Title: A bulk-boundary correspondence with factorization algebras

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Series: Mathematical Physics

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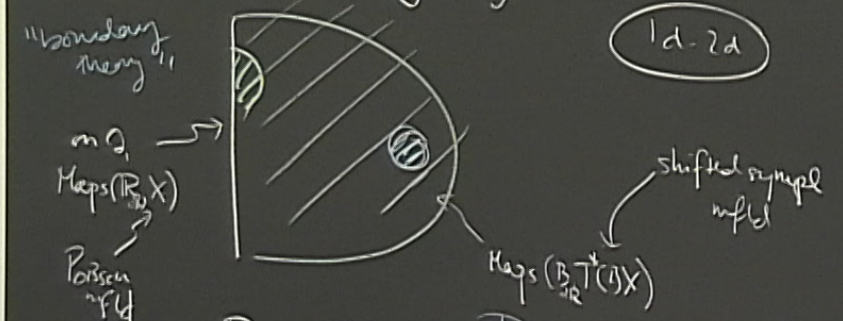
Abstract: Factorization algebras provide a flexible language for describing the observables of a perturbative QFT, as shown in joint work with Kevin Costello. In joint work with Eugene Rabinovich and Brian Williams, we extend those constructions to a manifold with boundary for a special class of theories that includes, as an example, a perturbative version of the correspondence between chiral U(1) currents on a Riemann surface and abelian Chern-Simons theory on a bulk 3-manifold. Given time, I'll sketch a systematic higher dimensional version for higher abelian CS theory on an oriented smooth manifold of dimension 4n+3 with boundary a complex manifold of complex dimension 2n+1. (This talk will be very informal.)

Sketch the kind of situation we'll formalize in this talk

[Joint work w/ Eugene Rabinovich & Brian Williams]

bulk - boundary systems:

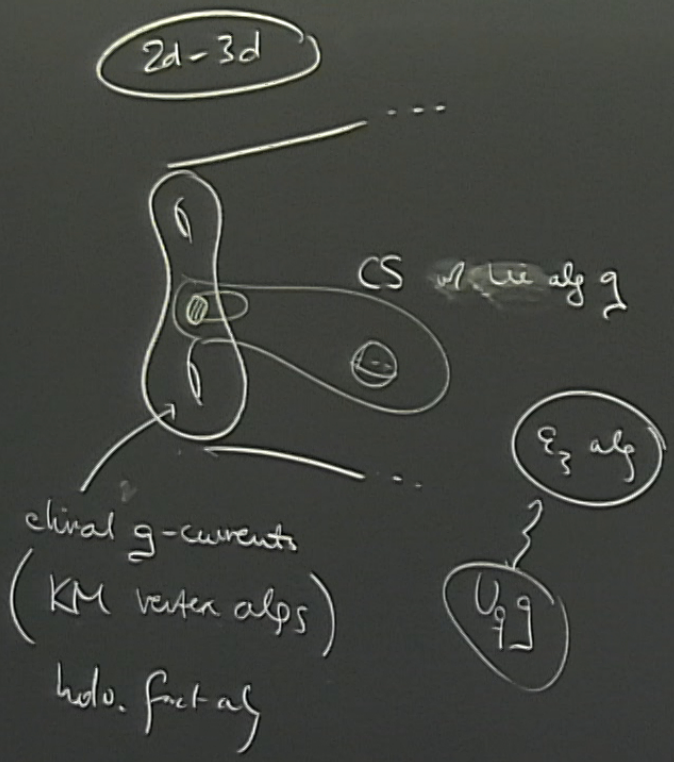


$\mathbb{R}_{\geq 0} \times \mathbb{R}$ Poisson σ-model

Kontsevich def quant 2d TFT

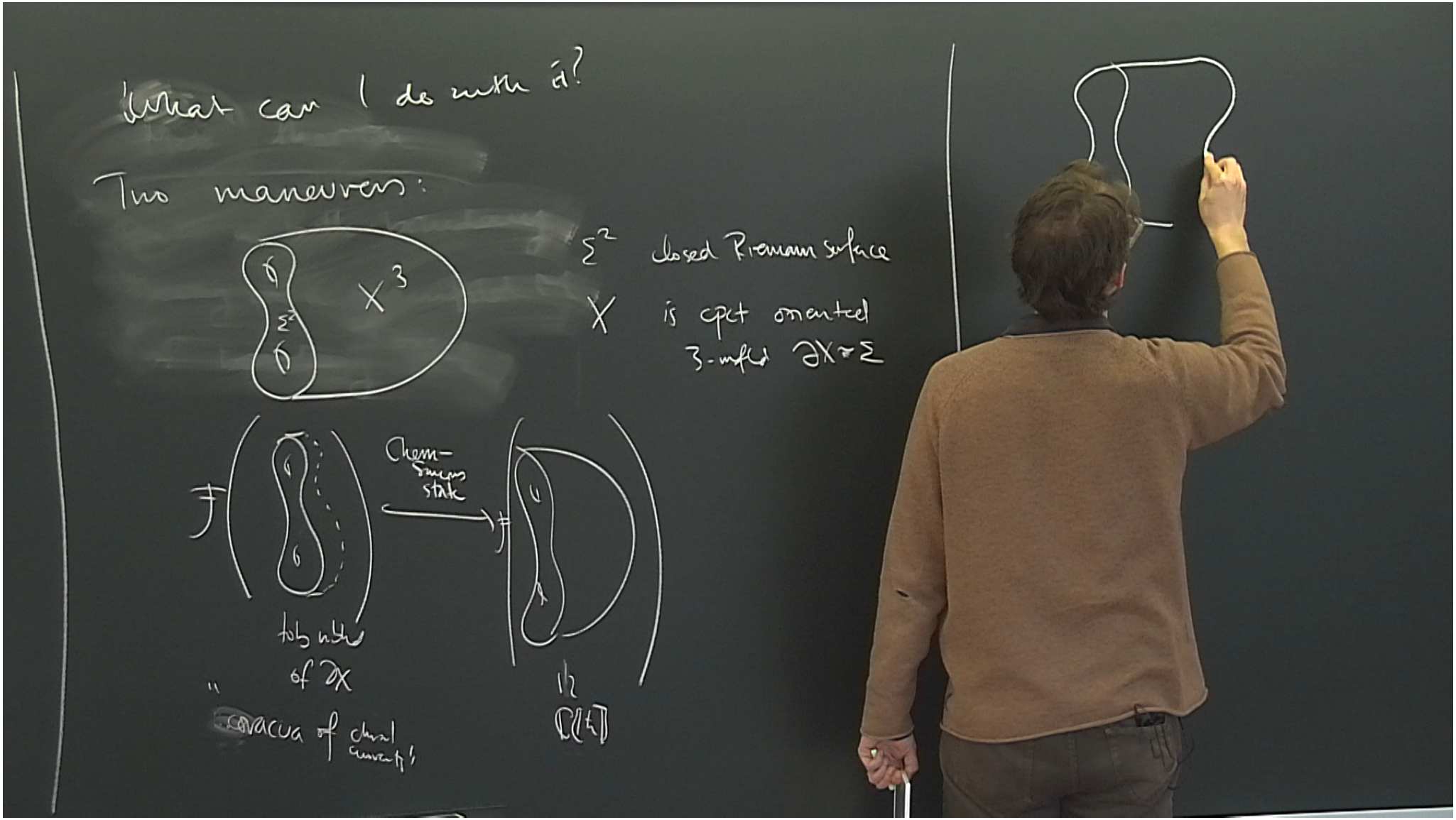
	$\mathcal{E}_1$ boundary	$\mathcal{E}_2$ bulk
classical	$(\mathcal{O}(X), \cdot)$ comm	$\mathrm{Hoch}^*(\mathcal{O}(X)) \simeq PV(X)$
Quantum	$\mathcal{O}(X)[[\hbar]], \star_\hbar$ def quant	$\mathrm{Hoch}^*(\mathcal{O}^q(X))$ $\mathcal{O}^q(X)$

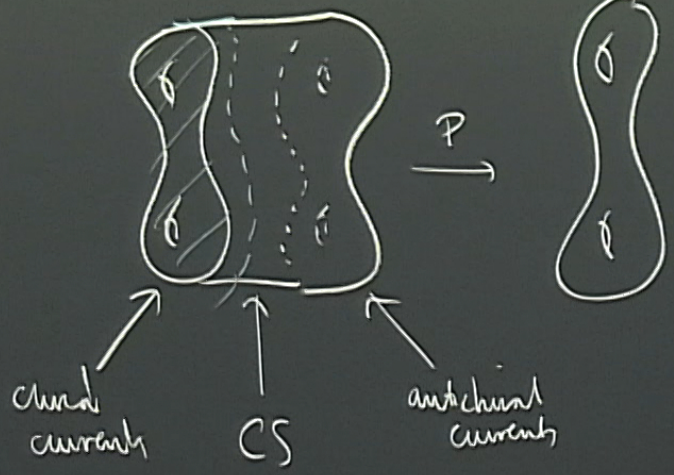
2d



"Meta" theorem

Given a bulk-boundary theory on a manifold M w/ boundary ∂M

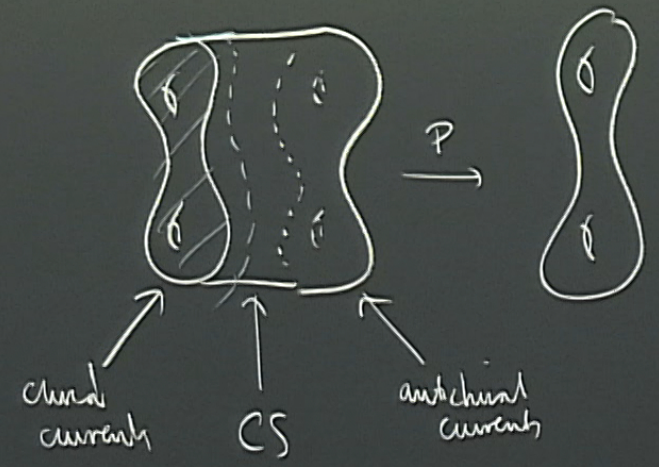

What can I do with it?
Two maneuvers:
Σ^2 closed Riemann surface
X is cpct oriented 3-mfld $\partial X = \Sigma$
X^3
Σ^2
Chern-Simons state
fob with ∂X
"
vacua of chiral currents
$\mathbb{C}(\!(t)\!)$



$P_*\left(\mathcal{T}\big|_{\Sigma\times[0,\varepsilon)}\right) \longrightarrow P_*\mathcal{T}$

chiral

fact alg for a pert. version of full WZW



$P_*\left(\mathcal{F}\big|_{\Sigma\times[0,\varepsilon)}\right) \longrightarrow P_*\mathcal{F}$

chiral

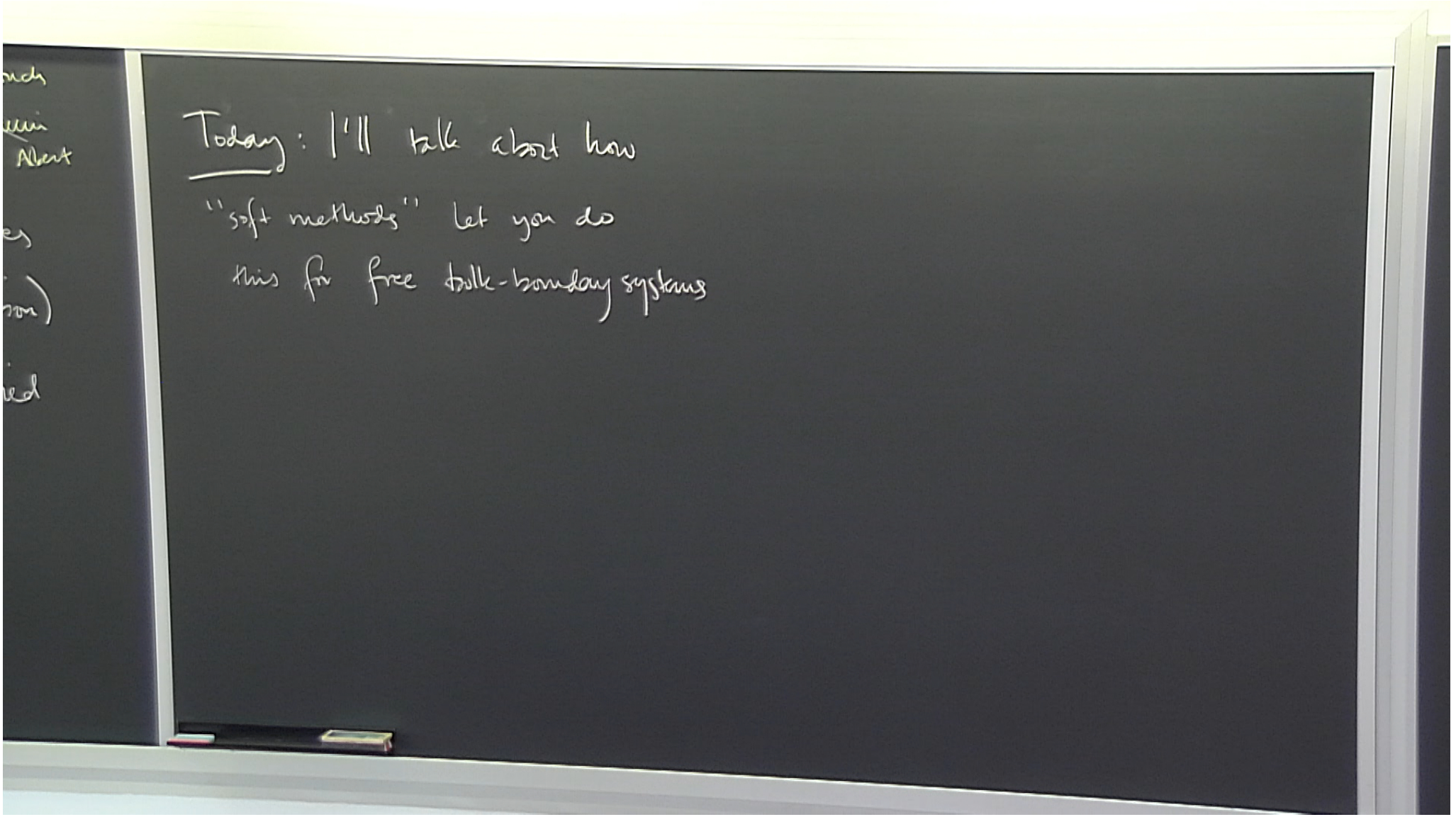
fact alg for a pert. version of full WZW

work in progress of E. Rabinovich & Benjamin Albert

Goal: to develop a class of bulk-boundary BV theories (modelled on Costello's bulk version) such that you obtain stratified fact algs $\mathcal{O}bs$

$$\mathcal{O}bs^{BB}\big|_{bulk} \simeq \mathcal{O}bs_{bulk}$$

$$\mathcal{O}bs^{BB}\big|_{\partial M} \simeq \mathcal{O}bs_{Boundary\ theory}$$


Today: I'll talk about how
"soft methods" let you do
this for free bulk-boundary systems

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Set up

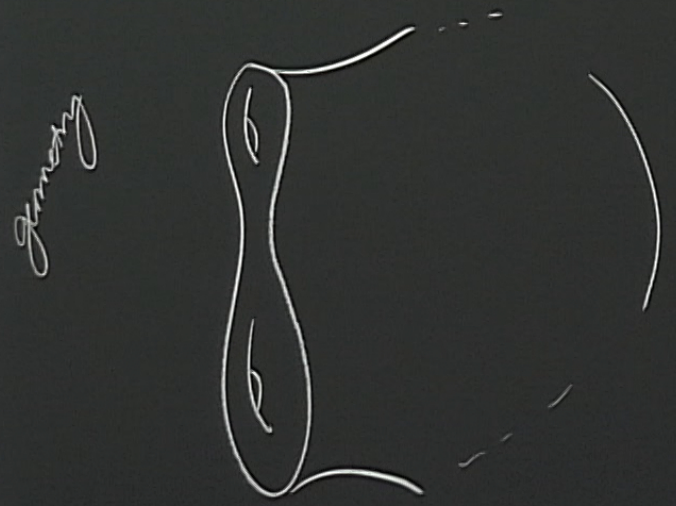
M – mfld w/ bdry ∂M

pick a diffeo.

$U \cong \partial M \times [0, R)$

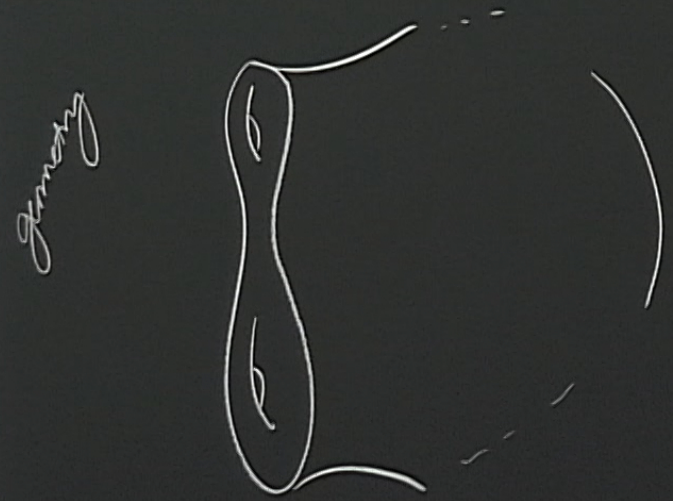
U: nbd of ∂M

collar coord



"soft methods" let you do this for free bulk-boundary systems

Set up



M – mfld w/ bdry ∂M

pick a diffeo.

$$U \cong \partial M \times [0, R)$$

U: nbhd of ∂M

collar coord

$\mathring{M} = M \setminus \partial M$ interior = bulk

ay: I'll talk about how

ft methods" let you do

his for free bulk-boundary systems

field theory: • pick a Costello-style BV theory on $\mathring{M}$

with the "property" that H is locally constant (or topological) in the normal direction

Set up

M – mfld w/ bdry ∂M

pick a diffeo.

$$U \cong \partial M \times [0, R)$$

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theory on $\mathring{M}$

with the "property" that

H is locally constant (or topological) in the normal direction

Set up

M – mfld w/ bdry ∂M

pick a diffeo.

$$U \cong \partial M \times [0,R)$$

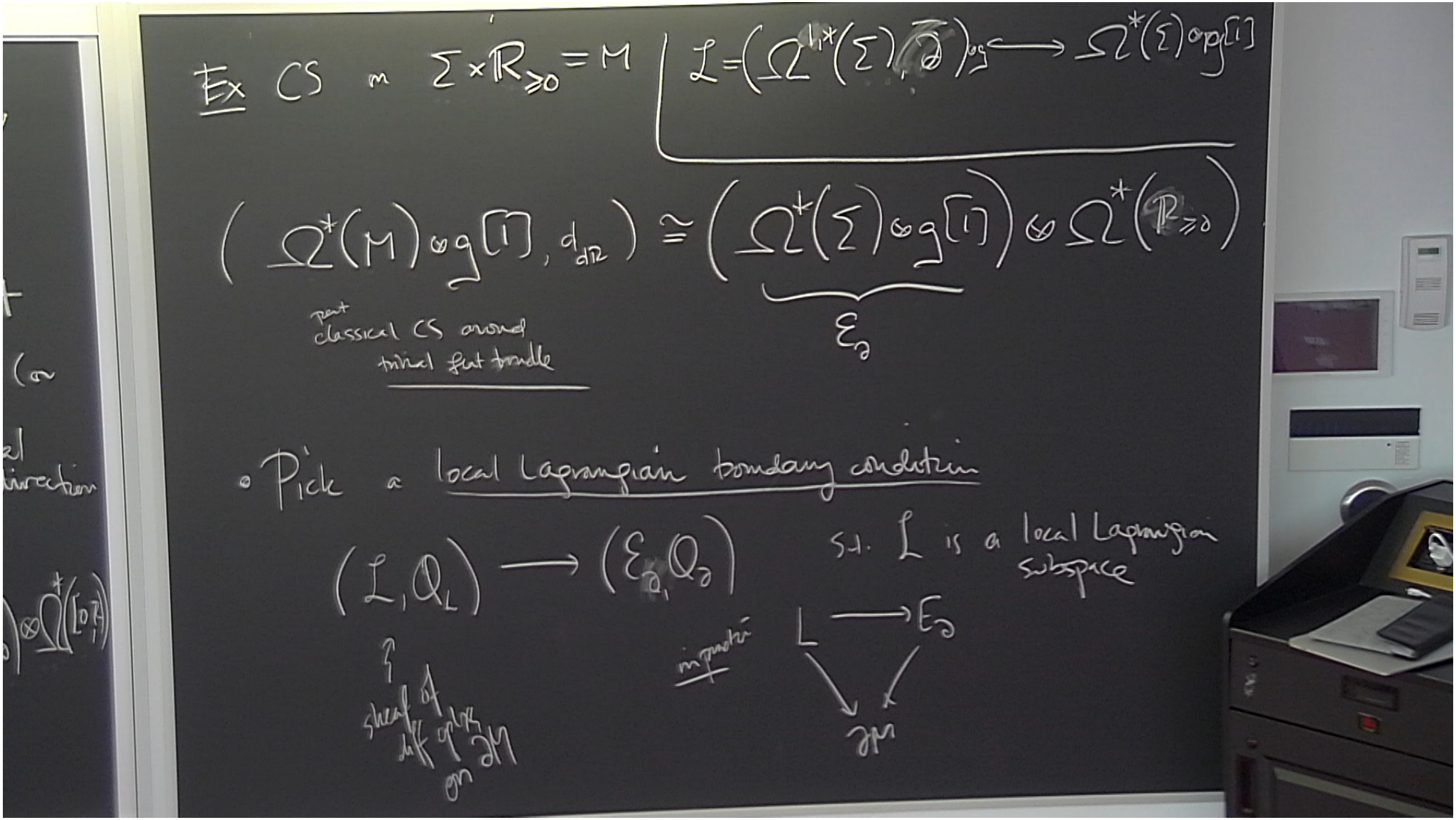
U: nbhd of ∂M; $[0,R)$: collar coord

$\mathring{M} = M \setminus \partial M$ interior = "the big bulk"

For free theories

$$\mathcal{E}(U), Q \cong \left(\mathcal{E}_\partial(\partial M), Q_\partial\right) \otimes \Omega^\bullet([0,R))$$

$\mathcal{E}(U), Q$: elliptic cplx on M; $\mathcal{E}_\partial(\partial M), Q_\partial$: elliptic cplx on ∂M


Ex CS on $\Sigma \times \mathbb{R}_{\geq 0} = M$
$\mathcal{L} = (\Omega^{1,*}(\Sigma, \mathfrak{g})_{cg} \longrightarrow \Omega^*(\Sigma) \otimes \mathfrak{g}[1]$
$(\Omega^*(M) \otimes \mathfrak{g}[1], d_{dR}) \cong (\Omega^*(\Sigma) \otimes \mathfrak{g}[1]) \otimes \Omega^*(\mathbb{R}_{\geq 0})$
$\mathcal{E}_\partial$
part classical CS around trivial flat bundle
• Pick a local Lagrangian boundary condition
$(\mathcal{L}, Q_L) \longrightarrow (\mathcal{E}_\partial, Q_\partial)$
s.t. $\mathcal{L}$ is a local Lagrangian subspace
sheaf of dgla on ∂M
important
$L \longrightarrow E_\partial$
∂M

The bulk-boundary field theory : take (homotopically correct)

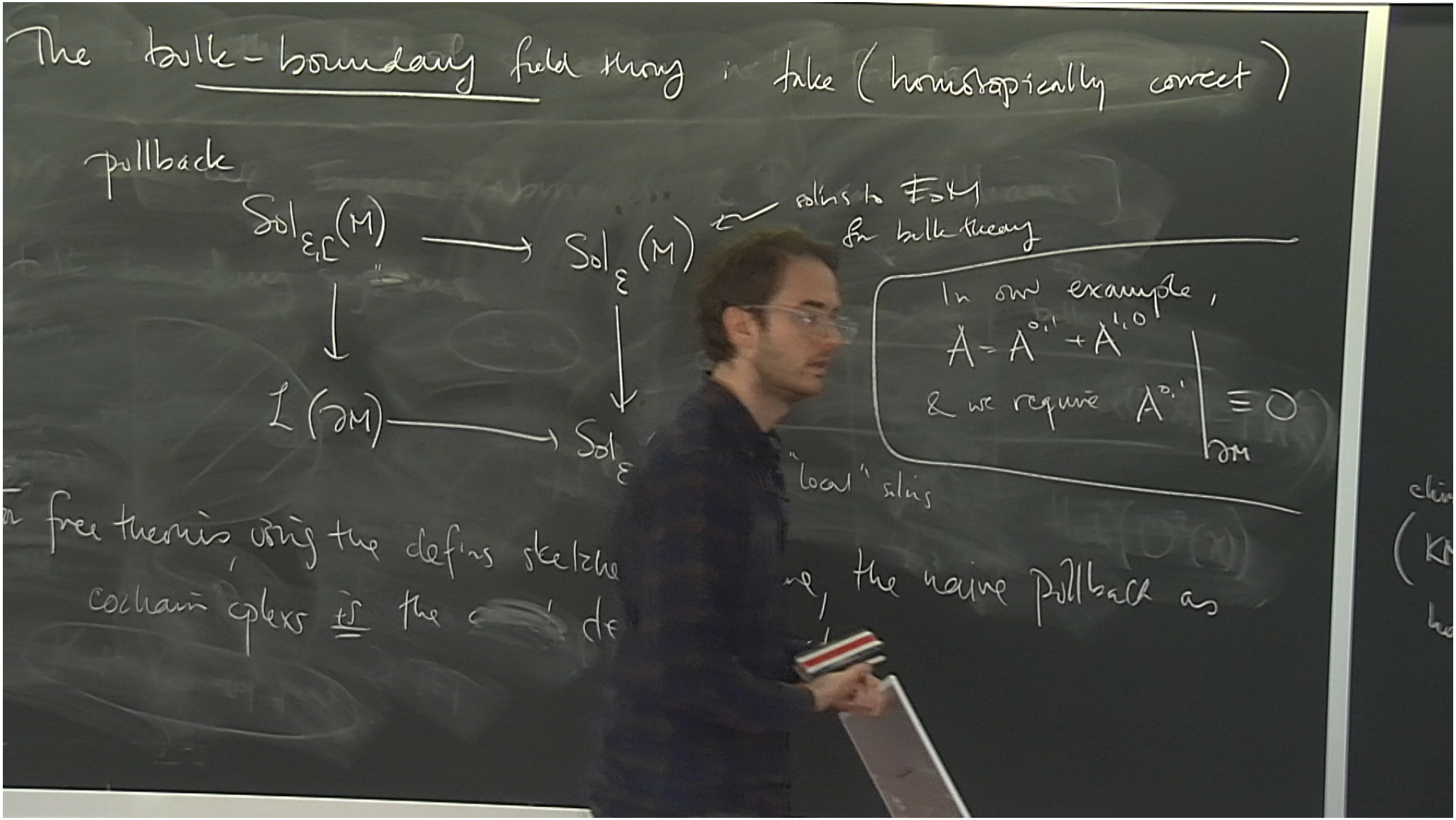
pullback

$$\begin{array}{ccc} Sol_{\mathcal{E},L}(M) & \longrightarrow & Sol_{\mathcal{E}}(M) \\ \downarrow & & \downarrow \\ \mathcal{L}(\partial M) & \longrightarrow & Sol_{\mathcal{E}_\partial}(\partial M) \end{array}$$

← solns to EoM for bulk theory

← "local" solns

For free theories, using the defns sketched over there, the naive pullback as cochain cplxs is the derived pullback.

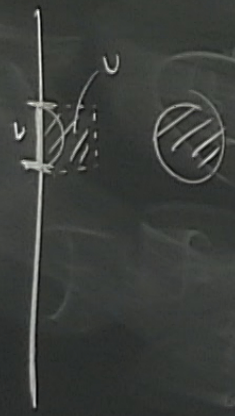

The bulk-boundary field theory : take (homotopically correct)
pullback
$Sol_{\mathcal{E},\mathcal{L}}(M)$
$Sol_{\mathcal{E}}(M)$
solns to EoM for bulk theory
$\mathcal{L}(\partial M)$
$Sol_{\mathcal{E}}$
In our example,
$\mathring{A} = A^{0,1} + A^{1,0}$
& we require $A^{0,1}\big|_{\partial M} \equiv 0$
"local" solns
free theories using the defns sketche
cochain complexes if the
the naive pullback as

if $U \subset \mathring{M}$

$$\mathcal{O}(Sol_{\mathcal{E},L}(U)) \simeq \mathcal{O}(Sol_{\mathcal{E}}(U))$$

if $U = V \times [0,\varepsilon)$, $V \overset{\text{open}}{\subset} \partial M$

$$\mathcal{O}(Sol_{\mathcal{E},L}(U)) \simeq \mathcal{O}(L(V))$$

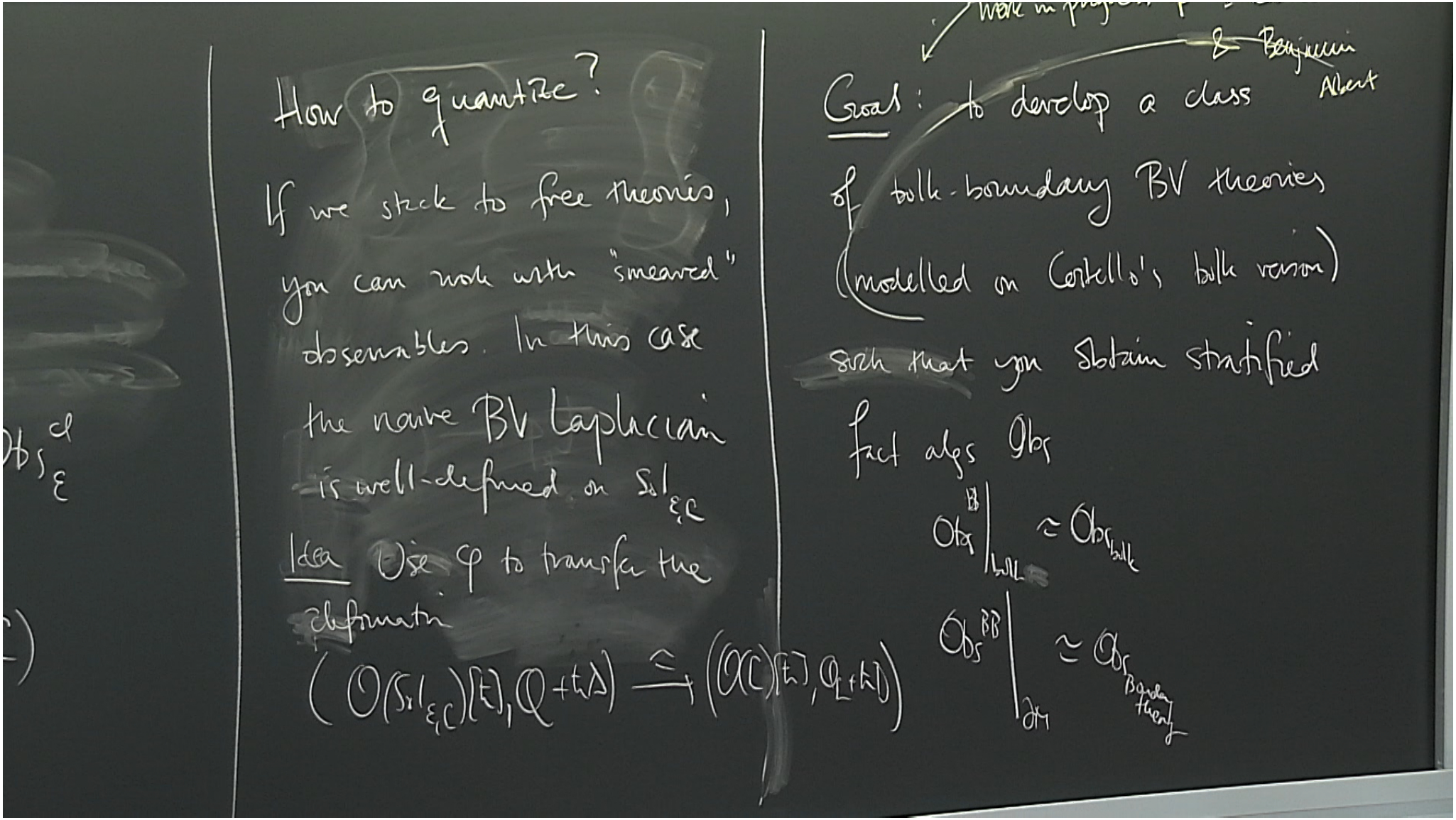


Classical level of our theorem:

$\mathcal{O}(Sol_{\mathcal{E},L}) = \mathcal{O}_{tot}^{cl}{}_{\mathcal{E},L}$ is stratified

where $\mathcal{O}bs^{cl}_{\mathcal{E},L}\big|_{\mathring{M}} \simeq \mathcal{O}bs^{cl}_{\mathcal{E}}$

$P_* \mathcal{O}bs^{cl}_{\mathcal{E},L} \simeq \mathcal{O}(L)$


How to quantize?
If we stick to free theories, you can work with "smeared" observables. In this case the naive BV Laplacian is well-defined on $Sol_{\mathcal{E},c}$
Idea Use φ to transfer the deformation
$(\mathcal{O}(Sol_{\mathcal{E},c})[\![\hbar]\!], Q+\hbar\Delta) \xrightarrow{\simeq} (\mathcal{O}(C)[\![\hbar]\!], Q_L+\hbar\Delta)$
Goal: to develop a class of bulk-boundary BV theories (modelled on Costello's bulk version) such that you obtain stratified fact algs Obs
$Obs^{BB}|_{bulk} \simeq Obs_{bulk}$
$Obs^{BB}|_{\partial M} \simeq Obs_{Boundary\ theory}$
work in progress
& Benjamin
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