Title: Gauge theory and mirror symmetry III

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Series: Mathematical Physics

Date: February 06, 2020 - 11:00 AM

URL: http://pirsa.org/20020054

Last time:

A-twist of pure gauge theory

$N =$ trivial action of $G \circlearrowright \mathrm{Vect}$

$D = G \circlearrowright \mathrm{Fuk}(T^*G)$

N has deformations

$$\begin{tikzcd} G \circlearrowright \mathrm{Vect} \arrow[r, Rightarrow, mapsto] & C_\bullet(\Omega G) \arrow[r] \arrow[d] & HH^\bullet(\mathrm{Vect}) = \mathbb{C} \arrow[r, Leftrightarrow] & Z(G^L) \\ & \mathbb{C}[\pi_1 G] \arrow[ur] & & \end{tikzcd}$$

3d mirror

...f pure gauge theory $\Longleftrightarrow$ B-twist of RW on

$= $ trivial action of $G \circlearrowright \mathrm{Vect}$

$$BFM(G^L) = \left(T^*_{reg} G^L_{reg} / G^L_{reg}\right)$$

$= G \circlearrowright \mathrm{Fuk}(T^*G)$

$$N^!_A = D_B = 1 \times \mathfrak{g}^*_{reg} / G = \text{“}T_1\text{”}$$

$$D^!_A = N_B \qquad C = \mathrm{Spec}\left(C_\bullet(\Omega G)\right)$$

deformations

$$G \circlearrowright \mathrm{Vect} \Longrightarrow C_\bullet(\Omega G) \to HH^\bullet(\mathrm{Vect}) = \mathbb{C} \qquad \Longleftrightarrow \qquad Z(G^L)$$

$$C_\bullet(\Omega G) \to \mathbb{C}[\pi_1 G] \to HH^\bullet(\mathrm{Vect})$$

...formations by

...(pt)

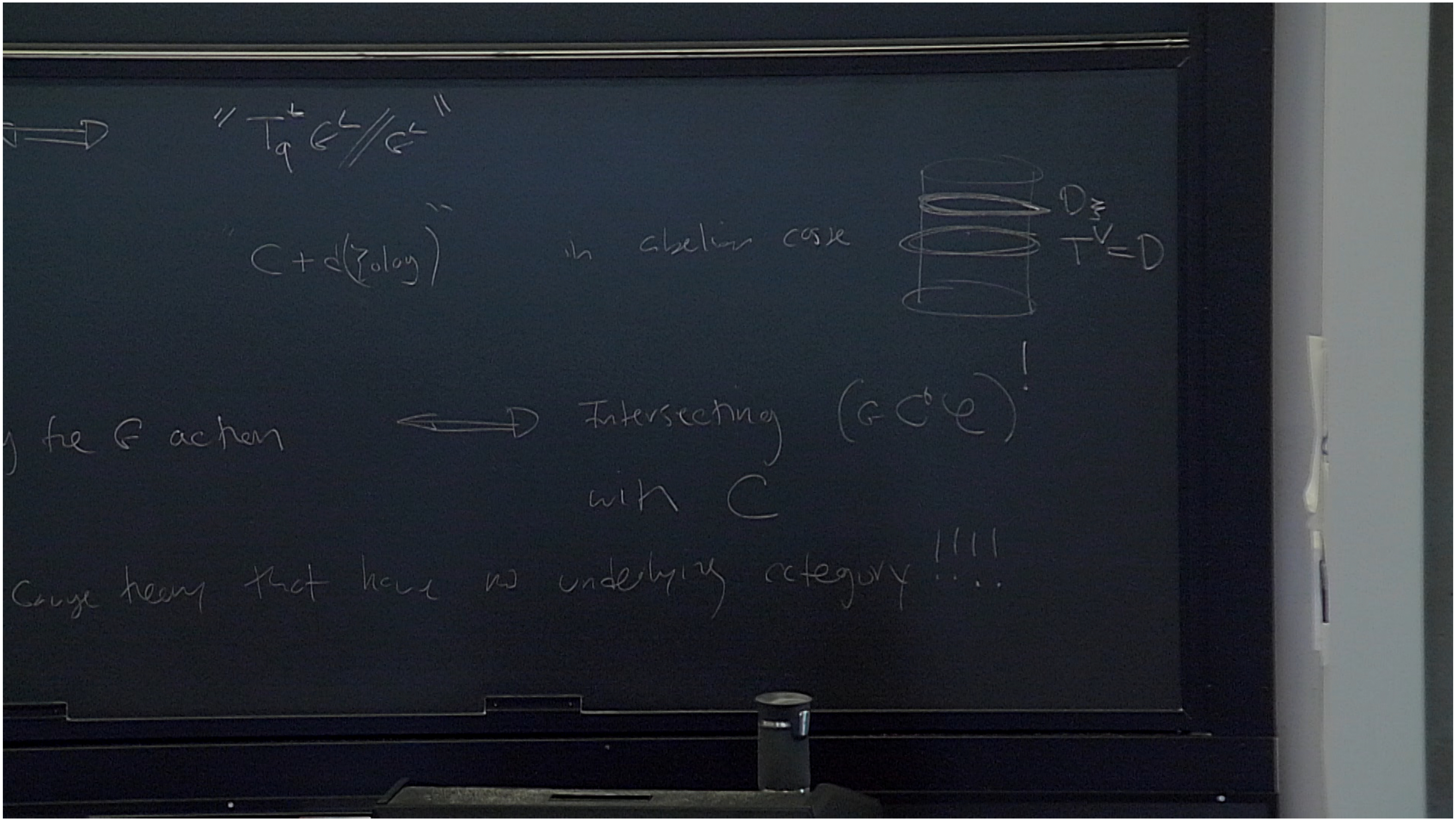
$\mathcal{N}_q = \mathrm{Vect}_q \qquad q \in Z(G^L) \qquad \Longleftrightarrow \qquad$ " $T^*_q G^L /\!/ G^L$ "

$D_\xi \qquad$ " $\mathbb{C} + d(\xi \circ \log)$ " in abe

Observations: $G \curvearrowright \mathcal{C}$ forgetting the G action $\Longleftrightarrow$ Interse

with

There must be b.c. for A-type Gauge theory that have no underlyi


in abelian case
Intersecting
with C
Gauge theory that have no underlying category!!!!

$\xi \in H_G^\bullet(pt)$ nice definition

$\mathbb{C}[\pi_1 G]$

$$BFM(G) = T^*_{reg} G^L // G^L = Z_{reg} / G^L = \mathrm{Spec}\ C_\bullet^G(\Omega G)$$

The moment map for the adjoint G^L action has 0-fiber given by

$$Z_{reg} = \{ (g, \xi) \mid g \xi g^{-1} = \xi,\ \xi \text{ regular} \}$$

Notice this space has two projections

$$\begin{array}{ccc} T^* G^L // G^L & \xrightarrow{\pi_h} & G^L / G \\ \downarrow \pi_h & & \\ \mathfrak{t}/W = \mathfrak{g}^*_{reg}//G & & \end{array}$$

$\mathrm{Spec}(H_G^\bullet(pt)) = \mathfrak{t}/W = \mathfrak{g}^*_{reg}//G$

$\mathbb{C}[\pi_1 G]$

$\xi \in H_G^\bullet(pt)$ mess definition

$$BFM(G) = T^*_{reg} G^L /\!/ G^L = Z_{reg} / G^L = \mathrm{Spec}\ H^{G}_{\bullet}(\Omega G),$$

The moment map for the adjoint G^L action has 0-fiber given by

$$Z_{reg} = \{ (g, \xi) \mid g \xi g^{-1} = \xi,\ \xi \text{ regular} \}$$

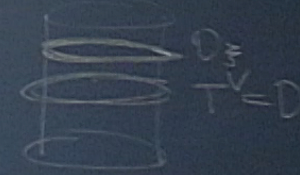
Notice this space has two projections

$$\begin{array}{ccc} T^* G^L /\!/ G^L & \xrightarrow{\pi_h} & G^L/G \\ \downarrow{\scriptstyle \pi_v} & & \\ \mathrm{Spec}(H_G^\bullet(pt)) = \mathfrak{t}/W = \mathfrak{g}^\vee_{reg}/\!/G & & \end{array}$$

$C = \pi_v^{-1}(1)$ $\qquad \Delta(g) = \pi_h^{-1}(g)$

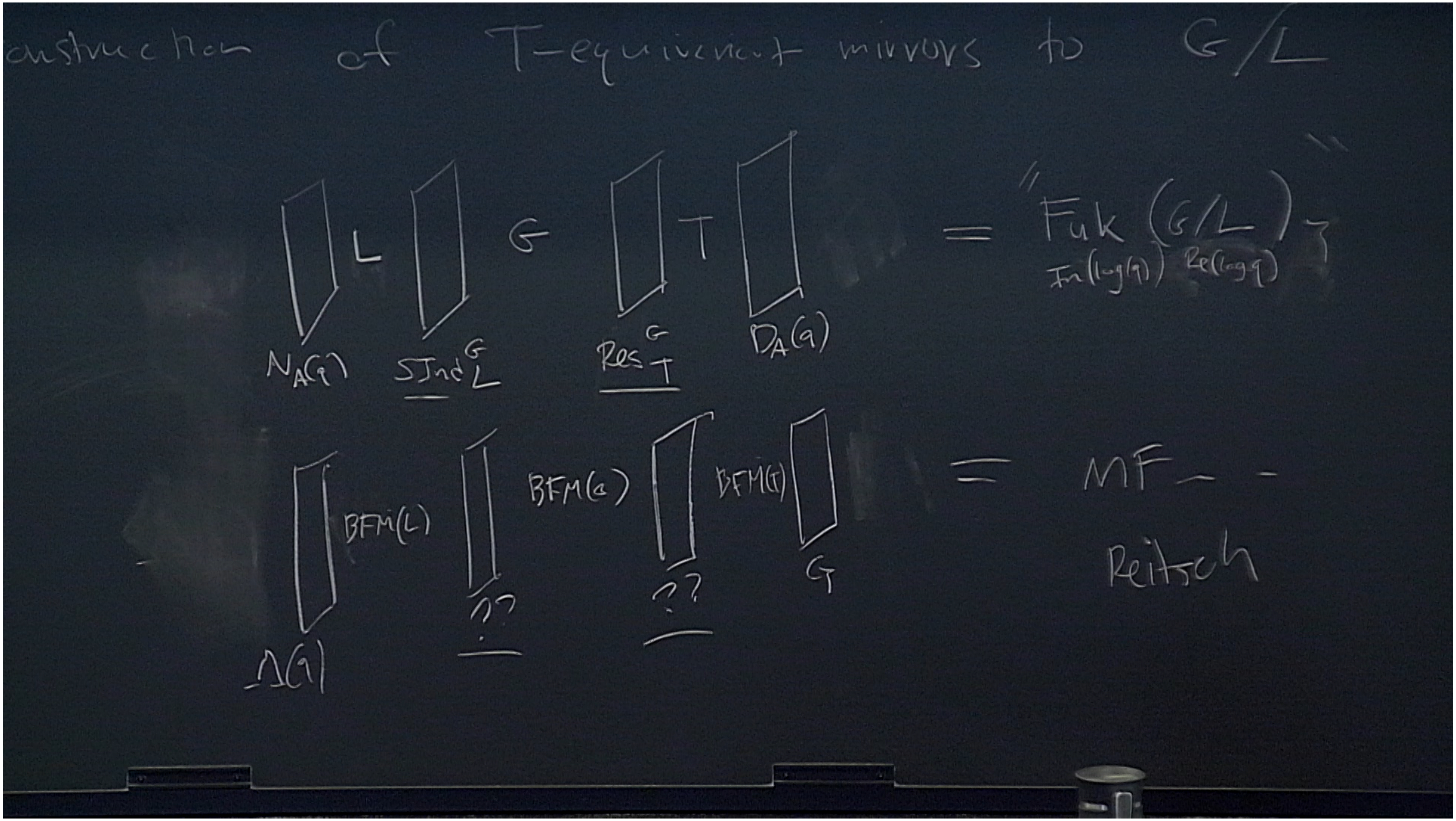
$\mathcal{W}_q = \mathrm{Vect}_q \qquad q \in Z(G^L) \qquad \Longleftrightarrow \qquad "T_q^L G^L /\!/ G^L" = \Lambda(q)$

$D_\xi \qquad \pi_V^{-1}(\xi) = "C + d(\zeta \circ \log)"$ in abelian case



<u>Observations</u> $\quad G \curvearrowright \mathcal{C}$ forgetting the G action $\Longleftrightarrow$ Intersecting $(G \curvearrowright \mathcal{C})^!$ with C

There must be b.c. for A-type Gauge theory that have no underlying category!!!!


onstruction of T-equivariant mirrors to G/L
L
G
T
= "Fuk(G/L)"
Im(log(q)) Re(log q)
N_A(q)
SInd^G_L
Res^G_T
D_A(q)
BFM(L)
BFM(G)
BFM(T)
= MF
Reitsch
Δ(q)
??
??
G

$\boxed{\mathrm{Ind}_T^G}$

$$T \to G \implies \Omega T \to \Omega G$$

$$\implies C_\bullet(\Omega T) \to C_\bullet(\Omega G)$$

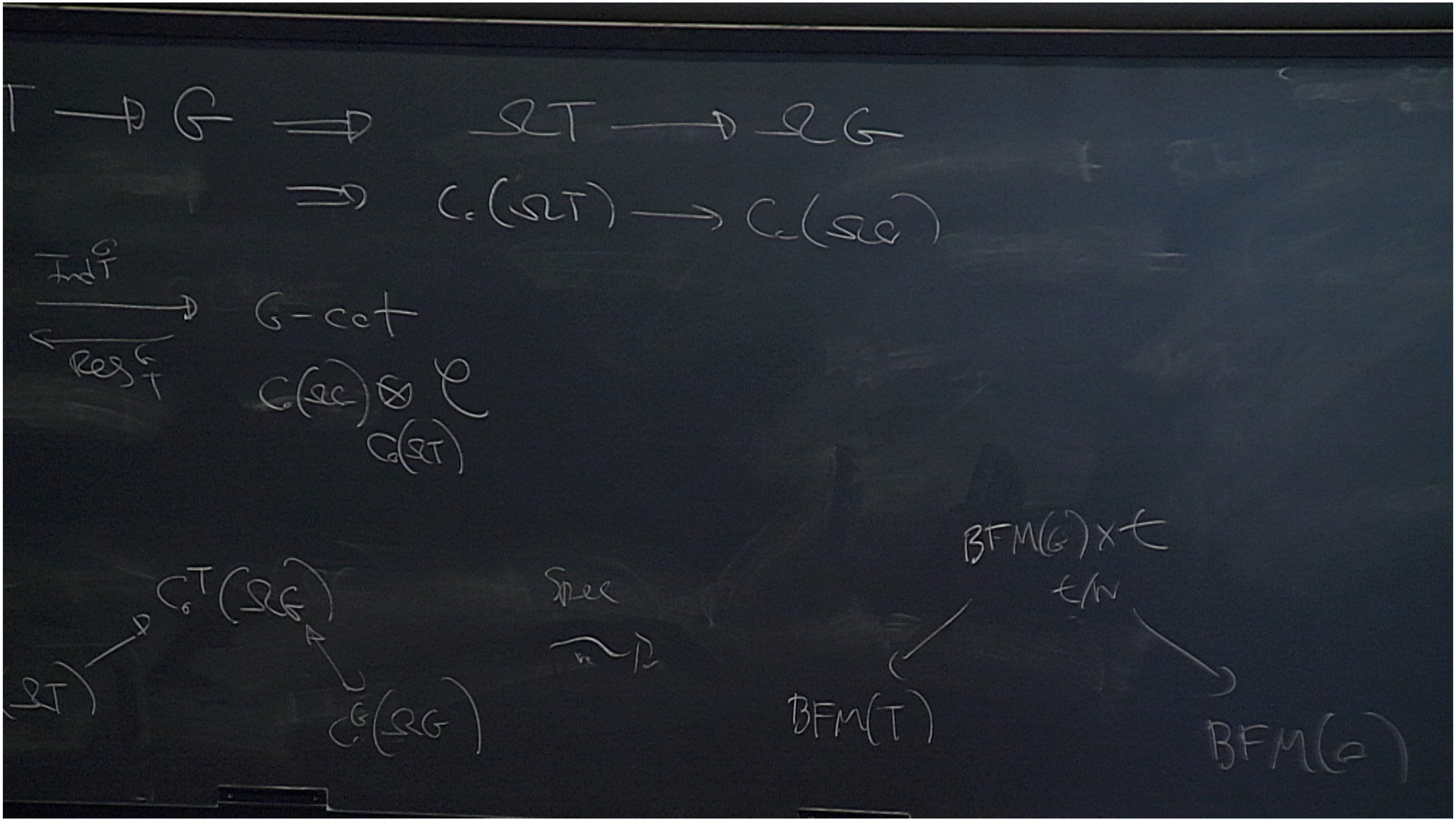
$$\begin{array}{ccc} T\text{-cat} & \underset{\mathrm{Res}_T^G}{\overset{\mathrm{Ind}_T^G}{\rightleftarrows}} & G\text{-cat} \\ \mathcal{C} & & C_\bullet(\Omega G) \otimes_{C_\bullet(\Omega T)} \mathcal{C} \end{array}$$

Conjecture Dual to

$$C_\bullet^T(\Omega T) \to C_\bullet^T(\Omega G) \leftarrow C_\bullet^G(\Omega G)$$

Spec $\sim$ BFM(T)

B


T → G ⟹ ΩT → ΩG
⟹ C•(ΩT) → C•(ΩG)
Ind T G
G-cat
Res T G
C•(ΩG) ⊗ C
C•(ΩT)
C•T(ΩG)
(ΩT)
C•G(ΩG)
Spec
BFM(G) × t
t/W
BFM(T)
BFM(G)

$\Omega T \longrightarrow \Omega G$

$C_c(\Omega T) \longrightarrow C_c(\Omega G)$

$\mathcal{C}$

(ΩT)

Spec

$\Omega G)$

Ind $\quad i: T_{\mathbb{C}} \longrightarrow G_{\mathbb{C}}$

$N^+ \Gamma B i$

$T^+ B T_{\mathbb{C}} \qquad T^+ B G_{\mathbb{C}}$

$N^+ \quad BT_{\mathbb{C}} \times B \quad BB_{\mathbb{C}}$

$T^+ B T_{\mathbb{C}} \qquad T^+ B G_{\mathbb{C}}$

SInd

$BFM(G) \times \mathfrak{t}$

$\mathfrak{t}/W$

$BFM(T) \qquad BFM(G)$

$C_0(\Omega T)$ $\quad$ $C^G_\cdot(\Omega G)$ $\quad$ $BFM(T) = T^*T^\vee_{\mathbb{C}}$ $\quad$ $BFM(G)$

nate model

$G \curvearrowright M \xrightarrow{\mu} \mathfrak{g}^\vee$

$$\begin{tikzcd} & Z_{reg}/B \arrow[dl] \arrow[dr] & \\ BFM(G) & & T^*_{reg} G^\vee // B^\vee_+ \end{tikzcd} \qquad \begin{tikzcd} & B^\vee_+ \times b^\vee_+ // B^\vee_+ \arrow[dl] \arrow[dr] & \\ T^*_{reg} G^\vee // B^\vee_+ & & T^* T^\vee \end{tikzcd}$$

$$\begin{tikzcd} & \mu^{-1}(0) \arrow[dl] \arrow[dr] & \\ M//G & & M \end{tikzcd} \qquad \begin{tikzcd} & B_+ \arrow[dl] \arrow[dr] & \\ G & & T \end{tikzcd}$$

$$\begin{tikzcd} & B_+ \times b_+ // B_+ \arrow[dl] \arrow[dr] & \\ T^*G // B_+ & & T^*T \end{tikzcd}$$

Conjecture Dual to

$$C_0^T(\Omega T) \to C_0^T(\Omega G) \leftarrow C_0^G(\Omega G)$$

Spec $\rightsquigarrow$

$$BFM(T) = T^* T^\vee_{\mathbb{C}} \leftarrow \mathfrak{t}/W \to BFM(G)$$

Alternate model

$$BFM(G) \leftarrow Z_{reg}/B_+^\vee \to T^*_{reg} G^\vee /\!/ B_+^\vee \leftarrow B_+^\vee \times \mathfrak{b}_+^\vee /\!/ B_+^\vee \to T^* T^\vee$$

$=$

$\boxed{\mathrm{SInd}_L^G}$

$P \to \tilde{X}$

V

VP

Telemon says

$$L\text{-cat} \xrightarrow{\mathrm{SInd}_L^G} G\text{-cat}$$

$$\mathcal{C} \longmapsto \underline{\mathrm{Fuk}}(G/L;\ \underline{\mathcal{C}_G})$$

$$\mathrm{Vect}_q \longmapsto \mathrm{Fuk}(G/L)_{\mathrm{Im}(\log(s)),\ \mathrm{Re}(\log(s))}$$

Conjectural dual

$$\mathrm{BFM}(L) \underset{i_L^G}{\overset{\text{open set}}{\longleftrightarrow}} \mathrm{BFM}(G)$$

$$\mathrm{BFM}(L) \leftarrow \Gamma_{i_L^G} \rightarrow \mathrm{BFM}(G)$$

Σ_{reg}/B_+ $\qquad$ $/B_+^L$ $\qquad$ $=$

$BFM(G)$ $\qquad$ $T^*_{reg} G^L // B^L_+$ $\qquad$ T^*T^L

The Toda isomorphism

$$T^*_{reg} G // G = N_\psi \backslash\backslash T^*G //_\psi N$$

$\psi : n \longrightarrow \mathbb{C}$

$B_+ , B \supseteq N$
opposite parabolics

$$\begin{pmatrix} 0 & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{pmatrix}$$

Pf $\quad$ $N \times N$ moment map fiber is

$$\mathcal{T} = \{ (g,\xi) \mid \pi(\xi) = \pi(g\xi g^{-1}) = \psi \}$$

$\pi : g^* \longrightarrow n^*$

Claim $\quad \mathcal{T} \subseteq T^*_{reg} G$

$B_+, B \supseteq N$

opposite parabolics

$N_\psi \backslash\backslash T^*G //_\psi N$

$\psi : n \longrightarrow \mathbb{C}$

$$\begin{pmatrix} 0 & * & * \\ & 0 & * \\ 0 & & 0 \end{pmatrix}$$

p fiber is

$\xi)\backslash \quad \pi(\xi) = \pi(g\xi g^{-1}) = \psi \; \xi$

$\pi : g^* \longrightarrow n^*$

$$\begin{tikzcd}
 & Z_{reg} \arrow[dl, hook] \arrow[dr] & & T \arrow[dl, hook] \arrow[dr] & \\
T^*_{reg} G // G & & T^*_{reg} G & & N_\psi \backslash\backslash T^* G //_\psi
\end{tikzcd}$$

eg G

Construct $BFM(L) \longrightarrow BFM(G)$

$$G = \coprod_{w \in W^P} UwP$$

minimal coset reps for W/W_L

$$U w_0 w_L^{-1} P = N_L w_0 w_L^{-1} L$$

$$FM(L) \longrightarrow BFM(G)$$

$$UwP$$

wP ← minimal coset reps for W/W_L

$$U w_0 w_L^{-1} P = N \overset{N_L}{\times} w_0 w_L^{-1} L \overset{N_L}{\times} N$$

$\Big\downarrow$ double reducing by $N \times N$

$$N_L \backslash\backslash_{\psi} \; L \; /\!/_{\psi} \, N_L$$

$$T^* U w_0 w_L^{-1} P$$

$N \backslash\backslash_{\psi} T^*G /\!/_{\psi} N$

To construct $BFM(L) \longrightarrow BFM(G)$

$$G = \coprod_{w \in W^P} U w P$$

minimal coset reps for W/W_L

$$U w_0 w_L^{-1} P = N \times^{N_L} w_0 w_L^{-1} \ldots$$

double reducing

$$N_L \backslash\backslash_{\psi} L /\!/_{\psi} N_L$$

So embedding is

$$BFM(L^\vee) \cong N_L \backslash\backslash_{\psi_L} T^*L /\!/_{\psi_L} N_L \cong N \backslash\backslash_{\psi} T^* U w_0 w_L^{-1} P /\!/_{\psi} N \subseteq N \backslash\backslash_{\psi} T^*G /\!/_{\psi} N = BFM(G^\vee)$$