Title: The Necromancy-Hardness of the SchrÃ¶dinger's Cat Experiment

Speakers: Scott Aaronson

Series: Perimeter Institute Quantum Discussions

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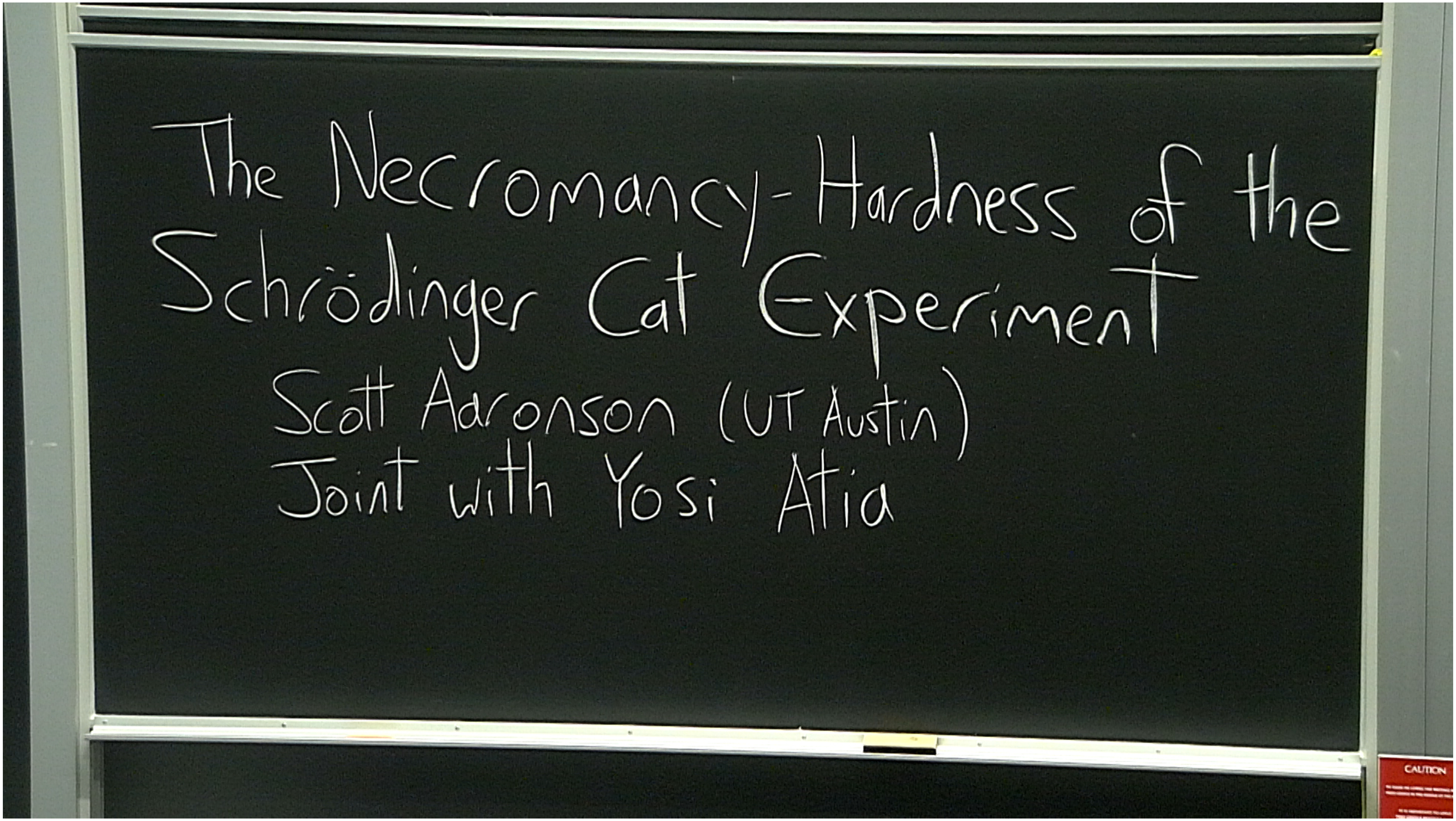
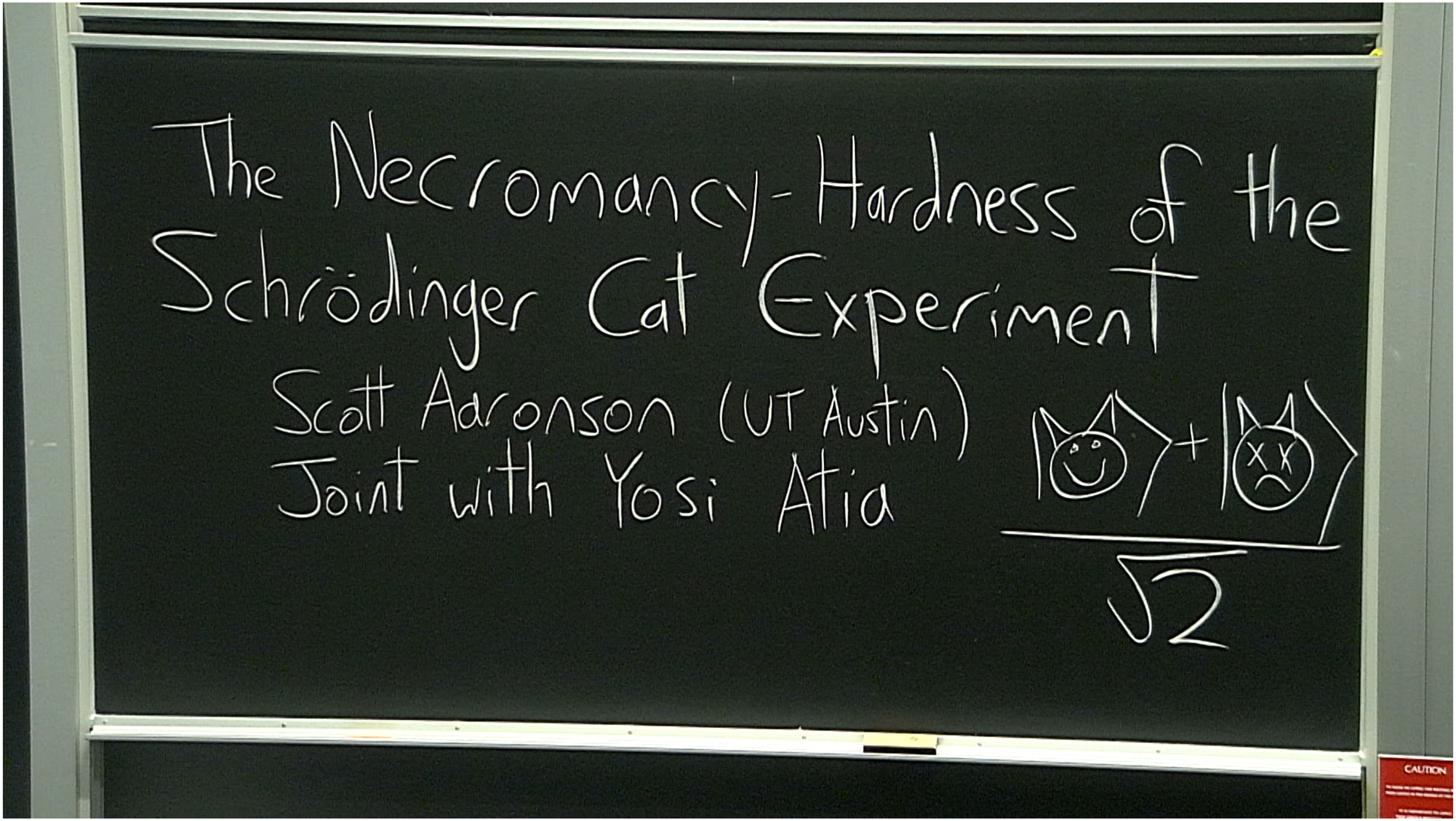
Abstract: Motivated by puzzles in quantum gravity AdS/CFT, Lenny Susskind posed the following question: supposing one had the technological ability to distinguish a macroscopic superposition of two given states |v> and |w> from incoherent mixture of those states, would one also have the technological ability to map |v> to |w> and vice versa? More precisely, how does the quantum circuit complexity of the one task relate to the quantum circuit complexity of the other? Here we resolve Susskind's question -- showing that the two complexities are essentially identical, even for approximate versions of these tasks, with the one caveat that a unitary transformation that maps |v> to |w> and |w> to -|v> need not imply any distinguishing ability. Informally, "if you had the ability to prove SchrÃ¶dinger's cat was in superposition, you'd necessarily also have the ability to bring a dead cat back to life." I'll also discuss the optimality of this little result and some of its implications.

Paper (with Yosi Atia) in preparation


The Necromancy-Hardness of the
Schrödinger Cat Experiment
Scott Aaronson (UT Austin)
Joint with Yosi Atia
CAUTION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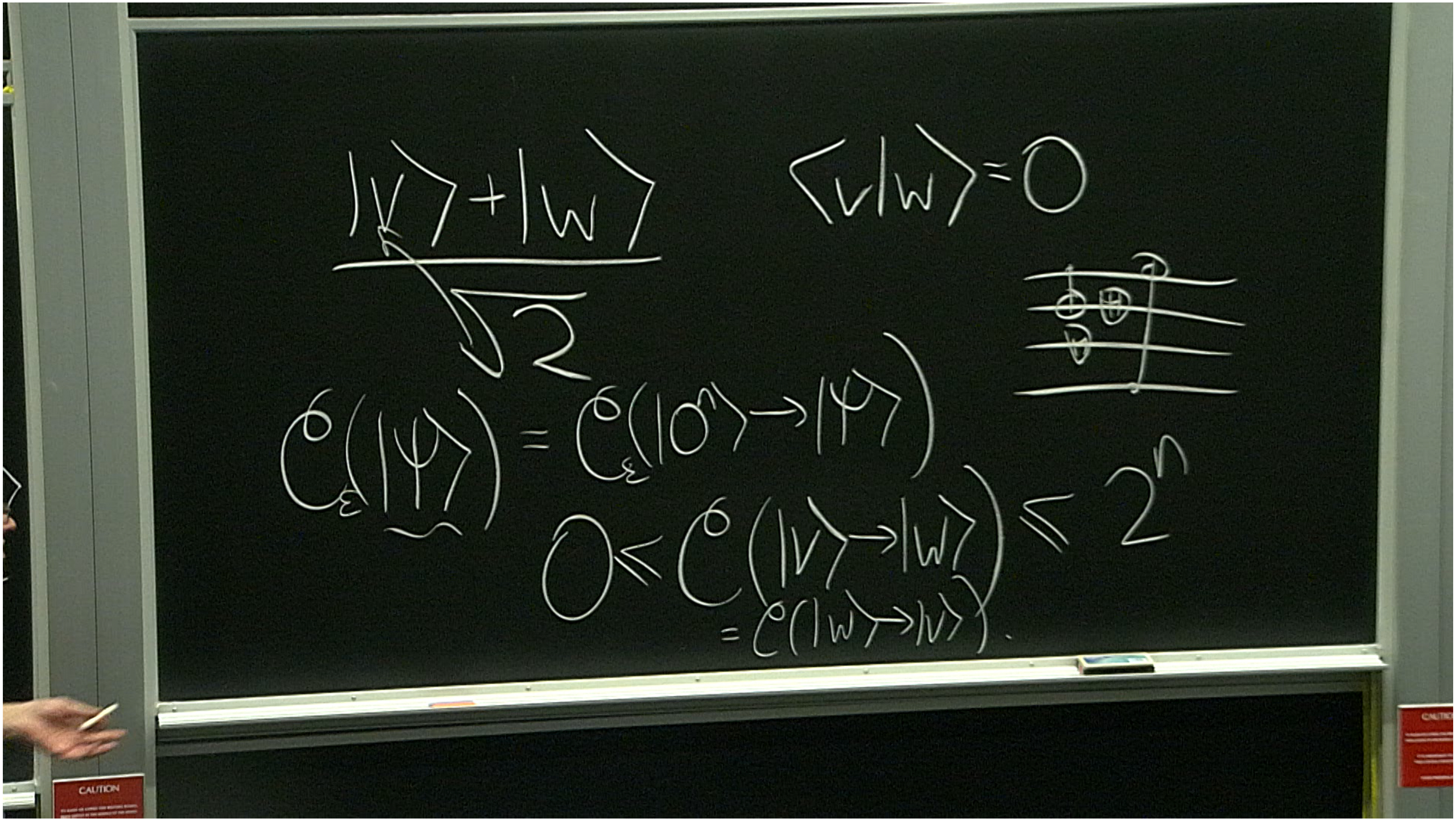

$\frac{|v\rangle + |w\rangle}{\sqrt{2}}$
$\langle v|w\rangle = 0$
$C_\varepsilon(|\psi\rangle) = C_\varepsilon(|0^n\rangle \to |\psi\rangle)$
$0 < C(|v\rangle \to |w\rangle) \leq 2^n$
$= C(|w\rangle \to |v\rangle)$.
CAUTION

For most $|\psi\rangle$, $\mathcal{E}(|\psi\rangle) \approx 2^n$

$$\frac{|v\rangle\langle v| + |w\rangle\langle w|}{2}$$

$$\frac{|v\rangle - |w\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{|v\rangle + |w\rangle}{\sqrt{2}}$$

$$U|v\rangle = |w\rangle$$

$$\Longleftarrow \quad U|w\rangle = |v\rangle$$

$$\Longrightarrow$$

$$|\phi\rangle = \frac{|v\rangle - |w\rangle}{\sqrt{2}}$$

$$\underline{|+\rangle}|u\rangle$$

$$|\psi\rangle = \frac{|v\rangle + |w\rangle}{\sqrt{2}}$$

$$|\phi\rangle = \frac{|v\rangle - |w\rangle}{\sqrt{2}}$$

$$A|\psi\rangle = |0\rangle|\cdots\rangle$$

$$\neq$$

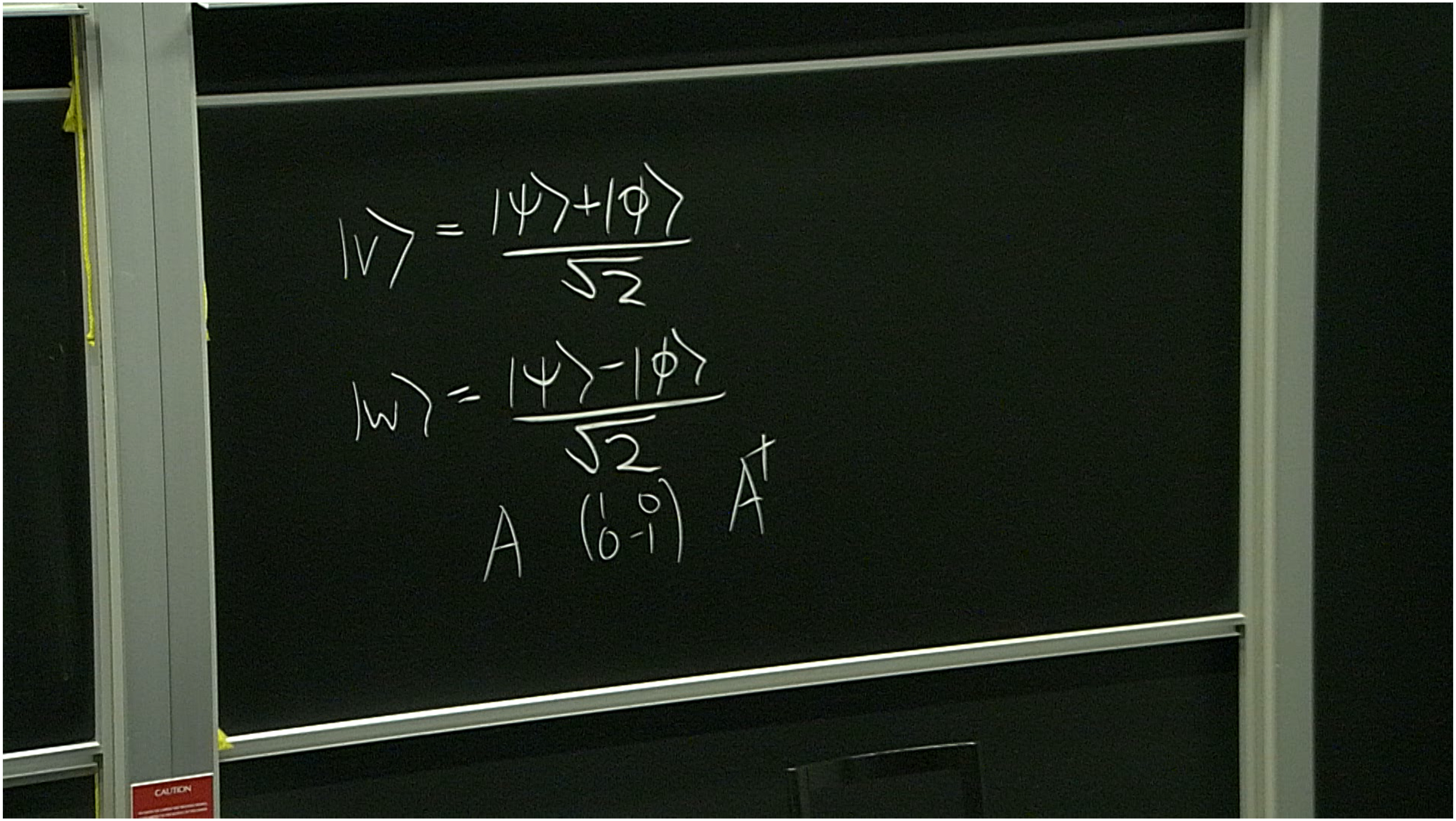
$$A|\phi\rangle = |1\rangle|\cdots\rangle$$

$$U|v\rangle = |w\rangle$$

$$\Longleftarrow \quad U|w\rangle = |v\rangle.$$

$$\Longrightarrow$$

$$|+\rangle|U\rangle$$


$|v\rangle = \frac{|\psi\rangle + |\phi\rangle}{\sqrt{2}}$
$|w\rangle = \frac{|\psi\rangle - |\phi\rangle}{\sqrt{2}}$
$A \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad A^\dagger$

$$|\psi\rangle = \frac{|v\rangle + |w\rangle}{\sqrt{2}}$$

$$|\phi\rangle = \frac{|v\rangle - |w\rangle}{\sqrt{2}}$$

$$A|\psi\rangle = |0\rangle|\cdots\rangle$$

$$A|\phi\rangle = |1\rangle|\cdots\rangle$$

$$U|v\rangle = |w\rangle$$

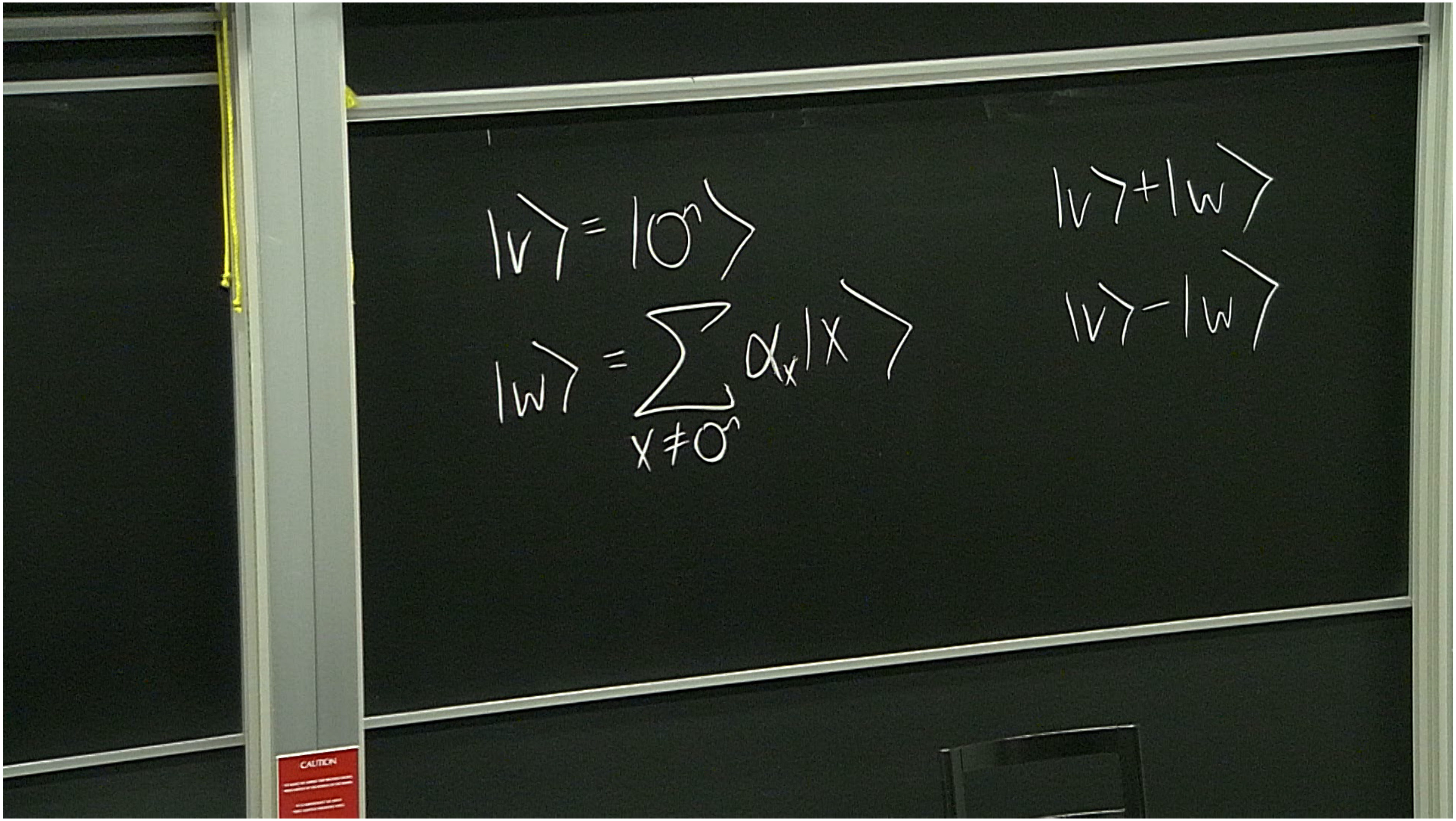
$$U|w\rangle = |v\rangle$$

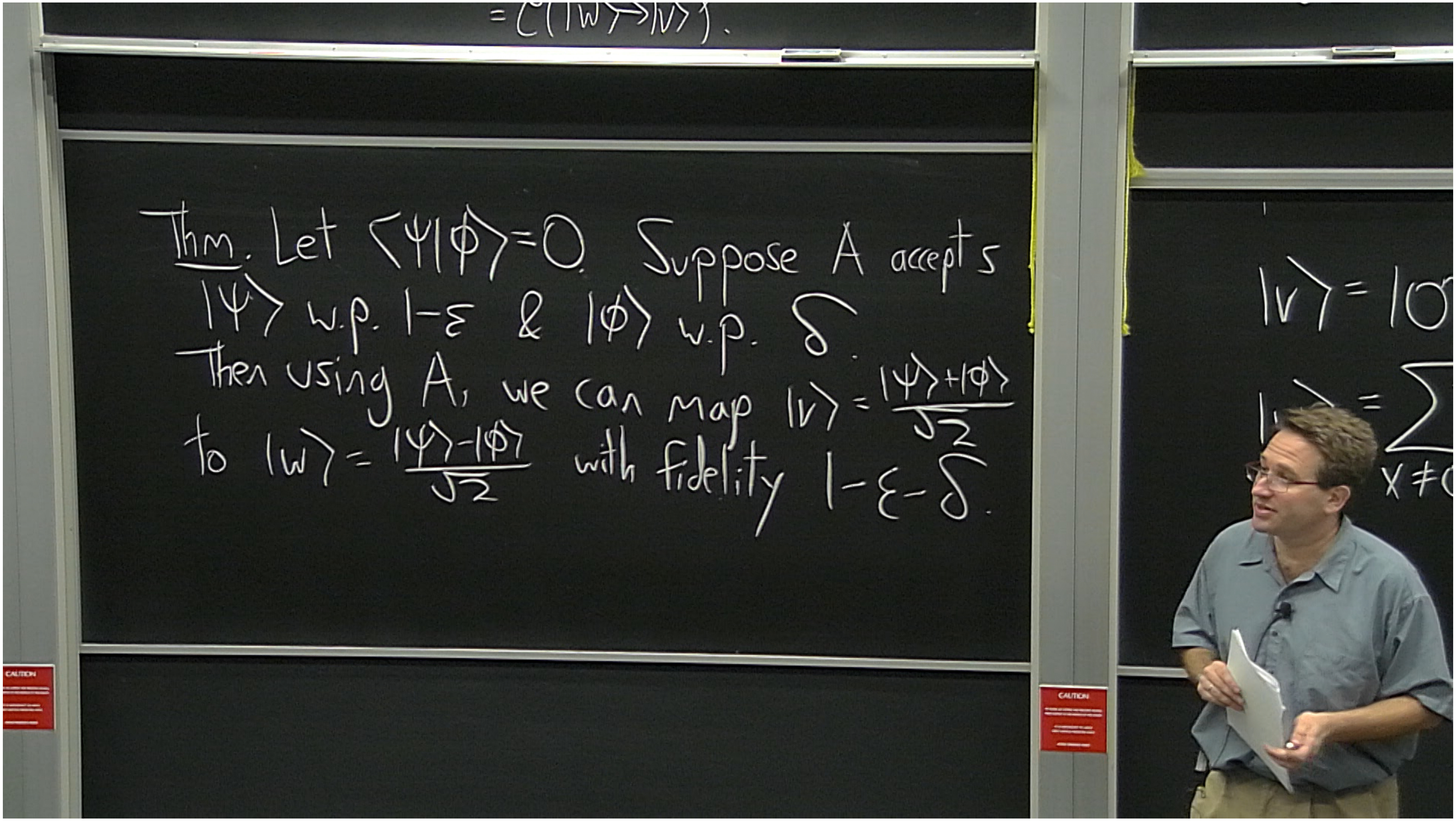
$$\Longleftarrow \quad \Longrightarrow$$

$$|+\rangle|u\rangle$$

$$C|0^n\rangle = |\psi\rangle$$

Thm. Let $\langle v|w\rangle = 0$, suppose $\langle w|U|v\rangle = a$, $\langle v|U|w\rangle = b$. Then using U, we can distinguish $|\psi\rangle = \frac{|v\rangle + |w\rangle}{\sqrt{2}}$ from $|\phi\rangle = \frac{|v\rangle - |w\rangle}{\sqrt{2}}$ with bias $\frac{|a+b|}{2}$.


$|v\rangle = |0^n\rangle$
$|w\rangle = \sum_{x \neq 0^n} \alpha_x |x\rangle$
$|v\rangle + |w\rangle$
$|v\rangle - |w\rangle$
CAUTION


Thm. Let $\langle\psi|\phi\rangle = 0$. Suppose A accepts $|\psi\rangle$ w.p. $1-\varepsilon$ & $|\phi\rangle$ w.p. δ.
Then using A, we can map $|v\rangle = \frac{|\psi\rangle + |\phi\rangle}{\sqrt{2}}$ to $|w\rangle = \frac{|\psi\rangle - |\phi\rangle}{\sqrt{2}}$ with fidelity $1-\varepsilon-\delta$.

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$\frac{|\mathrm{Re}(a+b)|}{2}$

Let $|v\rangle = \dfrac{|0\rangle|n\rangle + |1\rangle|0^n\rangle}{\sqrt{2}}$

$|w\rangle = \dfrac{i|0\rangle|n\rangle - i|1\rangle|0^n\rangle}{\sqrt{2}}$

$$U|v\rangle = |w\rangle$$

$$U|w\rangle = -|v\rangle$$

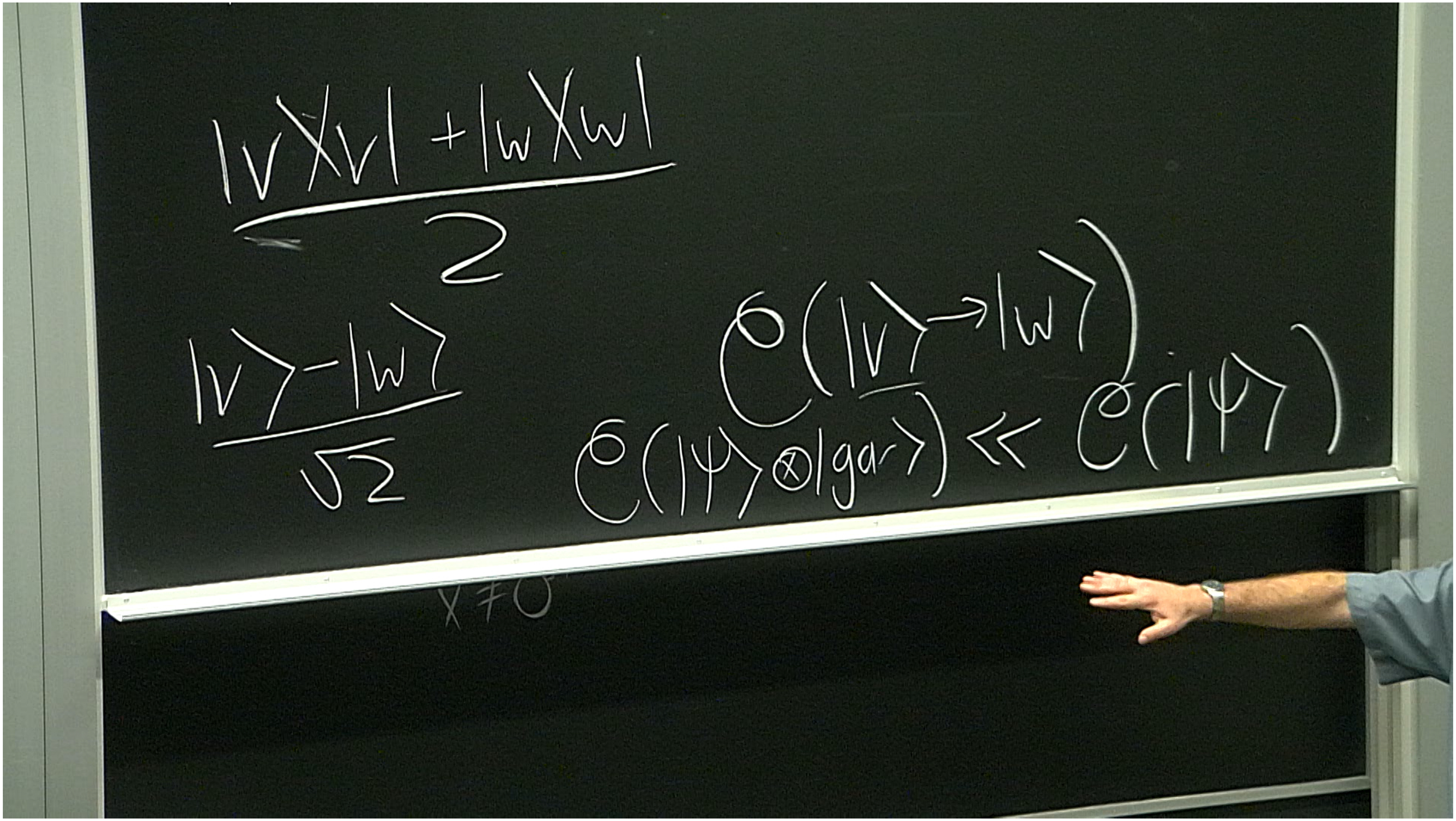
Suppose $U|v\rangle = |w\rangle$
$U|w\rangle = |v\rangle$

$$U|1\rangle|0^n\rangle = U\left(\frac{|v\rangle + i|w\rangle}{\sqrt{2}}\right)$$

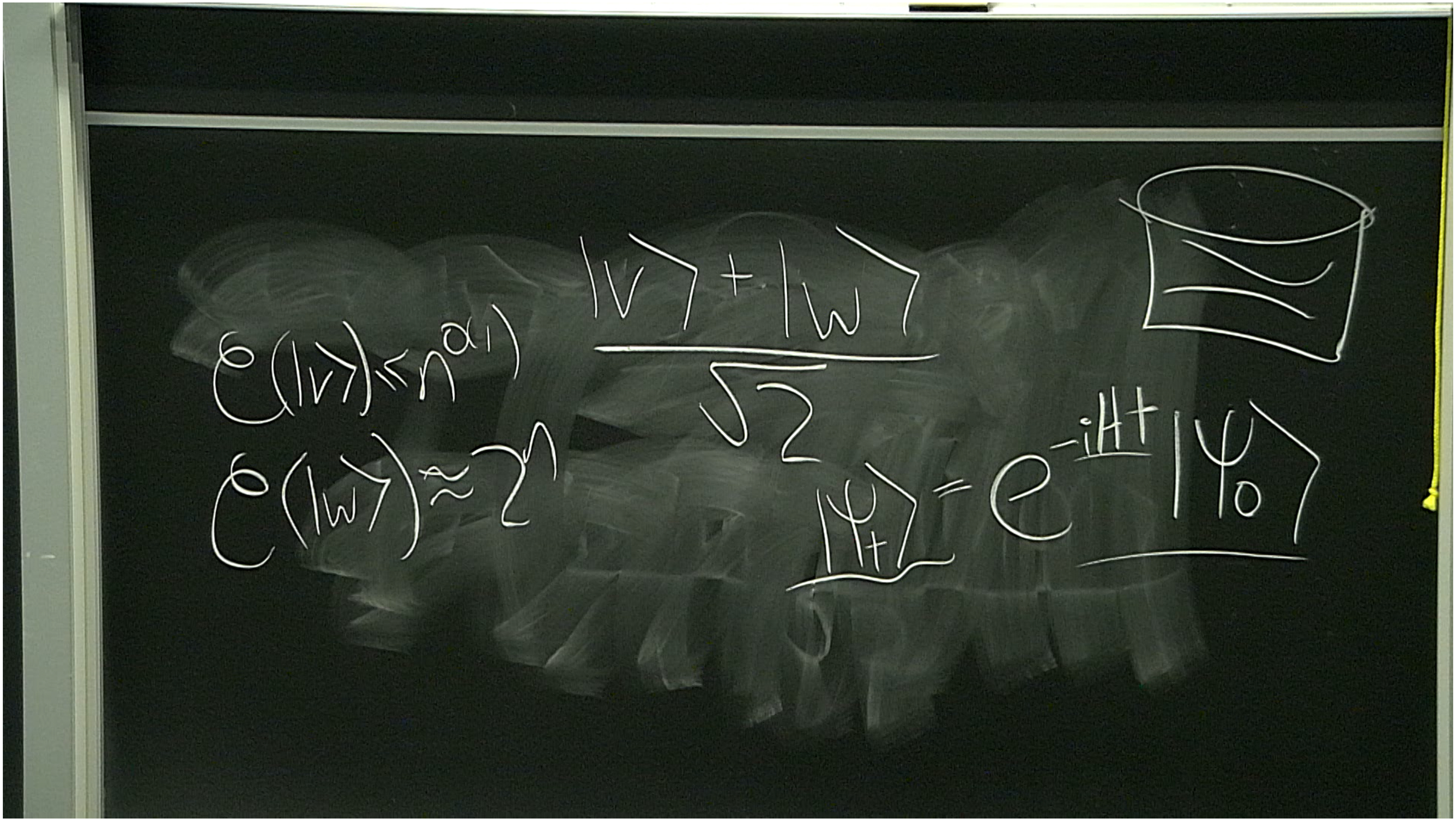
$$= \frac{|w\rangle + i|v\rangle}{\sqrt{2}}$$

$$= i\left(\frac{|v\rangle - i|w\rangle}{\sqrt{2}}\right)$$

$$= i|0\rangle|\Omega\rangle.$$



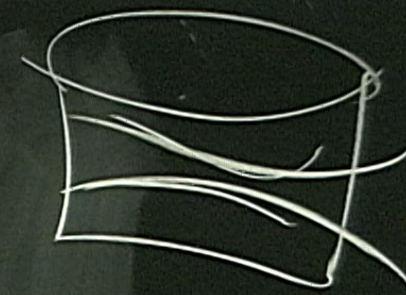
$\frac{|v\rangle\langle v| + |w\rangle\langle w|}{2}$
$\frac{|v\rangle - |w\rangle}{\sqrt{2}}$
$C(|v\rangle \to |w\rangle)$
$C(|\psi\rangle \otimes |gar\rangle) \ll C(|\psi\rangle)$
$x \neq 0$


$e(|v\rangle) \approx n^{\alpha_1}$
$e(|w\rangle) \approx 2^n$
$\frac{|v\rangle + |w\rangle}{\sqrt{2}}$
$|\psi_t\rangle = e^{-iHt}|\psi_0\rangle$

$\mathcal{C}(|v\rangle) \leq n^{\alpha_1}$

$\mathcal{C}(|w\rangle) \approx 2^n$

$$\frac{|v\rangle \oplus |w\rangle}{\sqrt{2}}$$



$$\boxed{|\psi_t\rangle} = \frac{e^{-iHt}|\psi_0\rangle}{\mathcal{C}(|\psi_t\rangle)}$$

$$\frac{|\psi_0\rangle \oplus |\psi_{2^n}\rangle}{\sqrt{2}}$$