Title: PSI 2019/2020 - Quantum Matter Part 2 - Lecture 1

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What is Quantum Complexity?

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UMASS BOSTON

PSI - Quantum Matter
February 2020

Work based on

Quantum Adiabatic Brachistochrone
A.T. Rezakhani, W.-J. Kuo, A. Hamma, D.A. Lidar, P. Zanardi
Phys. Rev. Lett. 103, 080502 (2009)

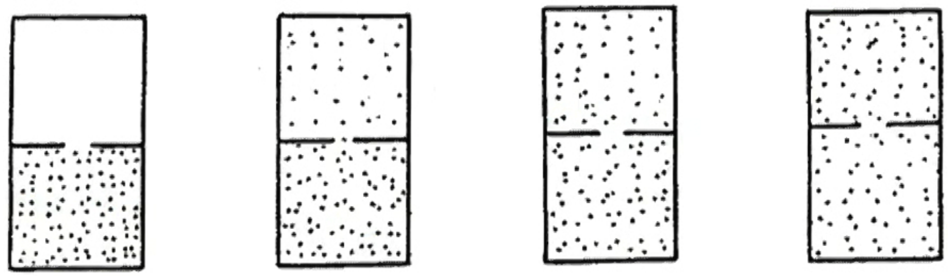
Emergent irreversibility and entanglement spectrum statistics
Claudio Chamon, A.H., Eduardo R. Mucciolo
Phys. Rev. Lett. 112, 240501 (2014)

Irreversibility and Entanglement Spectrum Statistics in Quantum Circuits
Daniel Shaffer, Claudio Chamon, A.H. Eduardo R. Mucciolo
J. Stat. Mech. (2014) P12007

Two-component Structure in the Entanglement Spectrum of Highly Excited States
Zhi-Cheng Yang, Claudio Chamon, A.H. , Eduardo R. Mucciolo
Phys. Rev. Lett. 115, 267206 (2015)

Entanglement Complexity, Thermalization, and Many-Body Localization
Zhi-Cheng Yang, A. H., Salvatore M. Giampaolo, Eduardo R. Mucciolo, Claudio Chamon
Phys. Rev. B 96, 020408 (2017)

Single T gate in a Clifford circuit drives transition to universal entanglement spectrum statistics
Shiyu Zhou, Zhi-Cheng Yang, Alioscia Hamma, Claudio Chamon
arXiv:1906:01079



- Why does the gas prefer to be in equilibrium, that is, to look like the snapshot on the right?
- This question is very subtle already in classical physics

WHY DOES ENTROPY INCREASE?

- Unitarity = Time Reversal
- Entropy is constant with unitary evolution
- Entanglement Entropy is not!
- Does increase in entropy mean irreversibility?
- Where does irreversibility come from?
- Irreversibility is a feature of Quantum Complexity

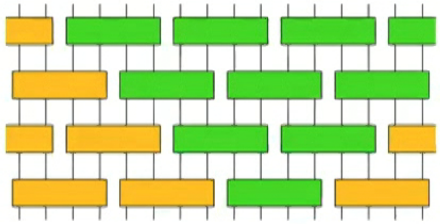
IRREVERSIBILITY IN QUANTUM PHYSICS

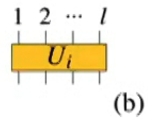
- There are two ways of approaching th notion of quantum complexity
- 1. (Bennett, Susskind) complexity relative to a fiducial state as the least number of elementary operations needed to make that state.
- If a quantum evolution is ergodic, it can eventually reach complex states: Gibbs POV
- 2. (this talk) The state is complex if it belongs to a large equivalence class, i.e., it is hard to tell it apart from many other states: Boltzmann POV

WHAT IS QUANTUM COMPLEXITY?

- The equivalence class is a Phase
- We consider equivalent all those states that can be connected by a (finite depth) quantum circuit
- This is a new way of classifying quantum phases

QUANTUM COMPLEXITY AS A PHASE





- If two states are connected by a quantum circuit of finite depth, they are in the same phase. This is called a *topological* phase
- If we constrain the gates to have a specific symmetry, the phase diagram splits in phases with different symmetry breakings

Quantum Phases and Symmetry Breaking

- We need to define constraints to the gates we use
- For instance, if we use only Clifford gates, these are *free* resources, and we have a certain quantum complexity class
- This way of classifying quantum phases is more general: a ferromagnetic state and a non trivial topological state can be connected by just Clifford gates. They both have low quantum complexity
- What are the complexity 'order parameters'?

QUANTUM COMPLEXITY PHASES

- Complexity of Entanglement
- t-Design
- Machine learning approach to classification
- Simulability and resources

COMPLEXITY ORDER PARAMETERS

RESOURCES

- **NOT** gate: flips the state of the qubit

$|0\rangle \rightarrow |1\rangle$ and $|1\rangle \rightarrow |0\rangle$

- **H** (Hadamard) gate:

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$ and $|1\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

- Phase gates $\mathbf{P}_\delta$: gives a state dependent phase

$|0\rangle \rightarrow |0\rangle$ and $|1\rangle \rightarrow e^{i\delta}|1\rangle$

The phase gate with $\delta = \pi/4$ is called **T**

The phase gate with $\delta = \pi/2$ is called **S**.

- **CNOT** gate or controlled-NOT gate:

$|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow |01\rangle$

$|10\rangle \rightarrow |11\rangle$, $|11\rangle \rightarrow |10\rangle$

- States are complex if their entanglem is complex
- This means that it is very difficult to distinguish them from many other entangled states
- The landscape of entanglement would have many local minima
- Trying to disentangle a state would be hard

QUANTUM COMPLEXITY AND ENTANGLEMENT

RANDOM QUANTUM CIRCUIT (RQC)

- CONSIDER A SYSTEM OF N SPINS 1/2 INITIAL A PRODUCT STATE

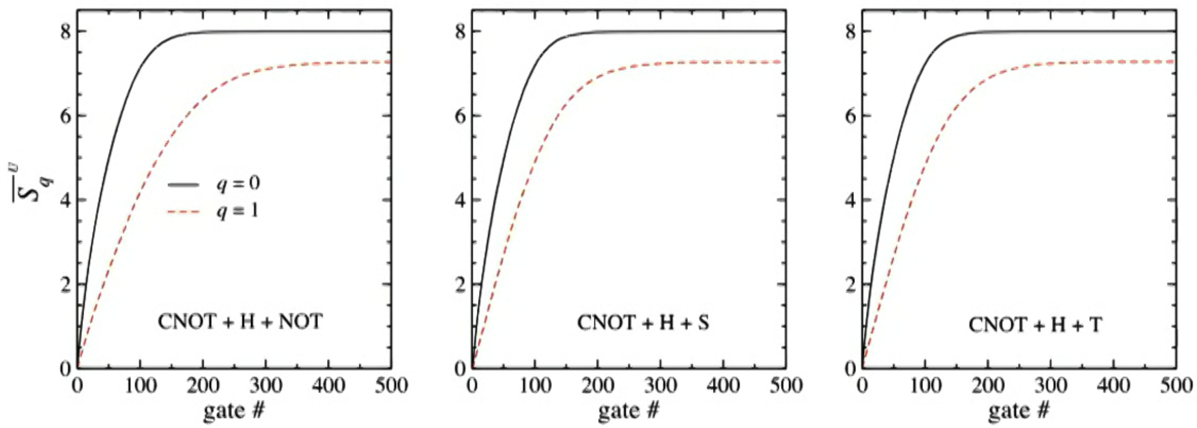
- WE DRAW SOME RANDOM GATES BETWEEN RANDOM QUBITS AND MONITOR THE ENTANGLEMENT (HEATING)

- AT SOME POINT THE ENTANGLEMENT BECOMES MAXIMAL AND EQUILIBRATES

- THEN WE TRY TO REVERSE THE EVOLUTION AND DISENTANGLE THE STATE (COOLING)

UNIVERSAL AND NOT SETS OF GATE

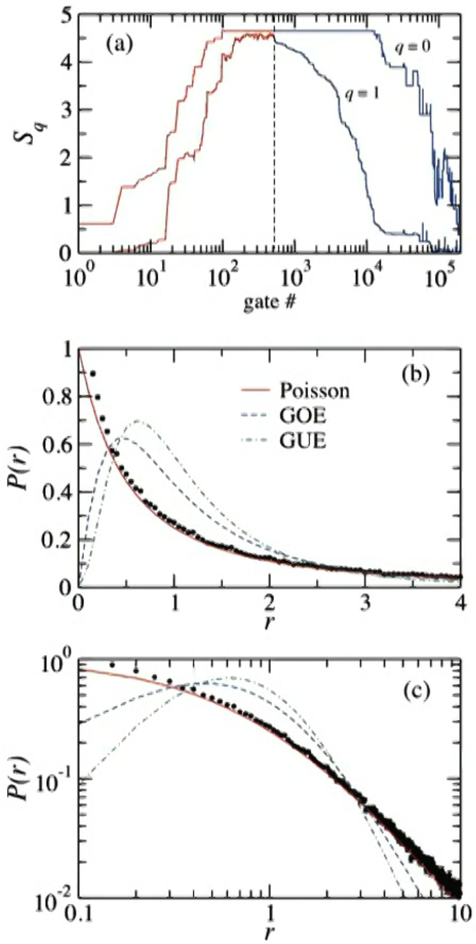
- (H,CNOT,S) dense in Clifford Group, **NO UNIVERSAL**
- (H,CNOT,NOT) = **NOT UNIVERSAL**
- (H,CNOT,T) = **UNIVERSAL**
- Entanglement Heating by RQC



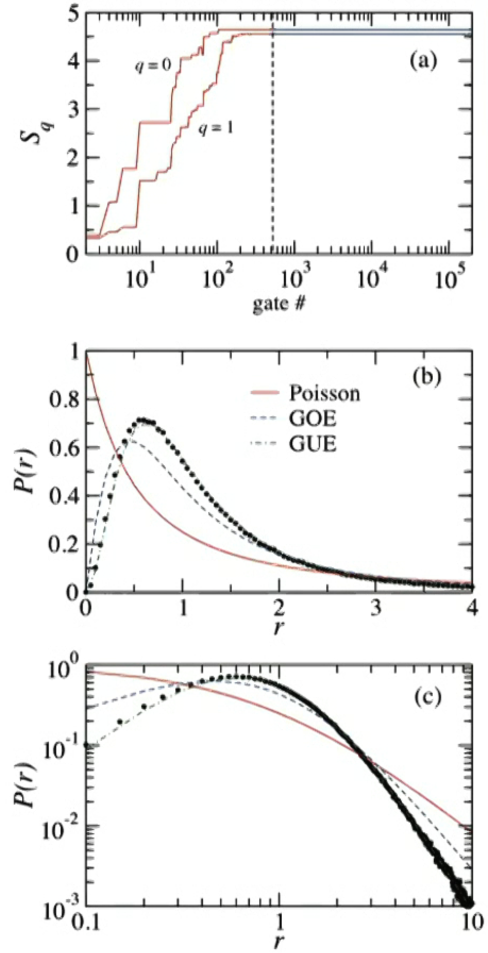
<u>Reversing the evolution</u>: *Disentangling the s*

Metropolis algorithm:

- (1) Compute the entanglement entropies (all bipartitions)
- (2) Choose a gate randomly
- (3) Apply the gate
- (4) Recompute the entanglement entropies
    - If entropies are reduced, add gate to reverse list
    - If entropies increase, keep gate with Boltzmann probability or discard it and go back to (2)
- (5) Stop when entanglement entropies are zero



CNOT+H+S



CNOT+H+T

STATISTICAL COMPLEXITY

Be $\{p_i\}$ the populations of the reduced density m

$$r_i = \frac{p_{i-1} - p_i}{p_i - p_{I+1}} \qquad P(r) = R^{-1} \sum_{I=1}^{R} \langle \delta(r - r_i) \rangle$$

- Complexity as adherence of the statistics of the gaps to random matrix theory

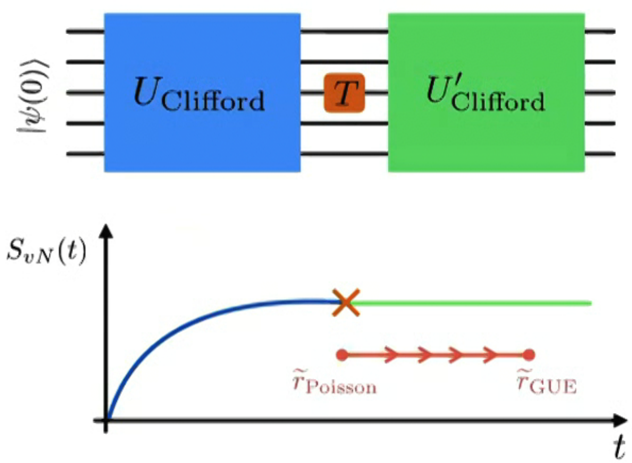
$$P_{WD} = \frac{1}{Z} \frac{(r + r^2)^{\beta}}{(1 + r + r^2)^{1+3\beta/2}} \qquad P_{\text{Poisson}}(r) = \frac{1}{(1+r)^2}$$

Operator Set	Level Spacing Statistics	Reversible?
$\{CNOT, H, NOT\}$	Poisson	Yes
$\{CNOT, H, S\}$	Poisson	Yes
$\{CNOT, H, T\}$	GUE	No

Irreversibility and Complexity

- Irreversibility arises not just because there are many ways of entangling vs few ways of disentangling.
- Even with active disentangling (Metropolis) it may be difficult
- Complex entanglement means that searching for a disentangler requires exhaustive search
- Complex entanglement is a property of the statistics of the entanglement gaps.
- Simple entanglement means that a disentangler can be found by some principle (annealing)
- This also characterizes universal computation as ergodic and non-universal as non complex

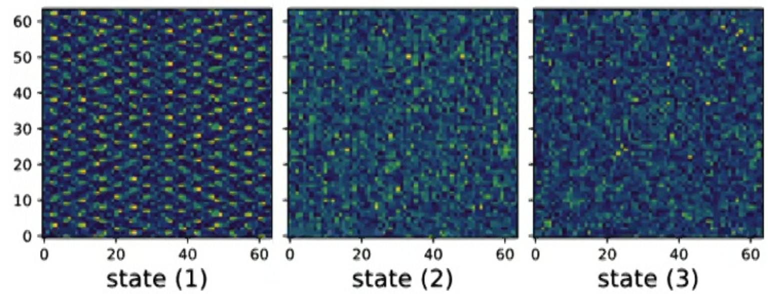
Transition to Quantum Complexity by Doping with T gates



- We place n (possibly just one) T gates sandwiched by Clifford circuits
- n=0 will be just a Clifford circuit: ESS will be Poisson
- Can we drive a transition to Universal (GUE) ESS?
- Is integrability immediately destroyed?
- What does Universal GUE correspond to?

$$|\Psi\rangle = \sum_{x_A, x_B} \Psi(x_A, x_B)|x_A x_B\rangle$$

$$|\Psi(x_A, x_B)|^2$$



- 1) Clifford gates
- 2) Cliffort gates + Sqrt(N) T gates
- 3) Random unitaries

CLASSIFICATION BY MACHINE LEARNING