Title: Gauge theory and mirror symmetry I

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Series: Mathematical Physics

Date: January 23, 2020 - 1:30 PM

URL: http://pirsa.org/20010096

Abstract: This talk will cover my interpretation of Teleman's article "Gauge theory and mirror symmetry."

3d mirror symmetry

$\Longleftrightarrow$

boundary conditions for B-twisted 3d $N=4$ σ-model with target $T^*G^L_{\mathbb{C}}/\!/G^L_{\mathbb{C}}$

$\quad = \mathrm{BFM}(G)$ Kapustin Rozansky Saulina

$\|$

holomorphic Lagrangians in $\mathrm{BFM}(G)$ equipped with coherent sheaf categories

Q: what is an action of a group G on a Fukaya category

boundary conditions for A-twisted 3d $N=4$ pure G-gauge theory

$\cup|$

A-type G actions on categories

3d mirror symmetry

$\Longleftrightarrow$

Q: what is an action of a group G on a Fukaya category

boundary conditions for A-twisted 3d $N=4$ pure G-gauge theory

$\cup|$

A-type G actions on categories $= G \circlearrowright$ A-twisted 2d $N=(2,2)$ theories $= \mathcal{T}_{2d}$

3d mirror symmetry $\Longleftrightarrow$

boundary con[...] for B-tw[...] 3d $N=4$ with targe[...]

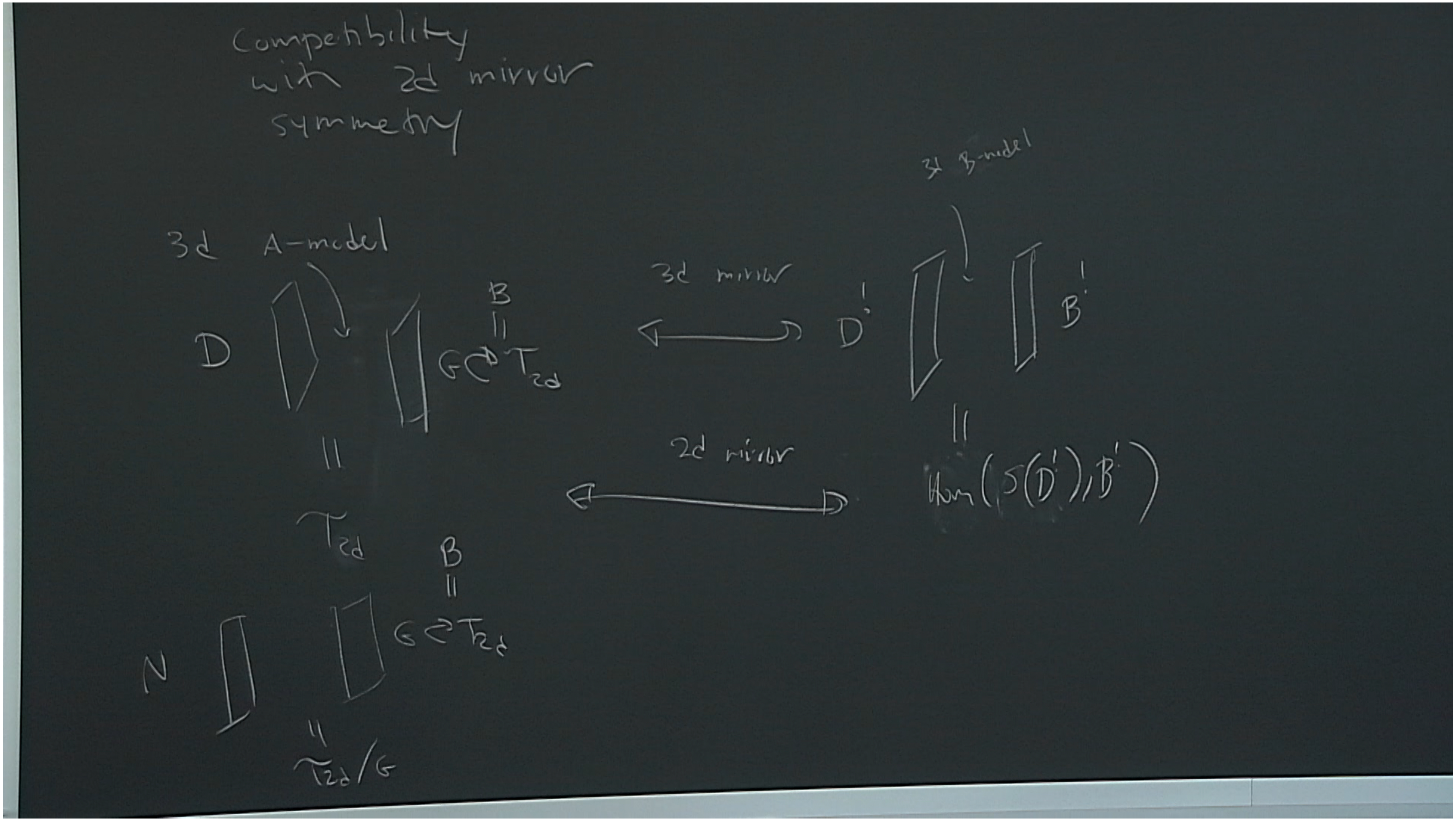
$\|$

holomorphic [...] in $BFM($[...] with coheren[...] categories

Special boundary conditions that exist in any gauge theory

Dirichlet, Neumann

$G \circlearrowright$ Vect trivial


Compatibility
with 2d mirror
symmetry
3d B-model
3d A-model
3d mirror
B
||
$G \circlearrowright T_{2d}$
D
D^!
B^!
||
2d mirror
$\mathrm{Hom}(S(D^!), B^!)$
||
T_{2d}
B
||
$G \circlearrowright T_{2d}$
N
||
T_{2d}/G

A-type actions on categories

2 Flavors: de Rham , $\boxed{\text{Betti}}$

Definitions

- Sheaves categories on BG equipped with a flat connection

$$\left[\mathrm{ShvCat}(BG_{dR}) \right]$$

$G \circlearrowright \mathcal{C}$ in a smooth way

- $\left[\mathrm{ShvCat}(BG) \right]$

Sheaves categories on BG equipped with an integrable connection

$$\left[\mathrm{ShvCat}(BG_B) \right]$$

Sheaves categories on BG equipped with an integrable connection

$$\left[\mathrm{ShvCat}(BG_B) \right]$$

you can differentiate the action. can ask that this be trivializable

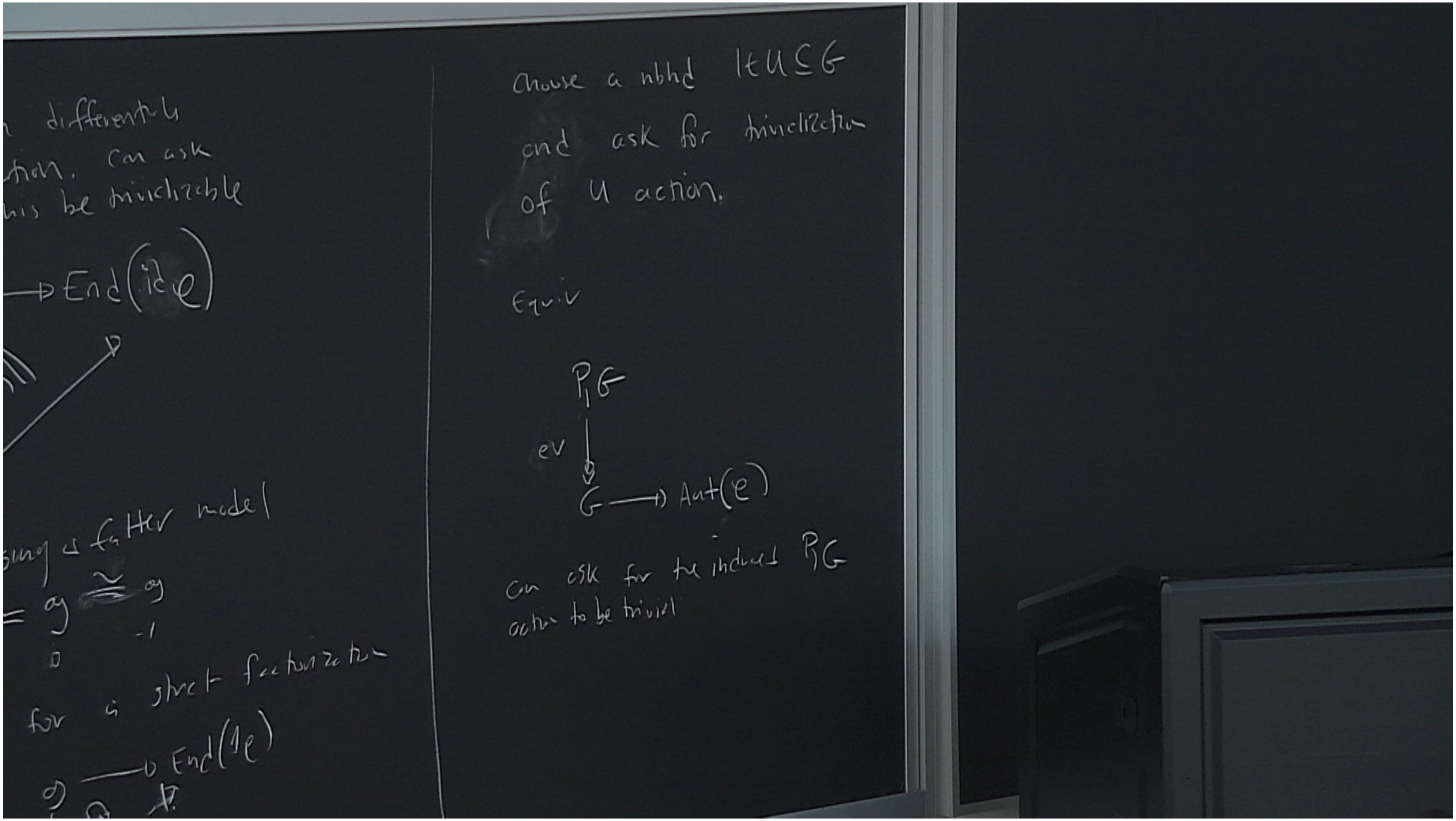
$$\begin{tikzcd} \mathfrak{g} \arrow[r] \arrow[d] & \mathrm{End}(\mathrm{id}_{\mathcal{C}}) \\ \mathcal{O} \arrow[ur, "\cong"] & \end{tikzcd}$$

By choosing a fatter model for $\mathcal{O} = \mathfrak{g}_0 \cong \mathfrak{g}_{-1}$

Can ask for a shuck factorization

$$\begin{tikzcd} \mathfrak{g} \arrow[r] \arrow[d] & \mathrm{End}(1_{\mathcal{C}}) \\ \mathfrak{g} \ast \mathfrak{g} \arrow[ur, dashed] & \end{tikzcd}$$

Choose a nbhd and ask for


differentials
tion. Can ask
this be trivializable
End(id C)
father model
for is strict factorization
End(1C)
Choose a nbhd 1∈U⊆G
and ask for trivialization
of U action.
Equiv
P1G
ev
G → Aut(C)
Can ask for the induced P1G
action to be trivial

dimension example

$G \circlearrowright X$

$\mathfrak{g} \circlearrowright \Omega^\bullet(X) \Longleftrightarrow$
L

$$v \longmapsto L_v$$

$$\begin{tikzcd} \mathfrak{g} \arrow[r, "L"] \arrow[d] & \mathrm{End}((\Omega^\bullet(X), d)) \\ \mathfrak{g} & \\ \mathfrak{g} \arrow[u] \arrow[uur, "L \oplus i"'] & \end{tikzcd}$$

$0 =$

Cartan magic formula

$$[d, i_v] = L_v$$

<u>Ex</u> There is a strong action of $\mathfrak{g}$ on $D\text{-mod}(X)$.

you can differentiate the action. Can ask that this be trivializable

$$\begin{tikzcd} \mathfrak{g} \arrow[r] \arrow[d] \arrow[dr, phantom, "\cong"] & \mathrm{End}(\mathrm{id}_{\mathcal{C}}) \\ \mathcal{O} \arrow[ur] & \end{tikzcd}$$

By choosing a fatter model for $\mathcal{O} = \mathfrak{g}_0 \simeq \mathfrak{g}_{-1}$

can ask for a strict factorization

$$\begin{tikzcd} \mathfrak{g} \arrow[r] \arrow[d] & \mathrm{End}(1_{\mathcal{C}}) \\ \cdot \arrow[ur, dashed] & \end{tikzcd}$$

Choose a nbhd $1 \in U \subseteq G$ and ask for trivialization of U action.

$\Longleftrightarrow$

$$\begin{tikzcd} \Omega G \arrow[r] & P_1 G \subseteq \mathrm{Maps}([0,1], G) \arrow[d, "ev"'] & \\ & G \arrow[r] & \mathrm{Aut}(\mathcal{C}) \end{tikzcd}$$

Can ask for the induced $P_1 G$ action to be trivial

taking difference of two trivializations gives

$$\Omega G \longrightarrow \mathrm{Aut}(\mathrm{id}_{\mathcal{C}})$$

$$\left(C_*(\Omega G) \longrightarrow \mathrm{End}(\mathrm{id}_{\mathcal{C}}) = HH^*(\mathcal{C}) \right.$$

$\mathrm{End}(1_e)$

has two compositions

loop composition $=$
$$\begin{tikzcd} e \arrow[r, bend left=60, "1_e"] \arrow[r, "1_e" description] \arrow[r, bend right=60] & e \arrow[l, phantom, shift right=3, "\alpha\Downarrow"] \arrow[l, phantom, shift left=3, "\beta\Downarrow"] \end{tikzcd}$$
or
$$\begin{tikzcd} e \arrow[r, bend left, "1_e"] \arrow[r, bend right, "1_e"'] \arrow[r, phantom, "\alpha\Downarrow"] \arrow[rr, bend left=70, "1_e"] \arrow[rr, bend right=70, "1_e"'] & e \arrow[r, bend left, "1_e"] \arrow[r, bend right, "1_e"'] \arrow[r, phantom, "\beta\Downarrow"] & e \end{tikzcd}$$
$\|$

group composition

$U \subseteq G$

...ivialization

...$, G)$

...$)$

...ed P_G

trivializations in

From now on our def will be an E_2-map

$$C_\bullet(\Omega G) \to HH^\bullet(\mathcal{C})$$

$$\|$$

$$\mathcal{C} \in \mathrm{ShvCat}\left(\mathrm{Spec}\, C_\bullet(\Omega_G)\right)$$

$\mathrm{End}(\mathbb{1}_{\mathcal{C}})$

has two compositions

loop composition $=$

$$\begin{tikzcd} \bullet \arrow[r, bend left, "\mathbb{1}_{\mathcal{C}}"] \arrow[r, "\mathbb{1}_{\mathcal{C}}" description] \arrow[r, bend right] & \bullet \end{tikzcd} \quad \alpha \Downarrow,\ \beta \Downarrow$$

or

$$\begin{tikzcd} \bullet \arrow[r, bend left, "\mathbb{1}_{\mathcal{C}}"] \arrow[r, bend right, "\mathbb{1}_{\mathcal{C}}"'] \arrow[r, phantom, "\Downarrow\alpha"] & \bullet \end{tikzcd} \quad \mathbb{1}_{\mathcal{C}},\ \mathbb{1}_{\mathcal{C}}$$

$\|$

group composition

From now on our def will be an E_2-map

$$C_*(\Omega G) \to HH^*(\mathcal{C}) = \mathrm{End}(1_{\mathcal{C}})$$

$$\|$$

$$\mathcal{C} \in \mathrm{ShvCat}\left(\mathrm{Spec}\, C_*(\Omega_G)^\vee\right)$$

$\mathrm{End}(1_{\mathcal{C}})$ has two compositions

loop composition $=$

$$\begin{tikzcd} \bullet \arrow[r, bend left=60, "1_{\mathcal{C}}"{name=A}] \arrow[r, "1_{\mathcal{C}}"{name=B, description}] \arrow[r, bend right=60, ""{name=C}] & \bullet \arrow[from=A, to=B, phantom, "\alpha\Downarrow"] \arrow[from=B, to=C, phantom, "\beta\Downarrow"] \end{tikzcd}$$

or

$$\begin{tikzcd} \bullet \arrow[r, bend left, "1_{\mathcal{C}}"{name=A}] \arrow[r, bend right, "1_{\mathcal{C}}"'{name=B}] \arrow[rr, bend left=70, "1_{\mathcal{C}}"] \arrow[rr, bend right=70, "1_{\mathcal{C}}"'] & \bullet \arrow[r, bend left, "1_{\mathcal{C}}"{name=C}] \arrow[r, bend right, "1_{\mathcal{C}}"'{name=D}] & \bullet \arrow[from=A, to=B, phantom, "\alpha\Downarrow"] \arrow[from=C, to=D, phantom, "\beta\Downarrow"] \end{tikzcd}$$

$\|$

group composition

$$\underline{BFM(G)} := \operatorname{Spec} H_\bullet^G(\Omega G)$$

Lagrangians $\cup\!|$

$$C := \operatorname{Spec} H_\bullet(\Omega G)$$

<u>Ex</u>

G acts Hamiltonianly on symplectic manifold X

$$\Longleftrightarrow$$

An action of symplectic groupoid T^*G on X

$$\text{act} = \{((g, \mu(x)), x, gx)\}$$

$$T^*G \times X \longleftarrow \text{act} \longrightarrow X$$

Q: what is

of a gr

a Fukay

boundary con

for A-twist

pure G-

$\cup\!|$

A-type G

Spec $H_0^G(\Omega G)$

... on symplectic

...plectic groupoid

...), x, gx) }

Assuming functoriality of Fuk^{wr} under Lagrangian correspondence

$\mathrm{Fuk}^{wr}(T^*G)$ monoidal

$\mathrm{Fuk}^{wr}(M)$

Pascaleff – Poisson ... on symplectic groupoids

3d mirror symmetry $\Longleftrightarrow$

boundary con...
for B-tw...
3d $N=4$
with targe...

||

holomorphic
in BFM(...
with coherent categories

$$\begin{array}{c} BFM(G) := \operatorname{Spec} H_\bullet^G(\Omega G) \\ \cup\! \mid \end{array}$$

Lagrangians

$$C := \operatorname{Spec} H_\bullet(\Omega G)$$

Ex

$$\begin{tikzcd} T^*G \arrow[d, shift right, "s"'] \arrow[d, shift left, "t"] \\ \mathfrak{g}^* \end{tikzcd}$$

G acts Hamiltonianly on symplectic manifold X

$\iff$

An action of symplectic groupoid T^*G on X

$$act = \{((g, \mu(x)), x, gx)\}$$

$$\begin{tikzcd} & act \arrow[dl] \arrow[dr] & \\ T^*G \times X & & X \end{tikzcd}$$

Assuming ...

of Fuk^{wr} ...

Lagrangian ...

$T_1^*G \in Fuk^{wr}(T^*G$ ...

monoidal ...

$Fuk^{wr}(M$ ...

Pascaleff – Poi...

$H_0^G(\Omega G)$

symplectic

groupoid

$, gx) \}$

Assuming functoriality of Fuk^{wr} under Lagrangian correspondence

$T_1^*G \in \mathrm{Fuk}^{wr}(T^*G) \cong \mathrm{End}(T_1^*G)$-mod

monoidal

$\mathrm{Fuk}^{wr}(M)$

de Rham

$D(G)$

$D(X)$

Pascaleff – Poisson ...
and
symplectic
groupoids

Assuming functoriality of Fuk^{wr} under Lagrangian correspondence

$$C_\bullet(\Omega G)$$
$$\shortparallel$$
$$T_1^* G \in \underbrace{\mathrm{Fuk}^{wr}(T^*G)}_{\text{monoidal}} \cong \mathrm{End}(T_1^*G)\text{-mod}$$

$$\begin{tikzcd} \mathrm{Fuk}^{wr}(M) \arrow[loop above] \end{tikzcd}$$

$$\left[\begin{array}{c} \text{de Rham} \\ D(G) \\ \circlearrowright \\ D(X) \end{array}\right]$$

$$\mathcal{M} \curvearrowleft \mathcal{C}$$
$$\downarrow$$
$$1_{\mathcal{M}}$$

$$\mathrm{End}(1_{\mathcal{M}}) \longrightarrow HH^\bullet(\mathcal{C})$$

Pascaleff – Poisson ...
and symplectic groupoids

...ming functoriality

Fuk^{wr} under

Lagrangian correspondence

$C_*(\Omega G)$

$\|$

$Fuk^{wr}(T^*G) \cong End(T^*_1 G)$ - mod

monoidal

$\circlearrowright$

$Fuk^{wr}(M)$

$$\left[\begin{array}{c} \text{de Rham} \\ D(G) \\ \circlearrowright \\ D(X) \end{array}\right]$$

coleff - Poisson ...

and

symplectic

groupoids

$\mathcal{M} \curvearrowright \mathcal{C}$

$\downarrow$

$1_{\mathcal{M}}$

$End(1_{\mathcal{M}}) \to HH(\mathcal{C})$

$Fuk^{wr}(T^*M)$

$\|$

$Shv_c(M)_M$

$Fuk^{wr}(T^*M, \Delta)$

$\|$

$Shv_c(M)_\Delta$

$G = U(1)$

$\Omega U(1) = \mathbb{Z}$

$\mathbb{C}^\times = \mathrm{Spec}\ \mathbb{C}(\Omega U(1))$

$\cdots \to HH(\mathcal{C})$

$= \mathrm{Loc}(M)$

$$\begin{array}{c} B^2\mathbb{Z} \\ \shortparallel \\ \mathrm{ShvCat}(BS^1) \end{array} \cong \mathrm{ShvCat}(\mathbb{C}^\times)$$

This a particular example of higher Cartier duality

$$\mathrm{ShvCat}(B^2 A) \cong \mathrm{ShvCat}(\hat{A})$$

$$\hat{A} = \mathrm{Hom}(A, \mathbb{G}_m)$$

$$\mathrm{QCoh}(BA) \cong \mathrm{QCoh}(A^\wedge)$$

$$\mathcal{O}_A \overset{\text{dual}}{\cong} \mathcal{O}_{A^\wedge}$$