Title: Rozansky-Witten theory via BV quantization II

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Series: Mathematical Physics

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02.03510v3)

$\otimes T_X[-1]$

$\det(\mathcal{D}^{hol}_X)$

$M^{\alpha\wedge\beta}$

$(I - \delta^{-1}(\omega))(\alpha)$

$(\alpha^{\otimes k}), \alpha\rangle$

$\mathrm{Obs}^{cl}(H_g) \simeq H^{\cdot}(X, (\Lambda^{\cdot} T_X)^{\otimes g})$. $H_g \subseteq M$ homeo to open g-handle

Today: Quantization

Set-up: let g be a Riemm metric on M

- $Q^{GF} = d_M^*$
- Heat kernel: for $t>0$, $\mathbb{K}_t \in \mathrm{Sym}^2(\mathcal{E})$ sth

$$\langle \mathbb{K}_t(x,y), \phi(y)\rangle = \left(\exp(-t[Q, Q^{GF}])\phi\right)(x)$$

factor as $\mathbb{K}_t(x,y) = K_t(x,y) \otimes \omega^{\vee}$.

- Scale t BV Laplacian: $\Delta_t : \mathcal{O}^{\cdot}(\mathcal{E}) \xrightarrow[\text{contract w/ } \mathbb{K}_t]{} \mathcal{O}^{\cdot}(\mathcal{E})$
- $\{\alpha, \beta\}_t = \Delta_t(\alpha\beta) - \Delta_t(\alpha)\beta \pm \alpha\Delta_t(\beta)$
- effective propogator: $\mathbb{P}_\varepsilon^L = \int_\varepsilon^L dt\,(Q^{GF} \otimes 1) K_t$

BV Quantization of Rozansky-Witten Model II

(Chan-Leung-Li 1502.03510v3)

Last time: classical Theory

(X, ω) hol sympl, M closed 3-mfld.

fields: $\mathcal{E} = \mathcal{A}^{\bullet}_{M} \otimes \mathfrak{g}_X[1]$, $\mathfrak{g}_X = \mathcal{A}^{\natural}_X \otimes T_X[-1]$

$C^{\bullet}(\mathfrak{g}_X) = \mathrm{dR}(\mathcal{J}^{hol}_X)$

pairing: $\langle -,- \rangle : \mathcal{E} \otimes_{\mathcal{A}_X} \mathcal{E} \longrightarrow \mathcal{A}_X$

$(\alpha \otimes f, \beta \otimes g) \longmapsto \omega(f,g) \int_M \alpha \wedge \beta$

action: $S(\alpha) = \int_M \omega((d_M + \nabla)\alpha, \alpha) + \rho(I - \delta^{-1}(\omega))(\alpha)$

$= \int_M d_M \alpha + \sum_{k \geq 0} \frac{1}{(k+1)!} \langle \ell_k(\alpha^{\otimes k}), \alpha \rangle$

$\mathrm{Obs}^{q}(H_g)$

Today: Qua...

Set-up: le...

- $Q^{GF} =$

- Heat ke...

- Scale ...

- $\{\alpha, \beta\}$...

- effective...

propogator: $\mathbb{P}_\varepsilon^L = \int_\varepsilon^L dt (Q^{GF} \otimes 1) K_t$

$\} \in \mathcal{O}^+(\mathcal{E})\}_{L \in \mathbb{R}_{\geq 0}}$

④ $\lim_{L \to 0} I[L] = I_a \mod \hbar$

Want: $I[L] = W(\mathbb{P}_0^L, I_a)$

but $\mathbb{P}_0^L$ is only smooth on $(M \times M) \setminus \Delta$

Approach: Compactify $(M \times M) \setminus \Delta$ and argue that $\mathbb{P}_0^L$ smoothly extends

L QME.

$= 0$

$\to$ present b/c $l_0 \neq 0$.

$[Q, Q^{GF}]])$

that satisfies the CME. A quantization is a collection of functionals $\{ I[L] \in \mathcal{O}^+(\mathcal{E}) \}_{L \in \mathbb{R}_{\geq 0}}$ s.th

① $I[L_2] = W(P_{L_1}^{L_2}, I[L_1])$

② Asymptotic locality

③ $\forall L > 0$, $I[L]$ satisfies the scale L QME.

$$\left(Q_L + \hbar \Delta_L + \frac{F(R)}{\hbar}\right) e^{I[L]/\hbar} = 0$$

$$Q_L = Q\left(-\nabla^2 \int_0^L dt\, Q^{GF} \exp(-t[Q, Q^{GF}])\right)$$

Present b/c $F_0 \neq 0$.

④ $\lim_{L \to 0} I$

Want: I

but

Approach

Compactified configuration spaces (Kontsevich - Axelrod - Singer)

let $V = \{1, \ldots, n\}$ $n \geq 2$, $S \subseteq V$ sth $|S| \geq 2$

$M^S = \{S \longrightarrow M\}$, $\Delta_S \subseteq M^S$ the small diagonal.

$Bl(M^S, \Delta_S)$ mfld w/ ∂.

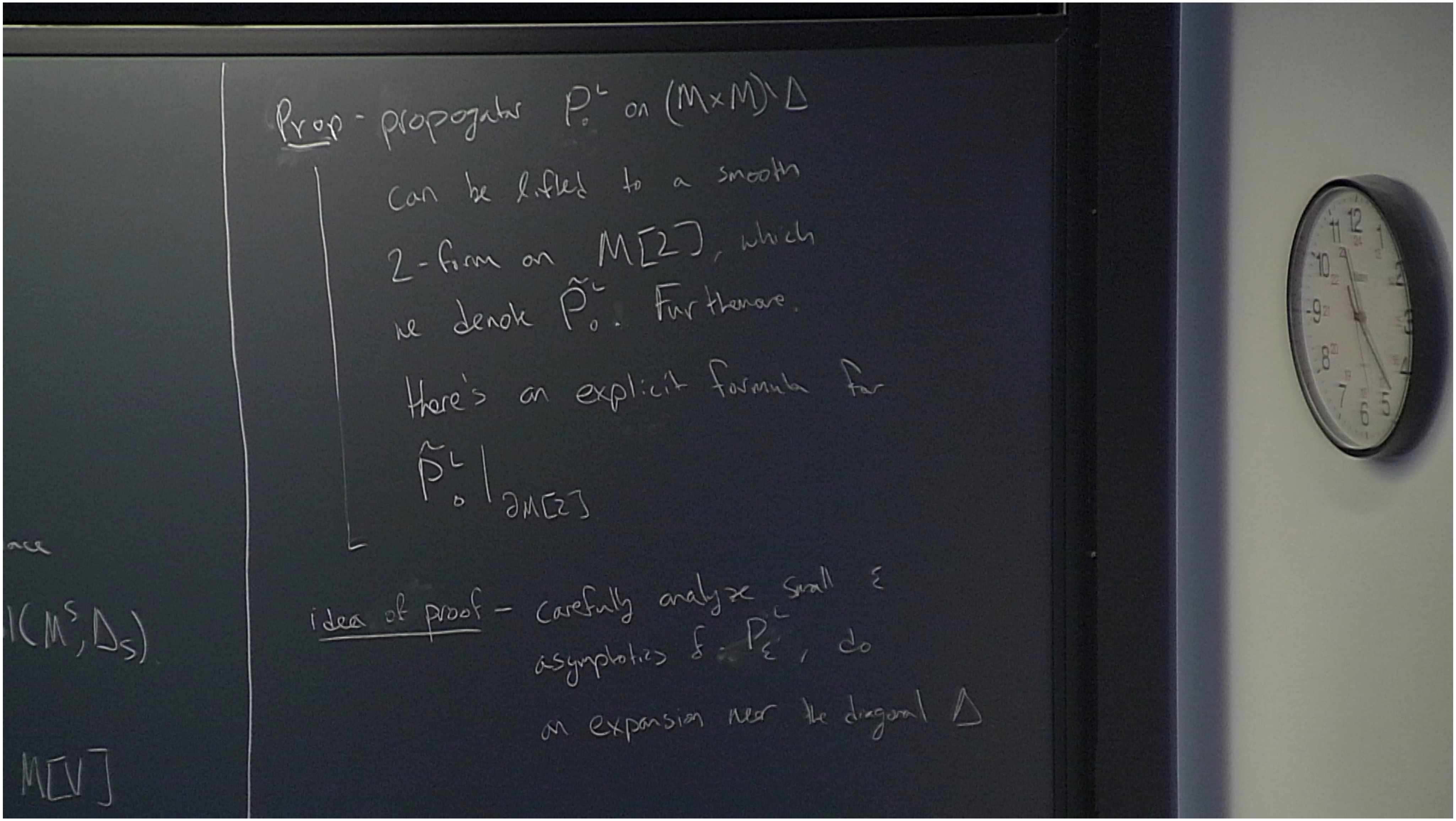
$\partial Bl \simeq S(N_{\Delta_S})$

$Bl^{int} \simeq M^S \setminus \Delta_S$

Defn - let M_0^V be the disjoint config space

$$M_0^V \hookrightarrow M^V \times \prod_{\substack{S \subseteq V \\ |S| \geq 2}} Bl(M^S, \Delta_S)$$

the compactified config space $M[V]$ is the closure of this embedding


Prop - propogator P_0^L on $(M\times M)\setminus\Delta$
can be lifted to a smooth
2-form on $M[2]$, which
we denote $\tilde{P}_0^L$. Furthermore,
There's an explicit formula for
$\tilde{P}_0^L|_{\partial M[2]}$
idea of proof - carefully analyze small ε
asymptotics of P_ε^L, do
an expansion near the diagonal Δ
ace
(M^S, Δ_S)
$M[V]$

Let $V(\gamma)$ denote the vertices of a graph γ

$S \subseteq V(\gamma)$, $M[V(\gamma)] \longrightarrow M[S]$.

$e \in E(\gamma)$, $M[V(\gamma)] \xrightarrow{\pi_e} M[\{s(e), t(e)\}]$

$v \in V(\gamma)$, $M[V(\gamma)] \xrightarrow{\pi_v} M$.

Defn - let γ be a connected, stable graph not containing tadpoles of tails, $|T(\gamma)| = k$.

$$W_\gamma(\tilde{\mathbb{P}}^L_0, I_{cl}) = \int_{M[V(\gamma)]} \prod_{e \in E(\gamma)} \pi_e^*(\tilde{\mathbb{P}}^L_0) \prod_{i=1}^{k} \phi_i$$

taining tadpoles

$\pi_2^k\left(\tilde{\mathbb{P}}_0^L\right) \prod^k \phi_i$

Combinatorial part of graph weight

$$w^{ij}\left(\ell_2(e_i \otimes e_j), \phi\right)$$

$$= -w^{ji}\left(\ell_2(e_j \otimes e_i), \phi\right)$$

BV QUANTIZATION OF ROZANSKY-WITTEN MODEL II

(Chan-Leung-Li 1502.03510v3)

Defn – naive quantization of I_4 is defined by

$$I_{naive}[L] = \sum_{\substack{\gamma \text{ connected} \\ \text{stable } \not\supset \\ \text{tadpoles}}} \frac{\hbar^{g(\gamma)}}{|Aut\,\gamma|} W_\gamma(\tilde{P}_0^L, I_4)$$

v3)

Quantum Master Eq^n:

$$\mathrm{QME} \iff \frac{1}{\hbar}\Big((d_M' + \nabla) I[L] + \hbar \Delta_L I[L] + \{I[L], I[L]\}\Big) e^{\frac{I[L]}{\hbar}}$$

$$= -\left(\nabla^2 \int_0^L dt\, Q^{GF} \exp(-t[Q, Q^{GF}]) + \frac{\rho(R)}{\hbar}\right) e^{\frac{I[L]}{\hbar}}$$

Lemma: $d_M(\tilde{\mathbb{P}}_0^L) = -\mathbb{K}_L$

Pf: suffices to check on $(M \times M) \setminus \Delta$.

$$d_M(\mathbb{P}_0^L) = \mathbb{K}_0 - \mathbb{K}_L$$

only supported on Δ

$\square$

$$d_M^V\left(W_\gamma(\tilde{\mathbb{P}}_0^L, I_{cl})(\phi_1, \dots, \phi_k)\right)$$

$$= \sum_{i=1}^{k} W_\gamma(\tilde{\mathbb{P}}_0^L, I_{cl})(\phi_1, \dots, d_M\phi_i, \dots, \phi_k)$$

$$= \int_{M[\bar{V}(\gamma)]} d\left(\prod_{e \in E(\gamma)} \pi_e^* \mathbb{P}_0^L \prod_{i=1}^{k} \phi_i\right) - \int_{M[\bar{V}(\gamma)]} \sum_{e_0 \in E(\gamma)} d(\pi_{e_0}^* \mathbb{P}_0^L) \prod_{e \in E(\gamma) - e_0} \pi_e^* \mathbb{P}_0^L \prod_{i=1}^{k} \phi_i$$

$$= \underbrace{\int_{\partial M[V(\gamma)]} \rule{2cm}{0.4pt} \text{''} \rule{2cm}{0.4pt}}_{\downarrow} + \underbrace{\int_{M[V(\gamma)]} \sum_{e_0 \in E(\gamma)} \mathbb{K}_L \rule{2cm}{0.4pt} \text{''} \rule{2cm}{0.4pt}}$$

need to understand boundary strata of $M[V(\gamma)]$

- if e_0 is nonseparating: cancels w/ the $\Delta_L I[L]$
- if e_0 is separating: cancels w/ the terms in $\{I[L], I[L]\}_L$ not involving e_0.

$M[V]$ is a disjoint union of open strata.

$$M[V] = \bigcup_{\mathcal{S}} M(\mathcal{S})^\circ$$

$\mathcal{S}$ is a collection of subsets of V s.th

- if $S_1, S_2 \in \mathcal{S}$ either $S_1 \cap S_2 = \emptyset$ or $S_1 \subseteq S_2$
- every $S \in \mathcal{S}$, $|S| \geq 2$

__Facts__: each $M(\mathcal{S})^\circ$ is of codim $|\mathcal{S}|$

for codim 1 strata $\mathcal{S} = \{S\}$

let $V = \{1, \ldots, n\}$, $S = \{1, \ldots, k\}$ $2 \le k \le n$

$E = S(N_{\Delta_S}) \xrightarrow{\pi} \Delta_S$

$M(S)^\circ = \{(e, x_1, \ldots, x_{|V(\gamma)|-|S|}) \in E \times M^{|V(\gamma)|-|S|}$ $\pi(e), x_1, \ldots, x_{|V(\gamma)|-|S|}$ not all equal $\}$

S_2

2

Assume that the vertices in S are connected in γ

(o/w integral vanishes for type reasons)

Step 1: if $|S| \geq 3$, then the integral over $M(S)^\circ$ vanishes.

Call the bounding structure w/ $S = V(\gamma)$ primitive.

<u>Idea</u>: can express sum of all bounding integrals as an RG flow of primitive bounding integrals.

let $\gamma' \subseteq \gamma$ be s.th.

$$V(\gamma') = S$$

$E(\gamma')$ = edges in γ incident to S

$T(\gamma')$ = half edges of γ incident to S but not part of internal edges.

let $W_{\gamma,s}\left(\tilde{\hat{\mathbb{P}}}^{L}_{0}, \bar{I}_a\right) = \int_{\mathcal{N}(s)^{\circ}} \prod_{e \in E} \pi_e^{k}\, \mathbb{P}_0^{L} \prod_{i=1}^{k} \phi_i$

$$W_{\gamma,s}\left(\tilde{\hat{\mathbb{P}}}^{L}_{0}, I_a\right) = W_{\gamma''}\left(\tilde{\hat{\mathbb{P}}}^{L}_{0}, I_a, W_{\gamma,s}\left(\tilde{\hat{\mathbb{P}}}^{L}_{0}, \bar{I}_a\right)\right).$$

S but

Quantum Master Eq^n

$$\text{QME} \iff \frac{1}{\hbar}\Big((d_M^\vee + \nabla) I[L] + \hbar \Delta_L I[L] + \{I[L], I[L]\}\Big) e^{\frac{I[L]}{\hbar}}$$

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only supported on Δ