Title: Lattice homotopy: the physics of moving things around and calling them the same

Speakers: Hoi Chun Po

Series: Condensed Matter

Date: February 25, 2020 - 3:30 PM

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Abstract: While the concept of topology is often introduced by contrasting oranges with bagels, the idea of topologically distinct quantum phases of matter is far more abstract. In this talk, we will focus on a more tangible form of topology that also arises in quantum condensed matter system: when the sites in a lattice are dragged around in a symmetric manner, what attributes of the lattice would remain unchanged? Such deformation can be viewed as the lattice analog of the familiar (mental) exercise of transforming a bagel into a coffee mug. We will first introduce the mathematical framework for identifying the topological classes of lattices, and then discuss how these invariants can constrain and inform the more abstract notion of topological phases of matter that emerge.

Lattice Homotopy:

moving things around & calling them the same

Adrian (Hoi Chun) Po
MIT

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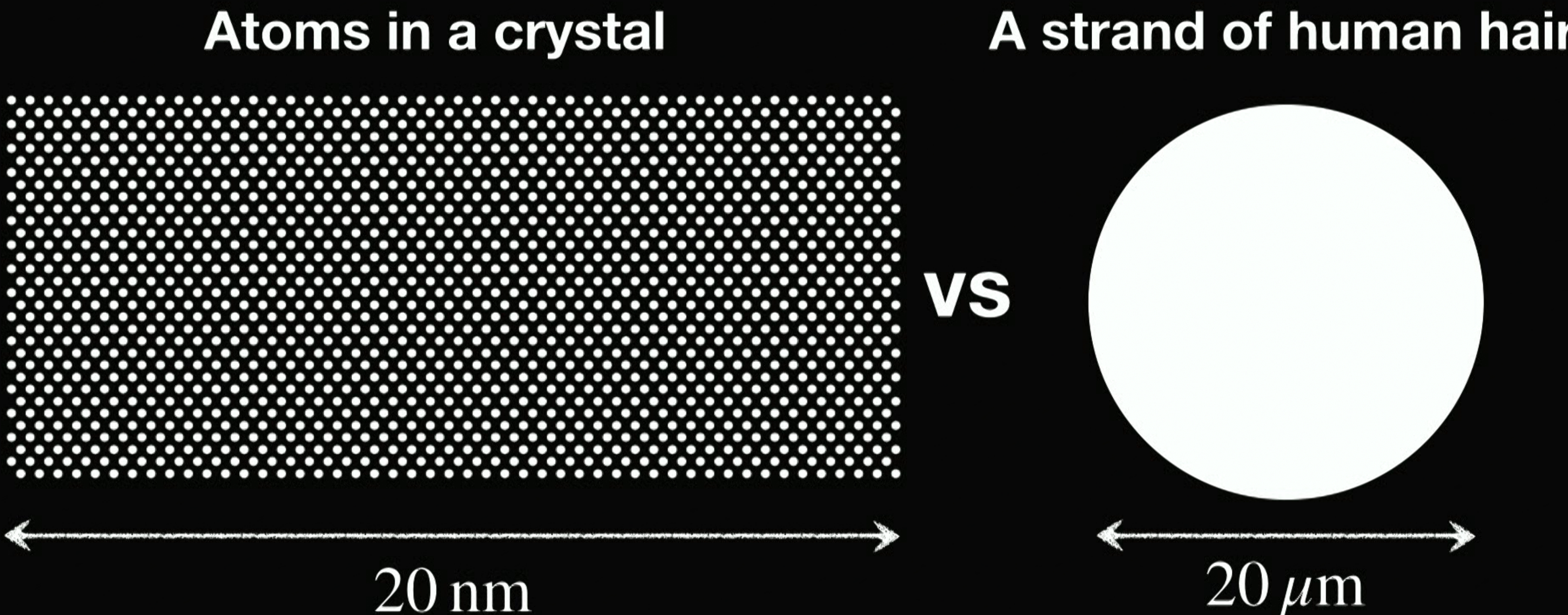
Perimeter Institute @Perimeter · Feb 14

On #ValentinesDay, how about some love for the fascinating fields of physics? We created dating profiles for a bunch of them.
hubs.ly/H0n0QQn0 #Physics #STEM #Love

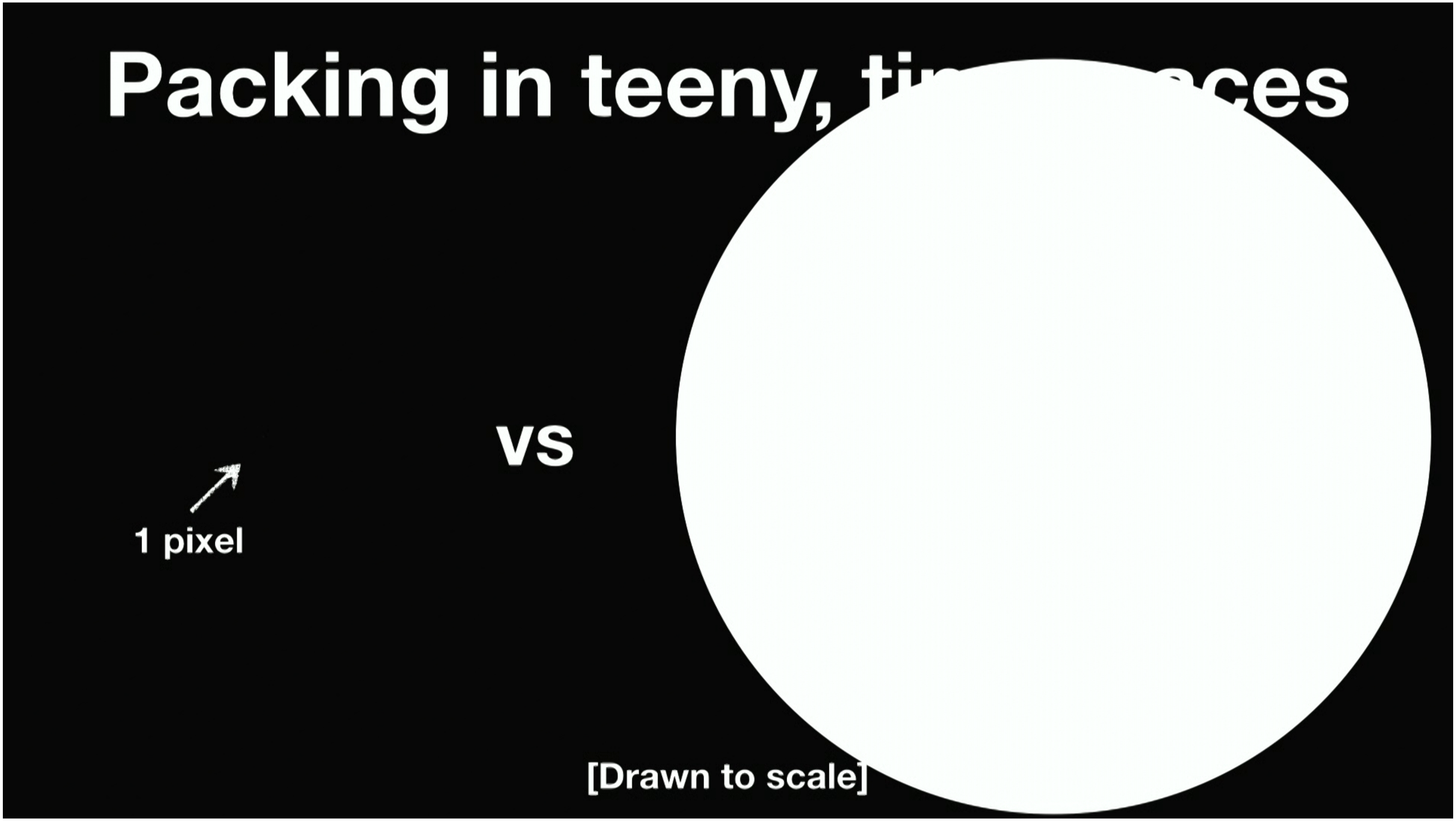
5 12

Confession: this slide inspired by Liujun's talk last week :)

Packing in teeny, tiny spaces



[Not drawn to scale]


Packing in teeny,
vs
1 pixel
[Drawn to scale]

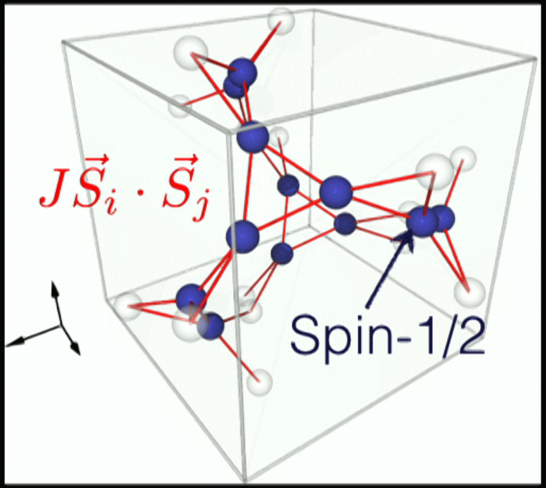
microscopic

Packing lots of stuff into teeny, tiny spaces,

just to see what happens.

Macroscopic

THE problem of quantum many-body problems

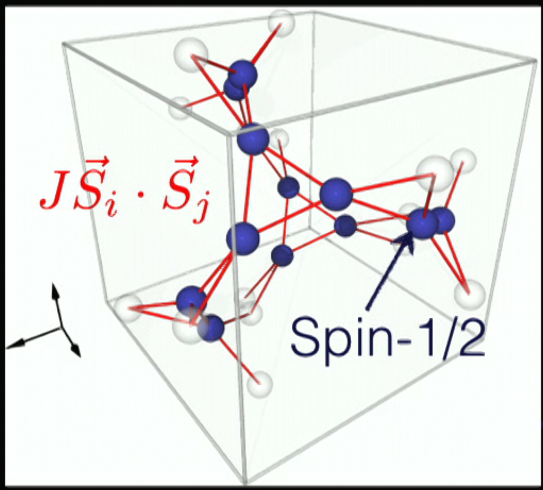


Consider the anti-ferromagnetic Heisenberg model on the hyperkagome lattice.

Find its ground state(s). [5 pts]

Simple to state, (almost) *impossible* to answer

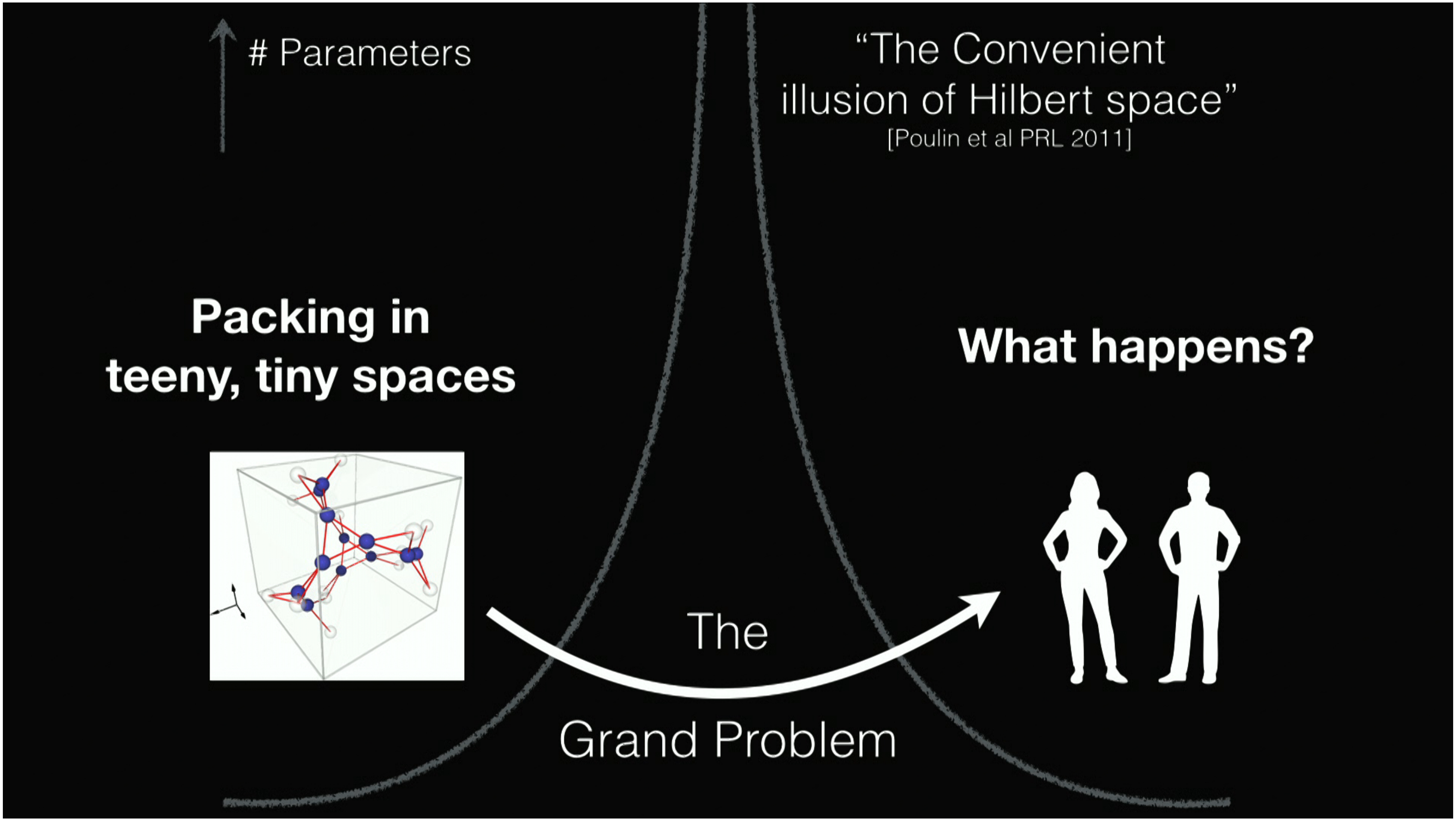
Why simple?



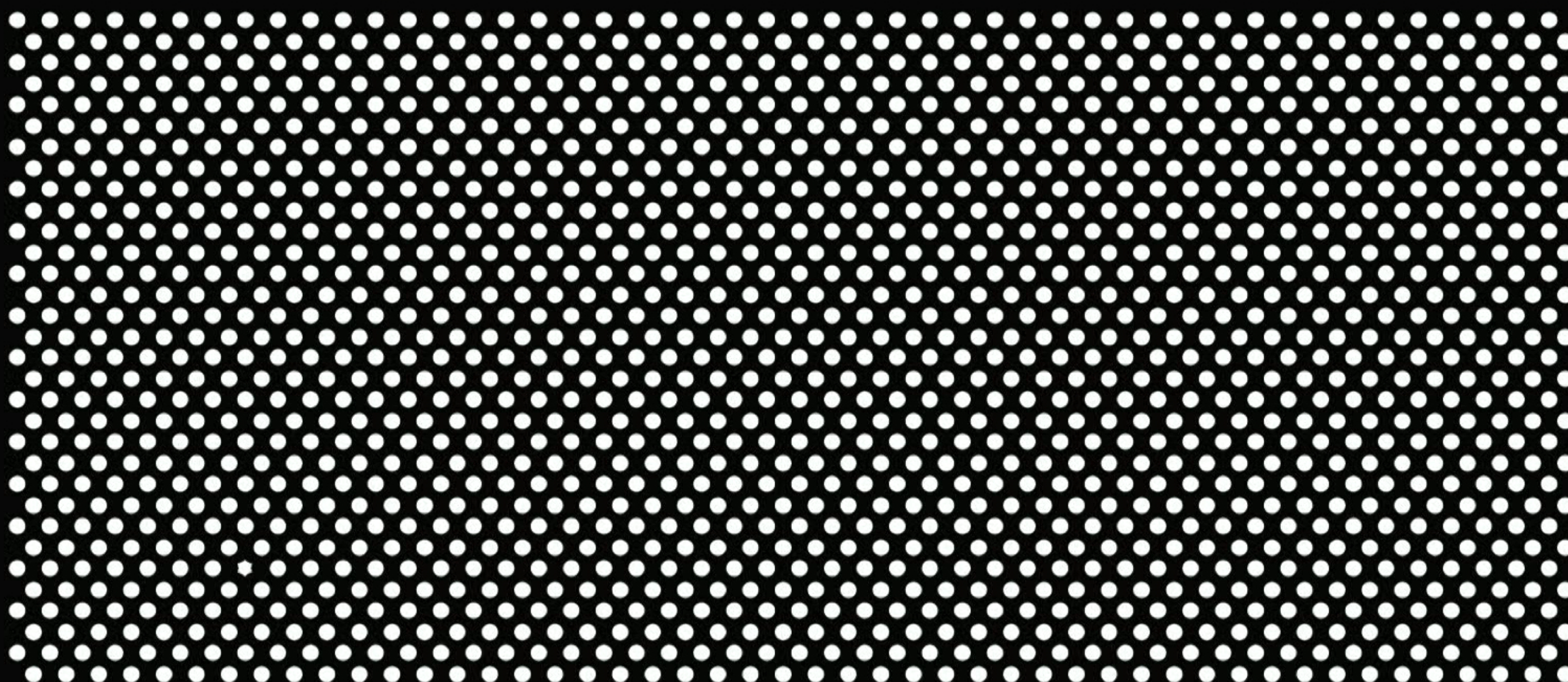
- O(1) number of parameters
  - What degrees of freedom
  - How they are arranged
  - How they are coupled

Why impossible?

- V spin-1/2 $\Rightarrow$ dimension = 2^V
- 1μm sample in 2D $\Rightarrow$ $2^{10^8} \simeq 2^{10^8}$ GB
- Total data "stored" in our world $\lesssim 2^{40}$ GB


Parameters
"The Convenient illusion of Hilbert space"
[Poulin et al PRL 2011]
Packing in teeny, tiny spaces
What happens?
The
Grand Problem

Packing in teeny, tiny spaces:
What information is important?



CONDENSED MATTER

I am complex, but don't let that scare you off. Sometimes that means I'm scattered, but under great pressure, I can do amazing things. Exotic and unusual, I'm looking for someone who can help harness my untapped potential. **maybe Psychology!**

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Sensorimotor play (Piaget)

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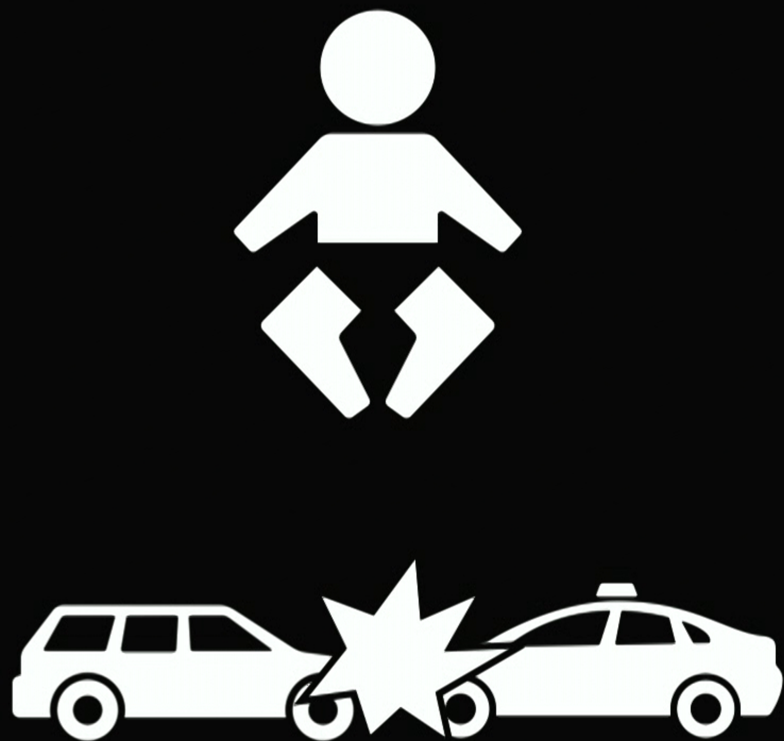
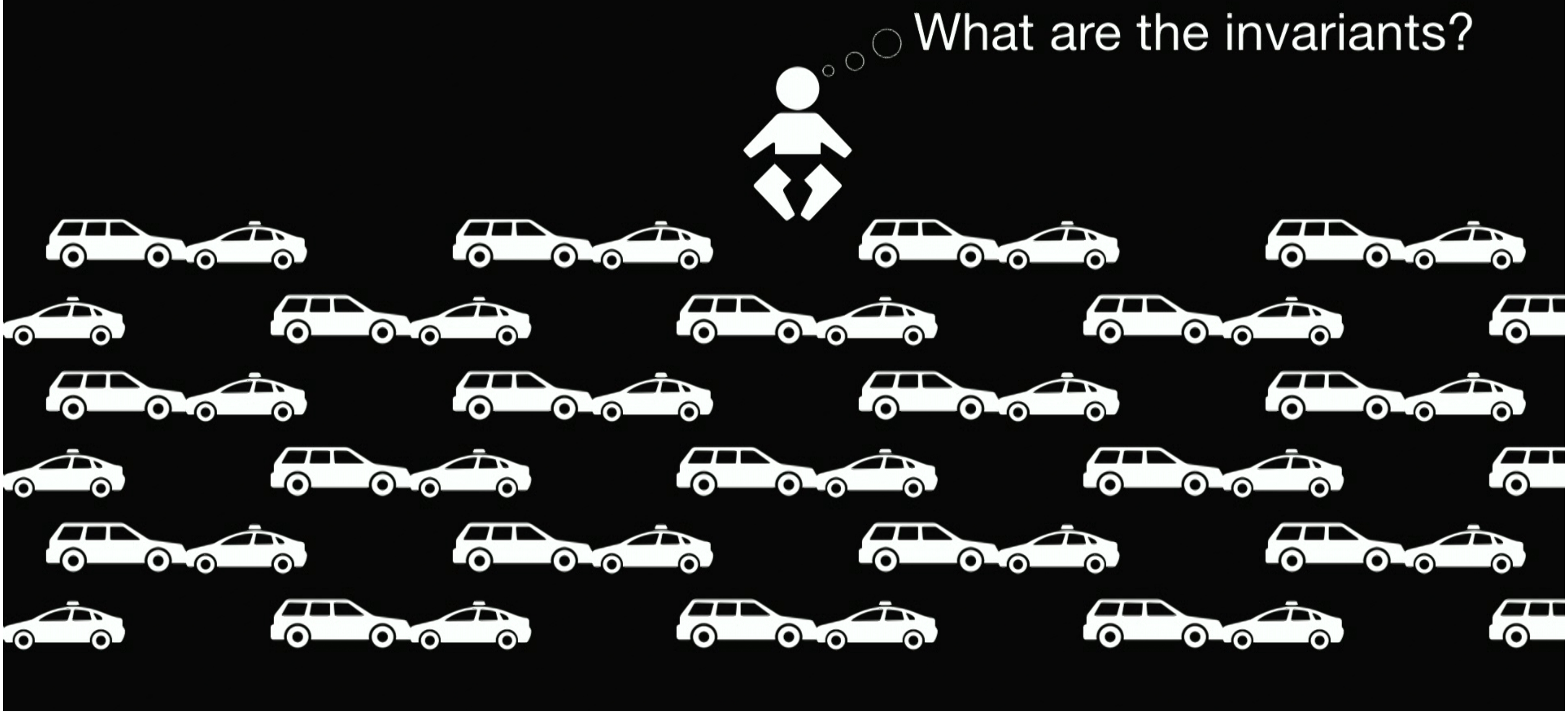


Back to condensed matter…



Overarching theme

How does the conventional notion of topology interact with our understanding on quantum ground states?

Lattice homotopy

What are the robust (topological) data in a lattice under smooth, symmetric deformation?

What are their implications on quantum phases of matter?

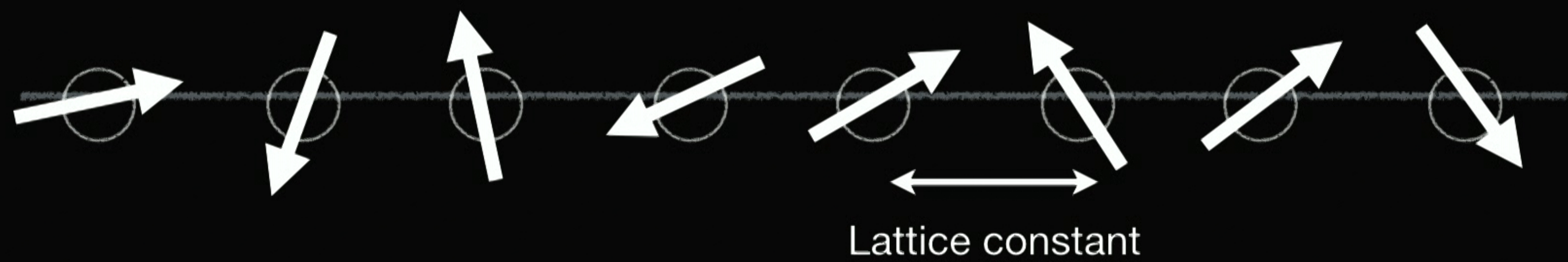
Outline aka take-homes

- Lieb-Schultz-Mattis theorem: no-go for “boring” phases
  - A motivating example of lattice constraints
- Lattice homotopy: general framework & quantum magnets
  - Generalization of Lieb-Schultz-Mattis to utilize the full space group
- Application to atomic insulators
  - Uncovering topologically nontrivial quantum materials

Lieb-Schultz-Mattis (LSM)

[1D result: Ann. Phys. 1961]
[Higher dimensions: Affleck PRB 1988; Oshikawa PRL 2000; Hastings, PRB 2004]

- Setup: 1D spin chain w/ translation & spin-rotation sym.



- Microscopic: a spin-1/2 per unit cell
- Macroscopic: ground state must be
  - (i) Symmetry breaking; or
  - (ii) Gapless

Gapped, unique ground state

Lieb-Schultz-Mattis (LSM)

[1D result: Ann. Phys. 1961]

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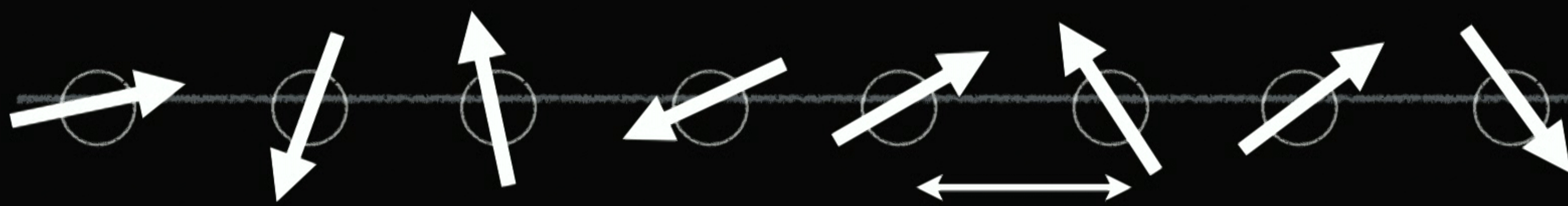

Lattice constant

- Microscopic: a spin-1/2 per unit cell
- Macroscopic: ground state must be
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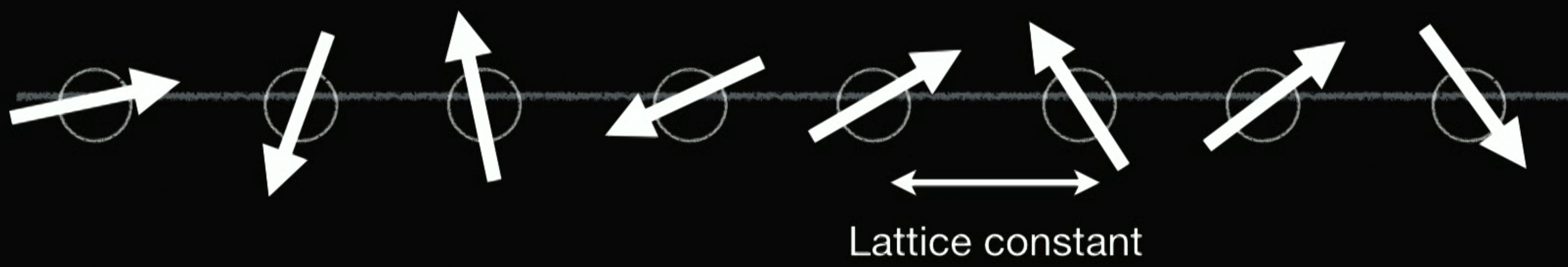
  - (ii) Gapless

Gapped, unique ground state

NON Lieb-Schultz-Mattis

- Setup: 1D spin chain w/ translation & spin-rotation sym.

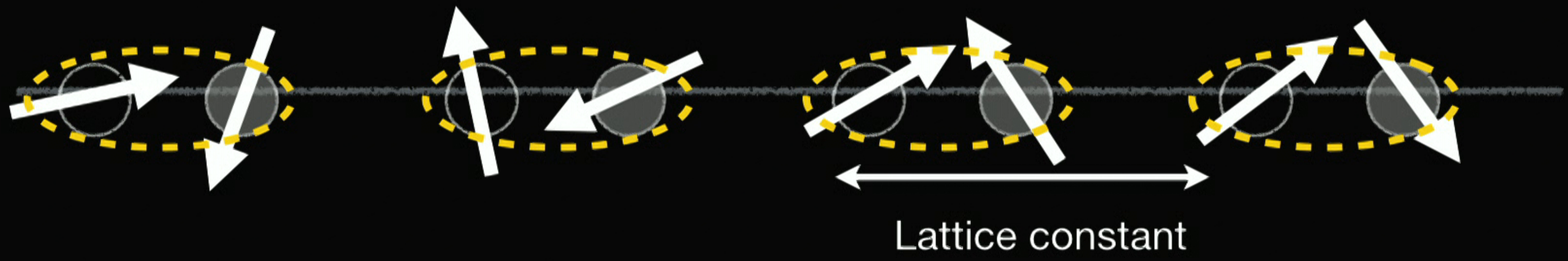


- Microscopic: a spin-1 per unit cell
- Macroscopic: ground state can be gapped and symmetric
  - Haldane/ AKLT chain, an early example of a symmetry-protected topological order

[Haldane PRL 1983, Affleck et al PRL 1987]

NON Lieb-Schultz-Mattis II

- Setup: 1D spin chain w/ translation & spin-rotation sym.

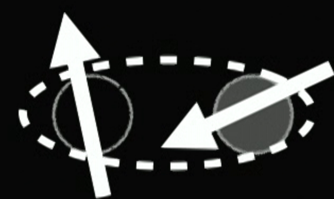


- Microscopic: two spin-1/2's per unit cell
- Macroscopic: ground state can be gapped and symmetric
  - Independent singlets

Heuristic explanation

- Consider a single unit cell

vs



Spin-1/2
two degenerate states

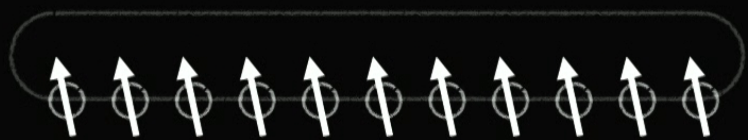
Two spin-1/2's

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$$

Non-degenerate w/ singlet

Heuristic explanation

- Translation sym.: extend to a ring of odd length L



vs

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \cdots \otimes \frac{1}{2} = \cdots \oplus \frac{L-2}{2} \oplus \frac{L}{2}$$

$$0 \otimes 0 \otimes \cdots \otimes 0 = 0$$

Always half-integer:
any state is degenerate

Non-degenerate
ground state is possible

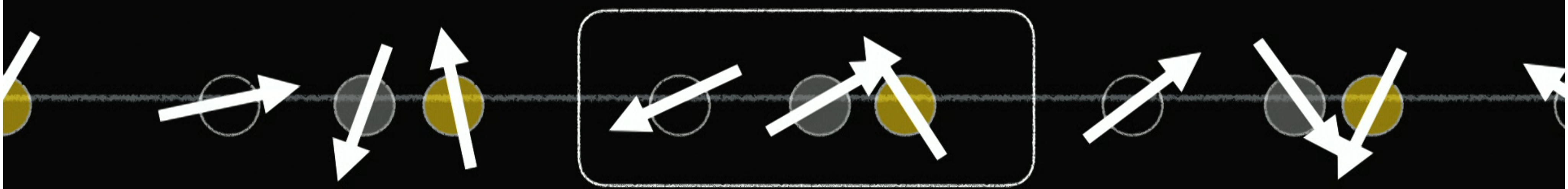
Lattice homotopy: a primer

Micro. data:

(i) Lattice translation and spin-rotation symmetries

(ii) Half-integer vs integer spin per unit cell

E.g. Three spin-1/2's in a unit cell



Lattice homotopy: a primer

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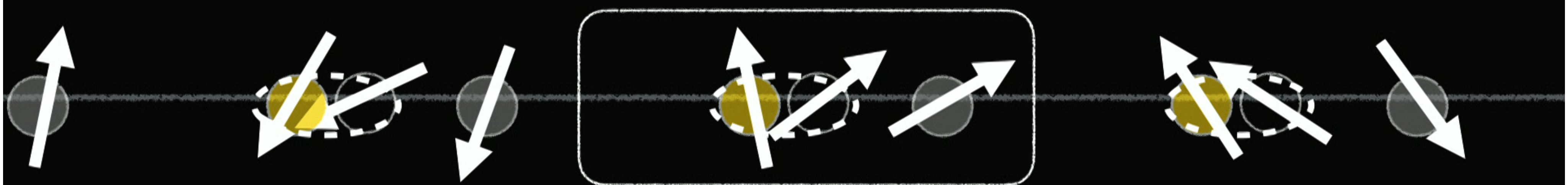


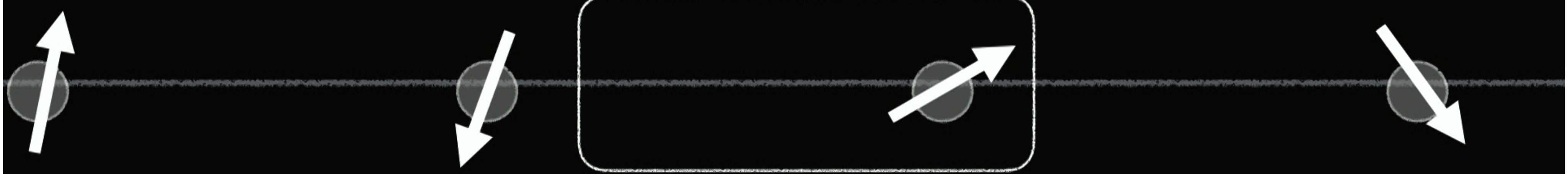
Lattice homotopy: a primer

Topological micro. data:

(i) Lattice translation and spin-rotation symmetries

(ii) Half-integer vs integer spin per unit cell

E.g. Half-integer spin in a unit cell



Is this the only invariant?

CONDENSED MATTER

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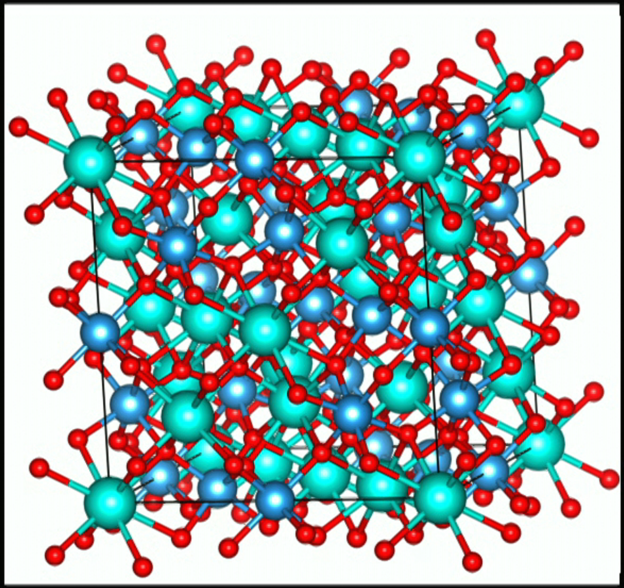
Materials can be complex…



$Yb_2Ti_2O_7$: spin liquid candidate
[Ross et al PRX 2011]

[crystal data from Springer Materials & Materials Project; visualization w/ VESTA]

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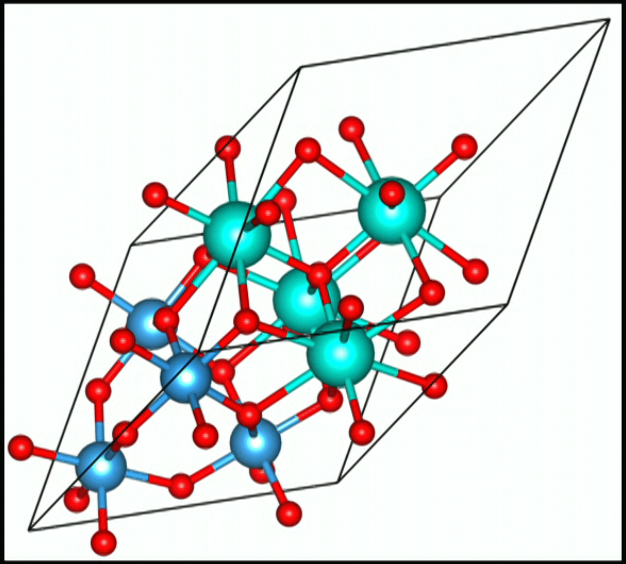


4 Kramers doublet per unit cell

$Yb_2Ti_2O_7$: spin liquid candidate
[Ross et al PRX 2011]

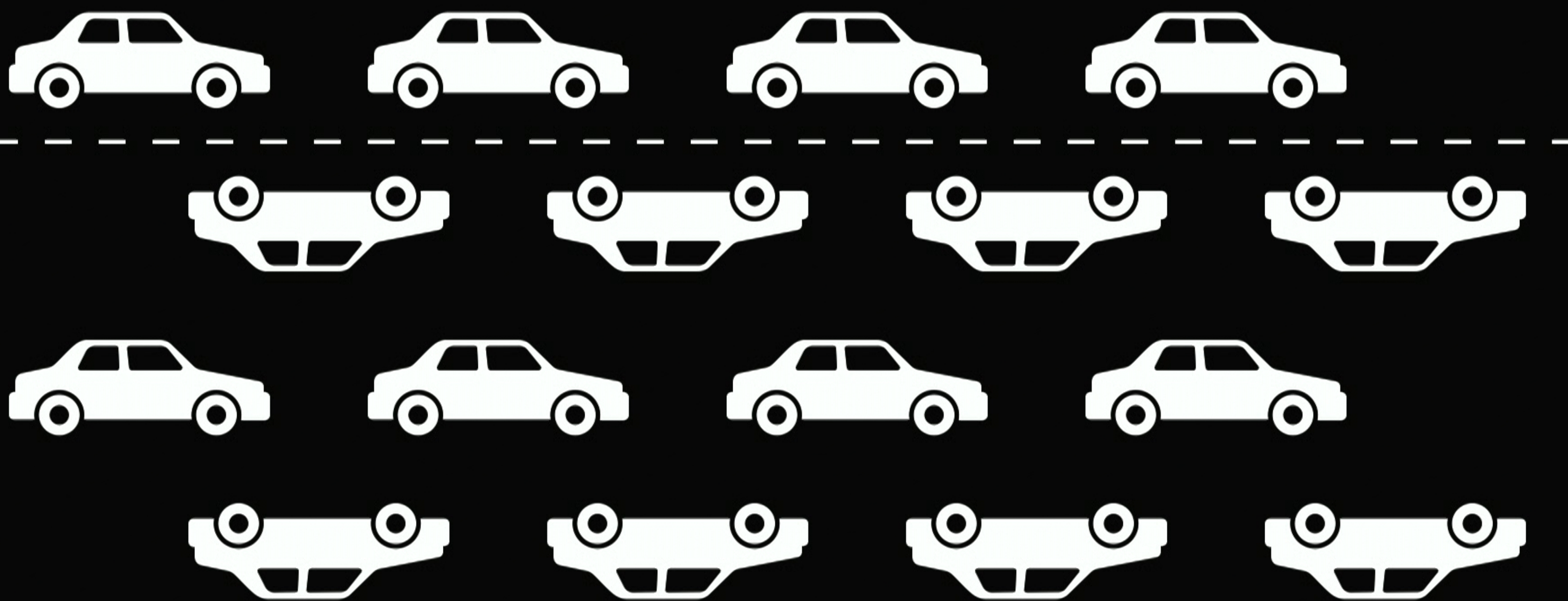
[crystal data from Springer Materials & Materials Project; visualization w/ VESTA]

Complex, but orderly:

make better use of spatial symmetries

Nonsymmorphic symmetry

E.g.: Glide = reflect & half-translation



Nonsymmorphic symmetry

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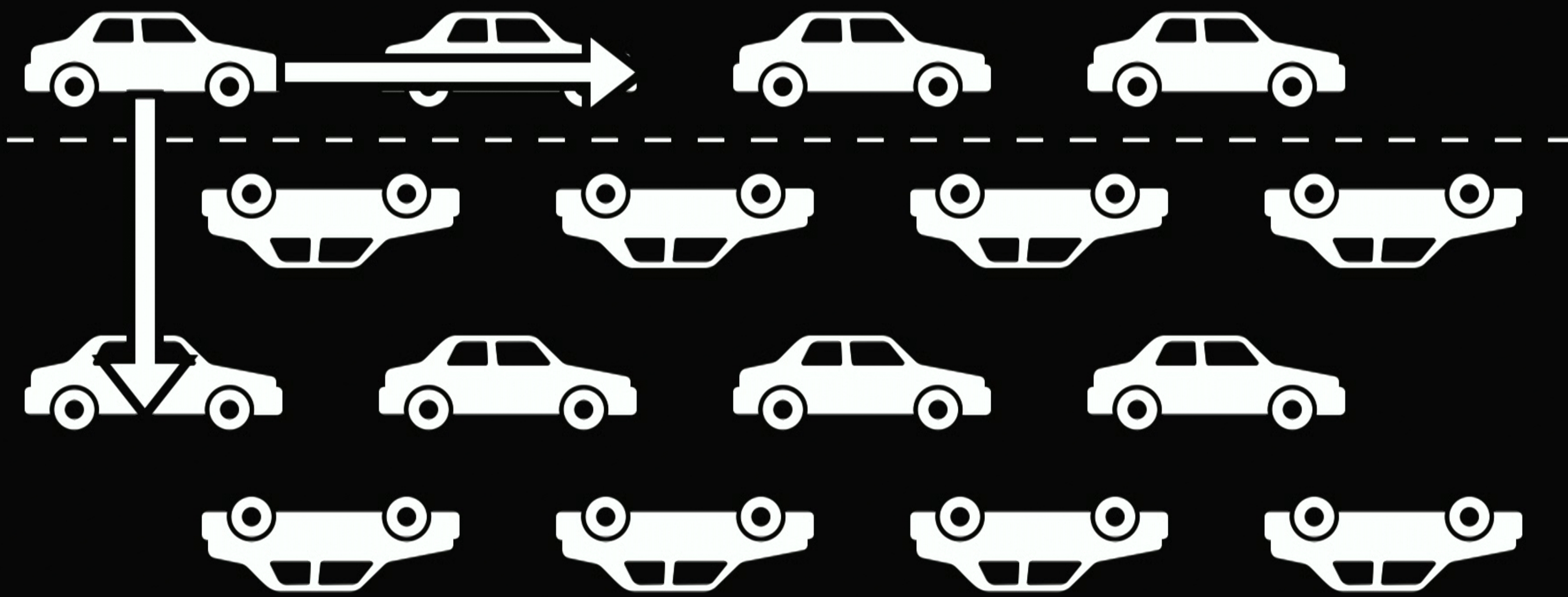

Nonsymmorphic symmetry

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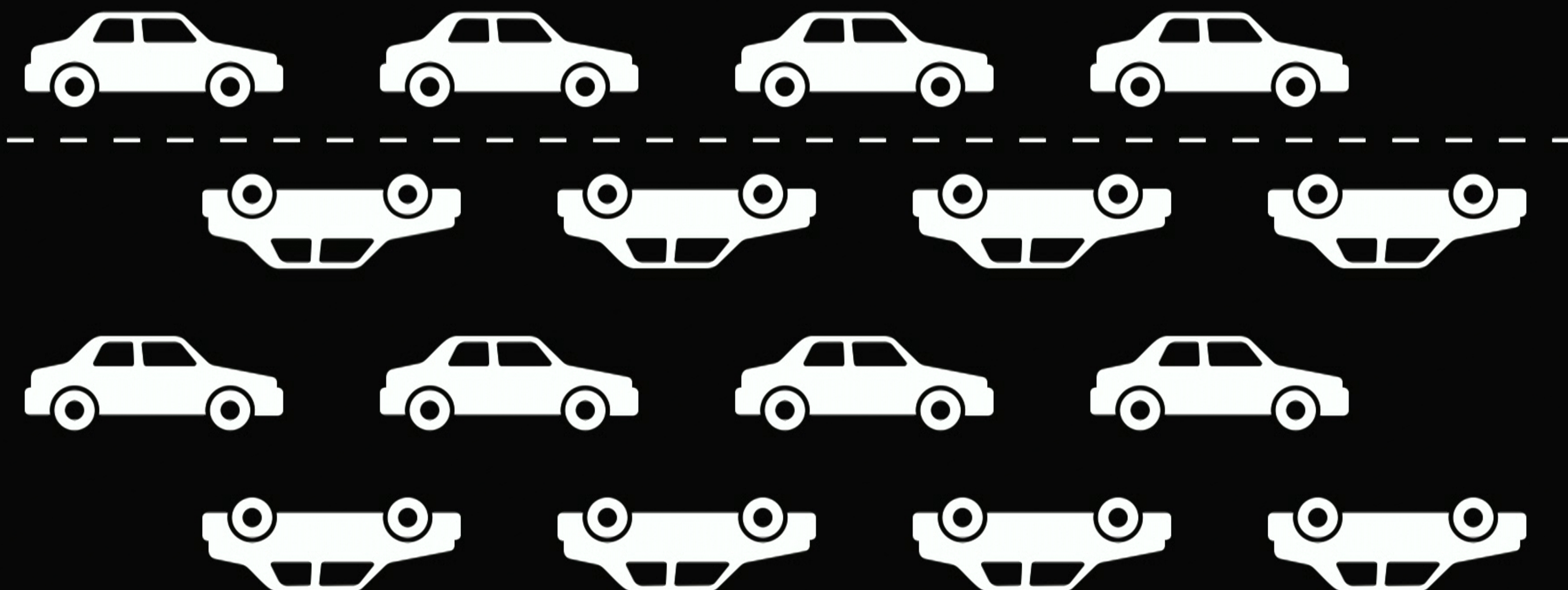
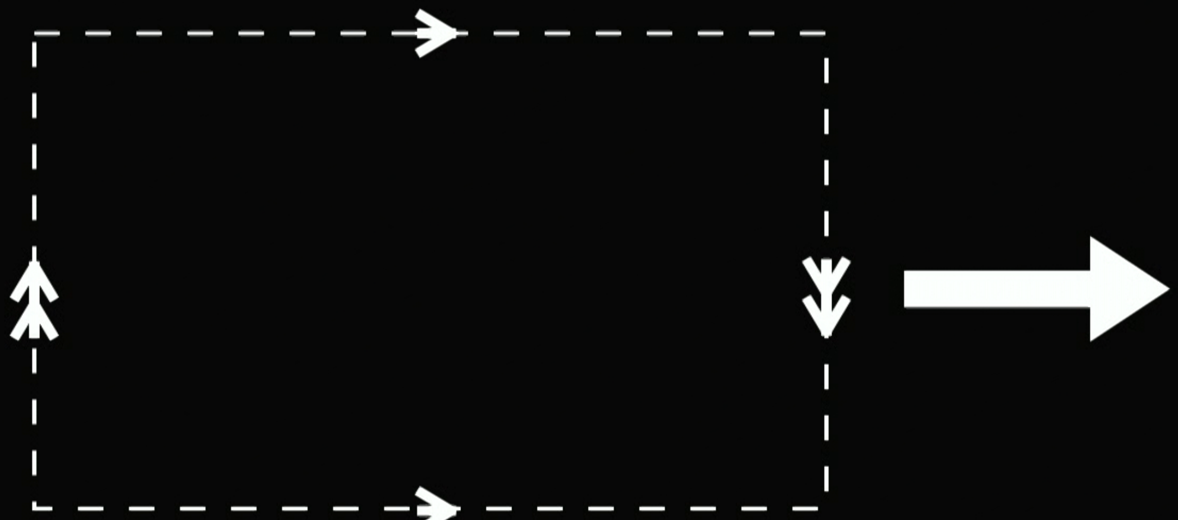
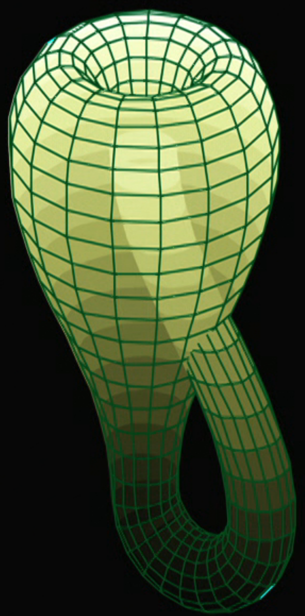


Nonsymmorphic LSMs

E.g.: Glide = reflect & half-translation





Klein bottle
[fig. from wiki]

Nonsymmorphic LSMs

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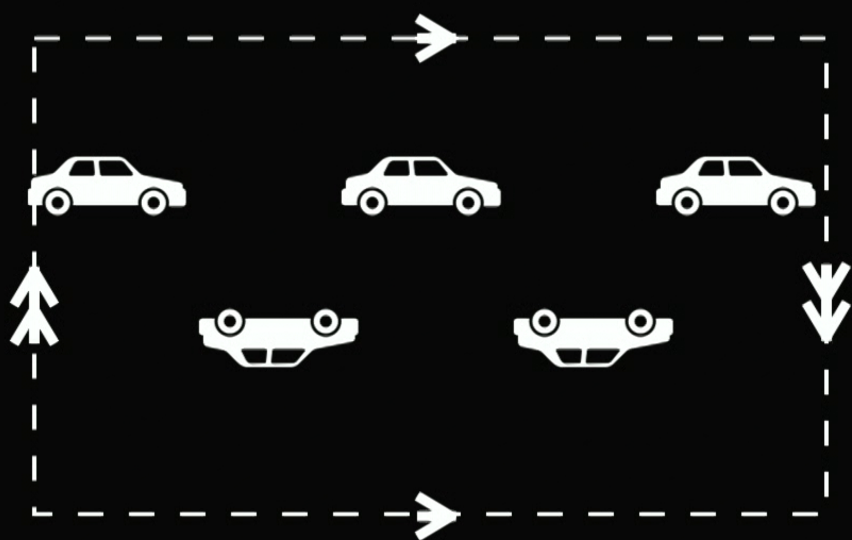
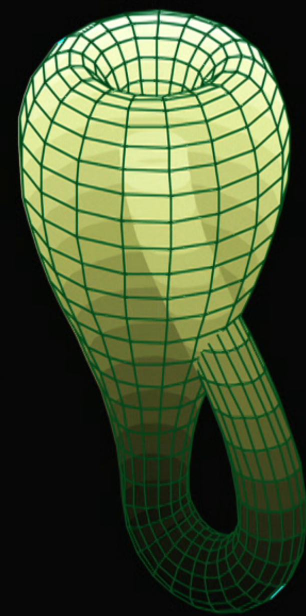


5 cars = $2\frac{1}{2}$ unit cells!



Klein bottle

[fig. from wiki]

Nonsymmorphic LSMs

- Tightened constraints by putting any 3D crystal onto one of the 10 "Bieberbach manifolds"

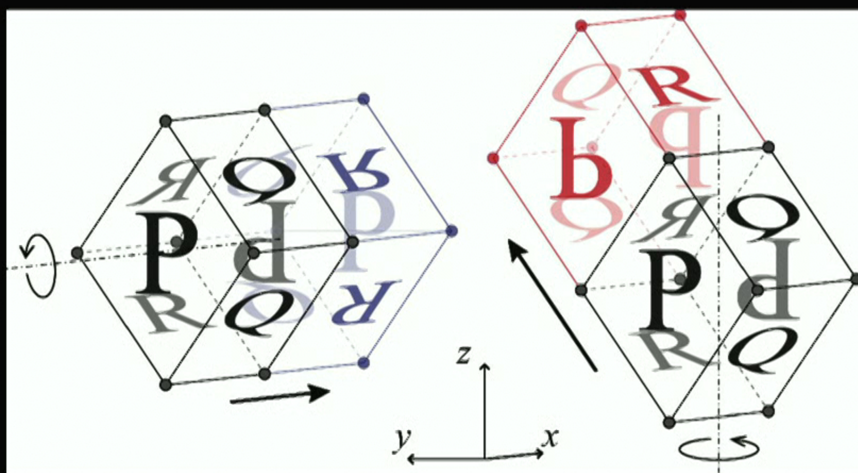


Table 1. Summary of ν_{min} for elementary space groups

ITC no.	Key elements	Minimal filling: Al*	Ent†	Bbb‡	Manifold name
1	(Translation)	2	2	2	Torus
4	2_1	4	4	4	Dicosm
144/145	$3_1/3_2$	6	6	6	Tricosm
76/78	$4_1/4_3$	8	8	8	Tetracosm
77	4_2	4	4	4	
80	4_1	4	4	4	
169/170	$6_1/6_5$	12	12	12	Hexacosm
171/172	$6_2/6_4$	6	6	6	
173	6_3	4	4	4	
19	2_1, 2_1	8	4	8	Didicosm
24	2_1, 2_1	4	2	4	
7	Glide	4	4	4	First amphicosm
9	Glide	4	4	4	Second amphicosm
29	Glide, 2_1	8	4	8	First amphidicosm
33	Glide, 2_1	8	4	8	Second amphidicosm

*The minimal filling required to form a symmetric atomic insulator.
†ν_{min} obtained in *Extension to 3D Symmorphic and Nonsymmorphic Crystals*. Bounds are not tight for nos. 19, 24, 29, and 33.
‡ν_{min} obtained in *Alternative Method: Putting Sym-SRE Insulators on Bieberbach Manifolds*. All bounds are tight.

[Watanabe, HCP, Vishwanath, Zaletel PNAS 2015]
[Also earlier: Parameswaran Nat Phys 2013, Roy 1212.2944]

LSM summary

Micro. topological data:

(i) Half-integer spin per “fundamental domain”

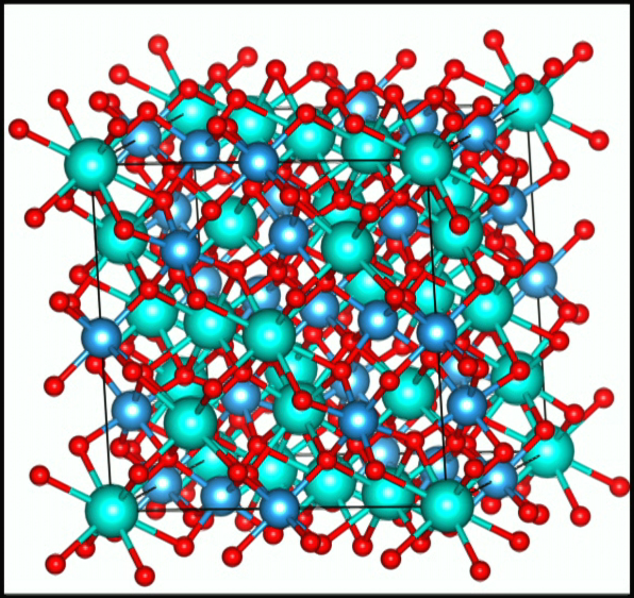
Macro. manifestation: quantum ground state must be

(i) Symmetry-breaking; or

(ii) Topologically ordered (e.g. gapped spin liquids); or

(iii) Gapless

Downside...



Not covered by this generalization

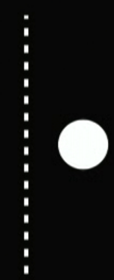
Outline aka take-homes

✓ Lieb-Schultz-Mattis theorem: no-go for "boring" phases

- A motivating example of lattice constraints

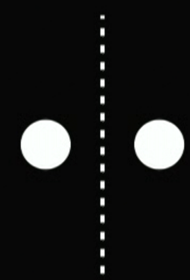
- Lattice homotopy: general framework & quantum magnets
  - Space-group generalization of Lieb-Schultz-Mattis
- Application to atomic insulators
  - Uncovering topologically nontrivial quantum materials

Some more crystalline sym.



Mirror

Some more crystalline sym.



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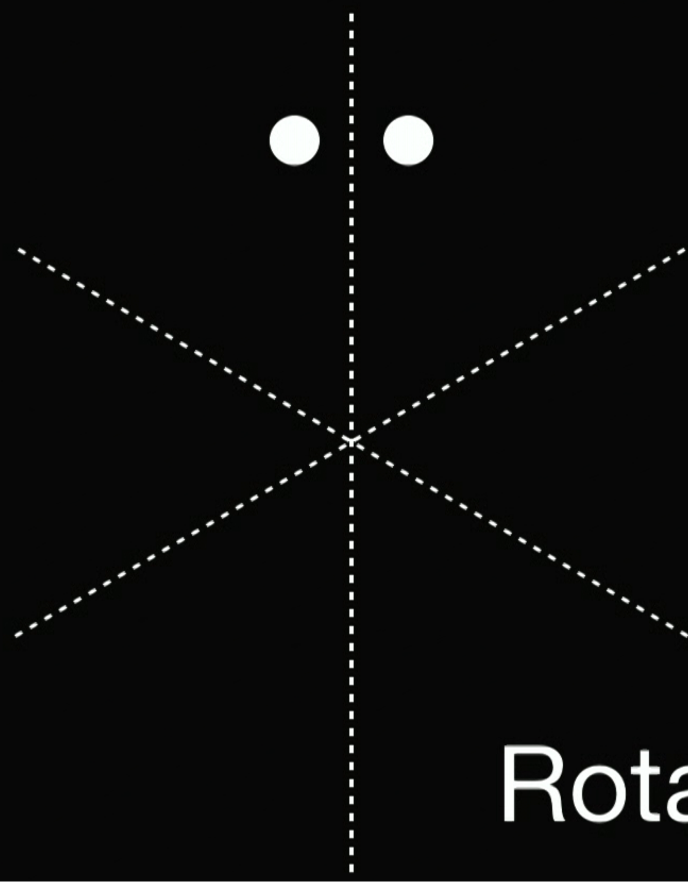
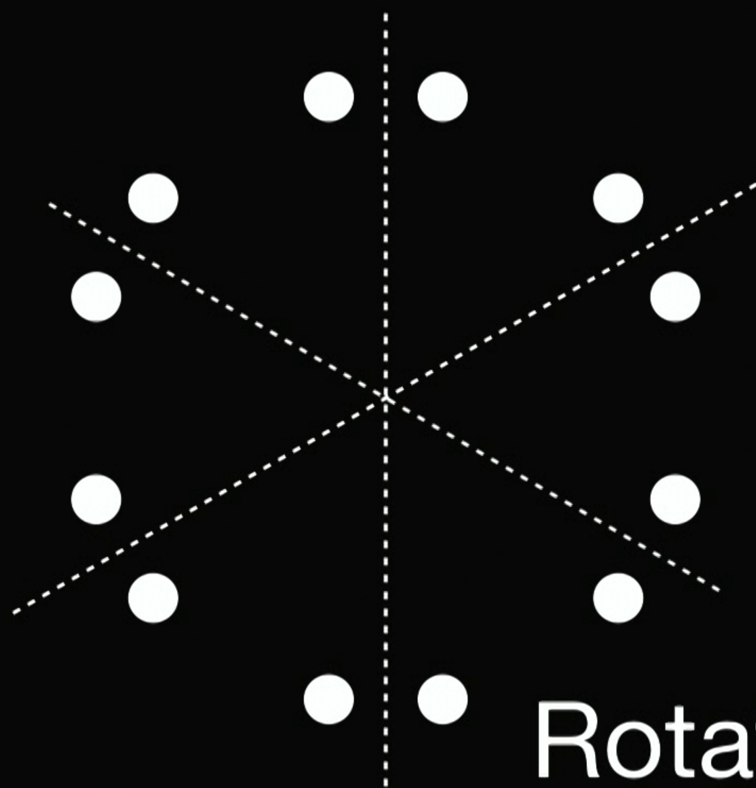


Rotation

Some more crystalline sym.



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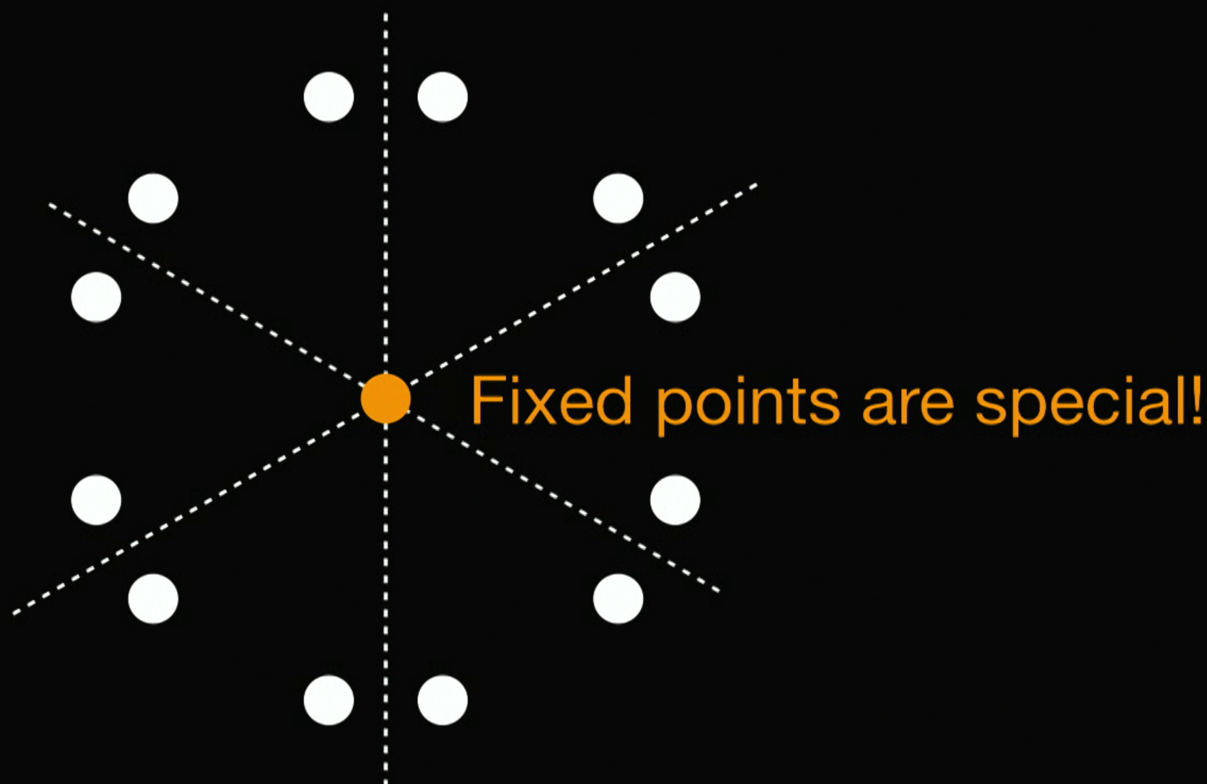
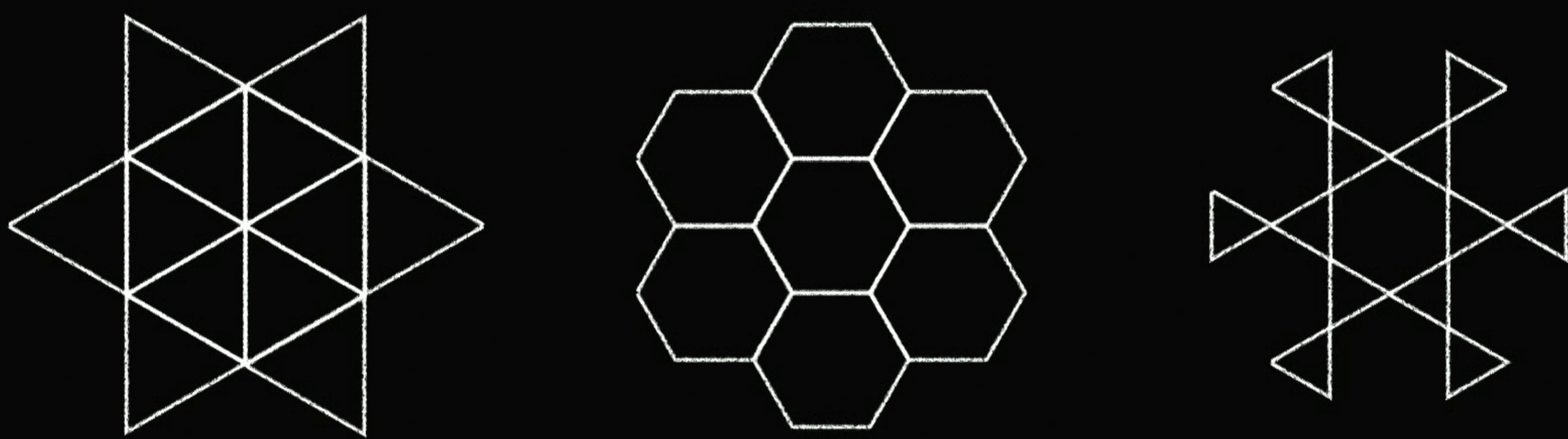


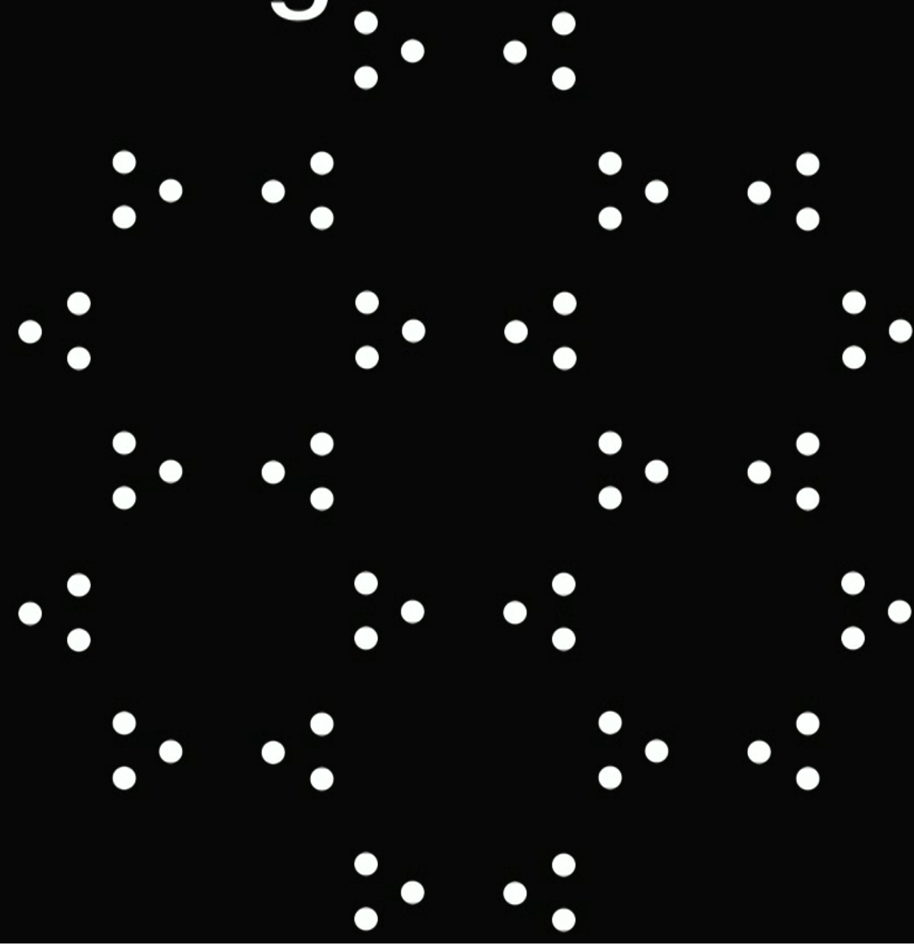
Lattices of fixed points

Same symmetries: C_6 rotation, mirror, and lattice translations



Triangular   Honeycomb   Kagome

Moving sites around



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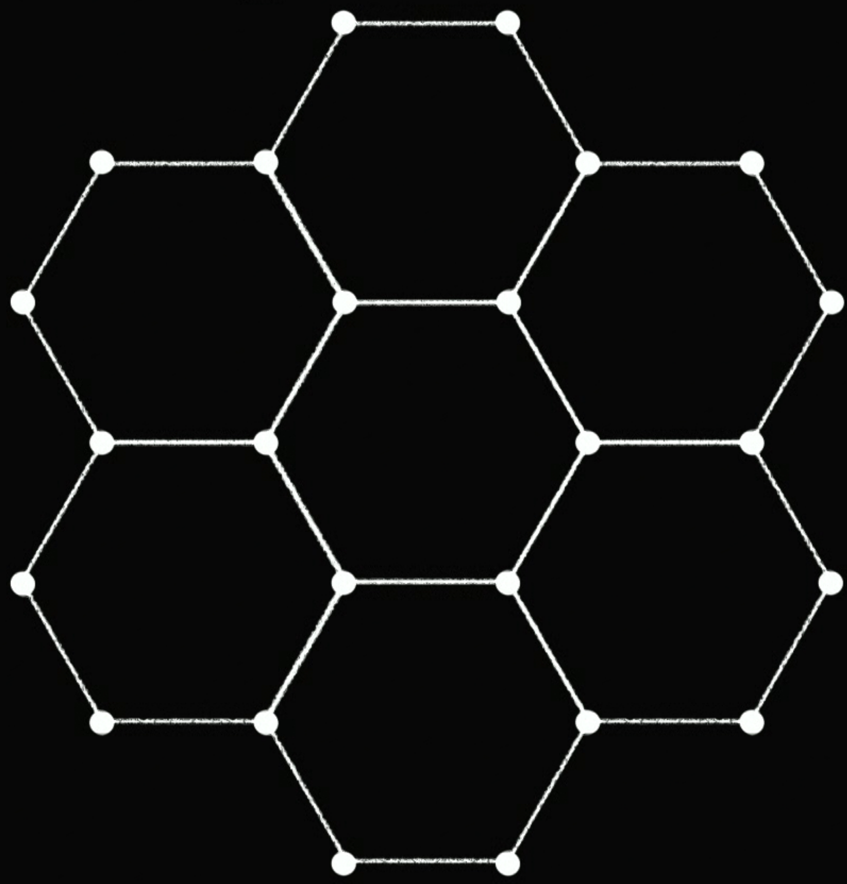

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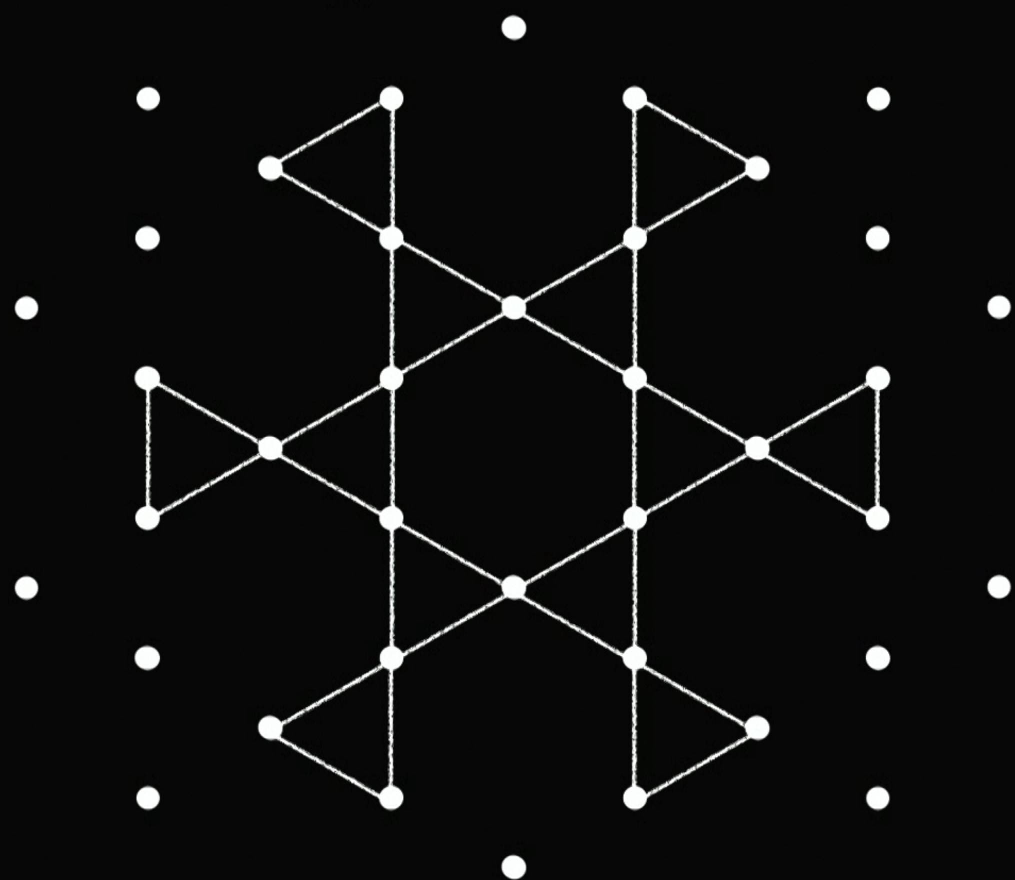
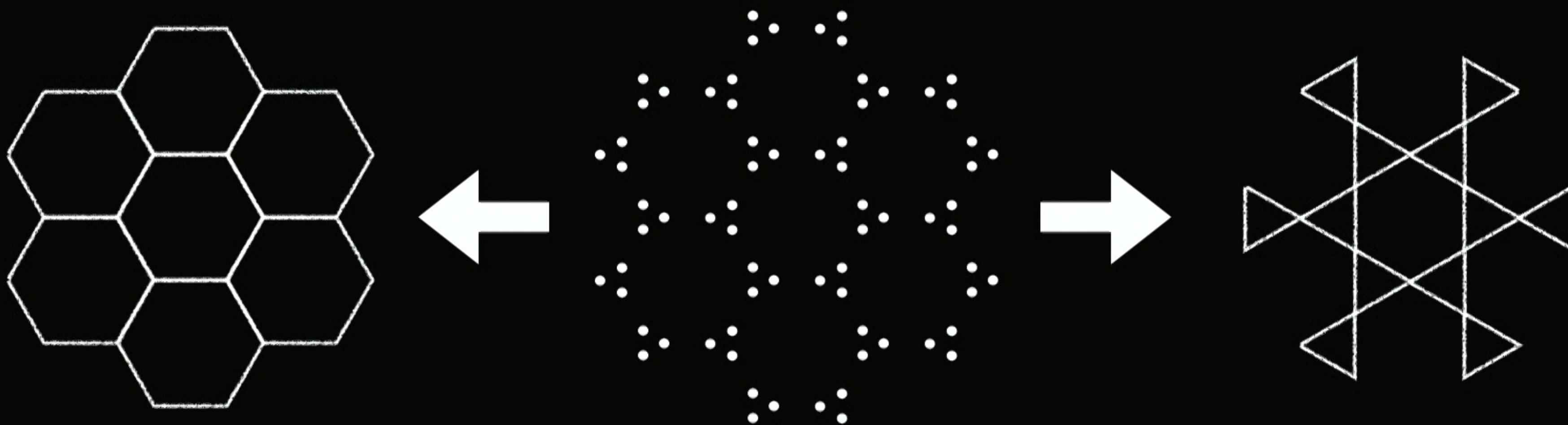
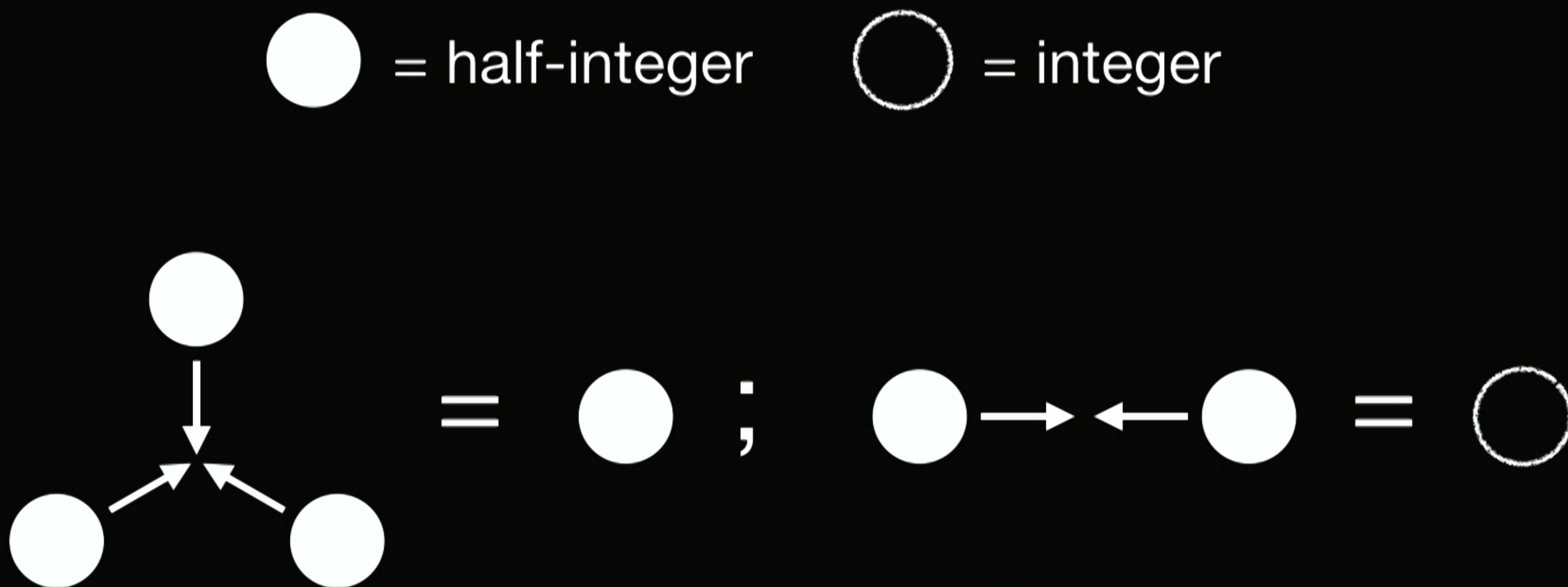


Moving sites around

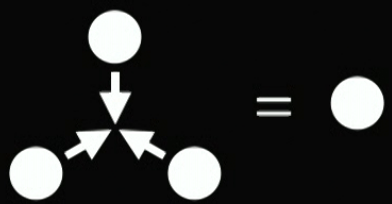


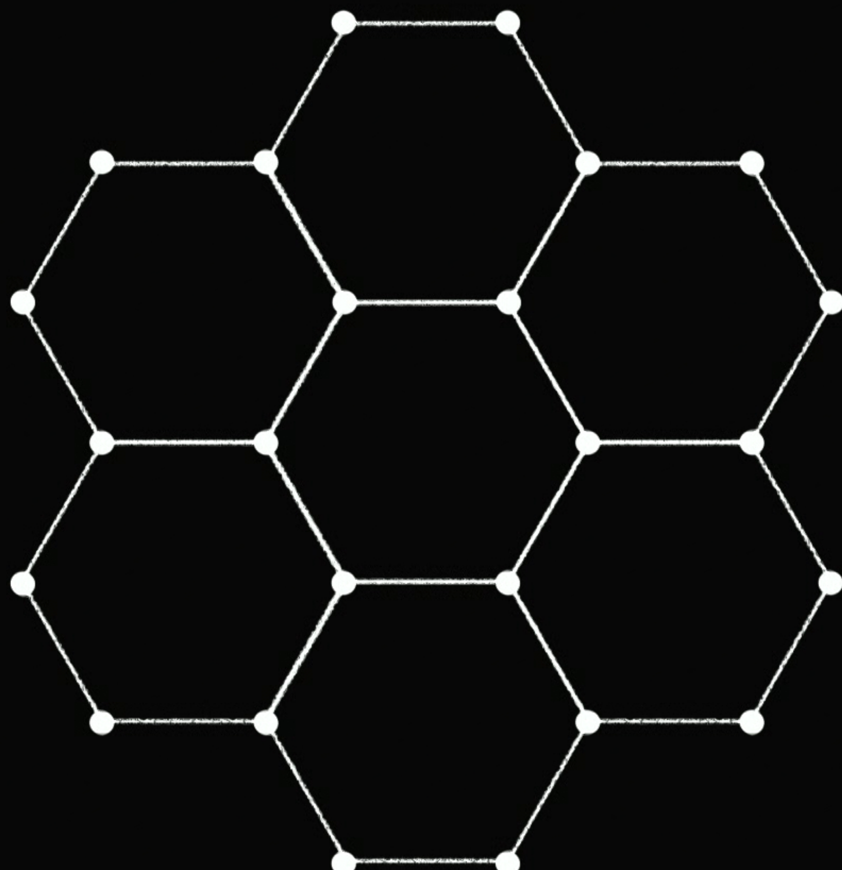
Are they the same lattice of *spins*?

Fusion of spins

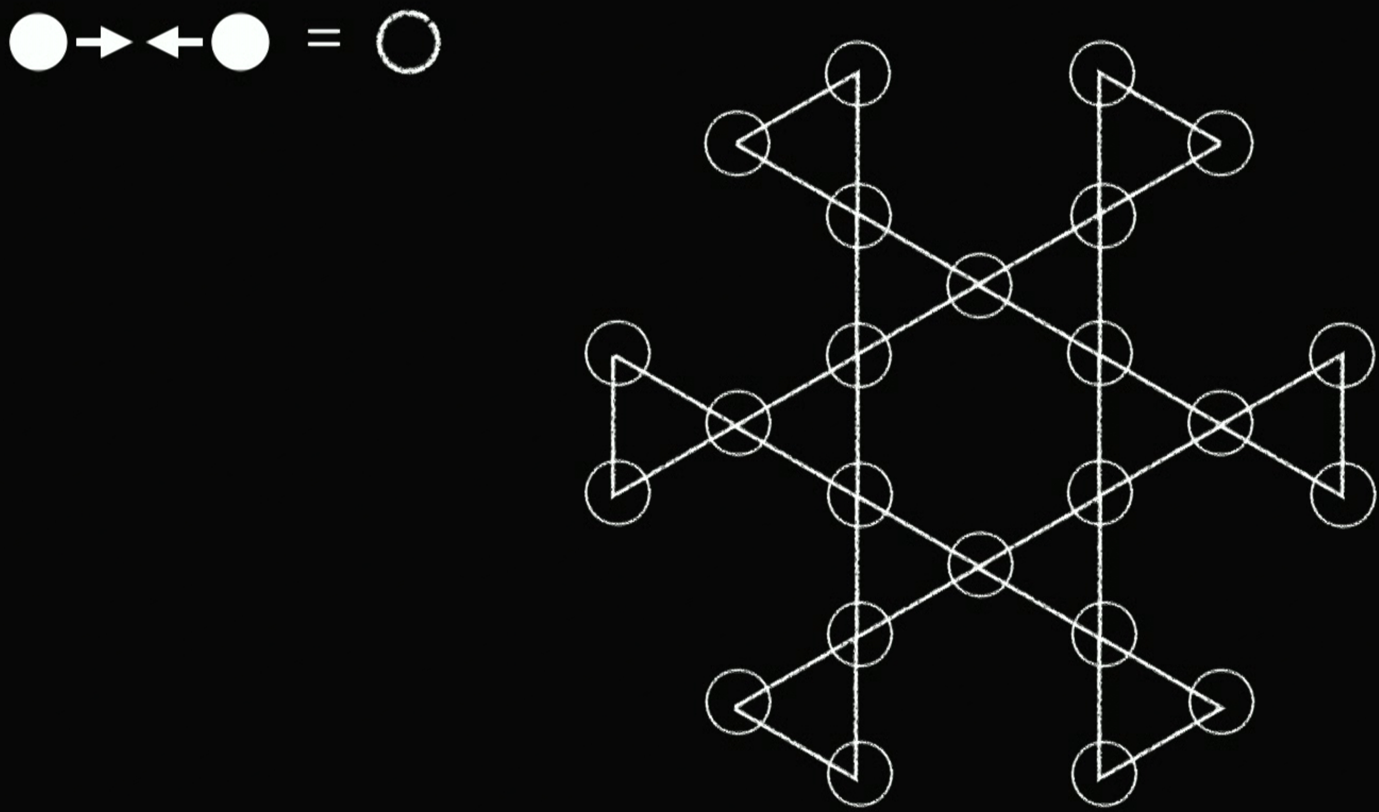


Moving *spins* around

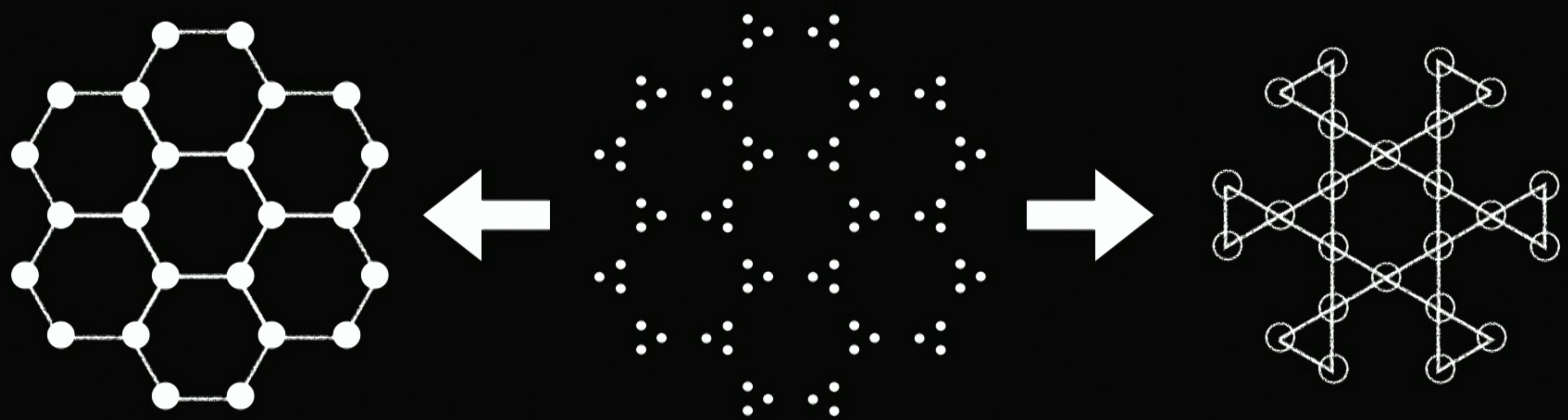




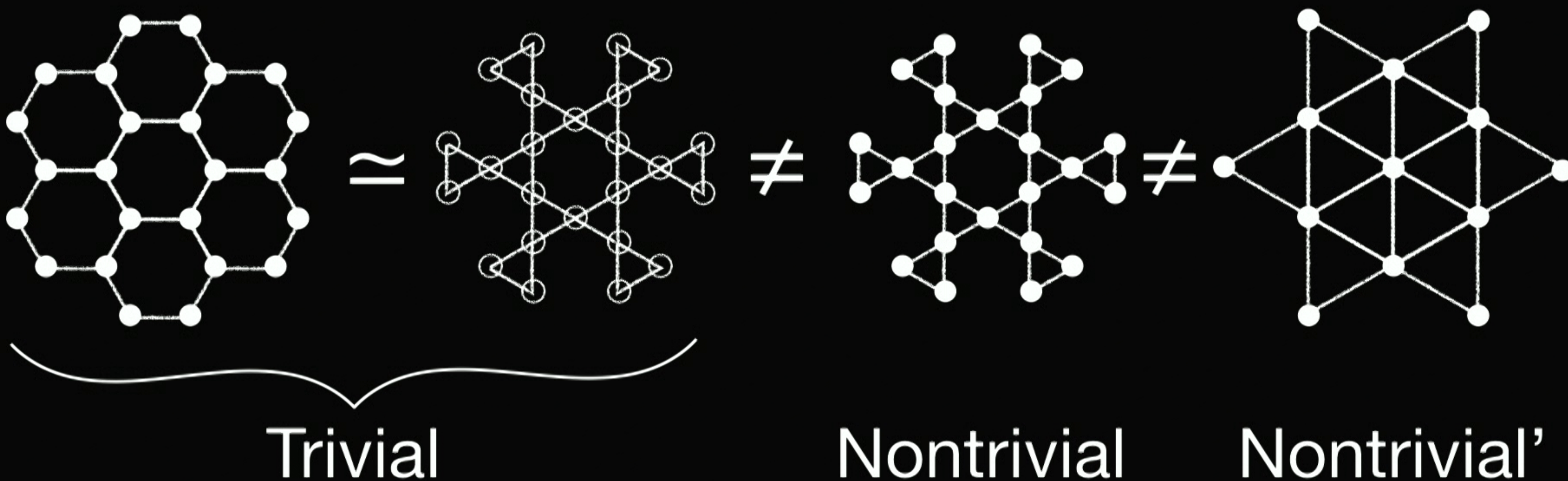
Moving *spins* around



Lattice homotopy equivalence



Lattice homotopy equivalence



Note: we’ve gone beyond the “total count”

Lattice homotopy: systematics

[HCP, Watanabe, Jian, Zaletel PRL 2017]

- Lattice data: $(\Lambda, \{\omega_{\mathbf{x}}\})$
  - Λ: a sym. set of points in space (i.e. lattice sites)
  - $\omega_{\mathbf{x}} \in \mathbb{Z}_2 = H^2(\mathrm{SO}(3), \mathrm{U}(1))$ proj. rep. (i.e. integer vs half-integer spin)
- Composite: $(\Lambda_1, \{\omega\}_1) + (\Lambda_2, \{\omega\}_2)$
- Special lattices of fixed points $\Rightarrow$ finite set of generators
- LH-related lattice $\Rightarrow$ equivalence relations

Lattice homotopy: 2D

[HCP, Watanabe, Jian, Zaletel PRL 2017]

Lattice homotopy	Wallpaper group No. [20]
$\mathbb{Z}_2$	1, 4, 5, 13, 14, 15
$(\mathbb{Z}_2)^2$	3, 8, 12, 16, 17
$(\mathbb{Z}_2)^3$	7, 9, 10, 11
$(\mathbb{Z}_2)^4$	2, 6

Generalized version: 3D with $\mathbb{Z}_n$ projective rep.

Generalized LSM

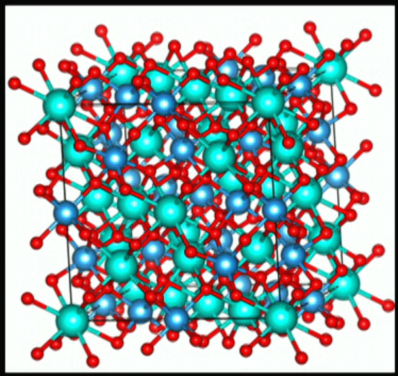
Conjecture: lattice homotopy nontrivial $\Rightarrow$ LSM-type constraint on the quantum many-body ground state

- (Physicists') proof for a more restricted setting

  [HCP, Watanabe, Jian, Zaletel PRL 2017]
- Proved for general quantum magnets

  [Else, Thorngren 1907.08204]

[Also, a large body of work on related problem of crystalline SPT: Cheng…Bonderson, Hermele, Xie, Fu, Else-Thorngren, Shiozaki, Xiong, Gomi,…]



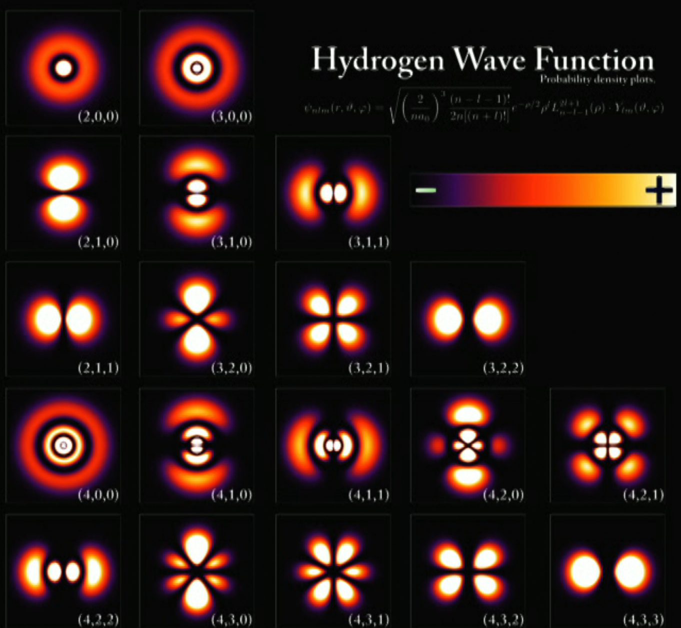
Upshot: this is lattice-homotopy nontrivial!

Outline aka take-homes

- ✓ Lieb-Schultz-Mattis theorem: no-go for "boring" phases
  - A motivating example of lattice constraints
- ✓ Lattice homotopy: general framework & quantum magnets
  - Space-group generalization of Lieb-Schultz-Mattis
- Application to atomic insulators
  - Uncovering topologically nontrivial quantum materials

Electronic insulators

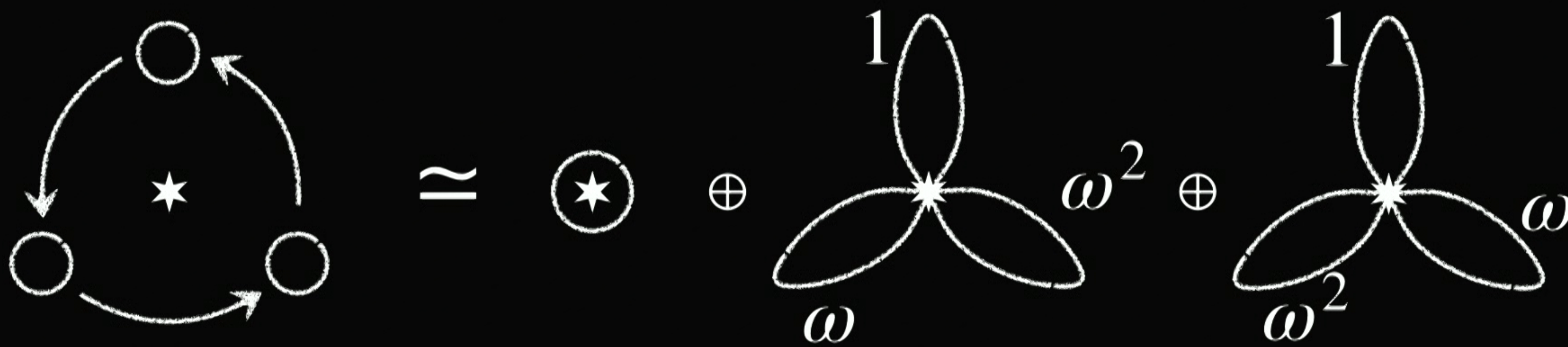
- Replace spin degrees of freedom by electronic orbitals



[Fig from Wiki]

Reduction in a crystal environment
(e.g. angular momentum defined mod n)

"Fusion" of atomic orbitals



$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} ; \quad \omega = e^{i\frac{2\pi}{3}}$$

Lattice homotopy for atomic insulators

[Else, HCP, Watanabe PRB 2019]
[Related idea: Zak PRB 1981, Bradlyn et al Nature 2017]

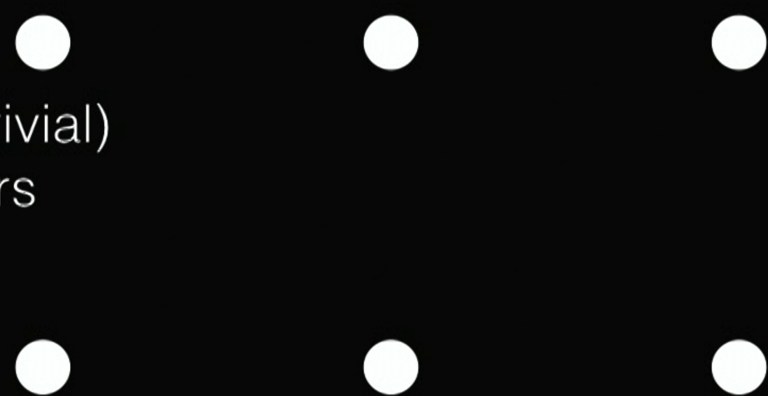
- Microscopic data: symmetries, lattice, and orbitals
- Equivalence from moving sites around and fusion (re-interpretation) of orbitals
- Output: finitely generated abelian group $\mathbb{Z}^d \times \mathbb{Z}_{n_1} \cdots$
  - Invariants for 1,651 magnetic space groups with or without spin-orbit: ~ 20 pages [HCP, unpublished]
    [see also: Liu et al PRX 2019, Song et al, Science 2020]

Atomic insulators are product states

- They don't have any inherent quantum entanglement…
- BUT they provide a gateway to those that do!
- Specialization to electronic band theory
  - Non-interacting electrons described in momentum space
  - Focus on symmetry representations
  - The group becomes $\mathbb{Z}^d$, which is like a vector space

“Bootstrapping” from the trivial

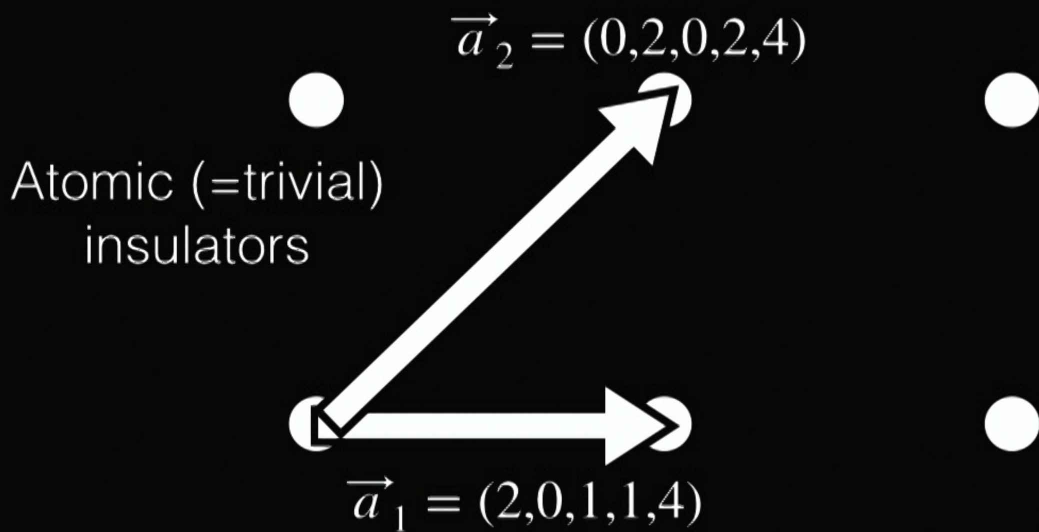
[HCP, Vishwanath, Watanabe Nature Comm. 2017]



Atomic (=trivial)
insulators

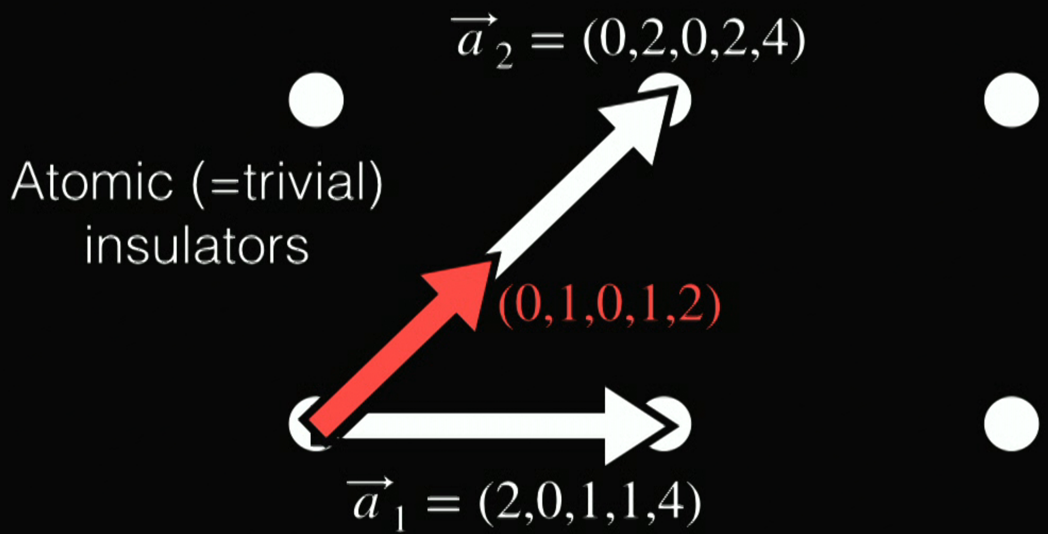
“Bootstrapping” from the trivial

[HCP, Vishwanath, Watanabe Nature Comm. 2017]



"Bootstrapping" from the trivial

[HCP, Vishwanath, Watanabe Nature Comm. 2017]

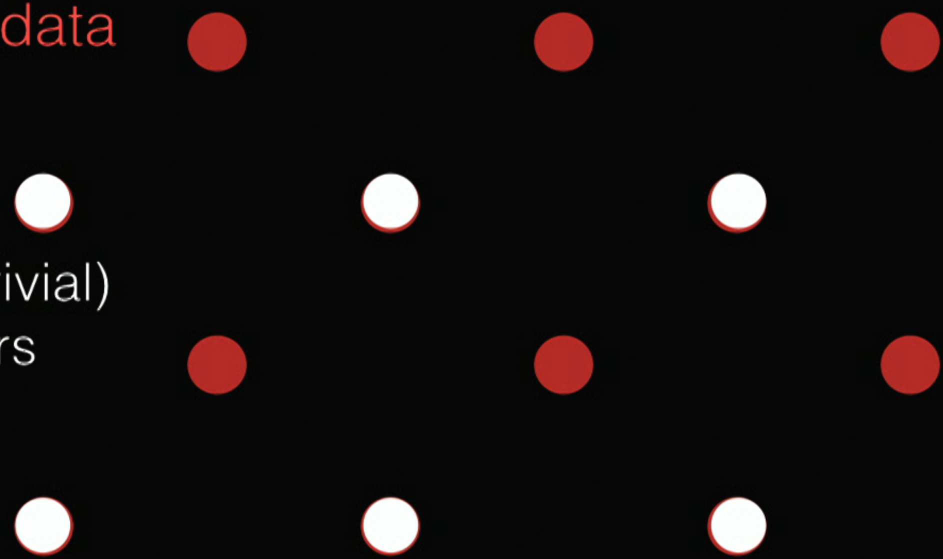


“Bootstrapping” from the trivial

[HCP, Vishwanath, Watanabe Nature Comm. 2017]

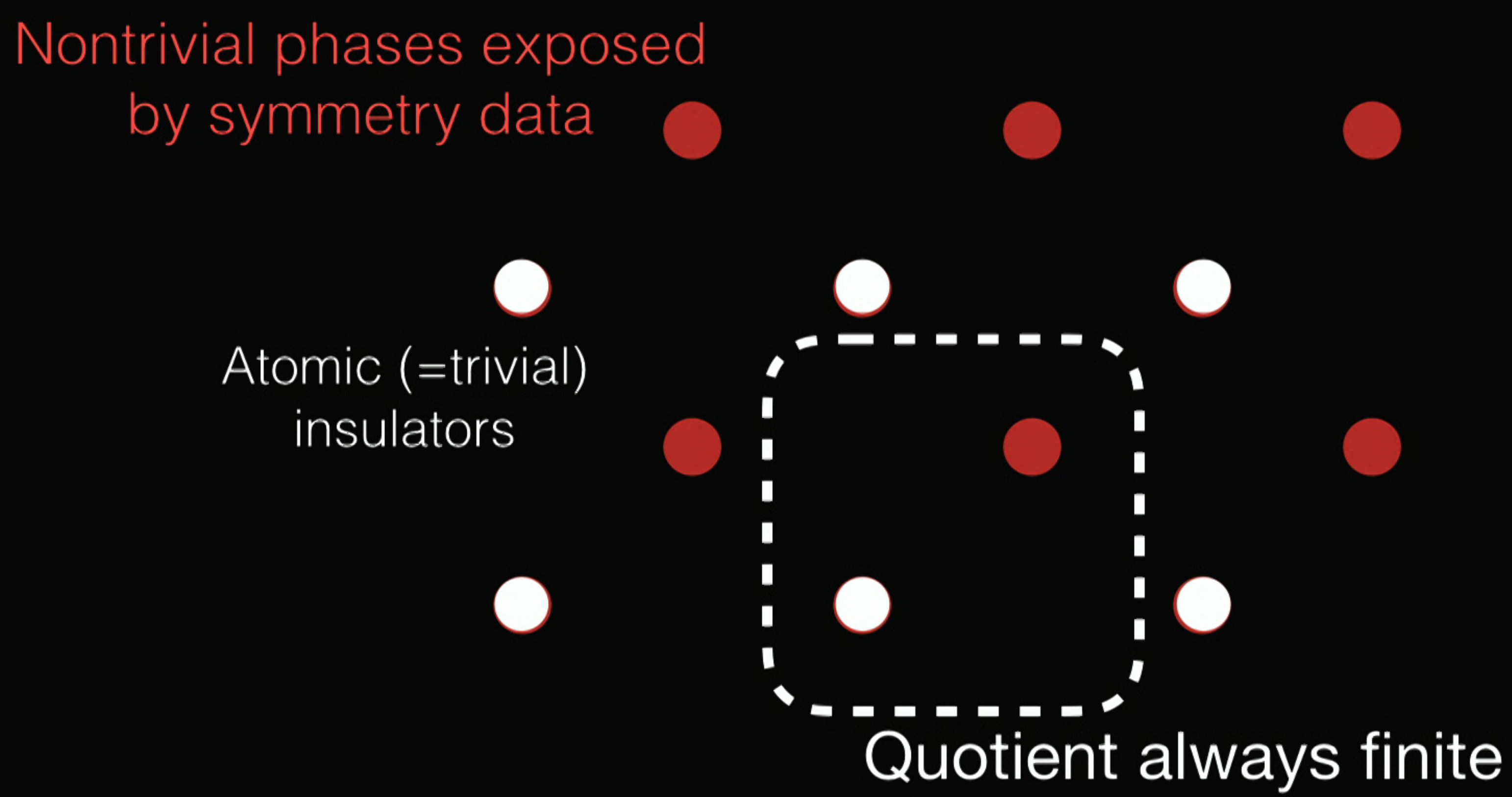
Nontrivial phases exposed by symmetry data

Atomic (=trivial) insulators



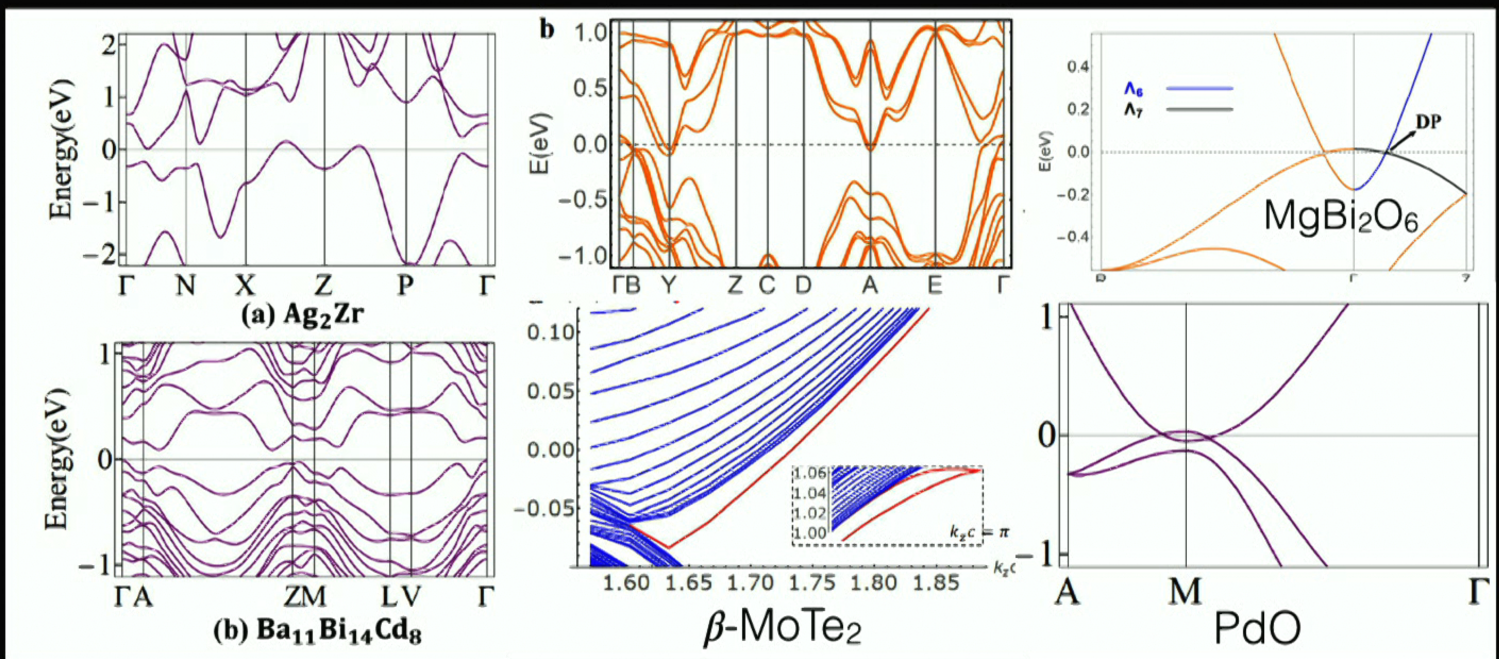
“Bootstrapping” from the trivial

[HCP, Vishwanath, Watanabe Nature Comm. 2017]



Uncovering topo. materials candidates

Chen, HCP, Neaton, Vishwanath, Nature Phys. 2018
Tang, HCP, Vishwanath & Wan, Nature Phys. 2019, Sci. Adv. 2019, Nature 2019
[also: Zhang et al., Nature 2019, Vergniory et al., Nature 2019]



Strong TIs   Higher-order TCIs   Dirac SM

Uncovering topo. materials candidates

Chen, HCP, Neaton, Vishwanath, Nature Phys. 2018

Tang, HCP, Vishwanath & Wan, Nature Phys. 2019, Sci. Adv. 2019, Nature 2019

ature 2019]

$\mathcal{SG}$	X_{BS}	Topological insulators
2	$\mathbb{Z}_3^2 \times \mathbb{Z}_4$	Ag_2F_5 …
11	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	BBeLi
12	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	Ag_4K_2 Ba_2Cd Bi_4Pb ,$PdSe_6$
14	$\mathbb{Z}_2 \times \mathbb{Z}_4$	Ag_2Te
15	$\mathbb{Z}_2 \times \mathbb{Z}_4$	MoP_4[
51	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	$AlPt_2$[
55	$\mathbb{Z}_2 \times \mathbb{Z}_4$	Bi_6In_2
58	$\mathbb{Z}_4$	Bi_2Hf[
59	$\mathbb{Z}_2 \times \mathbb{Z}_4$	Ag_3Sb
60	$\mathbb{Z}_4$	Au_2Pb
62	$\mathbb{Z}_4$	AsCdN 68],FS GeHfP Ge_2W[P_2Zr[8 94],SiS AlLaP
63	$\mathbb{Z}_2 \times \mathbb{Z}_4$	AgS_3T Ga[10 AlLa[1
64	$\mathbb{Z}_2 \times \mathbb{Z}_4$	As[115
65	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	Ag_3Te
69	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	Be_2Zn
71	$\mathbb{Z}_2 \times \mathbb{Z}_4$	AsTeT
72	$\mathbb{Z}_2 \times \mathbb{Z}_4$	Br_2Hg
87	$\mathbb{Z}_2 \times \mathbb{Z}_8$	Hf_5Te_4
121	$\mathbb{Z}_2$	Ag_2S_4
122	$\mathbb{Z}_2$	As_2Cd
123	$\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_8$	$BaGe_3$
127	$\mathbb{Z}_4 \times \mathbb{Z}_8$	B_4Y[1
129	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$HfSb_2$[
136	$\mathbb{Z}_4$	Ag_5Cs
139	$\mathbb{Z}_2 \times \mathbb{Z}_8$	Ag_2Zr[$CaGe_2$

$\mathcal{SG}$	X_{BS}	Topological crystalline insulators
2	$\mathbb{Z}_3^2 \times \mathbb{Z}_4$	CsHg[256]
11	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	$BaSb_2$[257],$MoTe_2$[258],
12	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	$Al_4Cl_{51}Zr_{12}$[262],Al_4Na $As_4Ba_3Zn_2$[269],Ba_3Cd_2 Cl_8NSc_5[277],NbP_2[278]
14	$\mathbb{Z}_2 \times \mathbb{Z}_4$	LaSbSe[285],AsSY[286],
51	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	$GaPt_2$[287]
55	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$Ca_5Ga_2Sb_6$[288]
58	$\mathbb{Z}_4$	Bi_3RbS_5[289]
59	$\mathbb{Z}_2 \times \mathbb{Z}_4$	BrNTi[290]
62	$\mathbb{Z}_4$	$HgSr_3$[291],LaSbTe[292]
63	$\mathbb{Z}_2 \times \mathbb{Z}_4$	BaGe[293],BaSi[294], B [301] $Cd_2FSb_5Sr_5$[302],PbSr[
64	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$Ca_5Ga_2N_4$[308],Li[309]
69	$\mathbb{Z}_2^2 \times \mathbb{Z}_4$	$Ge_6Li_2Sr_4$[310]
71	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$Ba_3Bi_4Li_4$[311],Ba_3Li_4S
87	$\mathbb{Z}_2 \times \mathbb{Z}_8$	Au_4Ti[312]
88	$\mathbb{Z}_4$	O_4PbPd_2[313]
123	$\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_8$	As_3CsZn_4[314],As_3RbZn
129	$\mathbb{Z}_2 \times \mathbb{Z}_4$	AsGeNb[316],GeHfS[317 GeSeZr[317],HfSSi[317,
137	$\mathbb{Z}_4$	Ba_2LiN[322]
139	$\mathbb{Z}_2 \times \mathbb{Z}_8$	$AsBa_2$[323],$AsCa_2$[324] Ba_2Bi[325],Ba_2Sb[326],
140	$\mathbb{Z}_2 \times \mathbb{Z}_8$	Bi[330],$GePt_3$[331]
148	$\mathbb{Z}_2 \times \mathbb{Z}_4$	Mo_3S_4[332]
164	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$BaSi_2$[333],BiTe[334],Bi
166	$\mathbb{Z}_2 \times \mathbb{Z}_4$	$AsNaTe_2Zr_2$[340],As_3Cd As_3Cd_4Na[341],As_3Cd_4 ClZr[348],Hg[349],P[350 BrLa[351]
187	$\mathbb{Z}_3 \times \mathbb{Z}_3$	CHf[352],$InNbSe_2$[353,
189	$\mathbb{Z}_3 \times \mathbb{Z}_3$	AgAsCa[4, 356],AgCaP[GeLiY[358],AsLuPd[35
191	$\mathbb{Z}_6 \times \mathbb{Z}_{12}$	B_2Zr[360]
193	$\mathbb{Z}_{12}$	Pb_3SZr_5[361]

$\mathcal{SG}$	Topological (semi-)metals
11	Br_9TeW[396],CBrHgNS[397],Li_7Sn_3[398],Mo_2S_2Sb[399]
15	BaI_4La[400]
51	AuCd[401],AuTi[402]
52	Ag_2BiO_3[403],Bi_3Sr_2[404]
55	Al_3Pd_5[405],Al_3Pt_5[406],BCl_6Sc_4[407], $Bi_9Ca_9Cd_4$[408], $Bi_9Ca_9Zn_4$[409],$Bi_9Cd_4Sr_9$[408],$In_5S_{13}Y_4$[410]
57	AlCaPd[411], BiK_2Sn[412]
58	$C_{60}K$[413]
59	Ag_3Sn[414]
60	F_4NaTi[415],O_2Re[416]
61	AgF_2[417]
62	$AgAuF_7$[418],AgF_3K[419],$AlPt_2$[420] Bi_3Ca_5[421],Bi_3Sr_5[422],Ca_5Sb_3[423] $GeNb_3Te_6$[424],$GePd_2Y$[425],N_3Nb_2[426],Nb_3Si[427] PdSbZr[428],$SiTa_3Te_6$[428],$AuLu_2$[429],GeLa[430],$GeLaPd_2$[430],$GeLuPd_2$[431],$LuPd_2Si$[432],LuPt[433], $LuPt_2Si$[434]
63	AgCa[435], AuCa[436], BiZr[437], Ga_3PdSr[438], Ga_5Zr_3[439], GeSc[440], GeY[441], HfSb[442],$K_3O_4Pd_2$[443],$K_3O_4Pt_2$[444],K_4P_3[445, 446], N_5NaTa_3[447],PdY[448],SiY[449], Sr_3Tl_5[450],LuSi[451],Al_6Re[452]
64	$AgCs_2F_4$[453],$Au_{10}Ca_4In_3$[454],Bi[455, 456],$LaSb_2$[457]
65	$C_7Re_2Sc_5$[458],$Ge_{10}La_7Li_8$[459]
71	$ReSi_2$[460]
74	Ag_2La[461],Au_2La[461],In_2La[462]
87	Ba_9In_4[463],$In_4Pd_{17}Se_4$[464],$Pt_{11}Zr_9$[465], $Pt_{12}Si_5$[466]
88	$Al_{21}Pt_8$[467],$CsFO_3S$[468],Ge_8Pd_{21}[469],LaO_4Pd_2[470]
123	$AgPPt_5$[471],$AlPPt_5$[471],$AsInPd_5$[472],$AsPd_5Tl$[473] $AsPt_5Tl$[473],As_2BaPd_2[474],BaP_2Pd_2[475],CaPb[476] CdPd[477],$CdPd_5Se$[478],CdPt[479],Cd_3Zr[480] $FKNb_4O_5$[481],HgPd[482],$HgPd_5Se$[483],HgPt[484],Hg_2Pt[484] $InPPd_5$[485],$InPPt_5$[485],PPd_5Tl[485],PPt_5Tl[485] PdTi[486],Pd_5SeZn[487],SiSr[488],Sr[489],C_2Re[490],AsLa[491],LaSb[492]
127	$AlSc_2Si_2$[493],Au_2Ca_2Pb[494],Au_2InY_2[495],B_2Ta_3[496] B_4W[497],C_2B_2Y[498],Ga_2MgSc_2[499],Ca_3Hg_2[500] Ga_2Nb_3[501],Ga_2Ta_3[502],Ge_2Hf_3[503],Hg_2Sr_3[504, 505] $InPd_2Y_2$[506],In_5Ti_2[507],$LiSi_2Y_2$[508],$PbPd_2Y_2$[509],N_2Re[510]
129	AgMgSb[511],AsNbSi[512],AsSiTa[513], BaMgSi[514],BiKMg[515],GeTeZr[516],MoNTa[517]

Outline aka take-homes

✓ Lieb-Schultz-Mattis theorem: no-go for "boring" phases

- A motivating example of lattice constraints

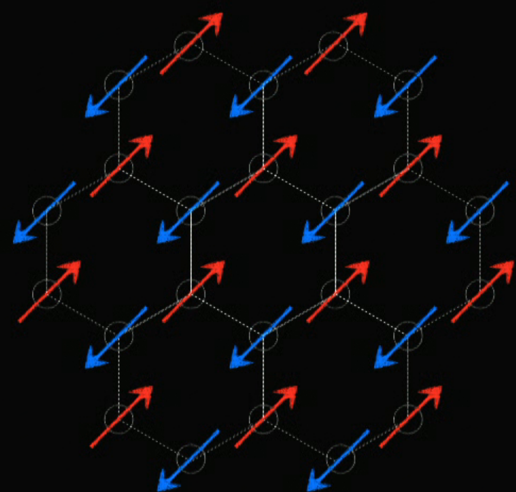
✓ Lattice homotopy: general framework & quantum magnets

- Space-group generalization of Lieb-Schultz-Mattis

✓ Application to atomic insulators

- Uncovering topologically nontrivial quantum materials

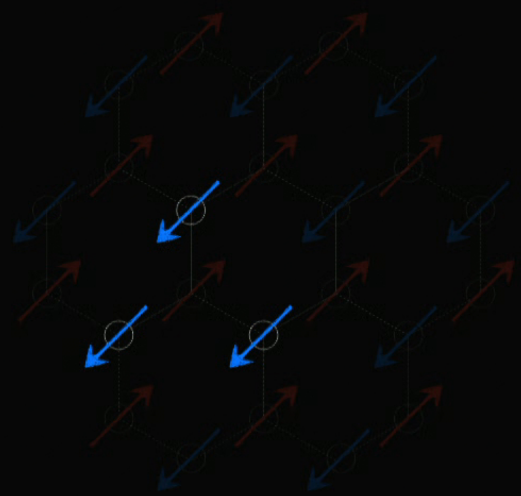
Topological materials:
interplay between symmetries and locality



Real-space description for 2D topological insulator
with time-reversal symmetry

[Soluyanov-Vanderbilt 2011]

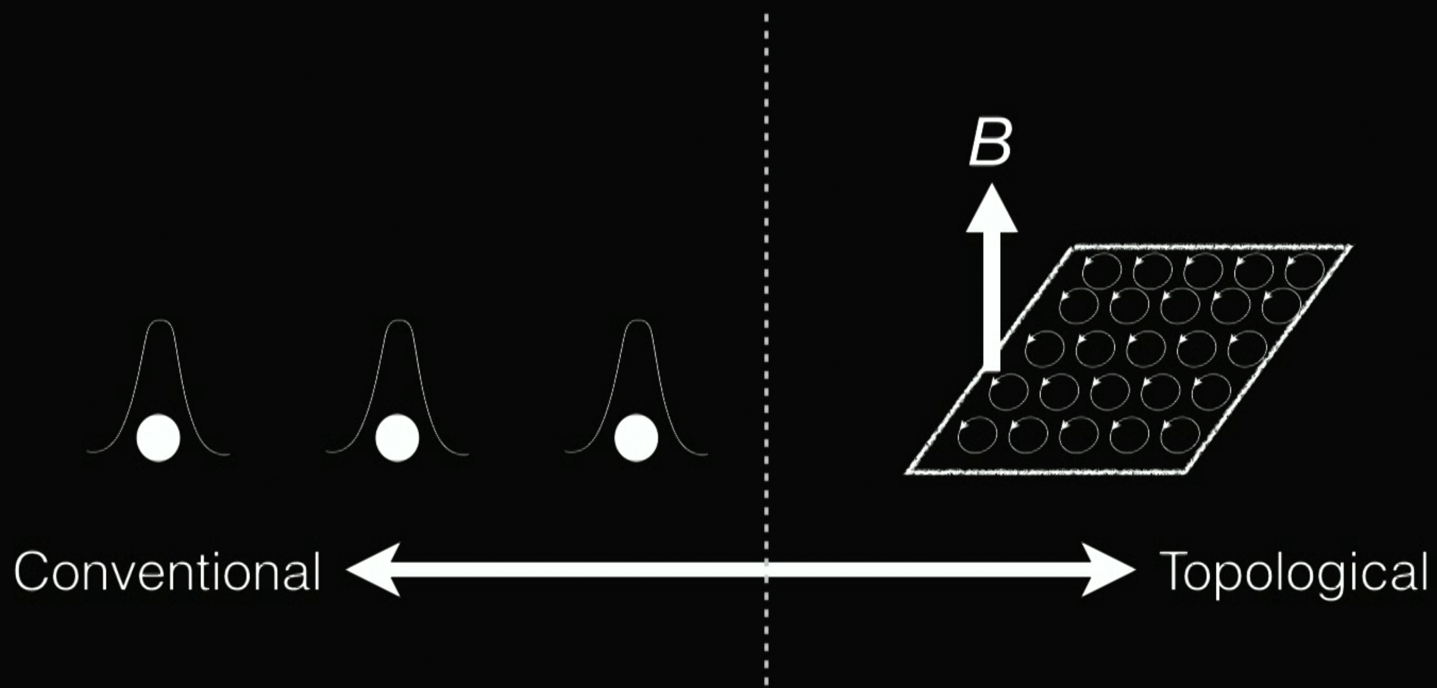
Topological materials:
interplay between symmetries and locality



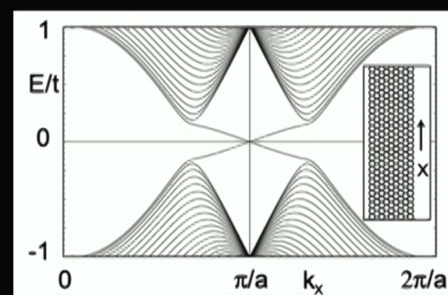
Real-space description for 2D topological insulator
with time-reversal symmetry

[Soluyanov-Vanderbilt 2011]

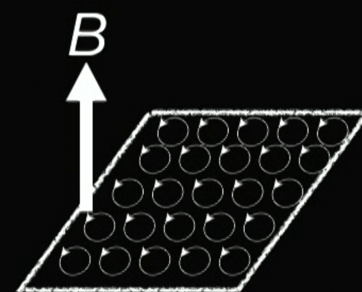
Lessons from the topo. materials



Lessons from the topo. materials



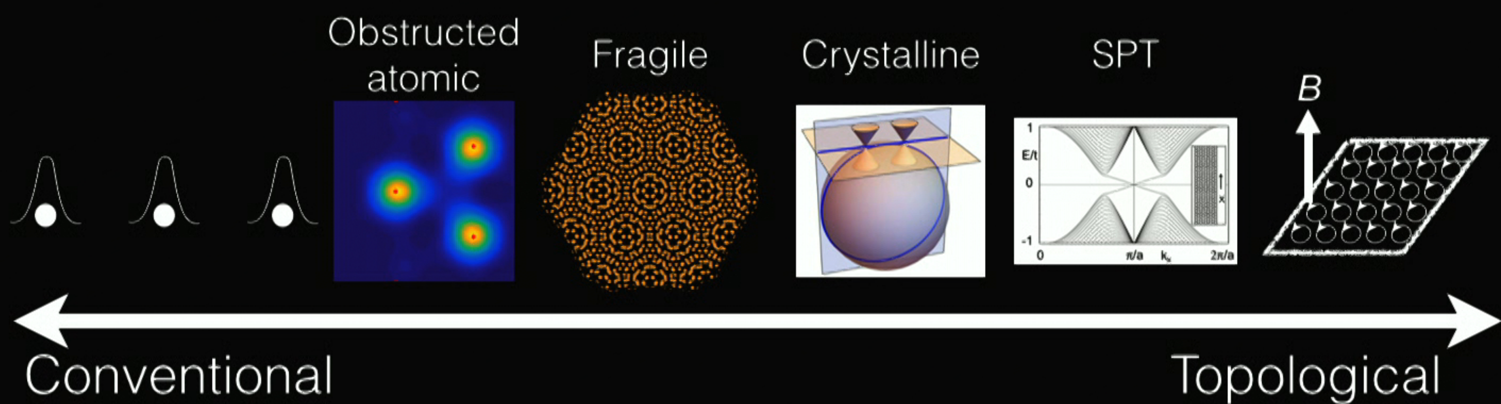
[Kane-Mele '04]

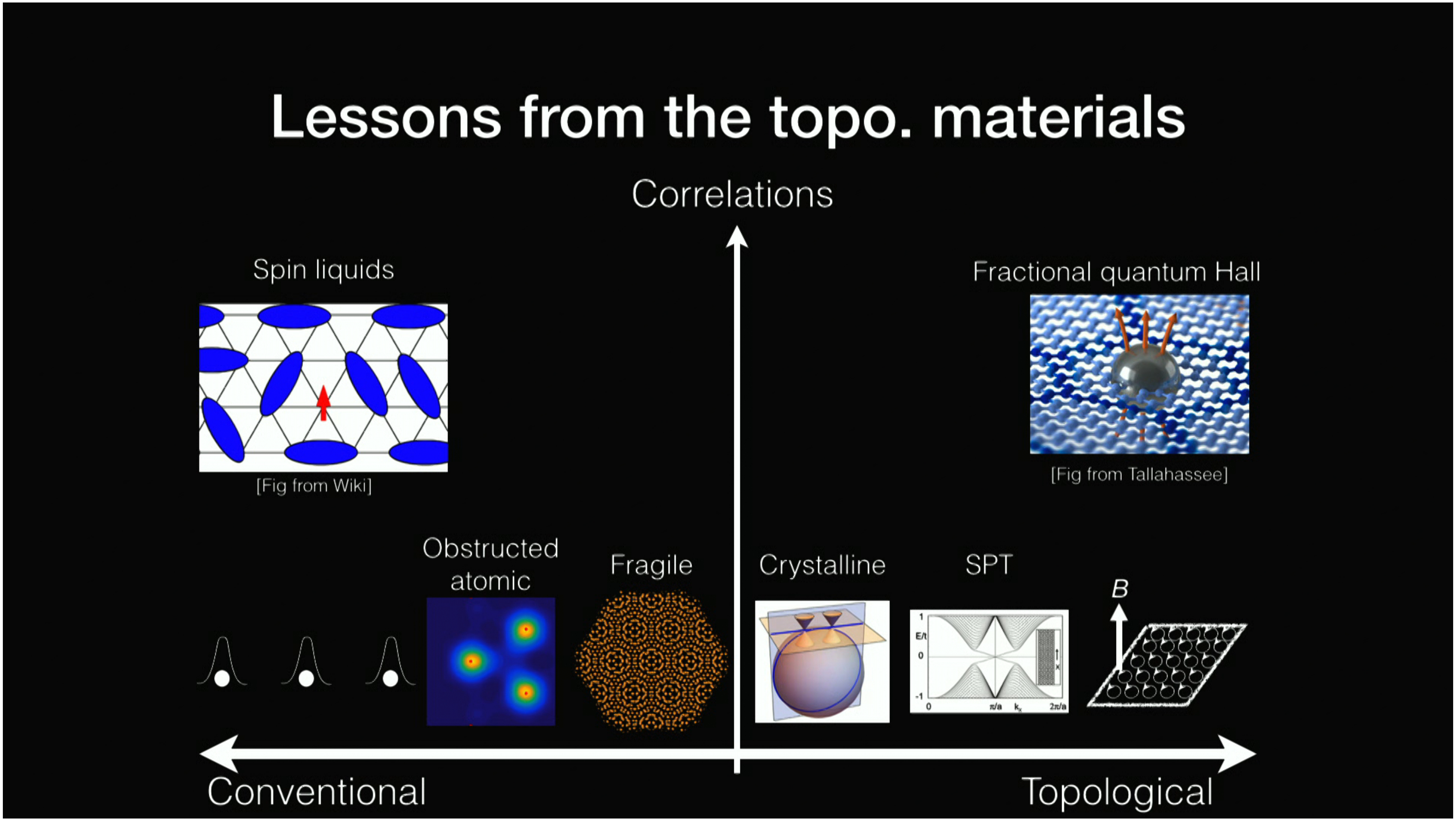


Conventional ⟷ Topological

Sym. protected topo.

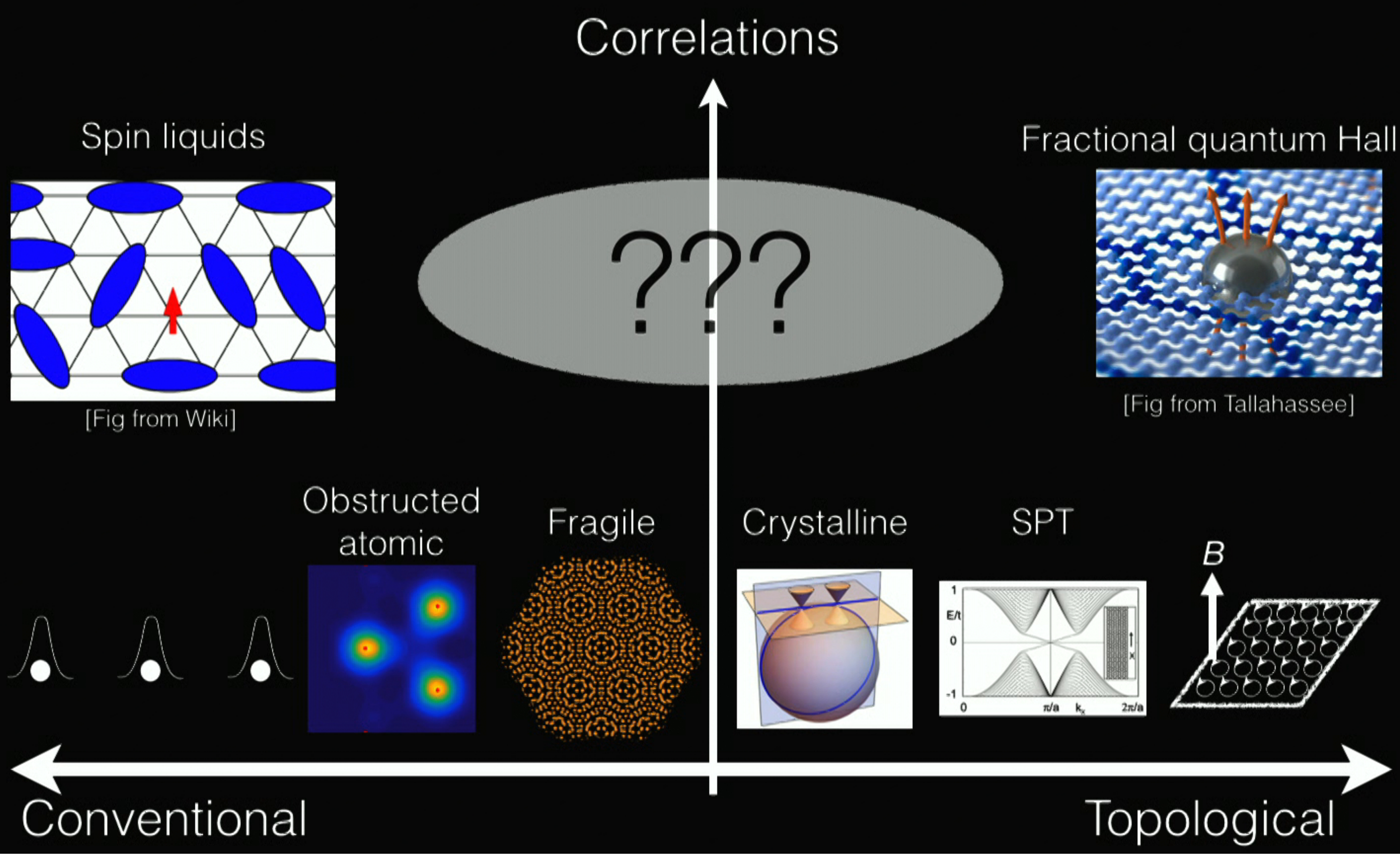
Lessons from the topo. materials





Lessons from the topo. materials
Correlations
Spin liquids
[Fig from Wiki]
Fractional quantum Hall
[Fig from Tallahassee]
Obstructed atomic
Fragile
Crystalline
SPT
B
Conventional
Topological

Lessons from the topo. materials



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Lattice homotopy — **Symmetry indicators** — **Materials discovery**

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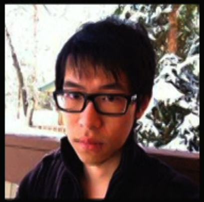
D. Else (MIT)



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Thanks!