Title: PSI 2019/2020 - Computational Physics - Lecture 7

Speakers: Erik Schnetter

Collection: PSI 2019/2020 - Computational Physics

Date: February 10, 2020 - 1:30 PM

URL: http://pirsa.org/20020014

PDEs

$$\left(\partial_t^2 - \partial_x^2\right) u = 0$$

$$f_t := \partial_t u$$

$$f_x := \partial_x u$$

$$\boxed{\begin{aligned} \partial_t u &= f_t \\ \partial_t f_t &= \partial_x f_x \\ \partial_t f_x &= \partial_x f_t \end{aligned}}$$

<u>PDEs</u>

$$\left(\partial_t^2 - \partial_x^2\right) u = 0$$

$$f_t := \partial_t u$$

$$f_x := \partial_x u$$

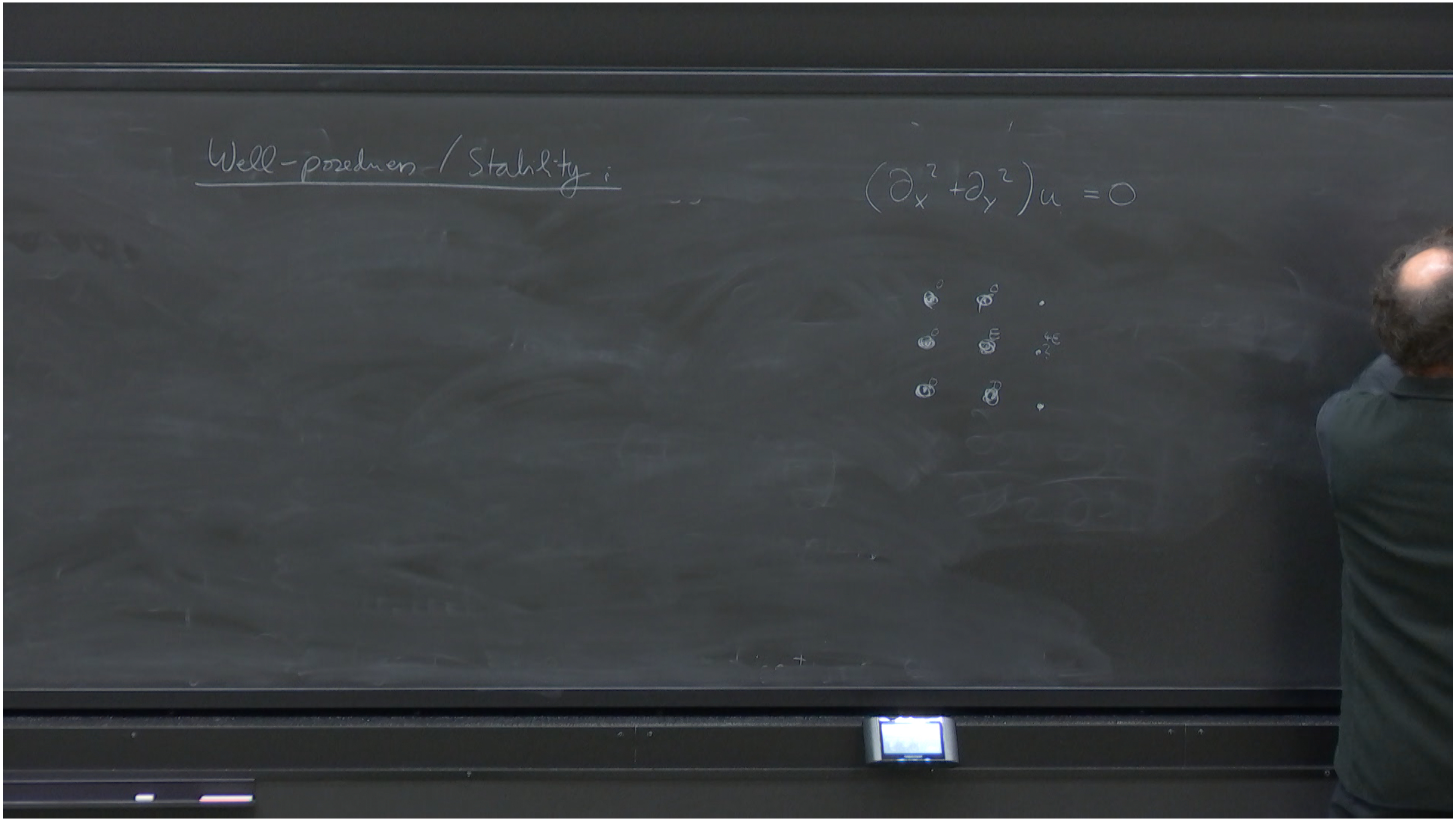
$$\boxed{\begin{aligned} \partial_t u &= f_t \\ \partial_t f_t &= \partial_x f_x \\ \partial_t f_x &= \partial_x f_t \end{aligned}}$$

"state vector"

$$f := \begin{pmatrix} f_t \\ f_x \end{pmatrix}$$

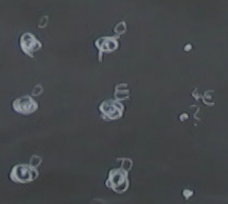
$$\left(A^t \partial_t + A^x \partial_x\right) f = 0$$

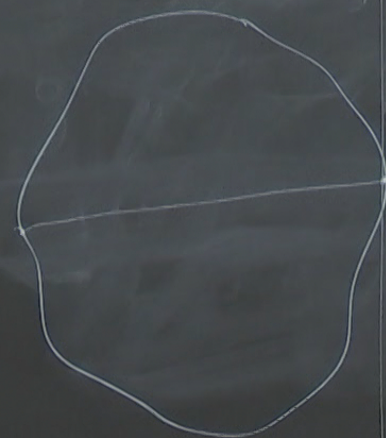
$$A^t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$


Well-posedness / Stability:
$(\partial_x^2+\partial_y^2)u = 0$

iptic:

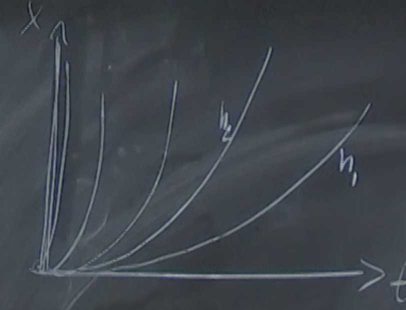
$$(\partial_x^2+\partial_y^2)u=0$$





hyperbolic: $(\partial_x^2-\partial_y^2)u=0$

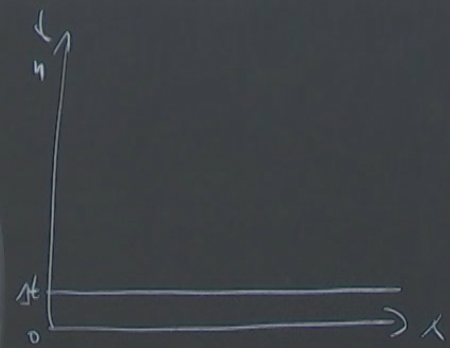
solution $\|u(t,x)\| \leq \|u(0,x)\| \cdot e^{\alpha t}$



h_2

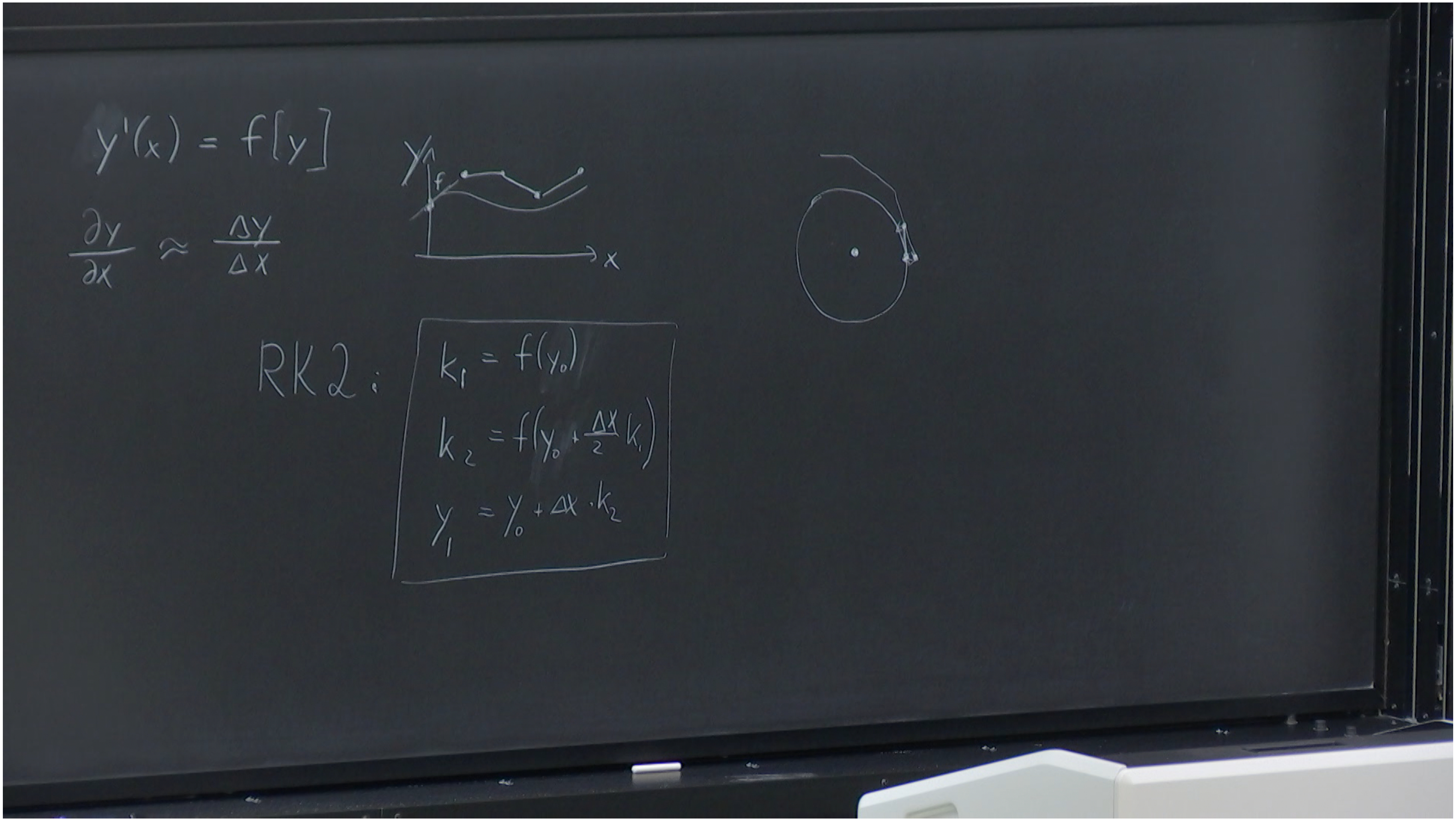
ill-posed (unstable)

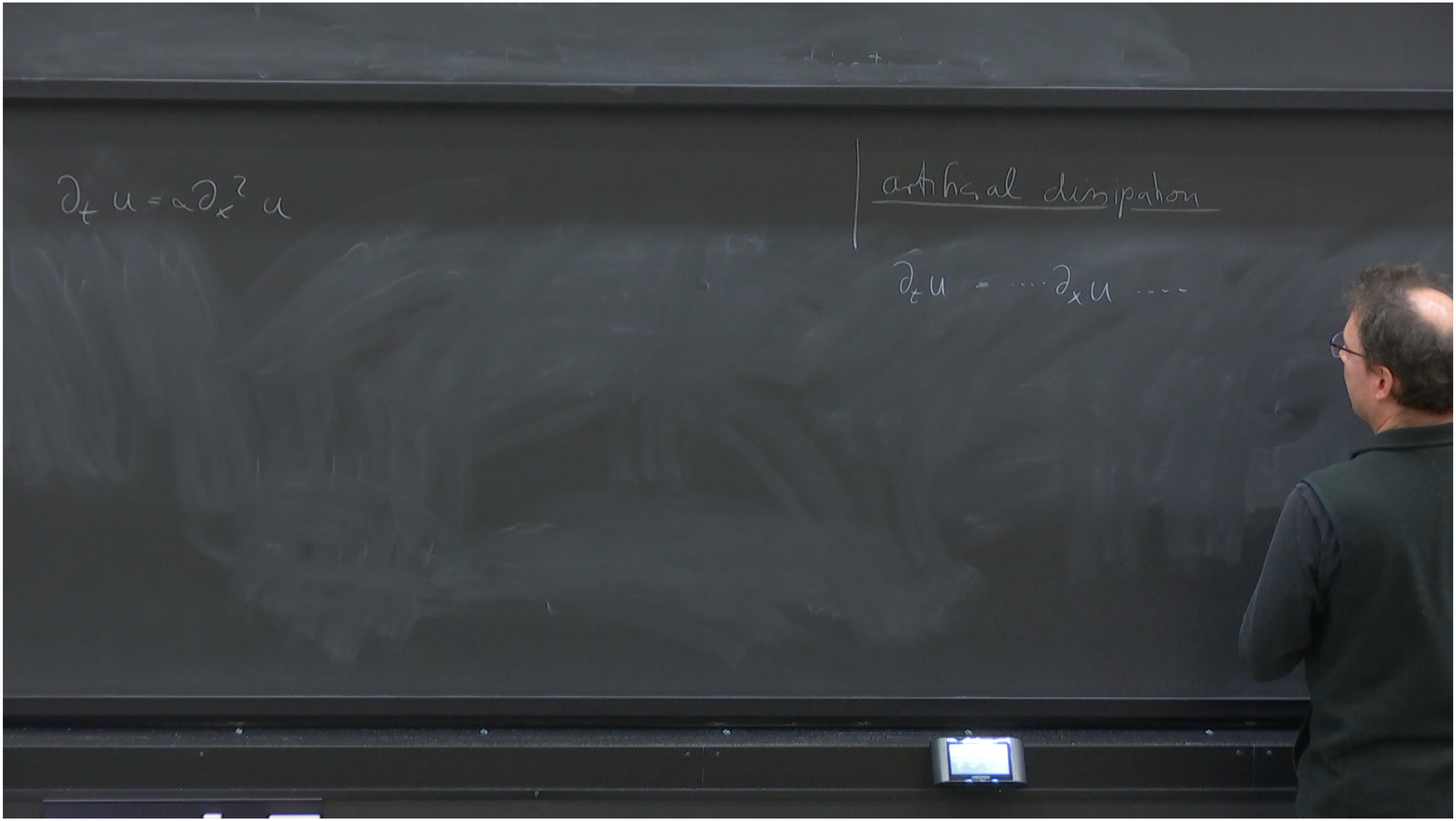
Method of Lines



$$\partial_t^2 u = \partial_x^2 u$$

$$d_t^2 u^i = \Delta^i_j u^j$$


$y'(x) = f[y]$
$\frac{\partial y}{\partial x} \approx \frac{\Delta y}{\Delta x}$
y
f
x
RK2:
$k_1 = f(y_0)$
$k_2 = f(y_0 + \frac{\Delta x}{2} k_1)$
$y_1 = y_0 + \Delta x \cdot k_2$


$\partial_t u = \alpha \partial_x^2 u$
artificial dissipation
$\partial_t u = \dots \partial_x u \dots$