Title: Entropy Variations and Light Ray Operators from Replica Defects

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Series: Quantum Fields and Strings

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Abstract: We study the defect operator product expansion (OPE) of displacement operators in free and interacting conformal field theories using replica methods. We show that as n approaches 1 a contact term can emerge when the OPE contains defect operators of twist dâˆ’2. For interacting theories and general states we give evidence that the only possibility is from the defect operator that becomes the stress tensor in the nâ†’1 limit. This implies that the quantum null energy condition (QNEC) is always saturated for CFTs with a twist gap. As a check, we show independently that in a large class of near vacuum states, the second variation of the entanglement entropy is given by a simple correlation function of averaged null energy operators as studied by Hofman and Maldacena. This suggests that sub-leading terms in the the defect OPE are controlled by a defect version of the spin-3 non-local light ray operator and we speculate about the possible origin of such a defect operator. For free theories this contribution condenses to a contact term that leads to violations of QNEC saturation.

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Ven Chandrasekaran

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Energy in Classical Field Theory

Classically the null energy is bounded from below (null energy condition)

$$T_{vv} \sim (\partial_v \phi)^2 \geq 0$$

NEC is used crucially for important theorems in GR:

- Hawking's area theorem for black holes, $A'_{\mathsf{BH}} \geq 0$
- Classical focussing theorem for light rays, $A'' \leq 0$
- Singularity theorems

Energy in Quantum Field Theory

Quantum mechanically, local energy density cannot be positive. Follows from separating property of vacuum [Witten (2018)]

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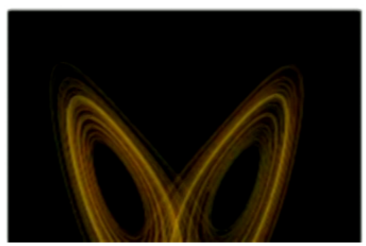
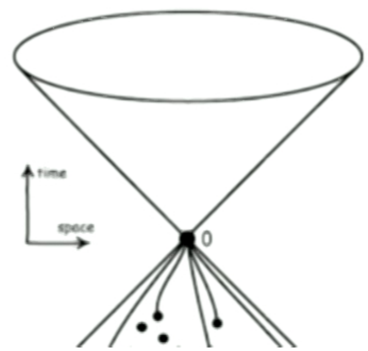
However, lots of recent progress has been made connecting constraints from causality, quantum information theory and chaos to energy conditions.





Averaged Null Energy Condition

Non-local bound on energy density

$$\int_{-\infty}^{\infty} dv \, \langle T_{vv}(y, u=0, v) \rangle \geq 0$$

Proven using a multitude of techniques

- Monotonicity of relative entropy [Faulkner et al. (2016)]

$$S_{\text{rel}}(\rho_A|\sigma_A) \geq S_{\text{rel}}(\rho_B|\sigma_B), \; A \supset B$$

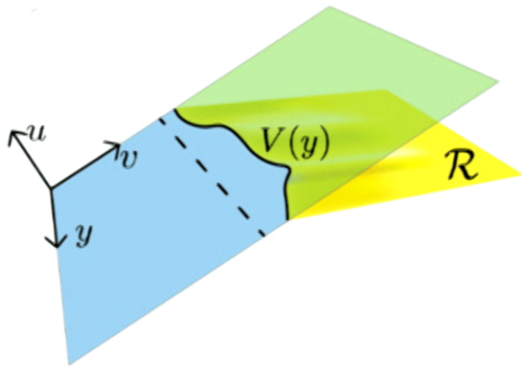
- Causality in the lightcone limit and reflection positivity (related to chaos bound) [Hartman et al. (2016)]

A Local Quantum Energy Condition

Can we do better? Yes, the quantum null energy condition!

$$\boxed{\langle T_{vv}(y_0)\rangle \geq \frac{1}{2\pi}\frac{d}{d\lambda}\left(\frac{\delta S}{\delta V(y_0)}\bigg|_{V(y;\lambda)}\right)}$$

$$S(\mathcal{R}(\lambda)) = -Tr[\rho_{\mathcal{R}}\log\rho_{\mathcal{R}}],\ \ \rho_{\mathcal{R}} = Tr_{\bar{\mathcal{R}}}\left|\psi\right\rangle\left\langle\psi\right|$$



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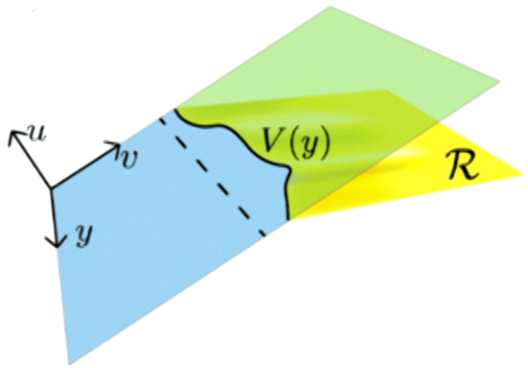
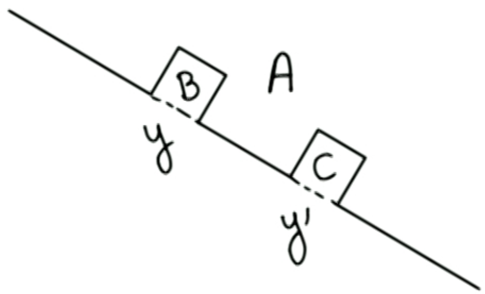


- Conjectured: [Bousso et al. (2015)]. Proofs: [Bousso et al. (2015)], [Koeller and Leichenauer (2015)], [Balakrishnan et al. (2017)]
- General proof unifies both ANEC proofs
- Connects energy and entanglement

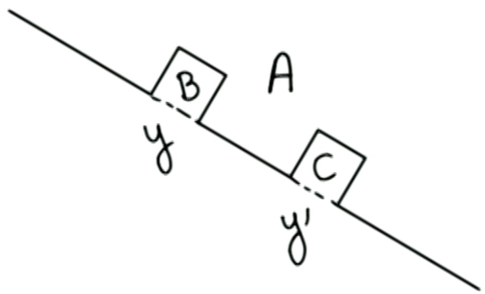
Second Variations of the Entanglement Entropy



$$\frac{d}{d\lambda}\left(\left.\frac{\delta S}{\delta V(y_0)}\right|_{V(y;\lambda)}\right) = \int d^{d-2}y' \frac{\delta^2 S}{\delta V(y_0)\delta V(y')} \frac{d}{d\lambda} V(y';\lambda)$$

Let's look at second variations of the entanglement entropy with respect to the entangling surface position:

Second Variations of the Entanglement Entropy



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$$\frac{\delta^2 S}{\delta V(y)\delta V(y')} = S''_{vv}(y')\delta^{d-2}(y-y') + \text{off-diagonal}$$

- S'' stands for the "diagonal" (local) variation of the entropy.

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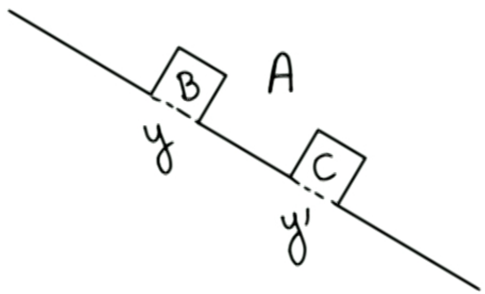

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- S'' stands for the "diagonal" (local) variation of the entropy.
- Strong sub-additivity, $S(A) - S(AB) \leq S(AC) - S(ABC)$, implies that the off-diagonal second variations are non-positive.
- Diagonal QNEC: $\langle T_{vv}\rangle \geq \frac{1}{2\pi}S''_{vv}$.

Saturation of the Diagonal QNEC

$$\boxed{S''_{vv} = 2\pi \langle T_{vv} \rangle}$$

Result: The *diagonal* (local) QNEC is saturated for all states in all QFTs with an *interacting* UV fixed point.

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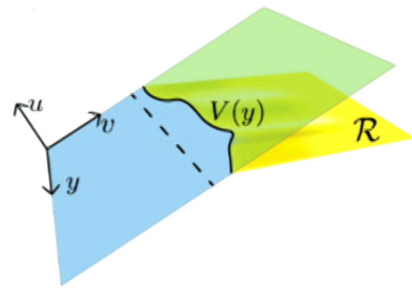
- *Interactions are key*: not saturated for free fields! [Bousso et al. (2015)]

Outline

- Review of QNEC ✓
- Vacuum modular Hamiltonians and the first law
- Defect CFTs
- Proof of saturation
- Light ray operators and interplay with free theories
- Future directions

Vacuum Modular Hamiltonians



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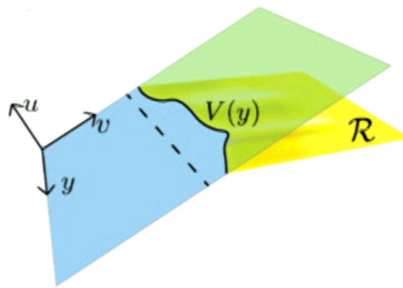

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Modular Hamiltonian: $\rho_{\mathcal{R}} \equiv \frac{1}{Z} e^{-K_{\mathcal{R}}} \implies K_{\mathcal{R}} = -\log \rho_{\mathcal{R}}$

[Casini, Teste and Torroba, (2017)]

$$\langle K_{\mathcal{R}}^{\text{vac}} \rangle_\psi = 2\pi \int d^{d-2}y \int_{V(y)}^{\infty} dv \, (v - V(y)) \, \langle T_{vv}(u = 0, v, y) \rangle_\psi$$

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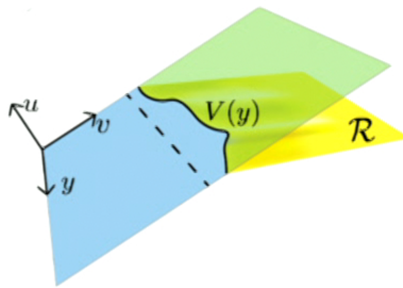

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First Law of Entanglement Entropy

Two facts

- Second variation of modular Hamiltonian:

$$\frac{\delta^2}{\delta V(y)\delta V(y')}\left\langle K_{\mathcal{R}}^{\text{vac}}\right\rangle_\psi = 2\pi \left\langle T_{vv}(y)\right\rangle_\psi \delta^{(d-2)}(y-y')$$

- First law for near vacuum states:

$$\delta S = \delta \left\langle K_{\mathcal{R}}^{\text{vac}}\right\rangle$$

$\hookrightarrow$ linear in state piece of entropy is modular Hamiltonian

Implication for near vacuum states:

$$\delta S''_{vv} = 2\pi\delta \left\langle T_{vv}(y)\right\rangle \delta^{(d-2)}(y-y')$$

Can we use this as a guiding principle for a general proof?

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A Detour into Defect CFTs

A simple line of logic

- We want to take limit of two variations as they approach each other
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A simple line of logic

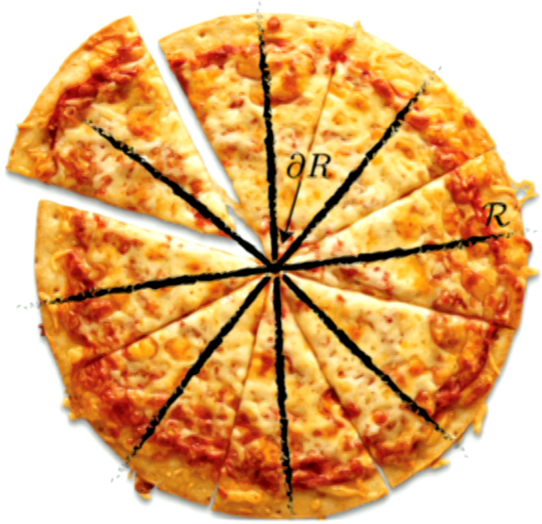
- We want to take limit of two variations as they approach each other
  $\hookrightarrow$ zooming in to arbitrarily small regions of a general state
- Intuitively, general state should look like near vacuum state when looking at local patches
- Can mock this up formally by looking at contact terms in operator product expansions (OPE)
  $\hookrightarrow$ OPE data is universal

Replica Trick

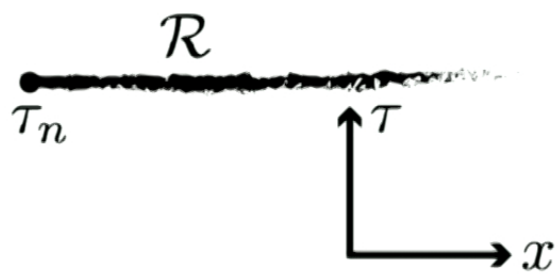
Use replica trick to compute entropy

$$S(\rho) = -\lim_{n\to 1} \partial_n Tr[\rho_\psi^n]$$

Compute partition function on n-sheeted branched (replica) manifold and analytically continue in n



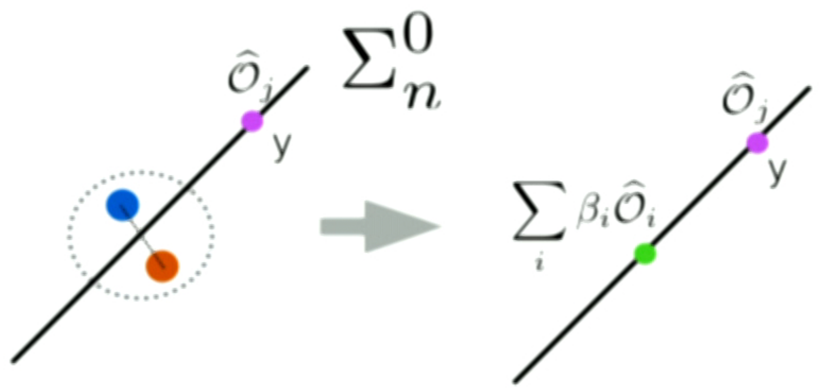
Twist Defect



Framework

- Branched manifold can alternatively be represented by a codimension-2 (non-local) twist defect τ_n on $\partial\mathcal{R}$ after orbifolding the CFT$^{\otimes n}$ under $\mathbb{Z}_n$
- This allows us to apply standard CFT considerations to the original manifold $\mathbb{R}^d$ but now in the presence of τ_n.

Defect OPE



[Balakrishnan, Faulkner, Khandker and Wang (2017)]

- Twist defect breaks full conformal symmetry down to $SO(2) \times SO(d-2)$
  $\hookrightarrow$ ambient operator close to the defect turns into infinite sum of local defect operators (bulk to defect OPE)

Defect OPE (cont.)

To be more explicit, choose complexified lightcone coordinates around the defect ($\bar{w} = v$ on Lorentzian section)

$$ds^2 = dw d\bar{w} + d\vec{y}^2$$

Then bulk to defect OPE is

$$\lim_{|w|\to 0} \sum_{k=0}^{n-1} \mathcal{O}^{(k)}(w, \bar{w}, y)\tau_n$$
$$= w^{-(\Delta_{\mathcal{O}}+\ell_{\mathcal{O}})}\bar{w}^{-(\Delta_{\mathcal{O}}-\ell_{\mathcal{O}})} \sum_j w^{(\hat{\Delta}_j+\ell_j)/2}\bar{w}^{(\hat{\Delta}_j-\ell_j)/2}\hat{\mathcal{O}}_j(y)\tau_n$$

We can extract each defect operator via a residue projection,

$$\hat{\mathcal{O}}_\ell(0)\tau_n = \lim_{|w|\to 0} \frac{|w|^{-\hat{\sigma}_\ell+\sigma_\alpha}}{2\pi i} \oint \frac{dw}{w} w^{-\ell+\ell_\alpha} \sum_{k=0}^{n-1} \mathcal{O}_\alpha^{(k)}(w, |w|^2/w, 0)\tau_n$$

Displacement Operator

Since we are interested in energy density, let's bring the stress tensor close to the defect. Its defect spectrum contains a spin 1 operator

$$\boxed{\hat{D}_{\bar{w}}(y) = \sum_{k=0}^{n-1} \oint d\bar{w}\; T^{(k)}_{\bar{w}\bar{w}}(\bar{w}, y)}$$

Displacement operator: defect local operator corresponding to entangling surface deformations

$\hookrightarrow$ follows from equivalent local definition

$$\nabla^{\mu}\langle \tau_n T_{\mu\nu}(y)\rangle = \delta_{\partial\mathcal{R}}(w,\bar{w})\langle \tau_n \hat{D}_{\nu}(y)\rangle$$

Computing Shape Variations of Entanglement Entropy

Displacement operator can be used to compute variations of the n-sheeted orbifold partition function:

$$\lim_{n\to 1}\frac{1}{(n-1)}\langle \tau_n^{\psi}\hat{D}_{\bar{w}}(y)\hat{D}_{\bar{w}}(y')\rangle \sim \langle \tau_n^{\psi}\rangle \frac{\delta^2 S}{\delta V(y)\delta V(y')}$$

Look at the OPE of $\hat{D}$ with itself:

$$[\hat{D}_{\bar{w}}] = d-1, \quad \hat{D}_{\bar{w}}(y)\hat{D}_{\bar{w}}(y') \sim \sum_{O}\frac{c_n\hat{O}_{\bar{w}\bar{w}}(y)}{|y-y'|^{d-2+(d-\Delta_O(n))}}$$

Where's the delta function?

General CFT Proof

Consider the following delta function representation:

$$\lim_{n\to 1} \frac{(n-1)}{|y-y'|^{d-2+\gamma'(n-1)}} \sim \delta^{(d-2)}(y-y')$$

Hence if $c_n \sim c'(n-1)^2 + \dots$ and $\Delta_O(n) \sim d + \gamma'(n-1) + \dots$

$$\Rightarrow \lim_{n\to 1} \frac{1}{n-1} \hat{D}_{\bar{w}}(y)\hat{D}_{\bar{w}}(0) \supset \hat{O}_{\bar{w}\bar{w}}(y)\delta^{(d-2)}(y-y')$$

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Eliminating the Possibilities

Leading order $\hat{\Delta}_j = \Delta_{\mathcal{O}} - \ell_{\mathcal{O}} + \ell_j + \mathcal{O}(n-1)$ [Balakrishnan, Faulkner, Khandker and Wang (2017)]

- Spin 2 defect operator induced by spin 1 primary saturating the unitarity bound, but charged operators cannot appear in entropy

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- Spin 2 defect operator induced by spin 1 primary saturating the unitarity bound, but charged operators cannot appear in entropy
- $\ell = 2$ higher spin displacement operators from lowest twist higher spin primaries

$$\hat{D}^J_{\bar{w}\bar{w}} = i \oint d\bar{w} \frac{\bar{w}^{J-3}}{|w|^{\gamma_J(n)}} \sum_{k=0}^{n-1} \mathcal{J}^{(k)}_{+\ldots+}$$

but $\hat{\Delta}_J = \tau_J + 2 + \mathcal{O}(n-1) > d$ for CFTs with a twist gap

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- Nonlocal defect operators arising from non-commutativity of $n \to 1$ and OPE limits . . . let's come back to this

Stress Tensor Prevails

Thus, only option in an interacting CFT with twist gap: $\boxed{T_{\bar{w}\bar{w}}}$

Use diffeomorphism Ward identity in the presence of replica defect:

$$\int d^{d-2}y' \langle \tau_n \hat{D}_{\bar{w}}(y') \hat{D}_{\bar{w}}(y) T_{ww}(w, \bar{w}, 0) \rangle = -\partial_{\bar{w}} \langle \tau_n \hat{D}_{\bar{w}}(y) T_{ww}(w, \bar{w}, 0) \rangle$$

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Computing both sides to $\mathcal{O}(n-1)$ we find desired behavior of c_n and $\Delta_O(n)$ for $T_{\bar{w}\bar{w}}$

Nonlocal Defect Operators

Directly compute leading spectrum of $\hat{D} \times \hat{D}$ OPE in $n \to 1$ limit:

$$\langle \tau_n \hat{D}_{\bar{w}}(y_1) \hat{D}_{\bar{w}}(y_2) \hat{D}_w(y_3) \hat{D}_w(y_4) \rangle \sim (n-1) \langle \mathcal{E}_w(y_1) \tilde{\mathcal{E}}_{\bar{w}}(y_2) \tilde{\mathcal{E}}_{\bar{w}}(y_3) \mathcal{E}_w(y_4) \rangle$$

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OPE of ANEC operators = sum over spin 3 non-local light ray operators:

$$\tilde{\mathcal{E}}_{\bar{w}}(y_1)\tilde{\mathcal{E}}_{\bar{w}}(y_2) \sim \sum_i \frac{c_i \mathbb{O}_i(y_2)}{|y_1 - y_2|^{2(d-2)-\tau^i_{\text{even},J=3}}}$$

where $\tau^i_{\text{even},J=3}$ is twist of even J primary on ith Regge trajectory analytically continued down to $J = 3$. [Hofman, Maldacena; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedev]

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No delta function contribution for CFTs with twist gap $\tau^i_{\text{even},J=3} > d - 2$

Further Evidence: Near Vacuum States

Consider near vacuum state $|\psi\rangle = |0\rangle + \varepsilon\mathcal{O}|0\rangle + \mathcal{O}(\varepsilon^2)$ and directly compute second derivative of relative entropy using perturbation theory:

$$\frac{\delta^2 S_{\mathsf{rel}}}{\delta V(y_1)\delta V(y_2)} \sim \varepsilon^2 \int_{-\infty}^{\infty} ds e^s \langle \mathcal{O}\mathcal{E}_v(y_1)\mathcal{E}_v(y_2)e^{isK^{\mathsf{vac}}}\mathcal{O}\rangle$$

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$\rightarrow$ Spin 3 light ray operator contributes to entropy variations

Free theories: unitarity bound is saturated $\mathcal{E}_v(y_1)\mathcal{E}_v(y_2) \sim \delta^{(d-2)}(y_1 - y_2)$

QNEC is *not* saturated in free theories! [Bousso et al. (2015)]

Weakly Interacting Theories

$$\left(S''_{vv}\right)_{\text{free}} = 2\pi \langle T_{vv} \rangle + Q$$

How does Q disappear from delta function piece as we turn on a small interaction with coupling λ?

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How does Q disappear from delta function piece as we turn on a small interaction with coupling λ?

We saw that in the free limit, Q comes from spin 3 light ray operator. This is not a protected operator, so it picks up an anomalous dimension $\Delta \sim d + O(\lambda)$ which smears it out!

Summary

- Defect CFT framework is a powerful tool for analyzing entanglement entropy variations
- $S''_{vv} = 2\pi \langle T_{vv} \rangle$ for general states in all interacting CFTs

Future Directions

- Relation to continuous spin ANEC [Simmons-Duffin et al, (2018)]
  - Is there a continuous spin QNEC?
- Non-null variations $\rightarrow$ saturation of quantum dominant energy condition [Wall, (2017)]?
- What is energetic part of entanglement entropy for general states in interacting theories?
- **Conjecture:** nonlocal defect operator $=$ infinite resummation of higher spin displacement operators

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- Relation to continuous spin ANEC [Simmons-Duffin et al, (2018)]
  - Is there a continuous spin QNEC?
- Non-null variations → saturation of quantum dominant energy condition [Wall, (2017)]?
- What is energetic part of entanglement entropy for general states in interacting theories?
- **Conjecture:** nonlocal defect operator = infinite resummation of higher spin displacement operators