C}^{2*} \otimes \mathbb{C}_{-1}$$

$$u_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_4 \qquad u_R^* = \mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}$$

$$d_R = \mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2} \qquad d_R^* = \mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_2$$

$$l_L = \mathbb{C} \otimes \mathbb{C}^2 \otimes \mathbb{C}_{-3} \qquad l_L^* = \mathbb{C} \otimes \mathbb{C}^{2*} \otimes \mathbb{C}_3$$

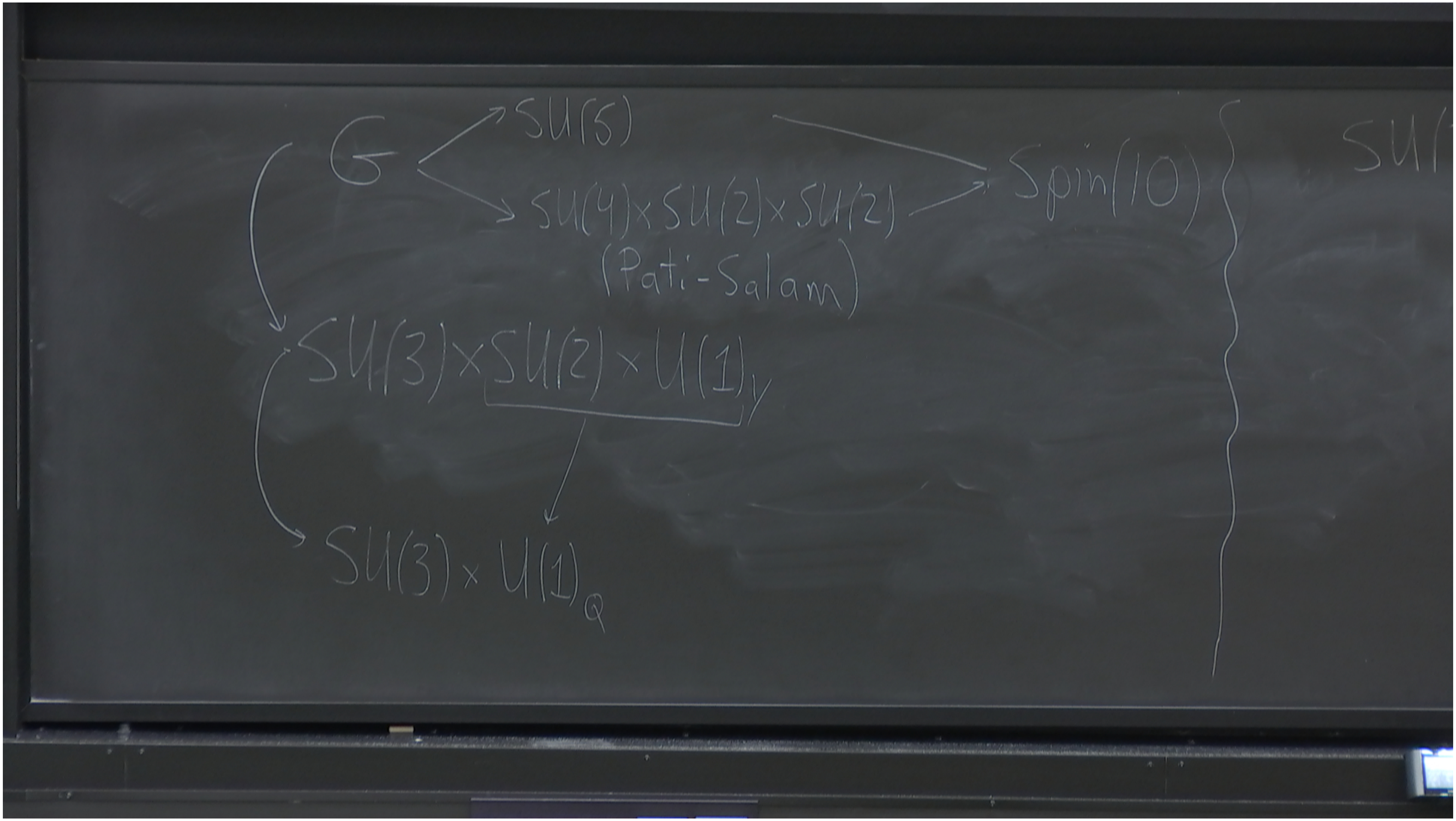
$$\nu_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0 \qquad \nu_R^* = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_0$$

$$e_R = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_{-6} \qquad e_R^* = \mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6$$

$V = \{v_1, \ldots, v_m\}$ m d

$W = \{w_1, \ldots, w_n\}$ n d

$V \oplus W = \{v_1, \ldots, v_m, w_1,$


G
SU(5)
SU(4)×SU(2)×SU(2)
(Pati-Salam)
Spin(10)
SU(3)×SU(2)×U(1)Y
SU(3)×U(1)Q

h	1	2	$1/2$

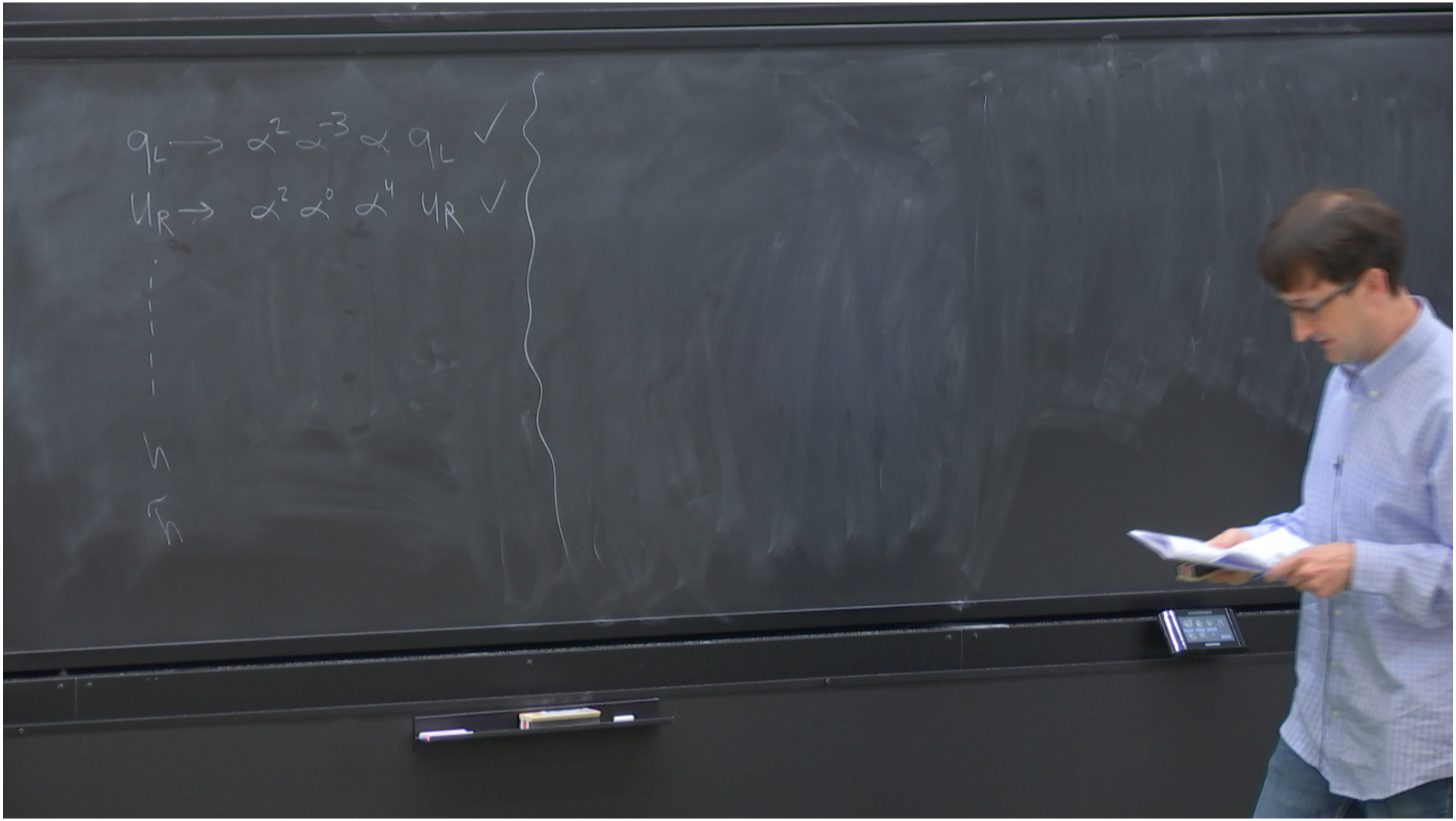
$$e_R = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_{-6} \qquad e_R^* = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_6$$

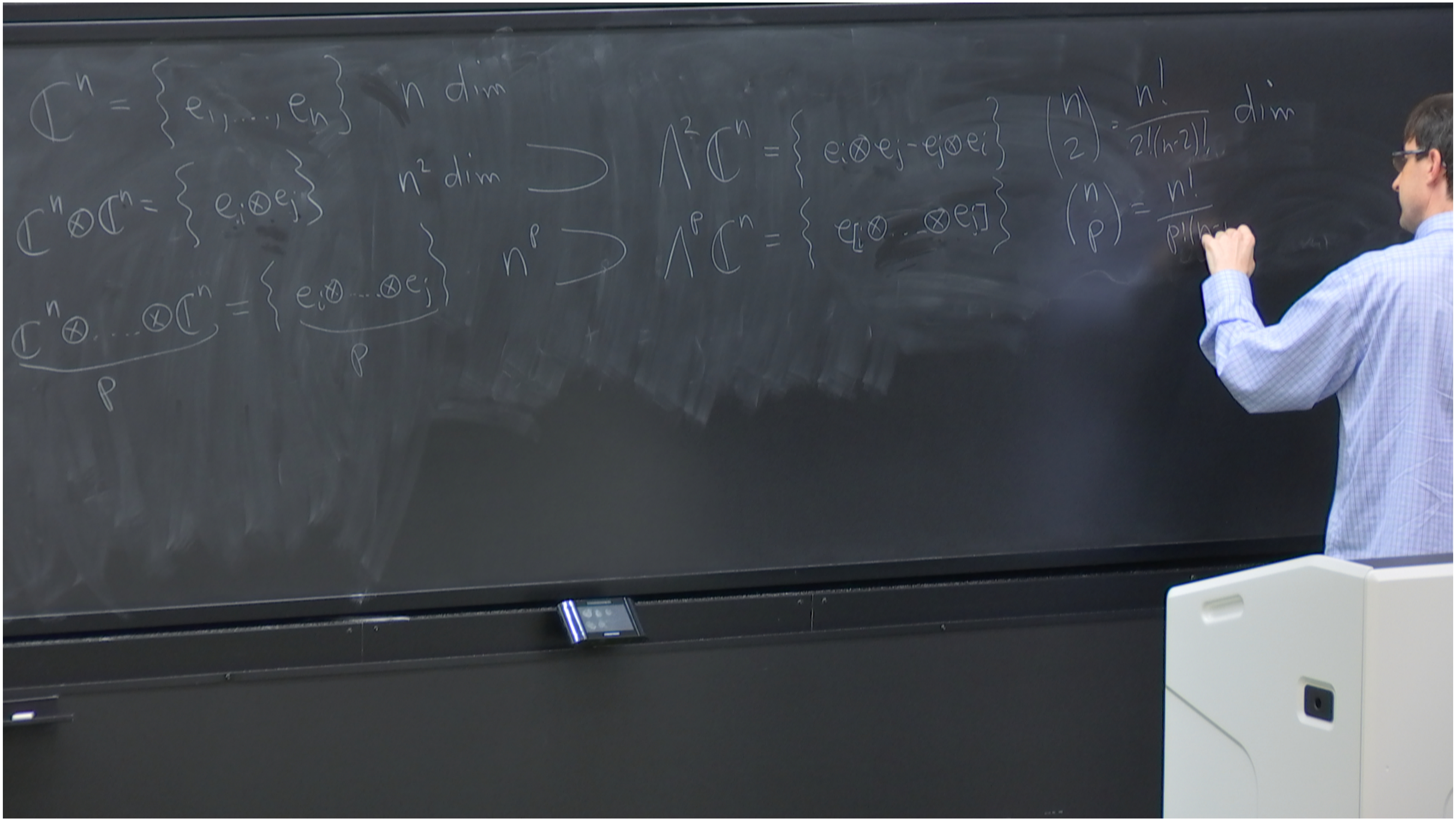
$$\left(\begin{array}{c|c} 1_2 & \\ \hline & U_3 \end{array}\right)\left(\begin{array}{c} \\ \hline \\ \end{array}\right)$$

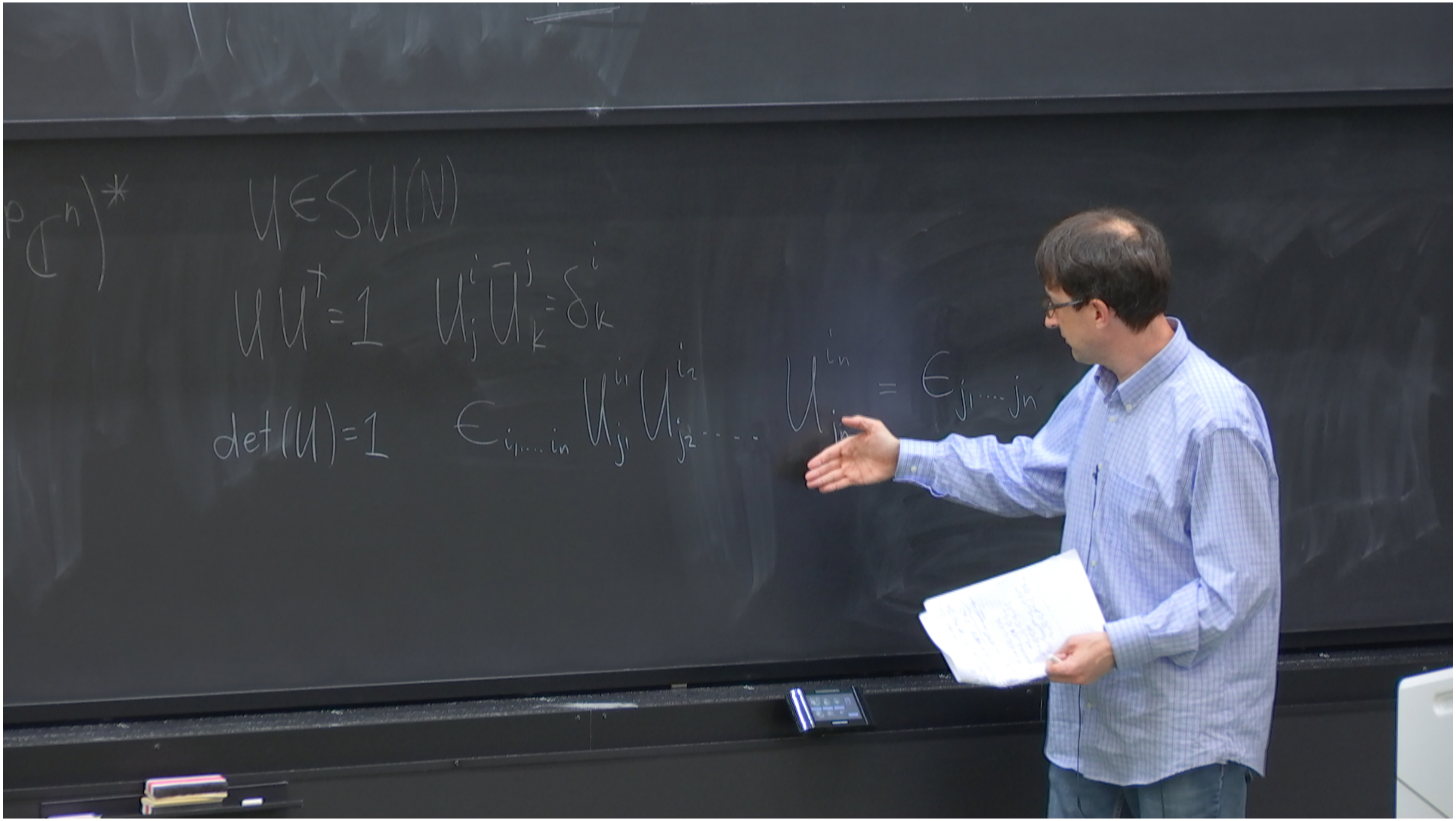
$$(U_3, U_2, \alpha) = \left(\begin{array}{c|c} \alpha^3 U_2 & \\ \hline & \alpha^{-2} U_3 \end{array}\right)$$

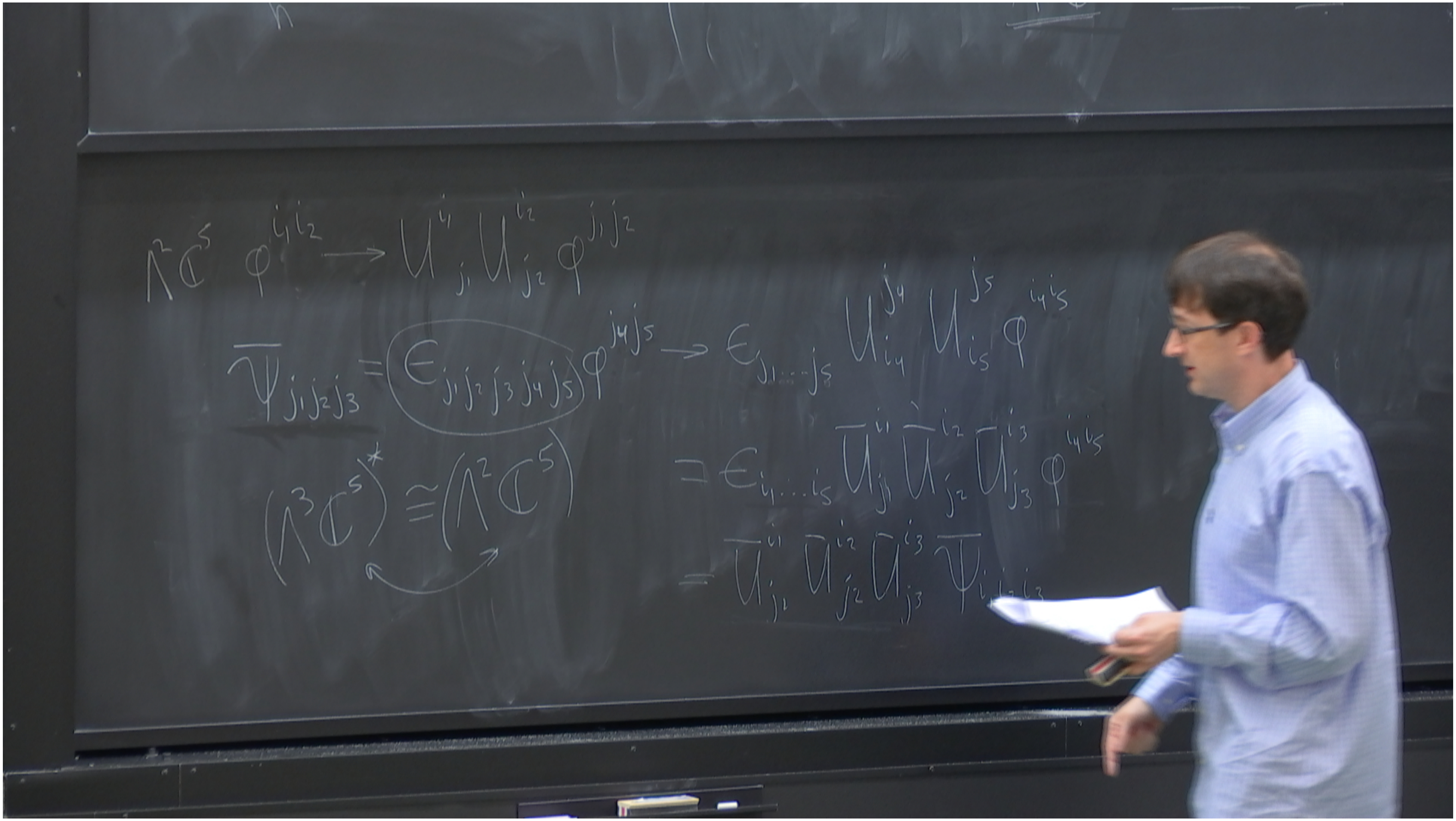
$$\left(\begin{array}{c|c} U_2 & \\ \hline & 1_3 \end{array}\right) \qquad \boxed{\alpha^6 = 1} \qquad (\alpha^2 I_3, \alpha^{-3} I_2, \alpha) \to I_S$$

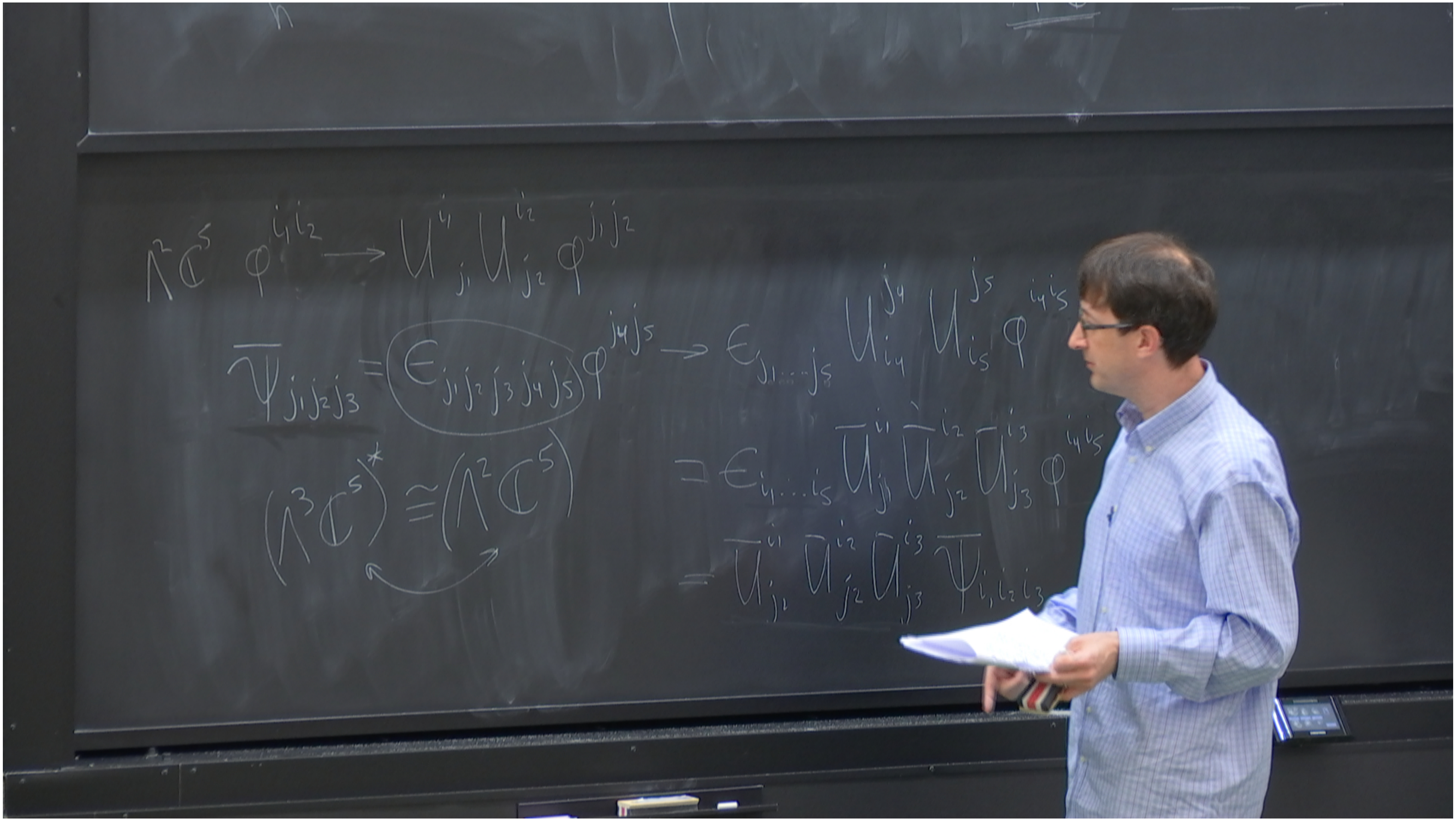
$$\left(\begin{array}{c|c} \alpha^p 1_2 & \\ \hline & \alpha^q 1_3 \end{array}\right) \qquad 2p+3q=0$$


q_L → α^2 α^-3 α q_L ✓
u_R → α^2 α^0 α^4 u_R ✓
h
h̃


$\mathbb{C}^n = \{e_1, \ldots, e_n\}$ n dim
$\mathbb{C}^n \otimes \mathbb{C}^n = \{e_i \otimes e_j\}$
n^2 dim
$\supset \Lambda^2 \mathbb{C}^n = \{e_i \otimes e_j - e_j \otimes e_i\}$
$\binom{n}{2} = \frac{n!}{2!(n-2)!}$ dim
$\underbrace{\mathbb{C}^n \otimes \cdots \otimes \mathbb{C}^n}_{p} = \{\underbrace{e_i \otimes \cdots \otimes e_j}_{p}\}$
n^p
$\supset \Lambda^p \mathbb{C}^n = \{e_{[i} \otimes \cdots \otimes e_{j]}\}$
$\binom{n}{p} = \frac{n!}{p!}$


$U \in SU(N)$
$UU^\dagger = 1 \quad U^i_j \bar{U}^j_k = \delta^i_k$
$\det(U) = 1 \quad \epsilon_{i_1 \dots i_n} U^{i_1}_{j_1} U^{i_2}_{j_2} \dots U^{i_n}_{j_n} = \epsilon_{j_1 \dots j_n}$


$\Lambda^2\mathbb{C}^5 \quad \varphi^{i_1 i_2} \longrightarrow U^{i_1}{}_{j_1} U^{i_2}{}_{j_2} \varphi^{j_1 j_2}$
$\overline{\Psi}_{j_1 j_2 j_3} = \epsilon_{j_1 j_2 j_3 j_4 j_5} \varphi^{j_4 j_5} \rightarrow \epsilon_{j_1 \dots j_5} U^{j_4}{}_{i_4} U^{j_5}{}_{i_5} \varphi^{i_4 i_5}$
$= \epsilon_{i_1 \dots i_5} \overline{U}^{i_1}{}_{j_1} \overline{U}^{i_2}{}_{j_2} \overline{U}^{i_3}{}_{j_3} \varphi^{i_4 i_5}$
$= \overline{U}^{i_1}{}_{j_1} \overline{U}^{i_2}{}_{j_2} \overline{U}^{i_3}{}_{j_3} \overline{\Psi}_{i_1 i_2 i_3}$
$(\Lambda^3\mathbb{C}^5)^* \cong (\Lambda^2\mathbb{C}^5)$

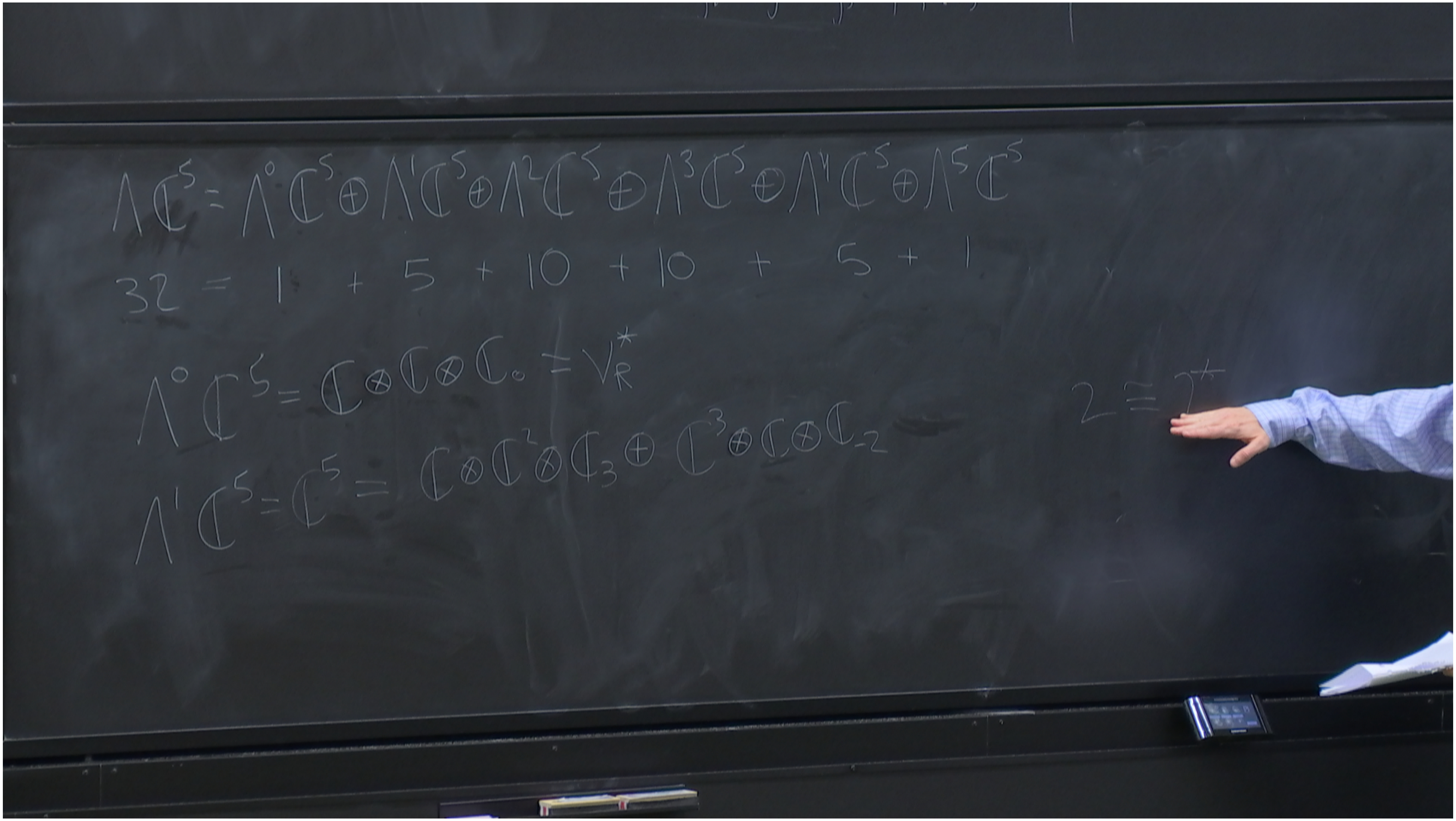

$\Lambda^2\mathbb{C}^5 \quad \varphi^{i_1 i_2} \longrightarrow U^{i_1}{}_{j_1} U^{i_2}{}_{j_2} \varphi^{j_1 j_2}$
$\overline{\Psi}_{j_1 j_2 j_3} = \epsilon_{j_1 j_2 j_3 j_4 j_5} \varphi^{j_4 j_5} \longrightarrow \epsilon_{j_1 \ldots j_5} U^{j_4}{}_{i_4} U^{j_5}{}_{i_5} \varphi^{i_4 i_5}$
$= \epsilon_{i_1 \ldots i_5} \overline{U}^{i_1}{}_{j_1} \overline{U}^{i_2}{}_{j_2} \overline{U}^{i_3}{}_{j_3} \varphi^{i_4 i_5}$
$= \overline{U}^{i_1}{}_{j_1} \overline{U}^{i_2}{}_{j_2} \overline{U}^{i_3}{}_{j_3} \overline{\Psi}_{i_1 i_2 i_3}$
$(\Lambda^3\mathbb{C}^5)^* \cong (\Lambda^2\mathbb{C}^5)$

$$\Lambda \mathbb{C}^5 = \Lambda^0\mathbb{C}^5 \oplus \Lambda^1\mathbb{C}^5 \oplus \Lambda^2\mathbb{C}^5 \oplus \Lambda^3\mathbb{C}^5 \oplus \Lambda^4\mathbb{C}^5 \oplus \Lambda^5\mathbb{C}^5$$

$$32 = 1 + 5 + 10 + 10 + 5 + 1$$

$$\Lambda^0\mathbb{C}^5 = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0 = V_R^*$$

$$\Lambda^1\mathbb{C}^5 = \mathbb{C}^5$$


$\Lambda\mathbb{C}^5 = \Lambda^0\mathbb{C}^5 \oplus \Lambda^1\mathbb{C}^5 \oplus \Lambda^2\mathbb{C}^5 \oplus \Lambda^3\mathbb{C}^5 \oplus \Lambda^4\mathbb{C}^5 \oplus \Lambda^5\mathbb{C}^5$
$32 = 1 + 5 + 10 + 10 + 5 + 1$
$\Lambda^0\mathbb{C}^5 = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0 \cong V_R^*$
$\Lambda^1\mathbb{C}^5 = \mathbb{C}^5 = \mathbb{C}\otimes\mathbb{C}^2\otimes\mathbb{C}_3 \oplus \mathbb{C}^3\otimes\mathbb{C}\otimes\mathbb{C}_{-2}$
$2 \cong 2^*$

$$\Lambda \mathbb{C}^5 = \Lambda^0\mathbb{C}^5 \oplus \Lambda^1\mathbb{C}^5 \oplus \Lambda^2\mathbb{C}^5 \oplus \Lambda^3\mathbb{C}^5 \oplus \Lambda^4\mathbb{C}^5 \oplus \Lambda^5\mathbb{C}^5$$

$$32 = 1 + 5 + 10 + 10 + 5 + 1$$

$$\Lambda^0\mathbb{C}^5 = \mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_0 \cong V_R^*$$

$$\Lambda^1\mathbb{C}^5 = \mathbb{C}^5 = \underbrace{\mathbb{C}\otimes\mathbb{C}^2\otimes\mathbb{C}_3}_{V} \oplus \underbrace{\mathbb{C}^3\otimes\mathbb{C}\otimes\mathbb{C}_{-2}}_{W} = l_L^* \oplus d_R$$

$$\Lambda^2\mathbb{C}^5 = \Lambda^2(V\oplus W) \cong \Lambda^2(V)\oplus\Lambda^2(W)\oplus(V\otimes W) = (\mathbb{C}\otimes\mathbb{C}\otimes\mathbb{C}_6)\oplus \mathbb{C}^{3*} \otimes \mathbb{C}\otimes\mathbb{C}_{-4}$$

$$\mathbb{C}^5 \oplus \Lambda^3\mathbb{C}^5 \oplus \Lambda^4\mathbb{C}^5 \oplus \Lambda^5\mathbb{C}^5$$

$$0 + 10 + 5 + 1$$

$$= V_R^*$$

$$\otimes \mathbb{C}_3 \oplus \underbrace{\mathbb{C}^3 \otimes \mathbb{C} \otimes \mathbb{C}_{-2}}_{W} = l_L^* \oplus d_R$$

$$\Lambda^2(V) \oplus \Lambda^2(W) \oplus (V \otimes W) = \overbrace{(\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}_6)}^{e_R^*} \oplus \overbrace{\mathbb{C}^{3*} \otimes \mathbb{C} \otimes \mathbb{C}_{-4}}^{u_R^*} \oplus \overbrace{\mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}_1}^{q_L}$$