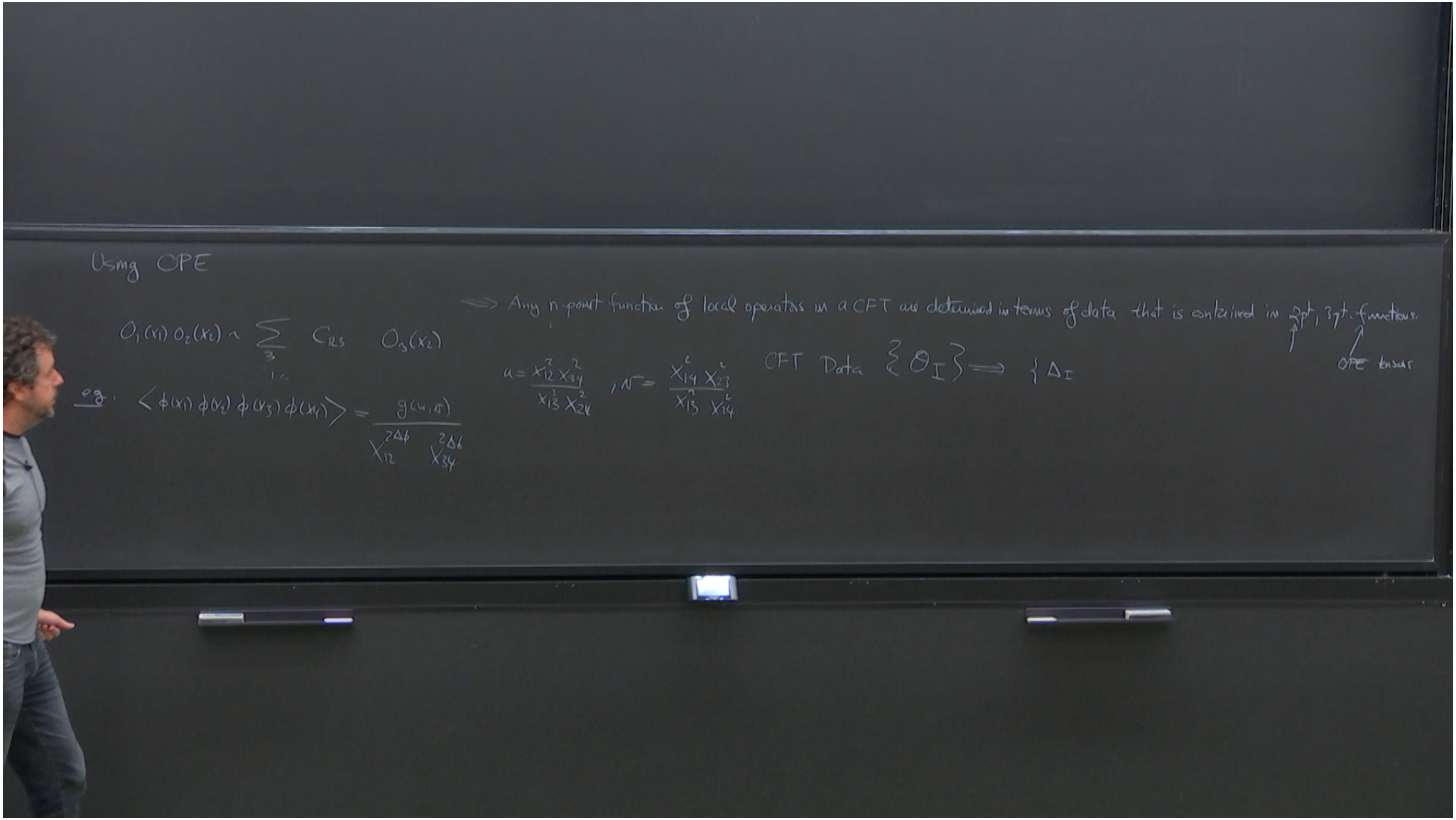
Title: PSI 2019/2020 - QFT III - Lecture 11

Speakers: Jaume Gomis

Collection: PSI 2019/2020 - QFT III

Date: February 25, 2020 - 11:15 AM

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Using OPE
$\mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2) \sim \sum_3 C_{123}\ \mathcal{O}_3(x_2)$
$\Longrightarrow$ Any n-point function of local operators in a CFT are determined in terms of data that is contained in 2pt, 3pt. functions.
2pt
3pt. functions
OPE tensors
$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$
CFT Data $\{\theta_I\} \Longrightarrow \{\Delta_I\}$
eg. $\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}}$

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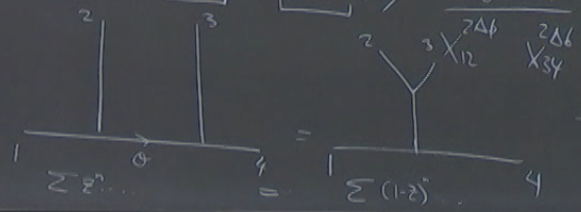
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CFT Data $\{\mathcal{O}_I\} \Longrightarrow \{\Delta_I,\ C_{IJK}\}$

CFT$_2$ (free field theory) + S ← relevant operator → CFT$_1$

$(x_1, x_2, x_3, x_4) = (0, z, 1, \infty)$, $z \in \mathbb{C}$

$\Longrightarrow$ Must obey nonlinear constraints



* D=2

Using OPE

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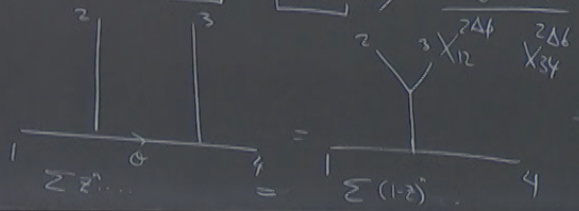
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crossing equation, conformal bootstrap

* D=2: $\Rightarrow$ Minimal Models, finite number of primaries. $\langle \phi(x_1) \cdots \mathcal{O}_n(x_n)\rangle$ are solutions of differential equations

* D=3: ∞ number of primaries $\Longrightarrow$ CFT's relevant for nature

- 3D Ising Model $\Longrightarrow \{\Delta_\sigma, C_{\sigma\sigma\varepsilon}\}$

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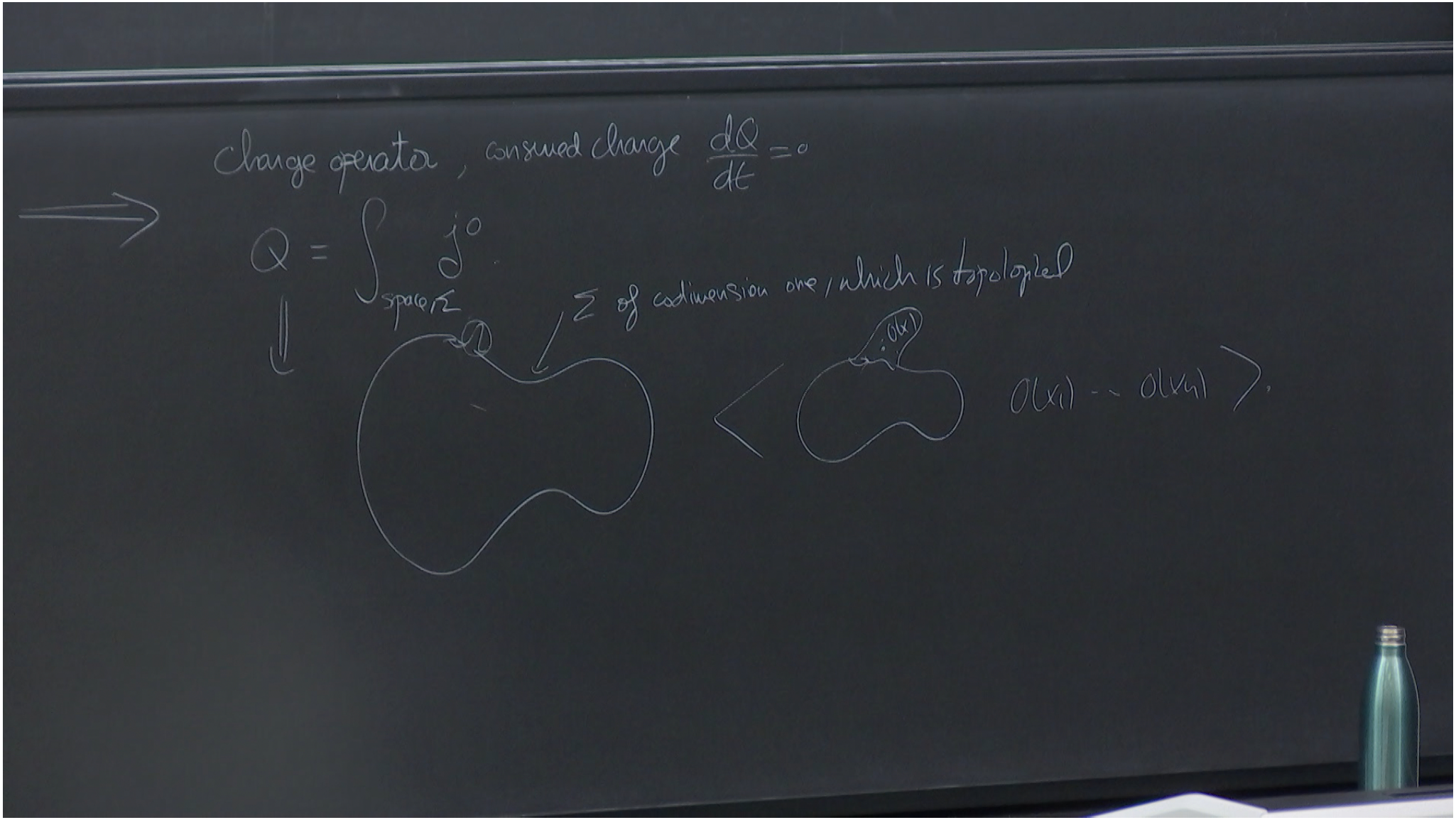
n pt. functions $n>4$ do not impose new constraints

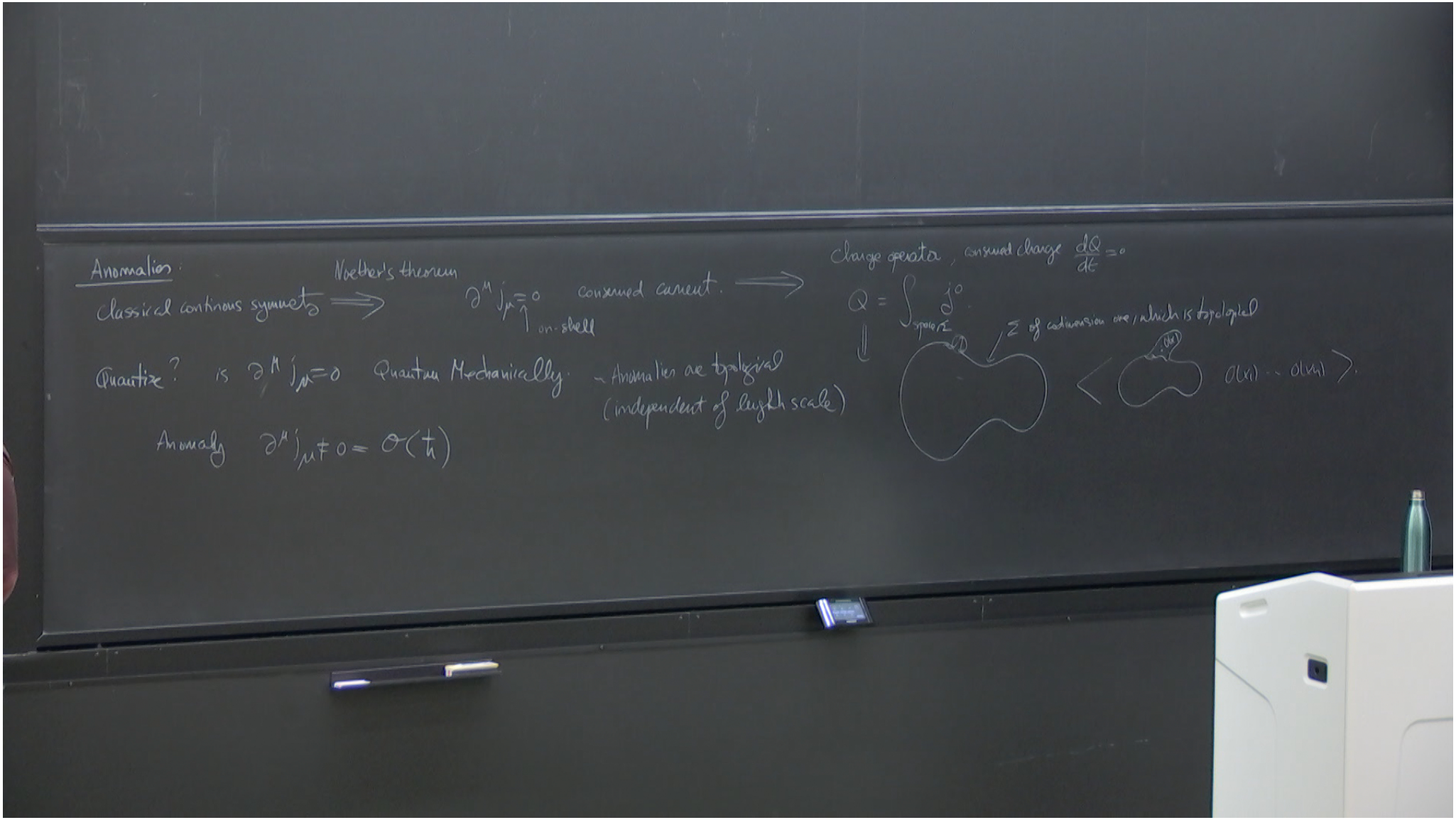
Anomalies:

Classical continous symmetry $\Longrightarrow$ (Noether's theorem) $\partial^\mu j_\mu = 0$ (on-shell) conserved current. $\Longrightarrow$ charg

Quantize? is $\partial^\mu j_\mu = 0$ Quantum Mechanically.

Anomaly $\partial^\mu j_\mu \neq 0 = \mathcal{O}(\hbar)$


Charge operator, conserved charge $\frac{dQ}{dt} = 0$
$Q = \int_{\text{space}/\Sigma} j^0$
Σ of codimension one, which is topological
$O(x_1)$
$\langle O(x_1) \cdots O(x_n) \rangle$


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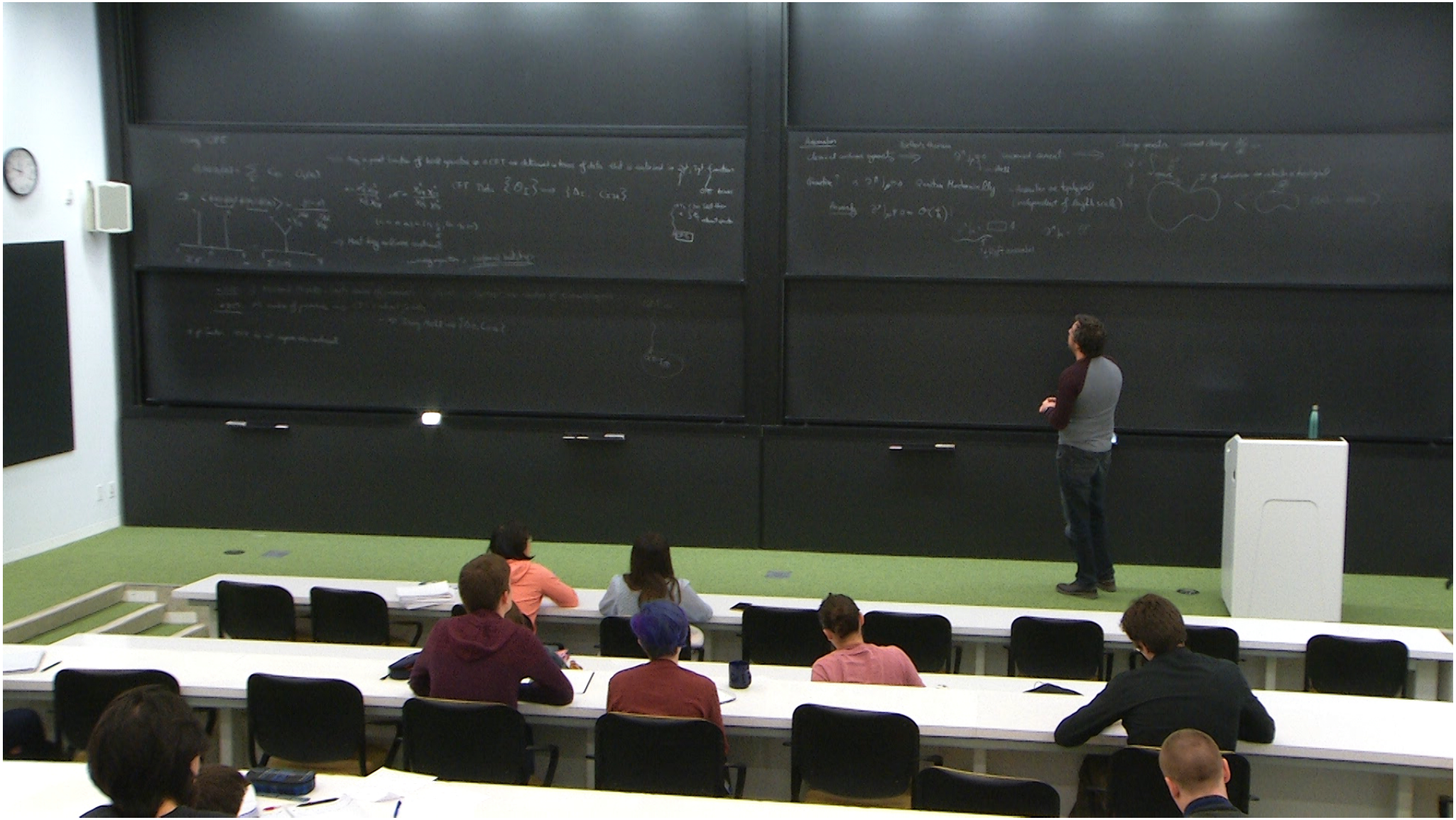
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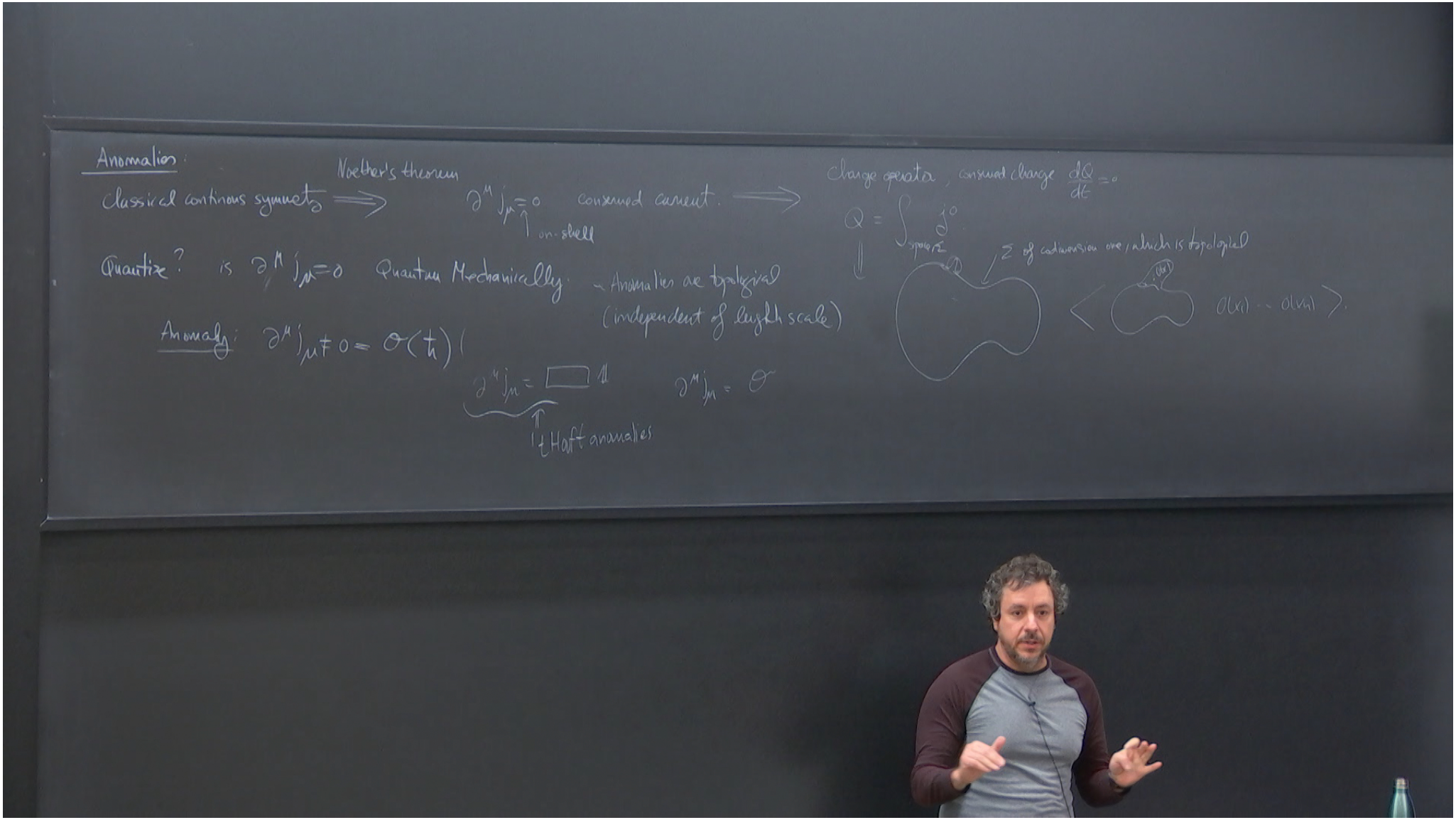
- 3D Ising Model $\Longrightarrow \{\Delta_z, C_{zzz}\}$

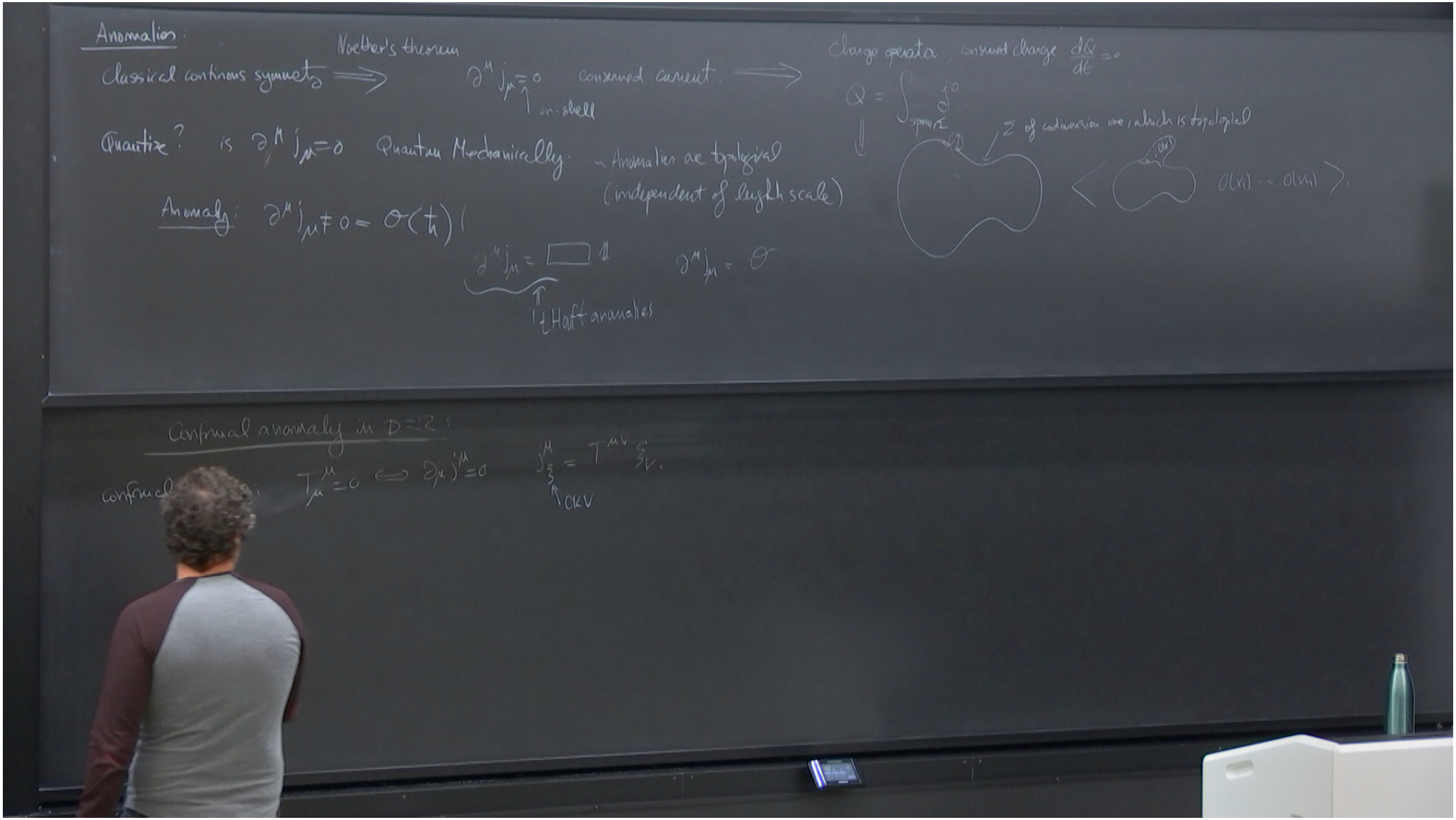
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CFT UV

QFT IR




Anomalies:
Noether's theorem
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⟨ O(x_1) ... O(x_n) ⟩
Conformal anomaly in D=2:
conformal: T^μ_μ = 0 ⟺ ∂_μ j^μ = 0
j^μ_ξ = T^μν ξ_ν
CKV

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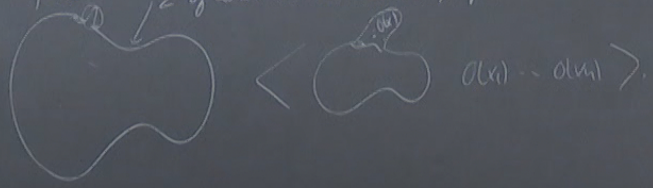
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$\partial^\mu j_\mu = \Box \mathbb{1}$ $\partial^\mu j_\mu = \mathcal{O}$

't Hooft anomalies



Conformal anomaly in D=2:

conformal invariance: $T^\mu_\mu = 0 \Longleftrightarrow \partial_\mu j^\mu_\xi = 0$ $j^\mu_\xi = T^{\mu\nu}\xi_\nu$

ξ: CKV

translational inv $\Longrightarrow$ $P^\mu T_{\mu\nu} = 0$

imposes 3 relations among 5 structures

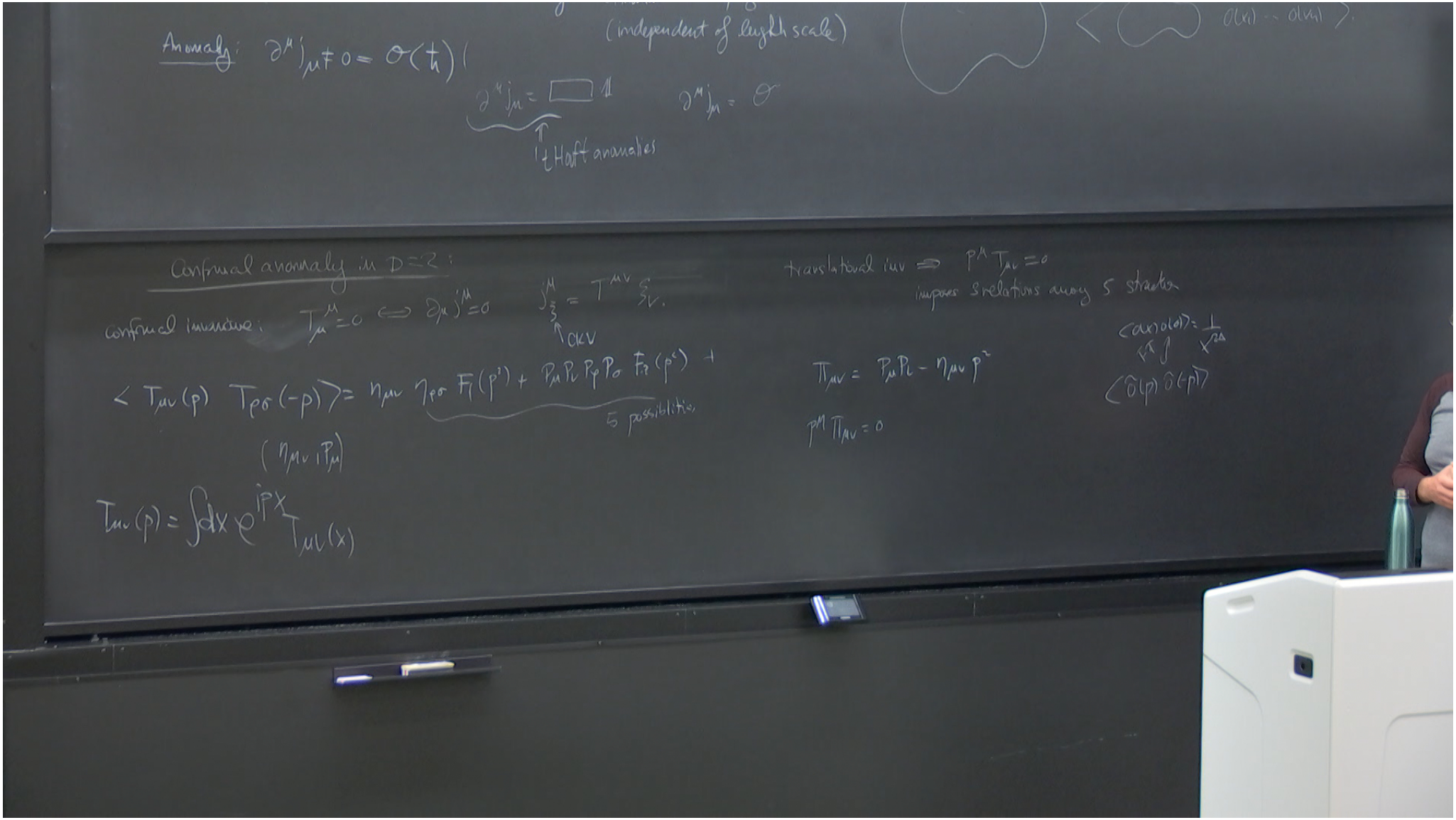
$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \eta_{\mu\nu}\eta_{\rho\sigma} F_1(p^2) + P_\mu P_\nu P_\rho P_\sigma F_2(p^2) + \cdots$

5 possibilities

$(\eta_{\mu\nu}, P_\mu)$

$\Pi_{\mu\nu} = P_\mu P_\nu - \eta_{\mu\nu} p^2$

$p^\mu \Pi_{\mu\nu} = 0$


Anomaly: $\partial^\mu j_\mu \neq 0 = \mathcal{O}(\hbar)$
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$\langle O(x_1) \cdots O(x_n) \rangle$
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5 possibilities
$(\eta_{\mu\nu}, P_\mu)$
$\Pi_{\mu\nu} = P_\mu P_\nu - \eta_{\mu\nu} p^2$
$P^\mu \Pi_{\mu\nu} = 0$
$\langle O(x) O(0)\rangle = \frac{1}{x^{2\Delta}}$
FT
$\langle \hat{O}(p)\, \hat{O}(-p)\rangle$
$T_{\mu\nu}(p) = \int dx\, e^{ipx} T_{\mu\nu}(x)$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \left[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right] F(p^2) \overset{0 \text{ in } D=2.}{\Big|}$$

$$+ \Pi_{\mu\nu}\Pi_{\rho\sigma}\, G(p^2)$$

$D=2$. Scale invariant theory. $\quad \Pi_{\mu\nu} = \hat{P}_\mu \tilde{P}_\nu \qquad \hat{P}_\mu = \epsilon_{\mu\nu} p^\nu$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = c\, \frac{\Pi_{\mu\nu}\Pi_{\rho\sigma}}{p^2} \qquad \delta^{(2)}(p+q)$$

$[T_{\mu\nu}(x)] = D.$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \Big[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \underbrace{\tfrac{2}{D-1}}_{\to 0 \text{ in } D=2}\,\Pi_{\mu\nu}\Pi_{\rho\sigma}\Big] F(p^2)$$

$$+\ \Pi_{\mu\nu}\Pi_{\rho\sigma}\, G(p^2)$$

$$\Pi_{\mu\nu} = P_\mu P_\nu - \eta_{\mu\nu} P^2$$

$$\eta^{\mu\nu}\Pi_{\mu\nu} \propto p^2$$

D=2. Scale invariant theory. $\quad \Pi_{\mu\nu} = \widehat{P}_\mu \widetilde{P}_\nu \qquad \widehat{P}_\mu = \epsilon_{\mu\nu} P^\nu$

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$$[T_{\mu\nu}(x)] = D.$$

$$\langle T^\mu_\mu(p)\, T_{\rho\sigma}(-p)\rangle \cdot \alpha\ c\,\Pi_{\rho\sigma} \Longleftarrow \text{anomaly}$$

$$\boxed{\langle T^\mu_\mu(x)\, T_{\rho\sigma}(0)\rangle \propto c\left(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2\right)\delta^2(x)}$$

$T^\mu_\mu \neq 0$ in correlation functions conformal. anomaly

$$\langle T^{\mu}_{\mu}(0)\dots\rangle_{\eta\to\eta+h} = \int dx\, \langle T^{\mu}_{\mu}(0)\, T_{\rho\sigma}(x)\rangle\, h^{\rho\sigma}(x) = c\,(\partial_\rho\partial_\sigma - \eta_{\rho\sigma}\partial^2)\, h^{\rho\sigma}(x)\,|_{x=0}$$

integrate by parts

$$e^{\int h^{\mu\nu} T_{\mu\nu}} \qquad (\partial\partial - \eta\,\partial^2)\,\delta(x)$$

$$T^{\mu}_{\mu} = \beta(g)\, F^2_{\mu\nu}$$

$\langle T^{\mu}_{\mu}(0) \dots \rangle_{\eta \to \eta + h}$ = $\int dx \, \langle T^{\mu}_{\mu}(0) \, T_{\rho\sigma}(x) \rangle \, h^{\rho\sigma}(x)$ = $c \, (\partial_\rho \partial_\sigma - \eta_{\rho\sigma} \partial^2) \, h^{\rho\sigma}(x) \, \mathbb{1}$

$e^{S h^{\mu\nu} T_{\mu\nu}}$

$\frac{1}{(\partial^2 - \eta \, \partial^2) \delta^2(x)}$

↑ integrate by parts

$\parallel$ Ricci scalar linearized

what is c?

c is known as the conformal anomaly

$T^{\mu}_{\mu} = \beta(g) F^2_{\mu\nu}$

$T^{\mu}_{\mu} \propto c \, R$

T Ricci scalar