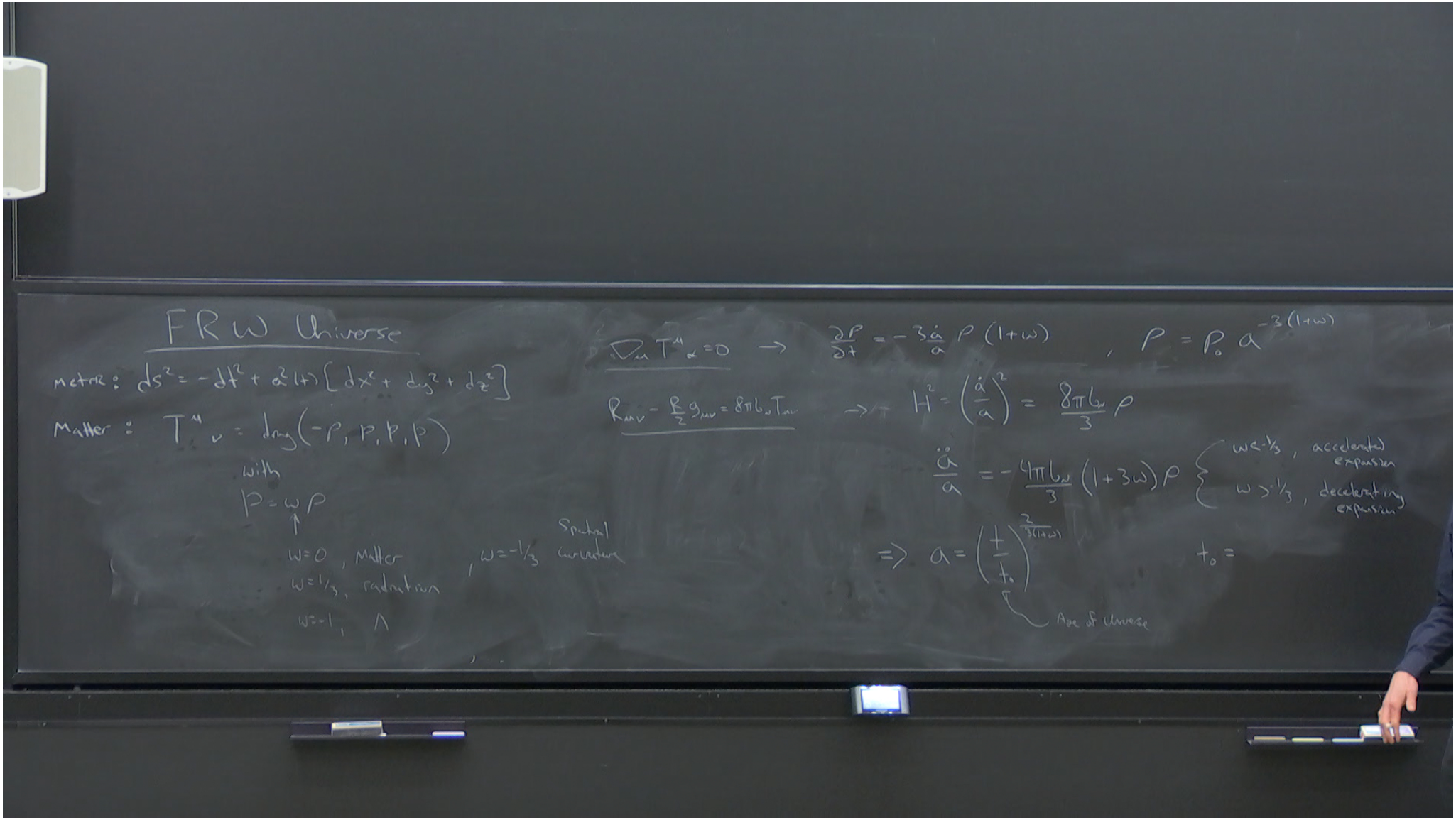
Title: PSI 2019/2020 - Cosmology Part 1 - Lecture 5

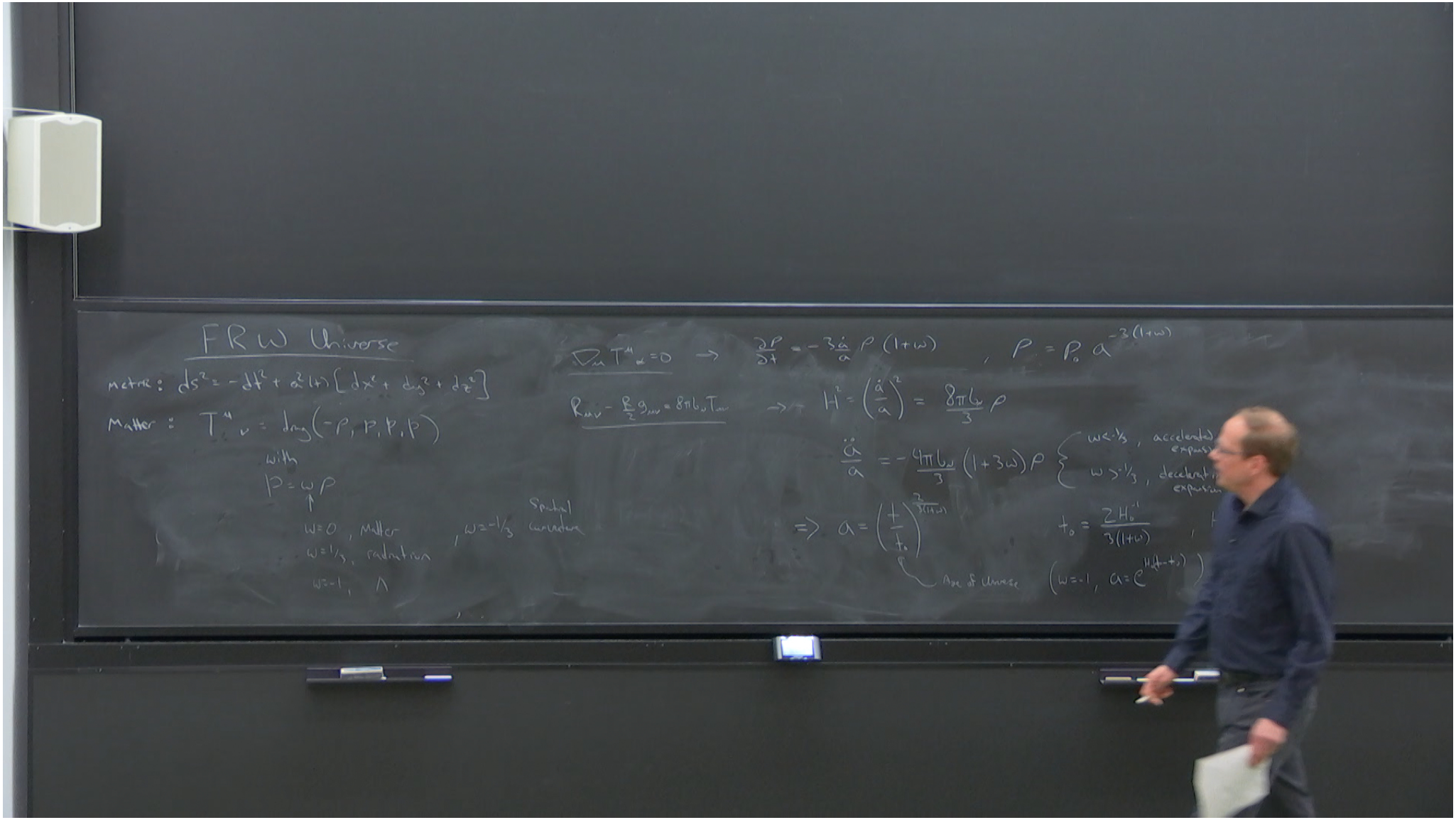
Speakers: Matthew Johnson

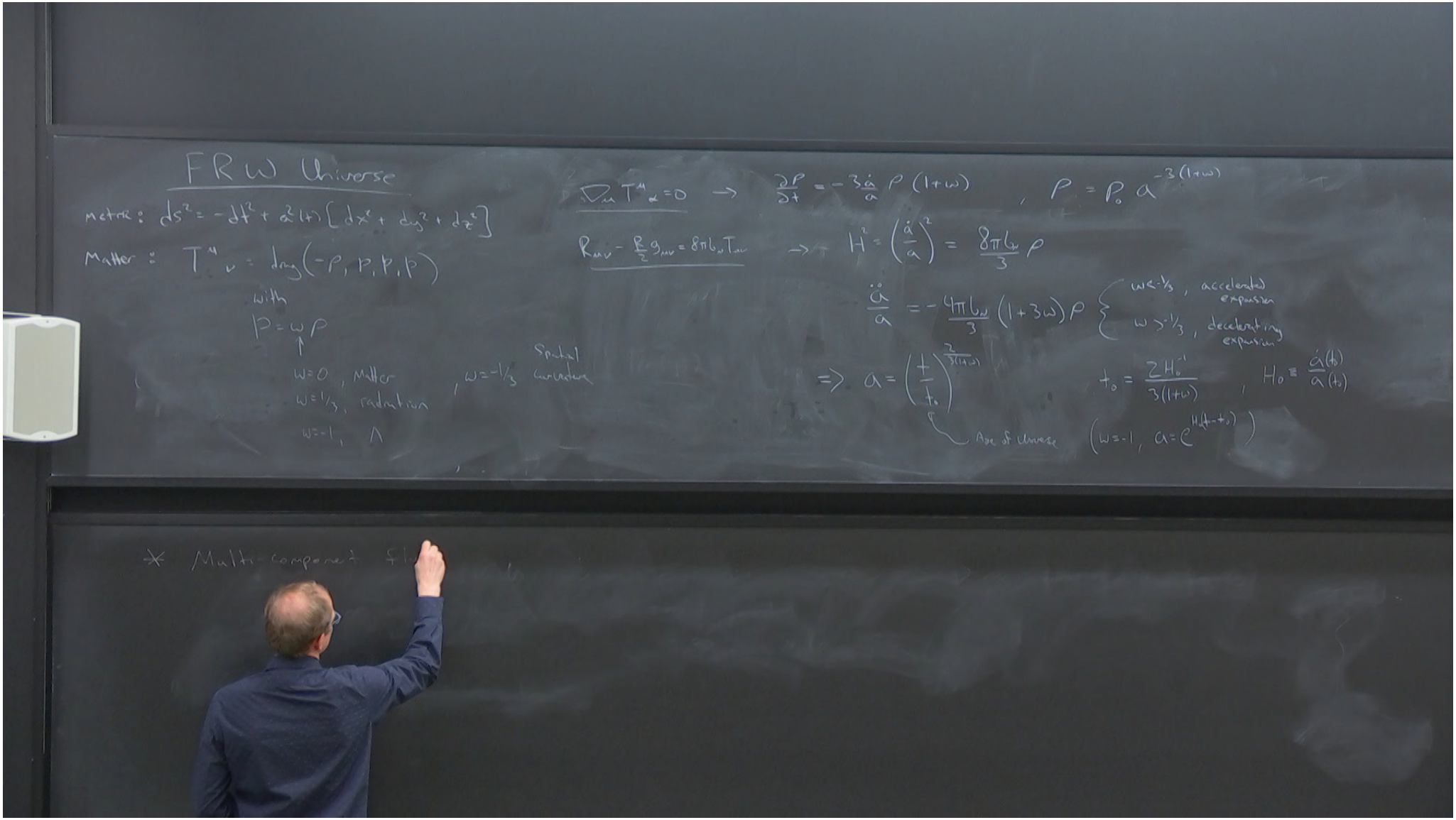
Collection: PSI 2019/2020 - Cosmology (Part 1)

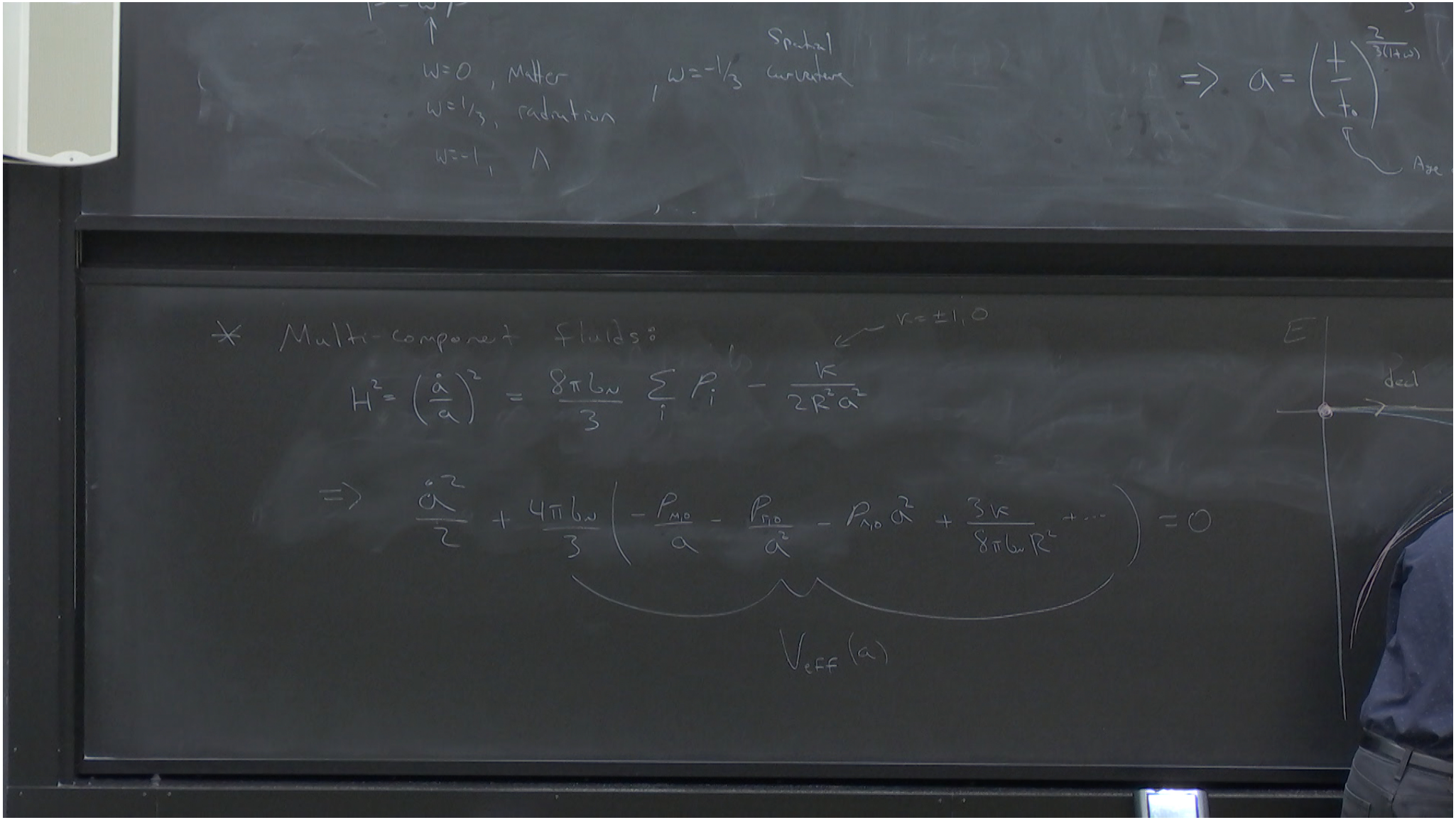
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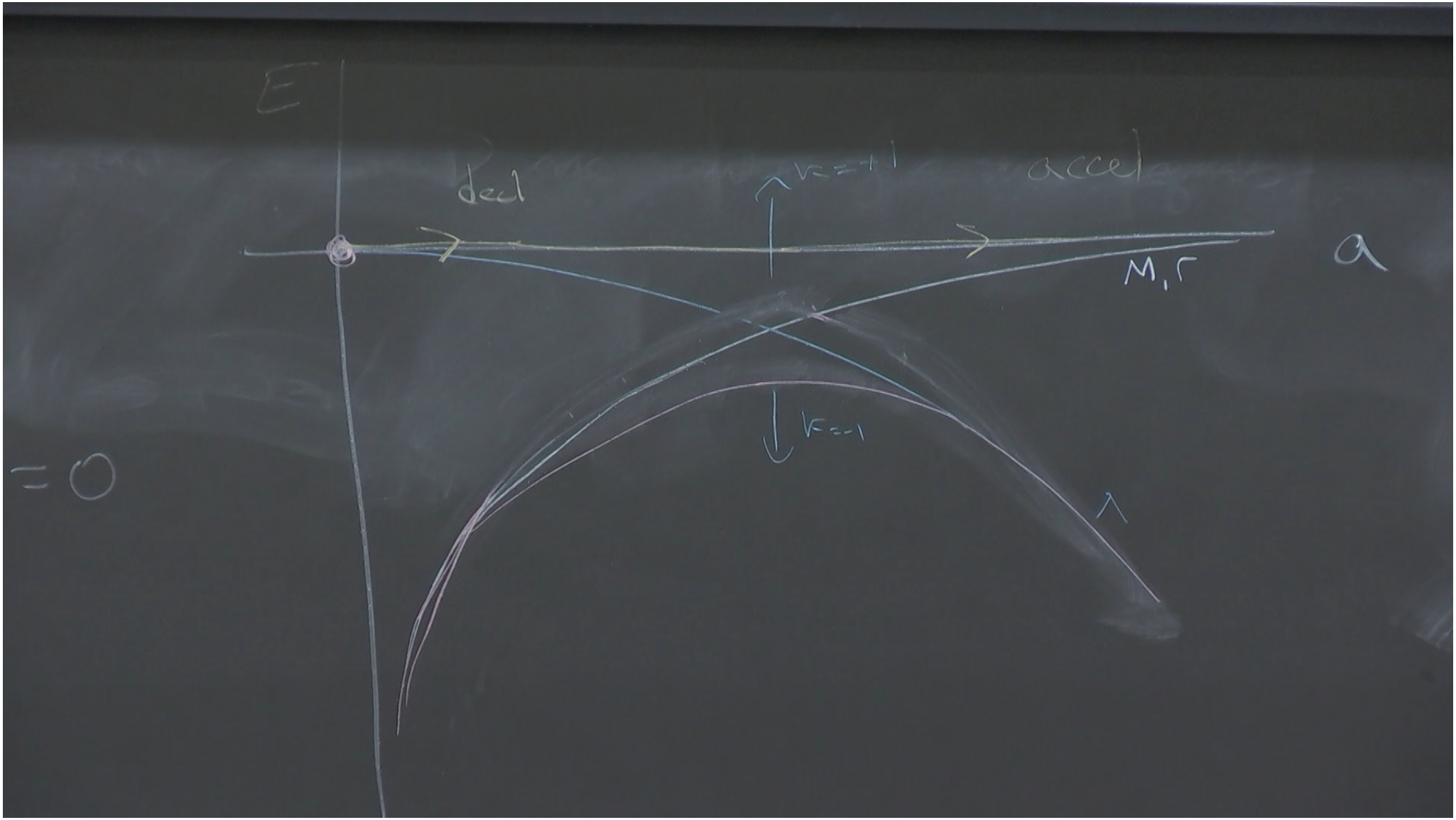
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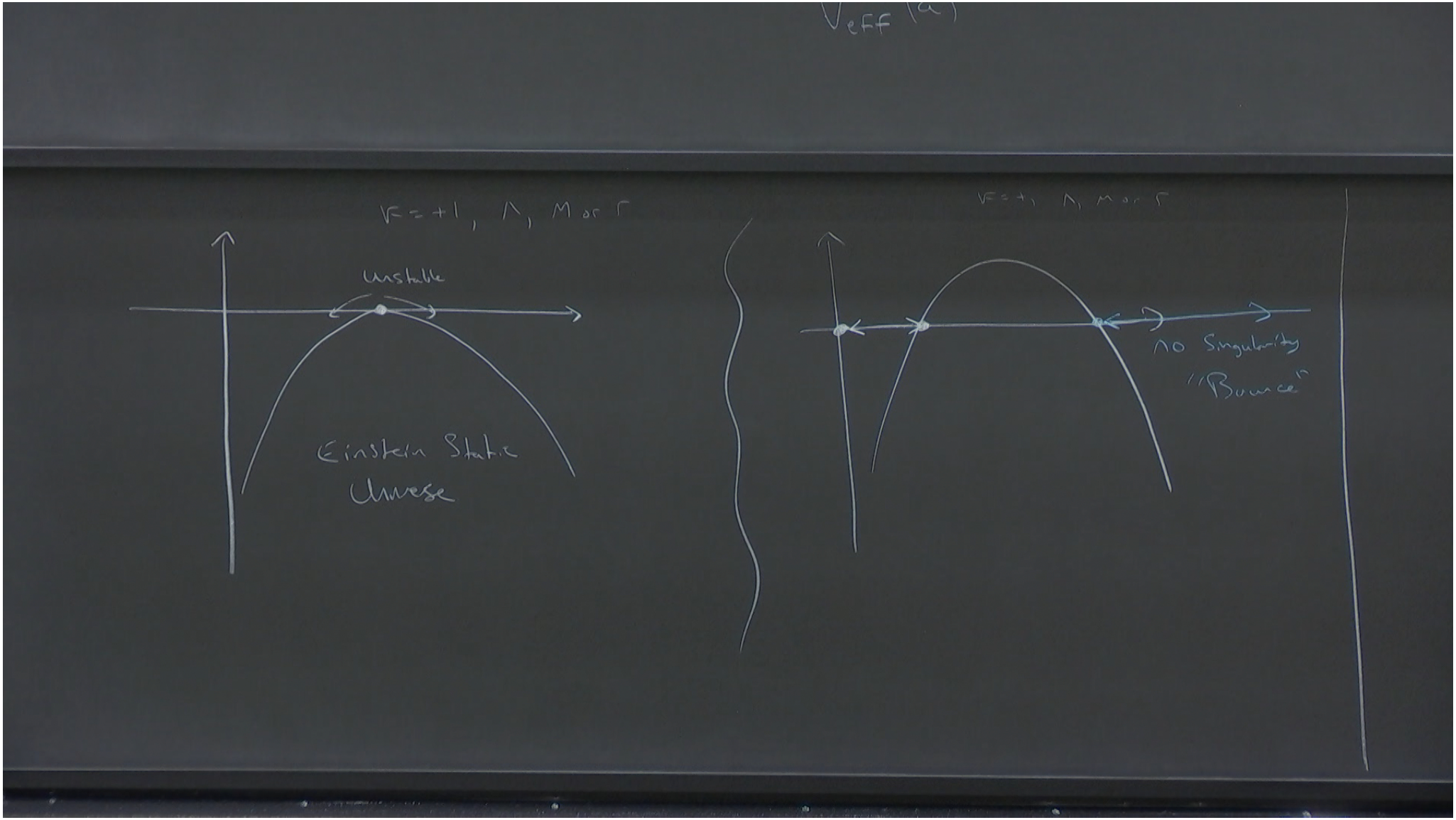

FRW Universe
metric : $ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$
Matter : $T^{\mu}{}_{\nu} = \mathrm{diag}(-\rho, p, p, p)$
with
$p = w\rho$
$w=0$, Matter
$w=1/3$, radiation
$w=-1$, Λ
Spatial curvature
$\nabla_\mu T^{\mu}{}_{\alpha} = 0 \rightarrow \frac{\partial \rho}{\partial t} = -3\frac{\dot{a}}{a}\rho(1+w)$, $\rho = \rho_0 a^{-3(1+w)}$
$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho$
$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(1+3w)\rho$
$w < -1/3$, accelerated expansion
$w > -1/3$, decelerating expansion
$\Rightarrow a = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$
$t_0 =$
Age of Universe


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$t_0 = \frac{2H_0^{-1}}{3(1+w)}$
$(w=-1,\ a = e^{H_0(t-t_0)})$


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$\Rightarrow a = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$
$t_0 = \frac{2H_0^{-1}}{3(1+w)}$, $H_0 \equiv \frac{\dot{a}(t_0)}{a(t_0)}$
Age of Universe
$(w=-1,\ a = e^{H_0(t-t_0)})$
* Multi-component fl


$w=0$, Matter
$w=1/3$, radiation
$w=-1$, Λ
$w=-1/3$ Spatial curvature
$\Rightarrow a=\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$
Age
* Multi-component fluids:
$k=\pm 1, 0$
$H^2=\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G_N}{3}\sum_i \rho_i-\frac{k}{2R^2a^2}$
$\Rightarrow \frac{\dot{a}^2}{2}+\frac{4\pi G_N}{3}\left(-\frac{\rho_{M0}}{a}-\frac{\rho_{R0}}{a^2}-\rho_{\Lambda 0}a^2+\frac{3k}{8\pi G_N R^2}+\cdots\right)=0$
$V_{eff}(a)$
E
decl


E
decl
k=+1
accel
a
M, r
k=-1
=0
Λ


$V_{eff}(a)$
$k=+1$, Λ, M or r
Unstable
Einstein Static Universe
$k=+1$, Λ, M or r
No Singularity
"Bounce"

Density Parameter

$$\Omega = \sum_i \Omega_i(t) = \sum_i \frac{\rho(t)}{\rho_{crit}(t)} \quad , \quad \rho_{crit} \equiv \frac{3H^2}{8\pi G_N}$$

$$\Rightarrow \Omega = 1 \quad , \quad k = 0$$

Friedmann eq'n: $\quad 1 = \Omega + \Omega_k \quad , \quad \Omega_k = -\frac{k}{H^2 R^2 a^2}$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{b,0} + \Omega_{c,0}}{a^3} + \Omega_{DE,0} + \frac{\Omega_{k,0}}{a^2}$$

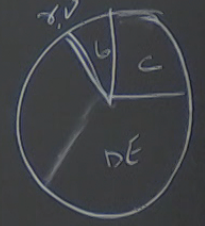
$\Omega_r = \Omega_\gamma + \Omega_\nu \simeq 9 \times 10^{-5}$

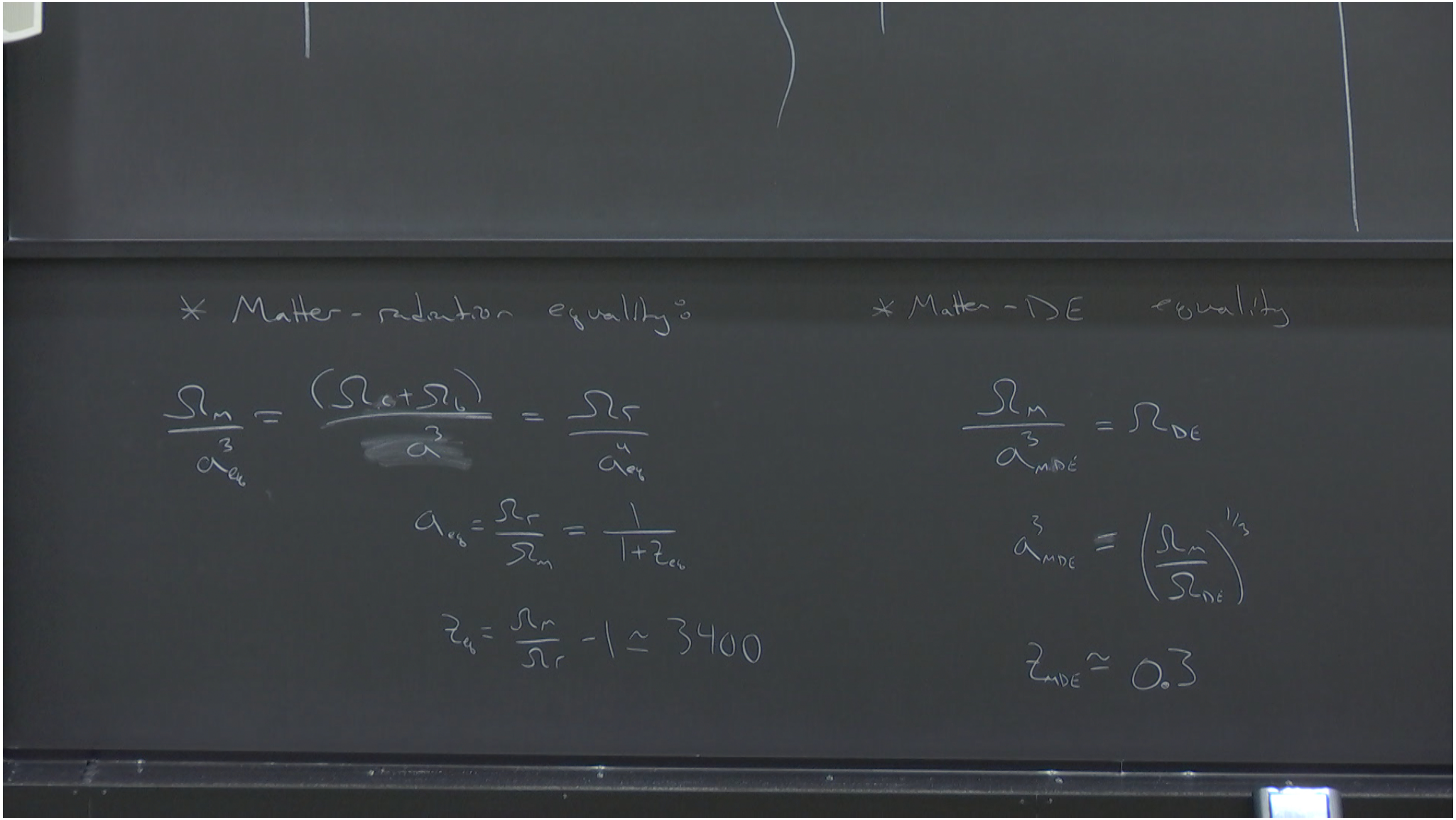
$\Omega_c = 0.264$

$\Omega_b = 0.0493$

$\Omega_{DE} = .685$

$|\Omega_k| < 0.0037$




* Matter - radiation equality:
$\frac{\Omega_M}{a^3_{eq}} = \frac{(\Omega_{c,0}+\Omega_b)}{a^3} = \frac{\Omega_r}{a^4_{eq}}$
$a_{eq} = \frac{\Omega_r}{\Omega_M} = \frac{1}{1+z_{eq}}$
$z_{eq} = \frac{\Omega_M}{\Omega_r} - 1 \simeq 3400$
* Matter - DE equality
$\frac{\Omega_M}{a^3_{MDE}} = \Omega_{DE}$
$a^3_{MDE} = \left(\frac{\Omega_M}{\Omega_{DE}}\right)^{1/3}$
$z_{MDE} \simeq 0.3$

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{b,0}+\Omega_{c,0}}{a^3} + \Omega_{DE,0} + \frac{\Omega_{k,0}}{a^2}$$

$\Omega_{DE} = .685$

$|\Omega_k| < 0.0037$

Thermal History

hot / dense

? Quantum gravity ?

??

Electroweak	-	TeV	Sort of understood
QCD	-	100 MeV	
Nuclear Physics	-	1 MeV	really well understood
atomic physics	-	eV	

Cold / diffuse

not understood

* Big Bang Nucleosynthesis (BBN)

1 MeV (p+n → nuclei)

* Recombination

eV (nuclei + e^- → atoms)

Coincides w/ birth of CMB

* Need Math to describe ensembles of interacting particles in an expanding Universe.

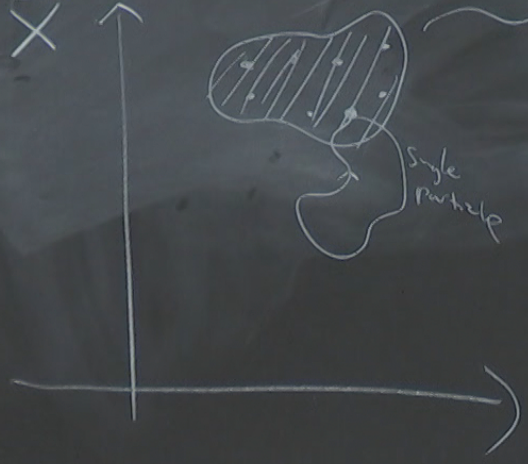
$$dN = f(\vec{x}, \vec{p}, t)\,\frac{d^3x\,d^3p}{(2\pi)^3}$$

dN: # of States; $f(\vec{x}, \vec{p}, t)$: distribution function

$\vec{x} = x^i$

$\vec{p} = g_{i\nu}\frac{dx^\nu}{d\lambda}$

$m^2 = E^2 - \vec{p}^2$



$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = -m^2$$

$$-E^2 + a^2\left(\frac{dx^i}{d\lambda}\right)^2 = -m^2$$

$$-E^2 + a^2\left(\frac{\vec{p}}{a^2}\right)^2 = -m^2$$

number den

$n(\vec{x}, t)$

Number density:

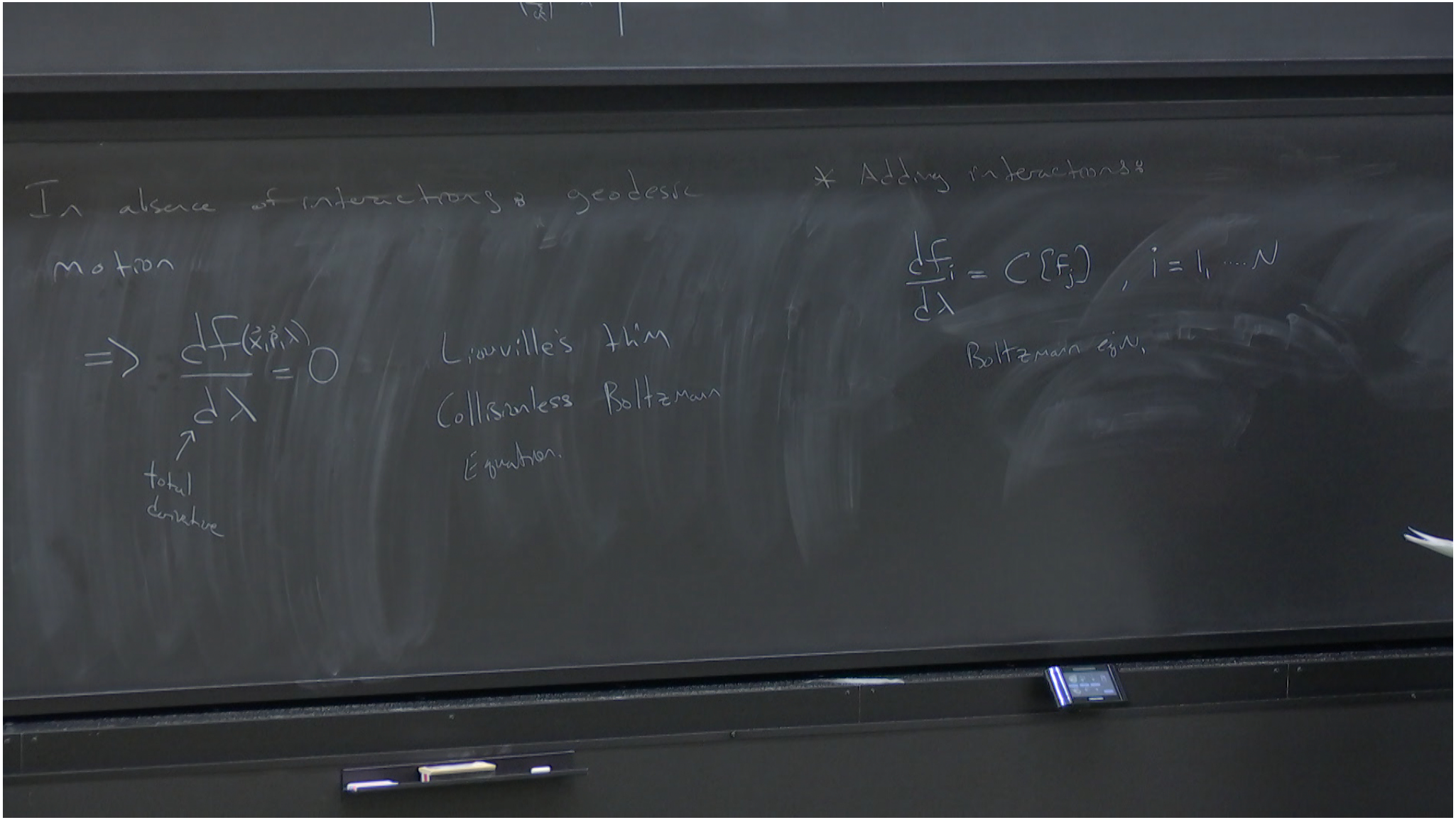
$$n(\vec{x},t) = \int \frac{dN}{d^3x} = \int \frac{d^3p}{(2\pi)^3} f(\vec{x},\vec{p},t)$$

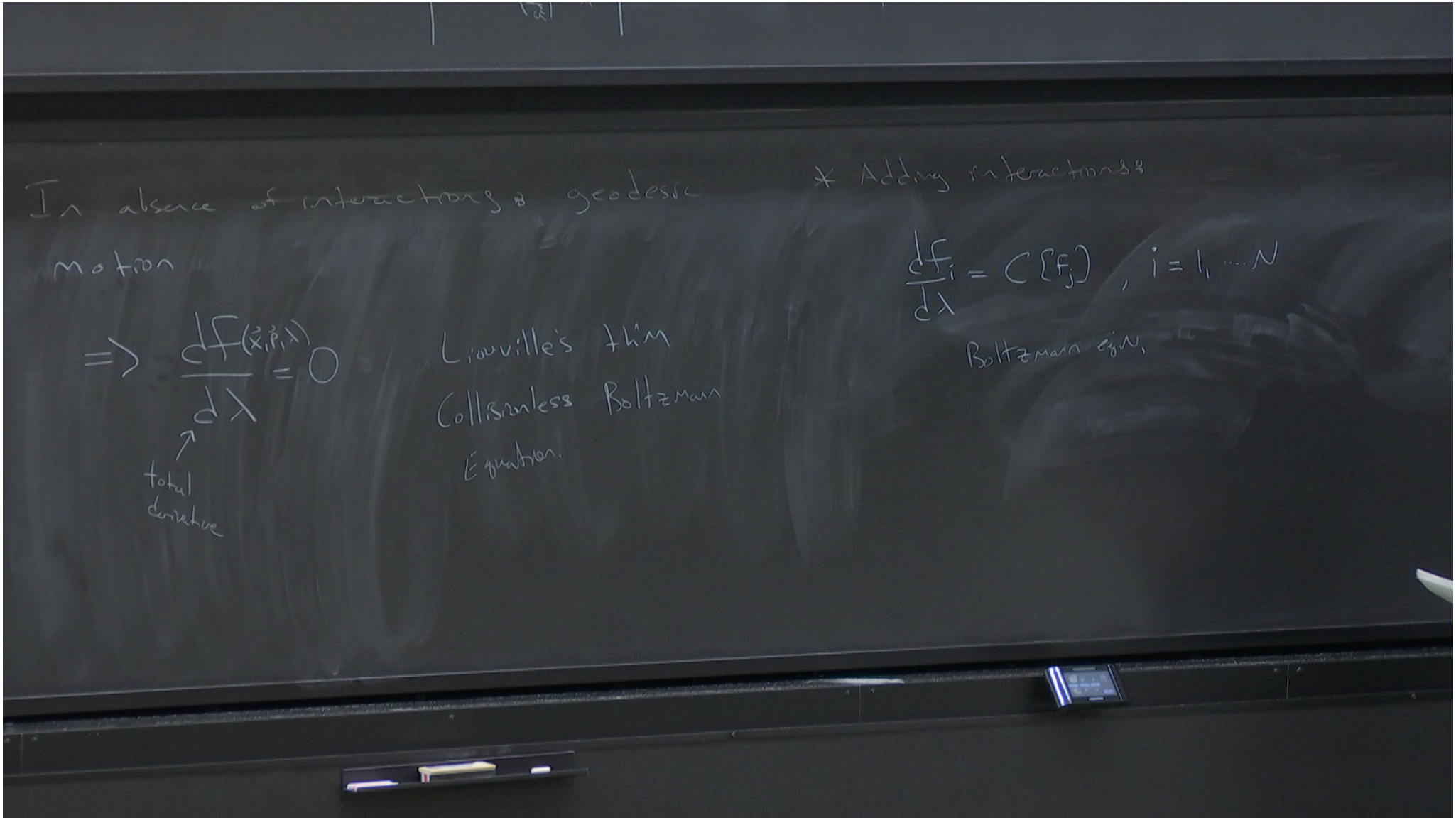
Energy density:

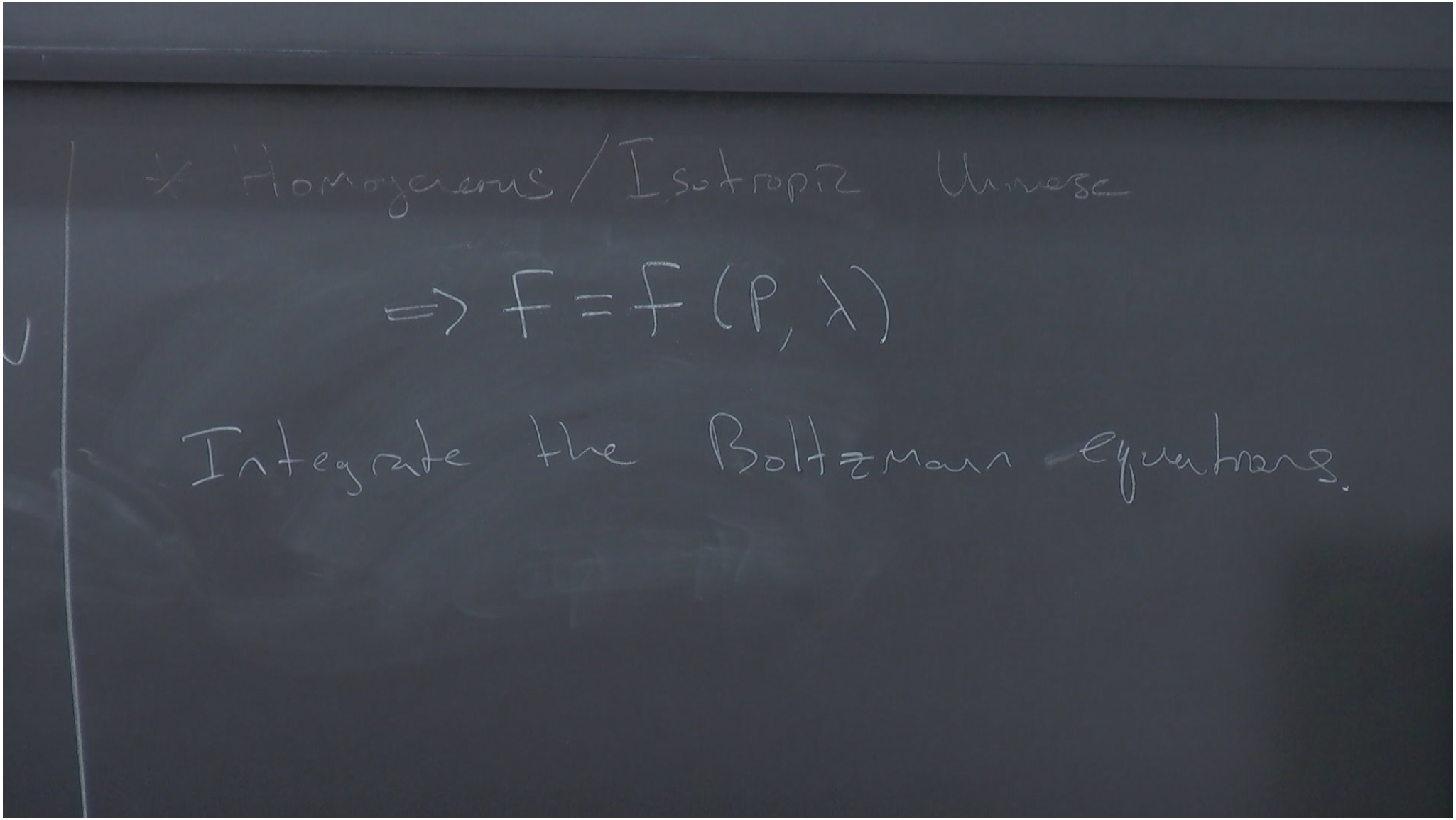
$$\rho(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} f(\vec{x},\vec{p},t)\, E(p)$$

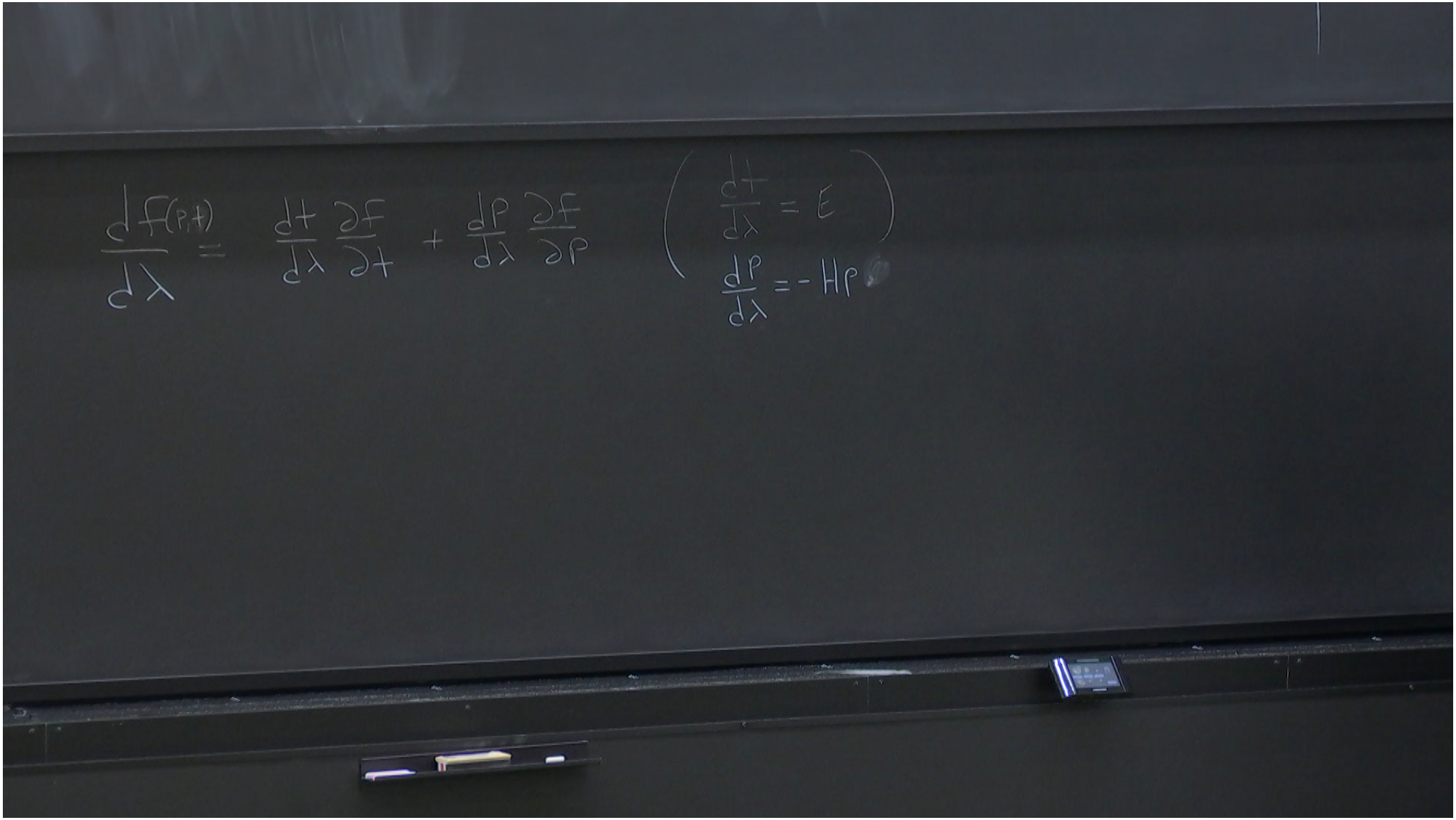
$i\nu \frac{dx^\nu}{d\lambda}$

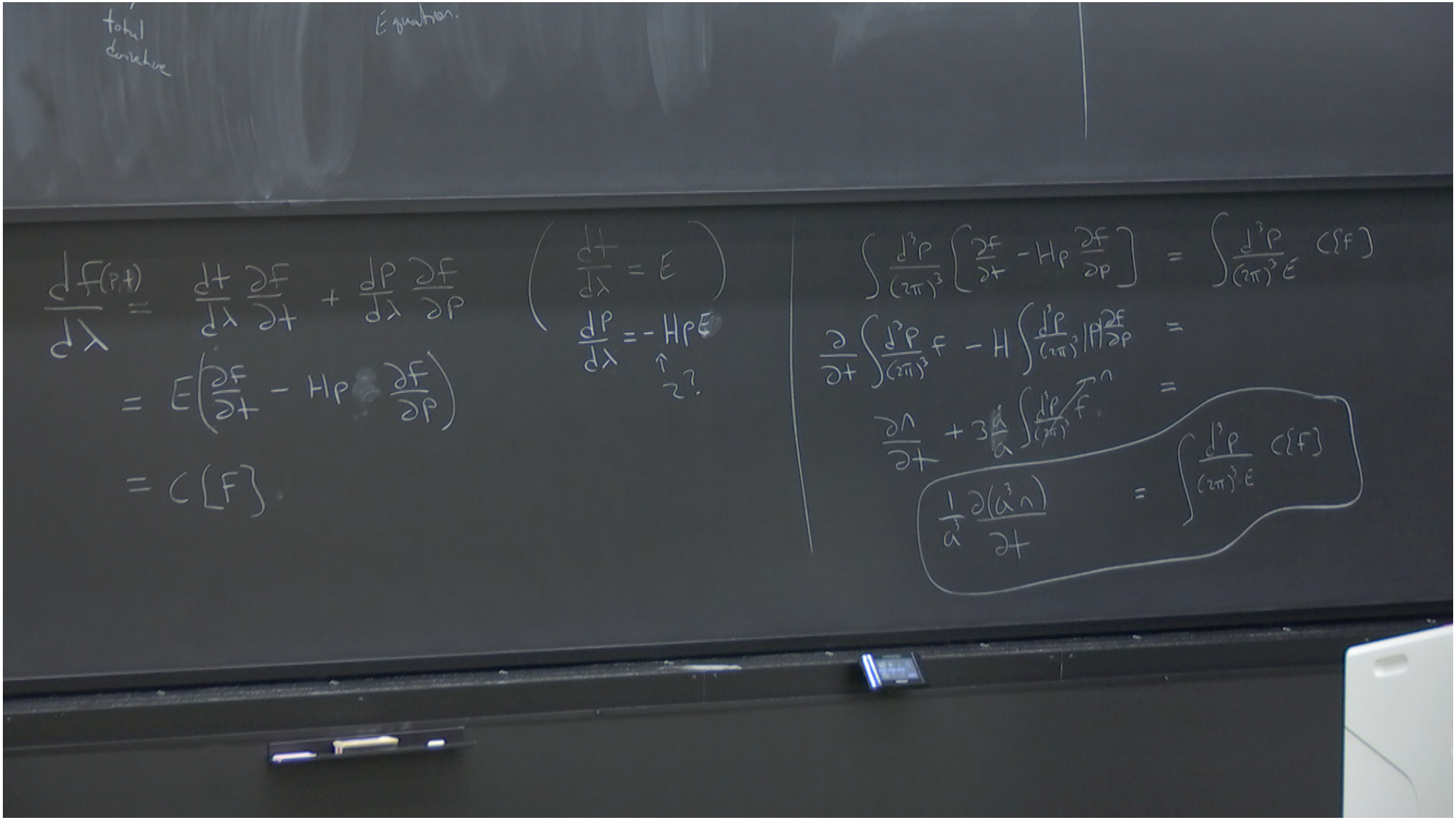
$\vec{p}^2$


In absence of interactions & geodesic motion
$\Rightarrow \frac{df(\vec{x},\vec{p},\lambda)}{d\lambda} = 0$
total derivative
Liouville's thm
Collisionless Boltzmann Equation.
* Adding interactions
$\frac{df_i}{d\lambda} = C[f_j]$, $i = 1, \cdots N$
Boltzmann eqn.


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$\Rightarrow \frac{df(\vec{x},\vec{p},\lambda)}{d\lambda} = 0$
total derivative
Liouville's thm
Collisionless Boltzmann Equation.
* Adding interactions
$\frac{df_i}{d\lambda} = C[f_j]$, $i = 1, \cdots N$
Boltzmann eqn.


Homogeneous/Isotropic Universe
$\Rightarrow f = f(p, \lambda)$
Integrate the Boltzmann equations.


$\frac{df(p,t)}{d\lambda} = \frac{dt}{d\lambda}\frac{\partial f}{\partial t} + \frac{dp}{d\lambda}\frac{\partial f}{\partial p}$
$\left(\frac{dt}{d\lambda} = E\right.$
$\left.\frac{dp}{d\lambda} = -Hp\right)$


total derivative
Equation.
$\frac{df(p,t)}{d\lambda} = \frac{dt}{d\lambda}\frac{\partial f}{\partial t} + \frac{dp}{d\lambda}\frac{\partial f}{\partial p}$
$= E\left(\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p}\right)$
$= C[f]$
$\left(\frac{dt}{d\lambda} = E\right)$
$\frac{dp}{d\lambda} = -HpE$
?
$\int \frac{d^3p}{(2\pi)^3}\left[\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p}\right] = \int \frac{d^3p}{(2\pi)^3 E} C[f]$
$\frac{\partial}{\partial t}\int \frac{d^3p}{(2\pi)^3} f - H\int \frac{d^3p}{(2\pi)^3}|p|\frac{\partial f}{\partial p} =$
$\frac{\partial n}{\partial t} + 3\frac{\dot{a}}{a}\int \frac{d^3p}{(2\pi)^3} f =$
$\frac{1}{a^3}\frac{\partial(a^3 n)}{\partial t} = \int \frac{d^3p}{(2\pi)^3 E} C[f]$

* Equilibrium : When interactions are frequent
(Thermal, or Kinetic equilibrium)

$$f = \frac{g}{e^{(E-\mu)/T} \pm 1}$$

g ← degeneracy factor

$\pm$: + - fermions
- - Bosons

μ : chemical potential
$\mu = 0$, chemical equilibrium

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{b,0}+\Omega_{c,0}}{a^3} + \Omega_{DE,0} + \frac{\Omega_{K,0}}{a^2}$$

$\Omega_{DE} = .68$

$|\Omega_K| < 0.0037$

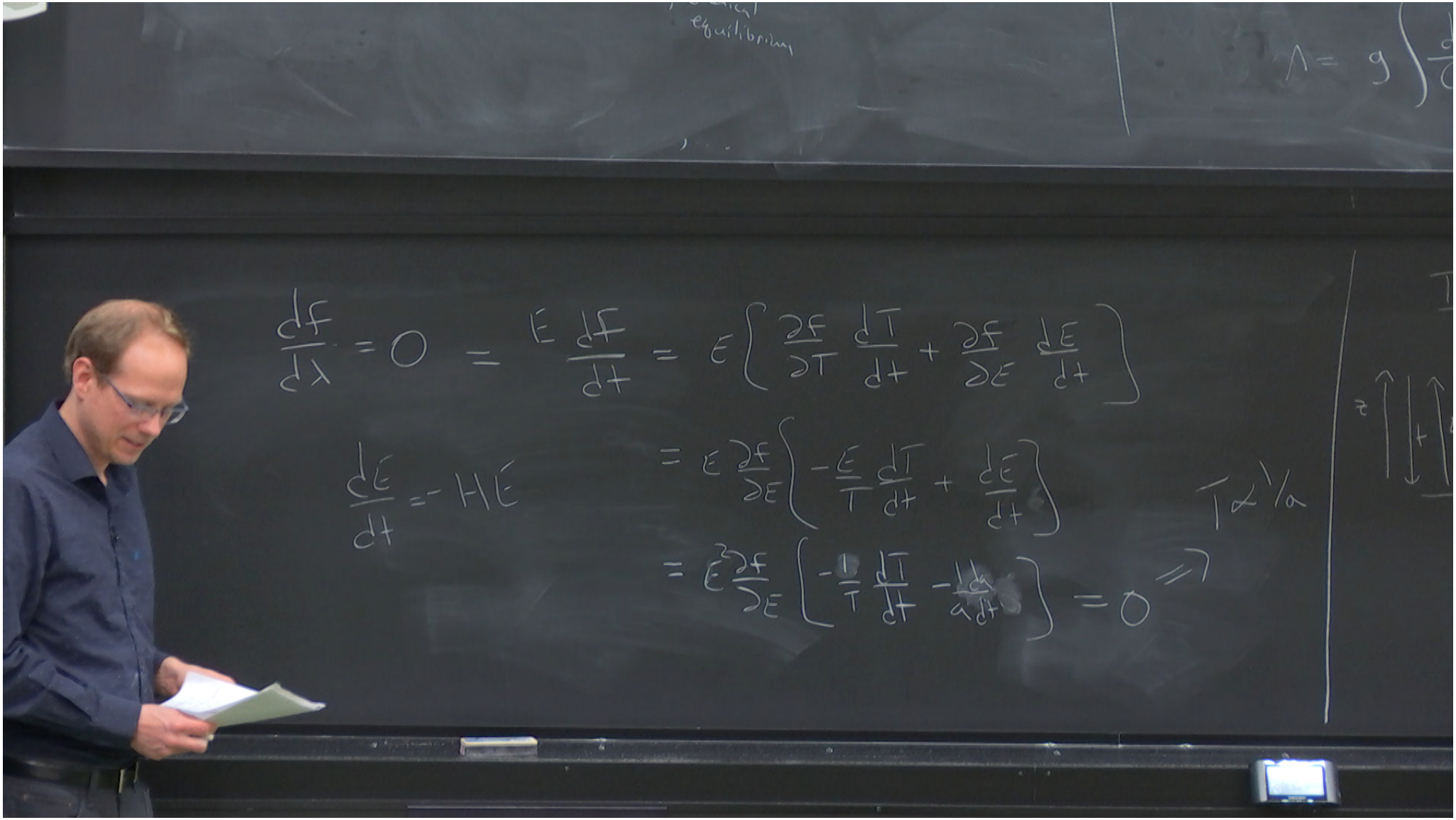
Number densities

relativistic - $E = p$

$$n = g\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}\mp 1} \simeq \frac{1.2g}{\pi}T^3 \propto 1/a^3 \rightarrow T \propto 1/a$$

Non-relativistic - $E \simeq M + p^2/2m$, $T \ll M$

$$n = g\int \frac{d^3p}{(2\pi)^3} e^{\mu/T} e^{-M/T} e^{-p^2/2mT} = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{\mu/T} e^{-M/T} \rightarrow T \propto 1/a^2$$


equilibrium
$\frac{df}{dt} = 0 = \frac{df}{dt} = \epsilon \left\{ \frac{\partial f}{\partial T} \frac{dT}{dt} + \frac{\partial f}{\partial \epsilon} \frac{d\epsilon}{dt} \right\}$
$\frac{d\epsilon}{dt} = -H\epsilon$
$= \epsilon \frac{\partial f}{\partial \epsilon} \left\{ -\frac{\epsilon}{T} \frac{dT}{dt} + \frac{d\epsilon}{dt} \right\}$
$= \epsilon \frac{\partial f}{\partial \epsilon} \left\{ -\frac{1}{T} \frac{dT}{dt} - \frac{1}{a} \frac{da}{dt} \right\} = 0 \Rightarrow$
$T \propto 1/a$