Title: Accurately modelling extreme-mass-ratio inspirals: beyond the geodesic approximation

Speakers: Adam Pound

Series: Strong Gravity

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Abstract: Recent observations of gravitational waves represent a remarkable success of our theoretical models of relativistic binaries. However, accurate models are largely restricted to binaries in which the two members have roughly equal masses; for binaries with more disparate masses, modelling is less mature. This is especially relevant for extreme-mass-ratio inspirals (EMRIs), in which a stellar-mass object orbits a supermassive black hole in a galactic core. EMRIs are uniquely precise probes of black hole spacetimes, and they will be key targets for the space-based detector LISA. They are best modelled by gravitational self-force theory, in which the smaller object generates a small gravitational perturbation that reacts back on it to exert a "self-force", accelerating the object away from geodesic motion. For LISA science, we must work to second order in this perturbative treatment. In this talk, I discuss the foundations of self-force theory, its application to EMRIs, and the current status of first- and second-order models.

Accurately modelling extreme-mass-ratio inspirals: beyond the geodesic approximation

Adam Pound

Perimeter Institute

16 January 2020

University of Southampton — STAG Research Centre — The Royal Society

Gravitational waves and binary systems

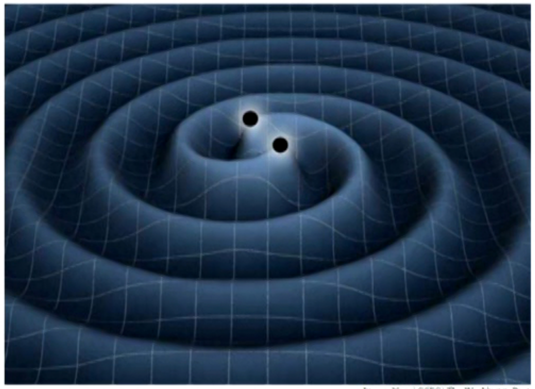


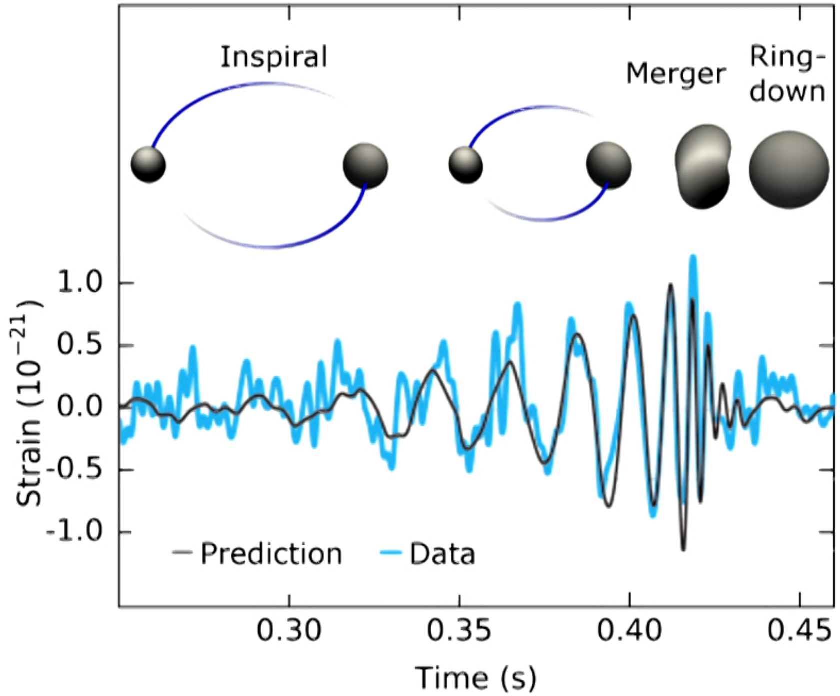
Image: Nasa/ GSFC/ The Washington Post

- compact objects (black holes or neutron stars) strongly curve the spacetime around them
- their motion in a binary generates gravitational waves, small ripples in spacetime

- waves propagate to detector
- to extract meaningful information from a signal, we require models that relate the waveform to the source

Compact binary detections

Four years ago, LIGO first detected the gravitational waves from a black hole binary merger...



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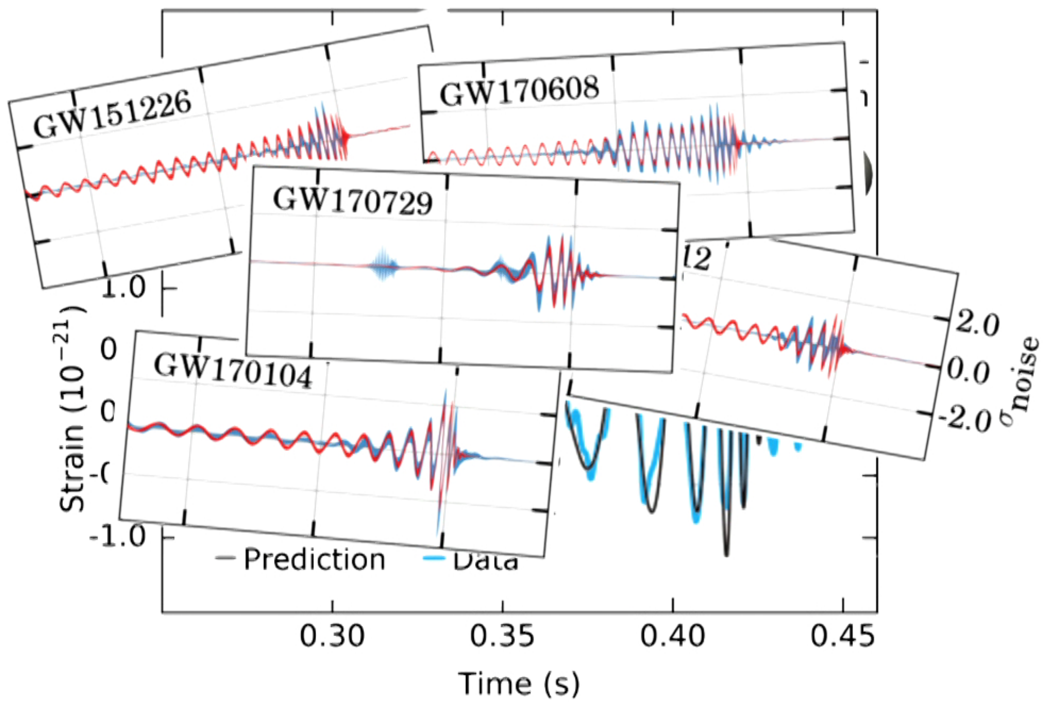

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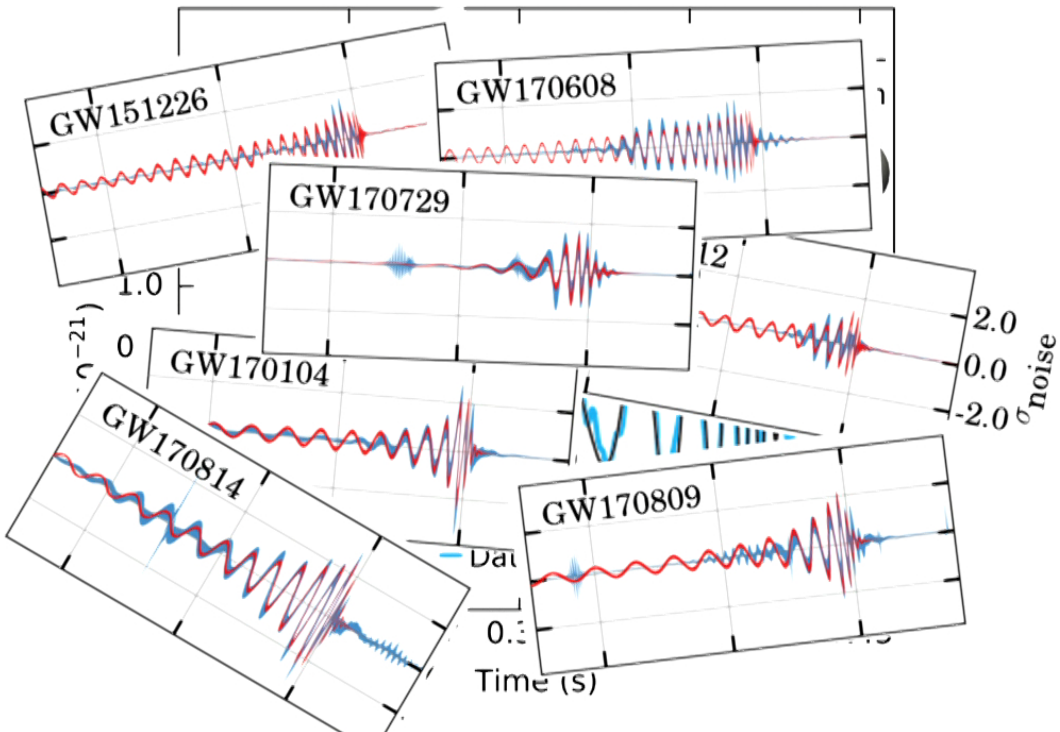
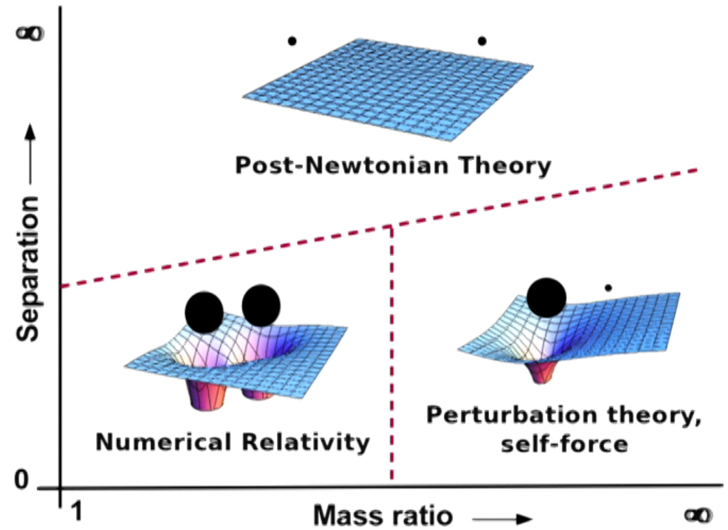


Many types of binaries

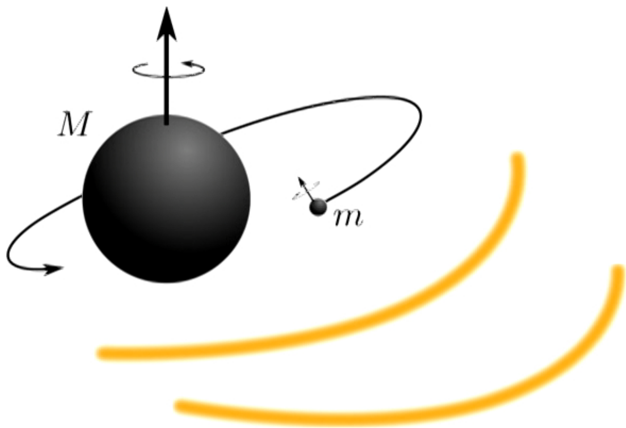
- LIGO is only sensitive to comparable-mass binaries
- different classes of binaries will be observed by different detectors and tell us different things

Binary parameter space



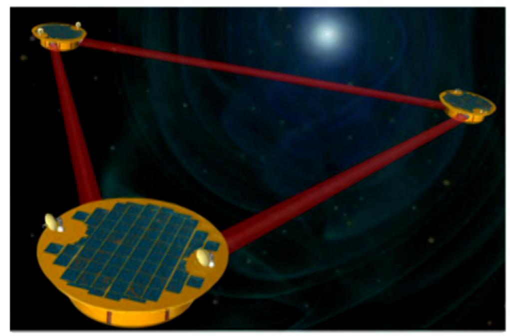
[Image courtesy of Leor Barack]

Extreme-mass-ratio inspirals (EMRIs)



- space-based detector LISA will observe extreme-mass-ratio inspirals of stellar-mass BHs or neutron stars into massive BHs
- small object spends $\sim M/m \sim 10^5$ orbits near BH $\Rightarrow$ unparalleled probe of strong-field region around BH

- emitted waveforms are intricate and long-lived in the LISA band
- contain a wealth of information

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EMRI science yield

Fundamental physics

- measure central BH parameters: mass and spin to $\sim .01\%$ error, quadrupole moment to $\sim .1\%$
  $\Rightarrow$ measure deviations from the Kerr relationship $M_l + iS_l = M(ia)^l$
  $\Rightarrow$ test no-hair theorem
- measure deviations from Kerr QNMs, presence or absence of event horizon, additional wave polarizations, changes to power spectrum
- constraints on modified gravity typically one or more orders of magnitude better than any other planned experiment

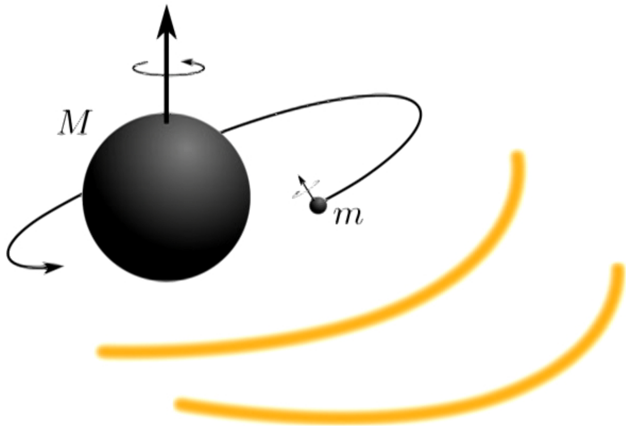
Astrophysics

- constrain mass function $n(M)$ (number of black holes with given mass)
- provide information about stellar environment around massive BHs

Cosmology

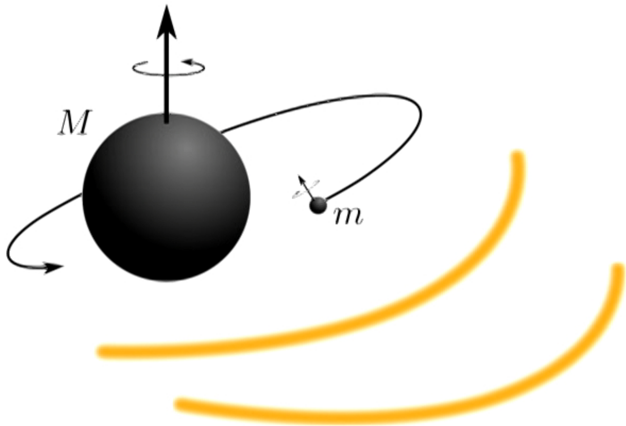
- measure Hubble constant to $\sim 1\%$

EMRI modeling: why self-force?



- highly relativistic, strong fields
- disparate lengthscales
- long timescale: inspiral is slow, produces $\sim \frac{M}{m} \sim 10^5$ wave cycles

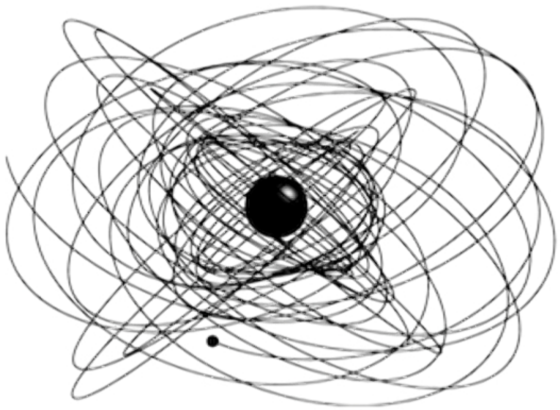
EMRI modeling: why self-force?



- highly relativistic, strong fields
  $\Rightarrow$ *can't use post-Newtonian theory*
- disparate lengthscales

- long timescale: inspiral is slow, produces $\sim \frac{M}{m} \sim 10^5$ wave cycles

Zeroth-order approx.: point mass on a geodesic in Kerr

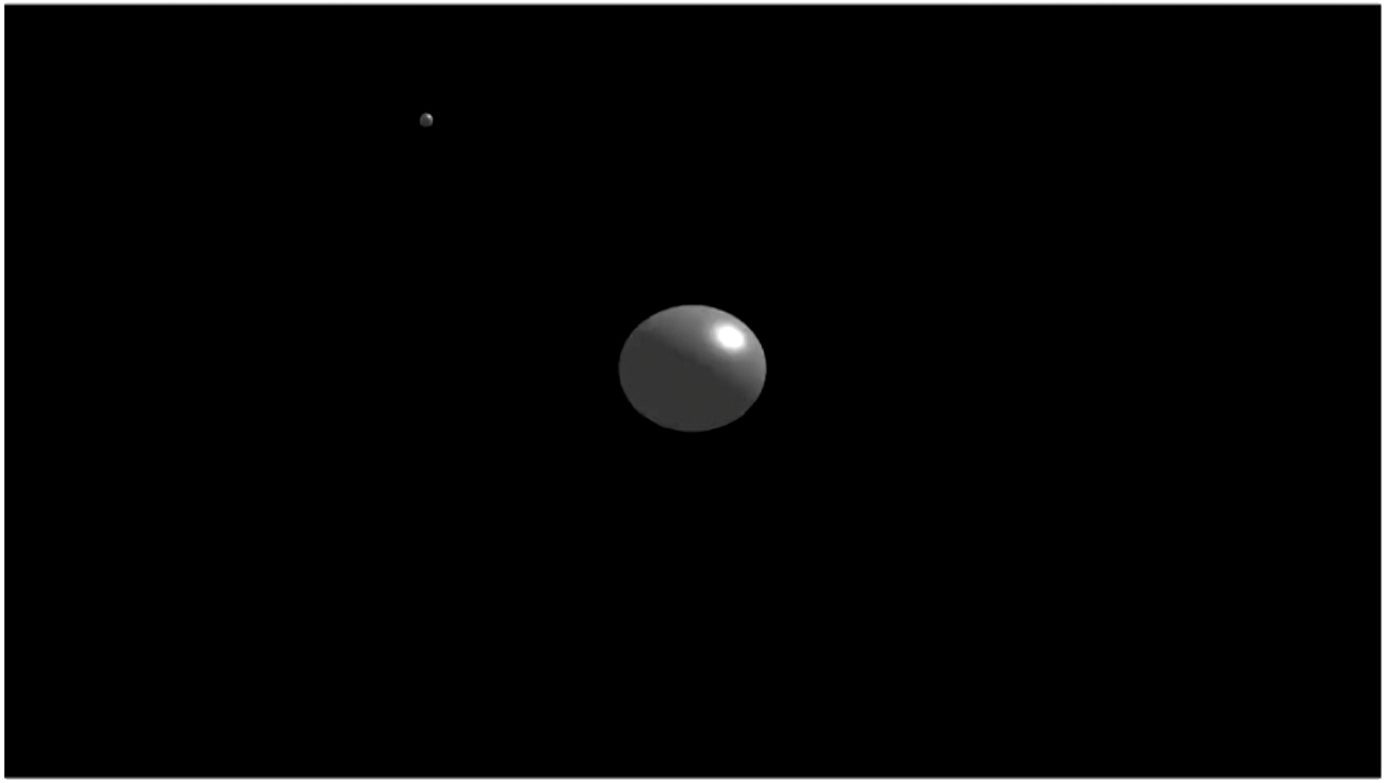


[image courtesy of Steve Drasco]

- geodesic characterized by three constants of motion:
  1. energy E
  2. angular momentum L_z
  3. Carter constant Q, related to orbital inclination

- E, L_z, Q related to frequencies of r, ϕ, and θ motion
- emitted waveform has all harmonics of these frequencies
- (and *resonances* occur when two of the frequencies have a rational ratio)

But GWs carry off energy and ang. momentum, and the small object slowly spirals into the black hole...



[animation courtesy of Steve Drasco]

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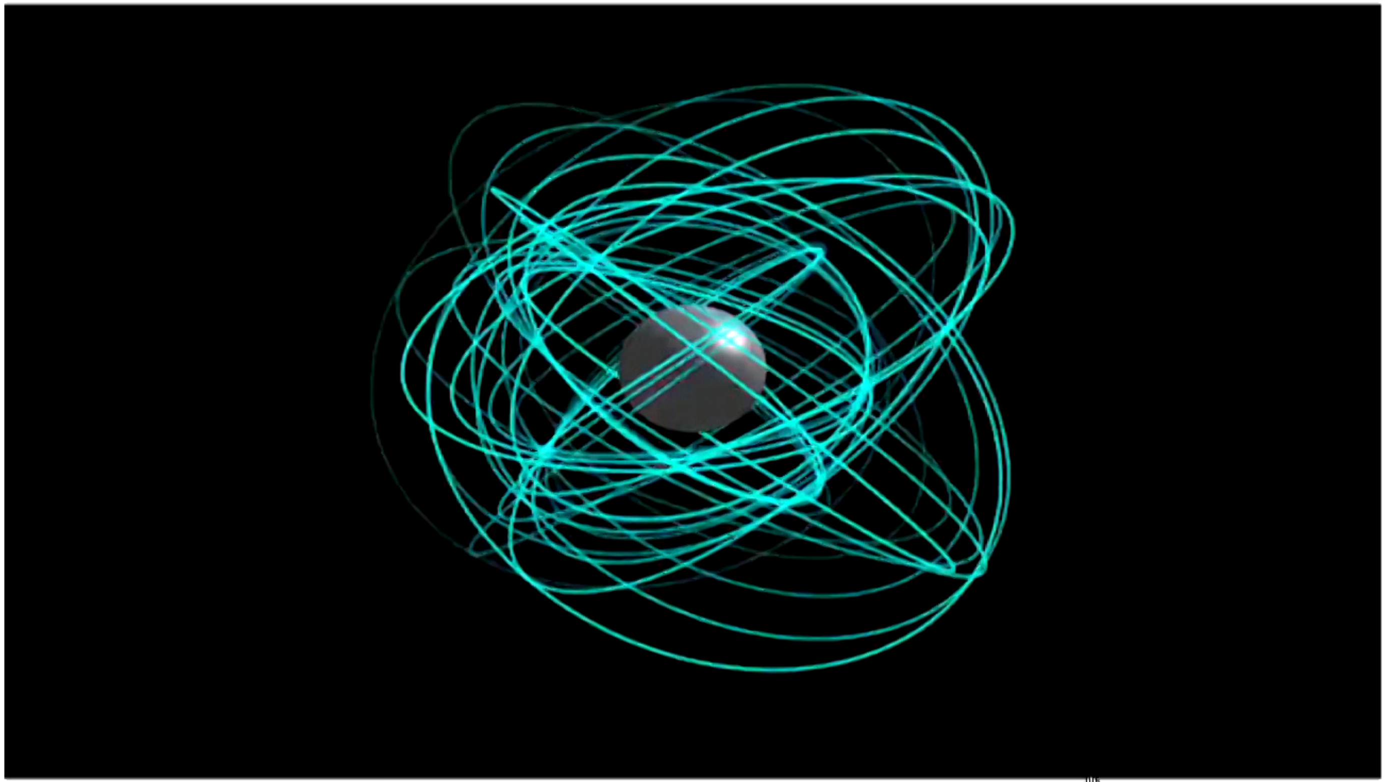

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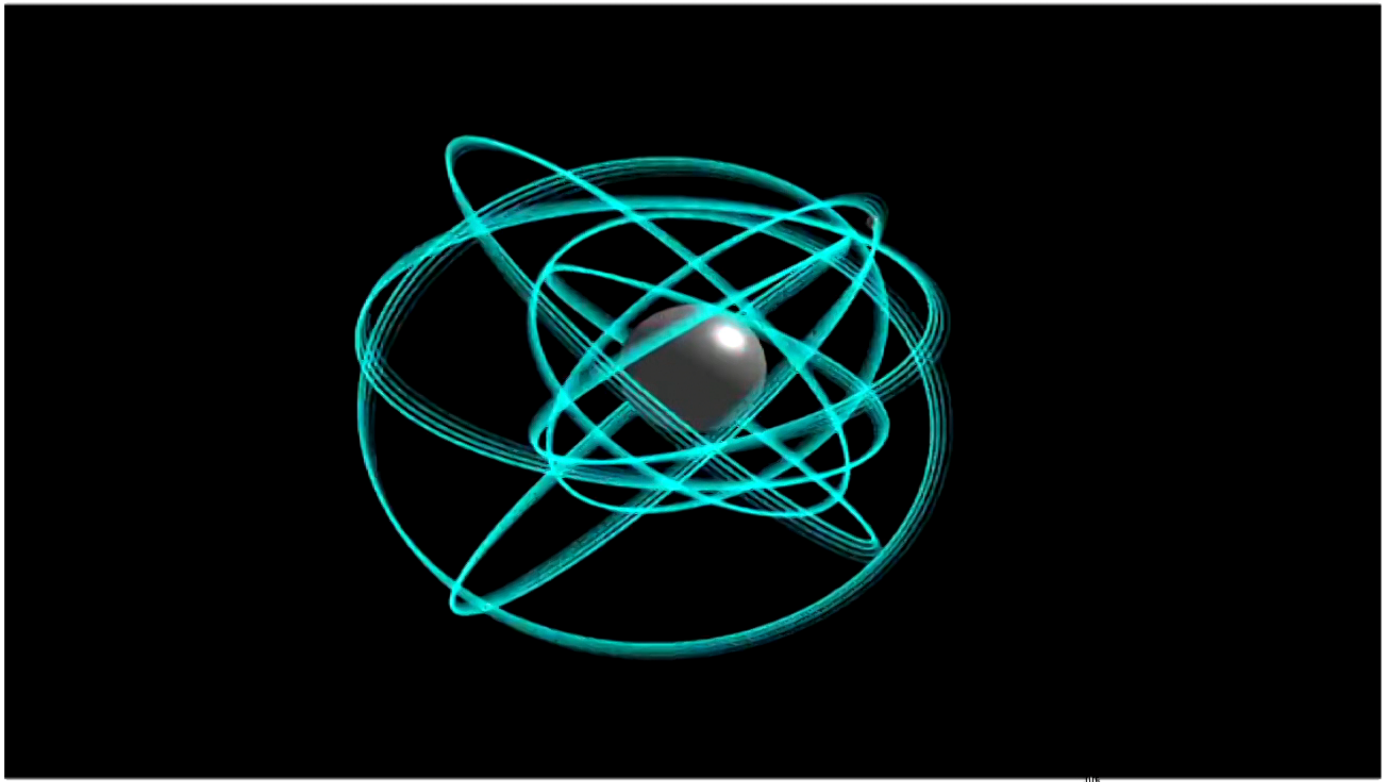

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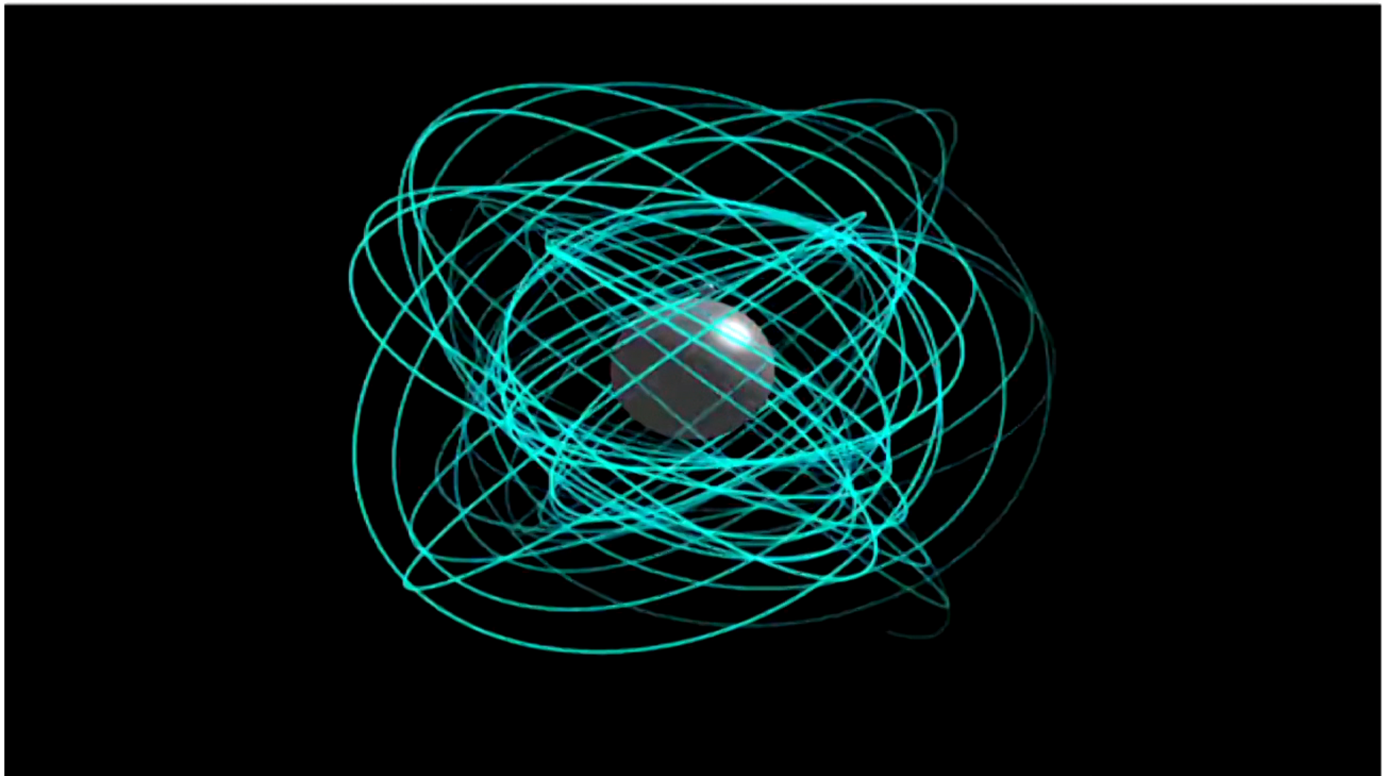

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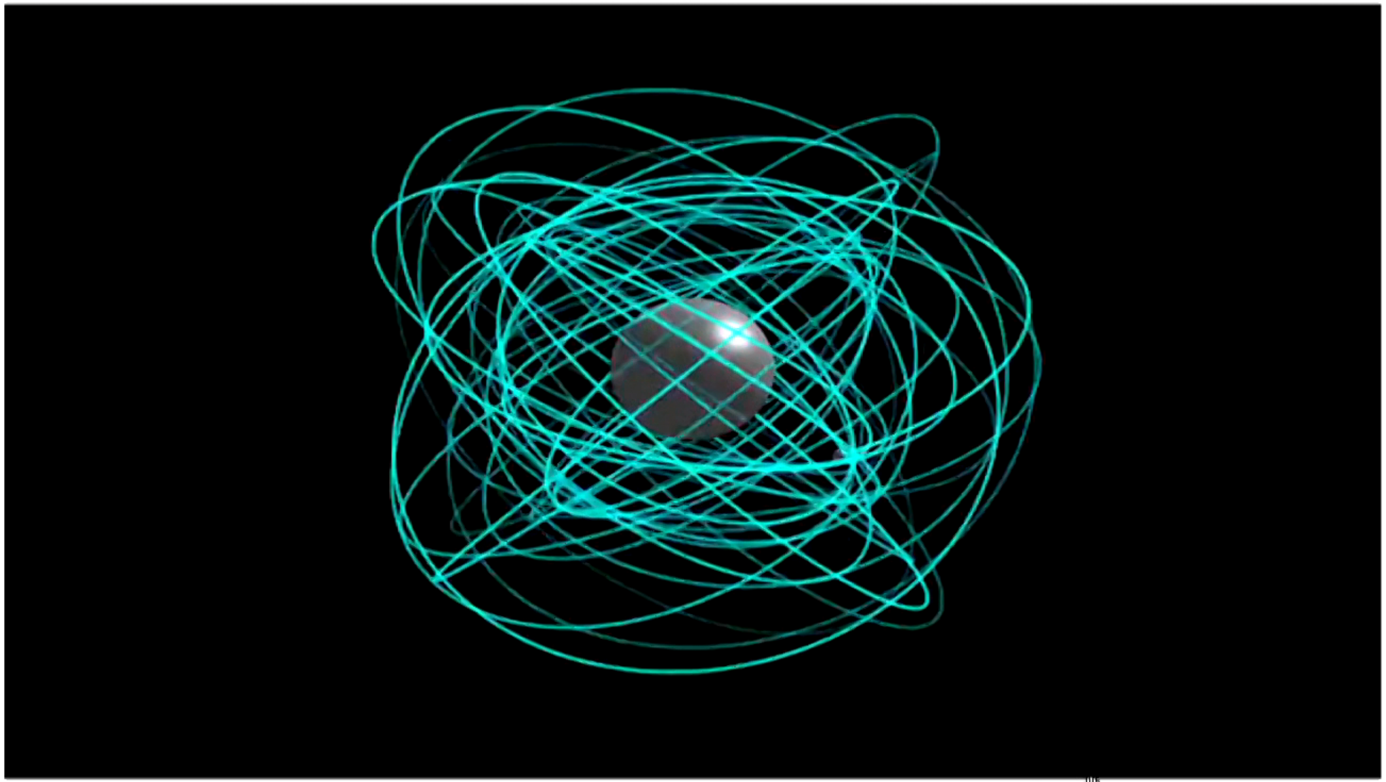

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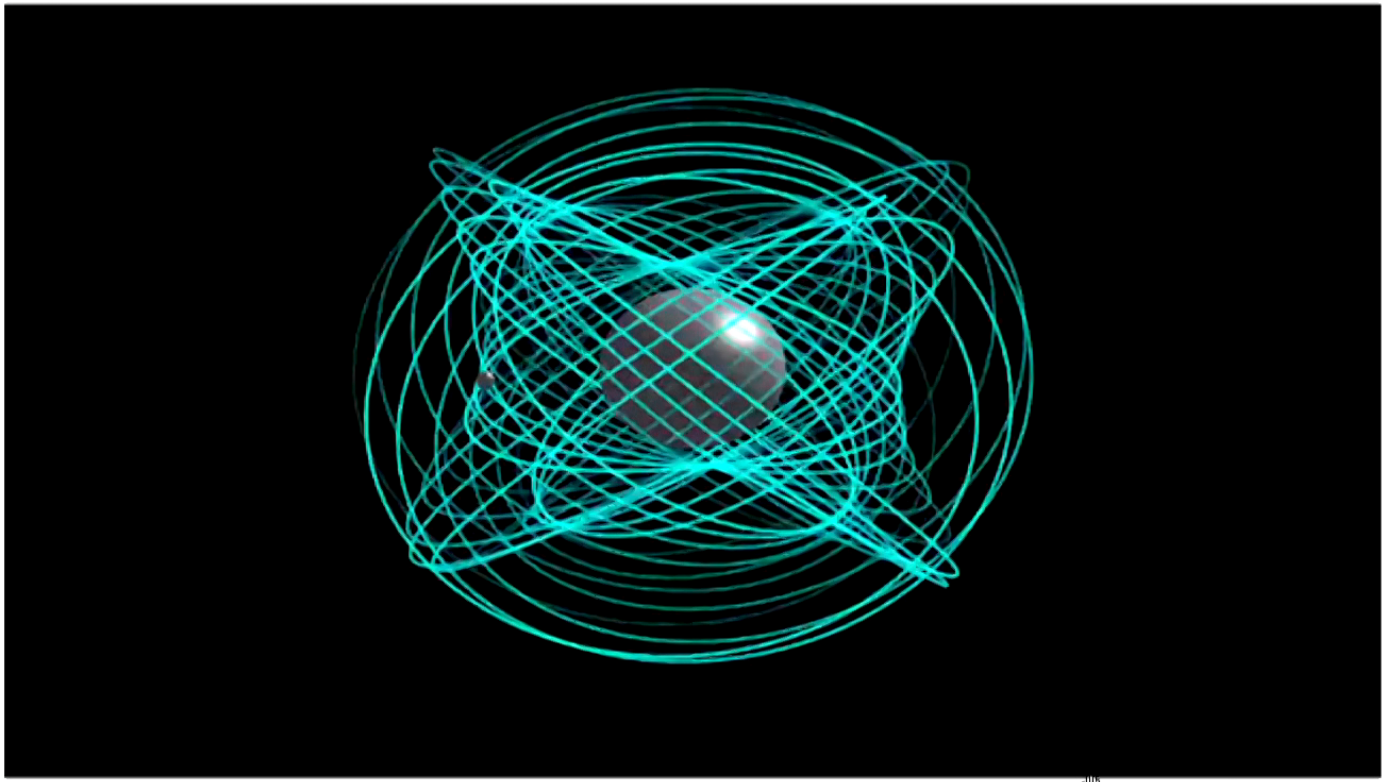

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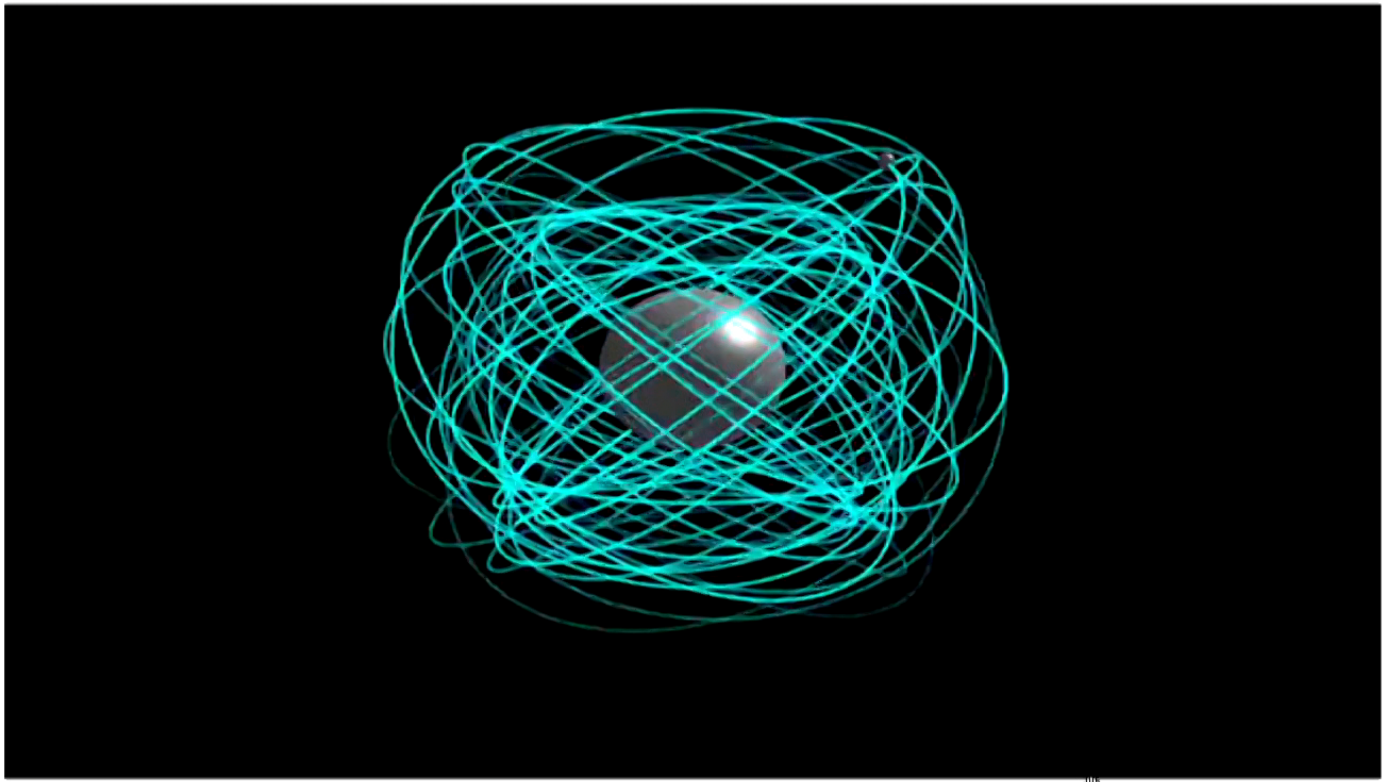

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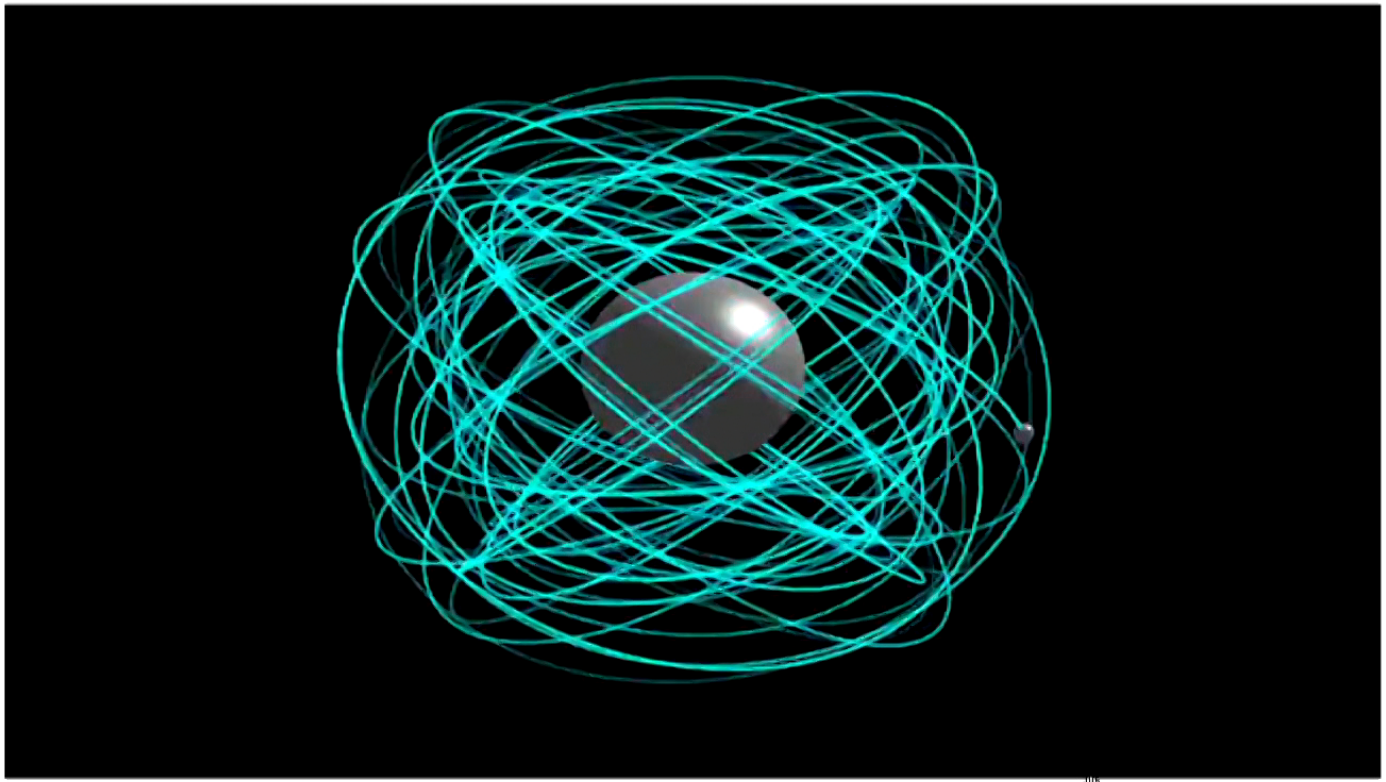

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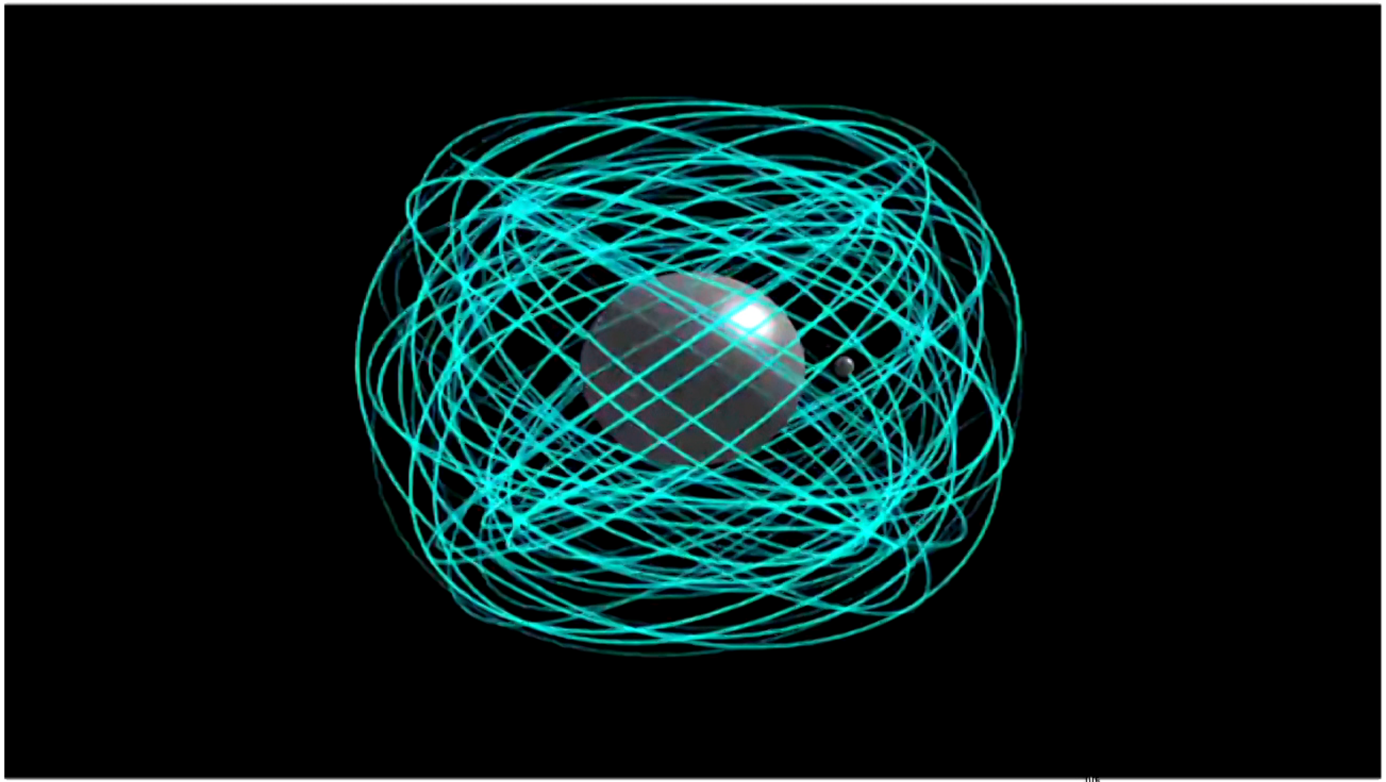

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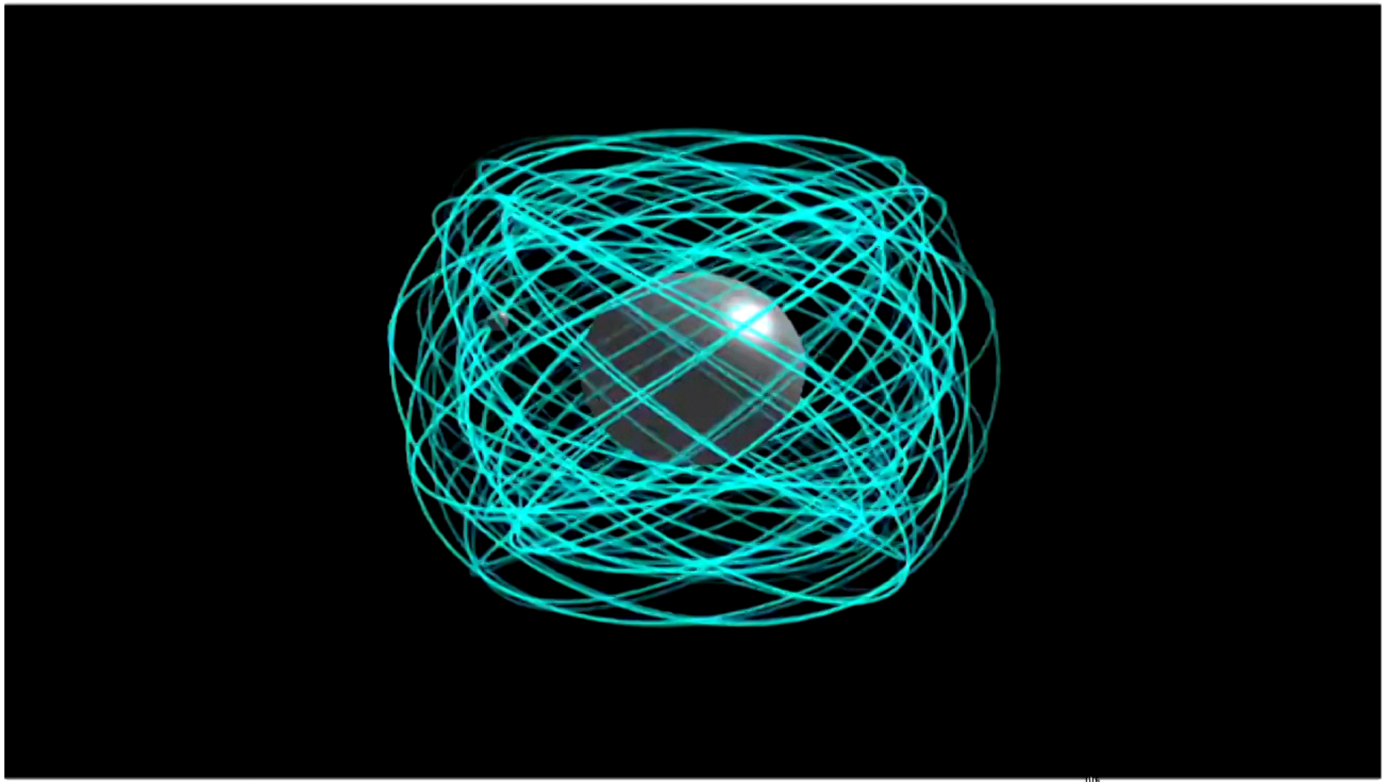

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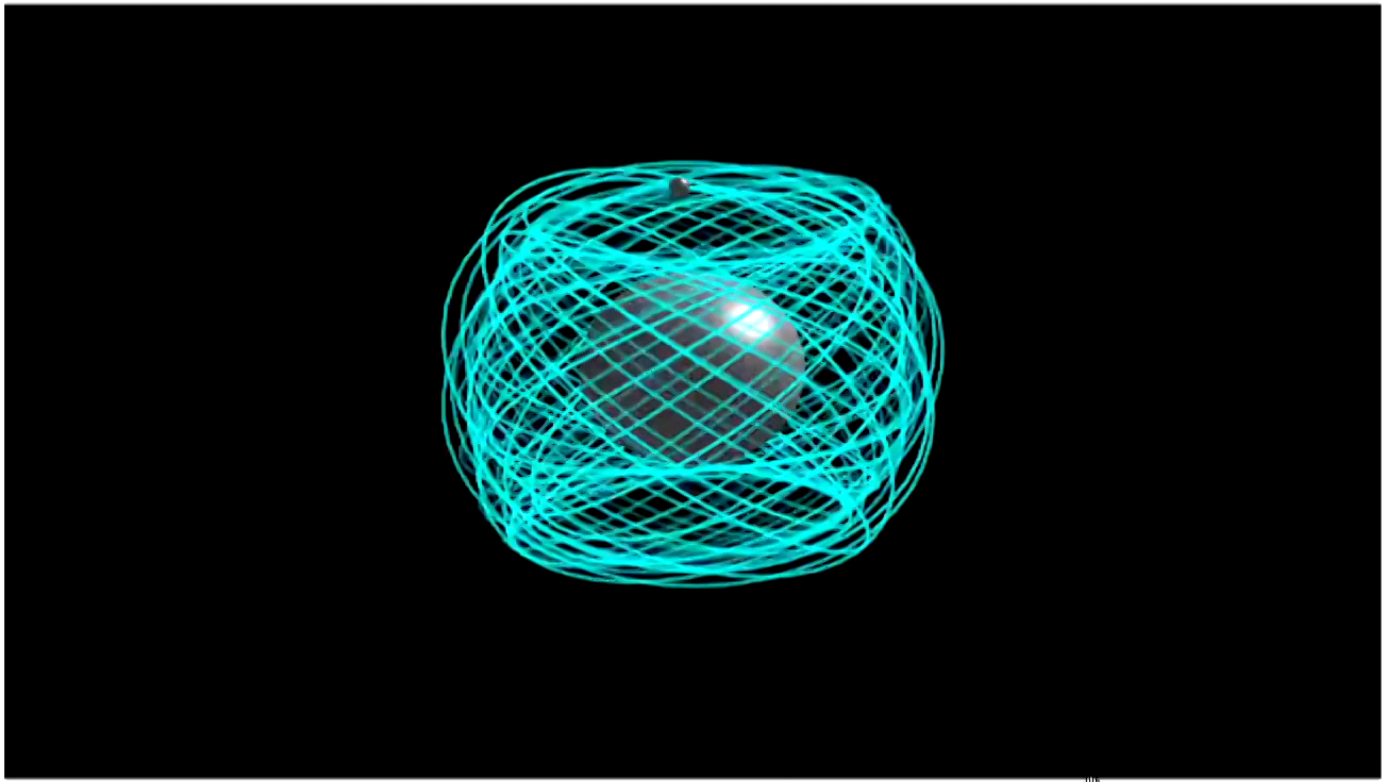

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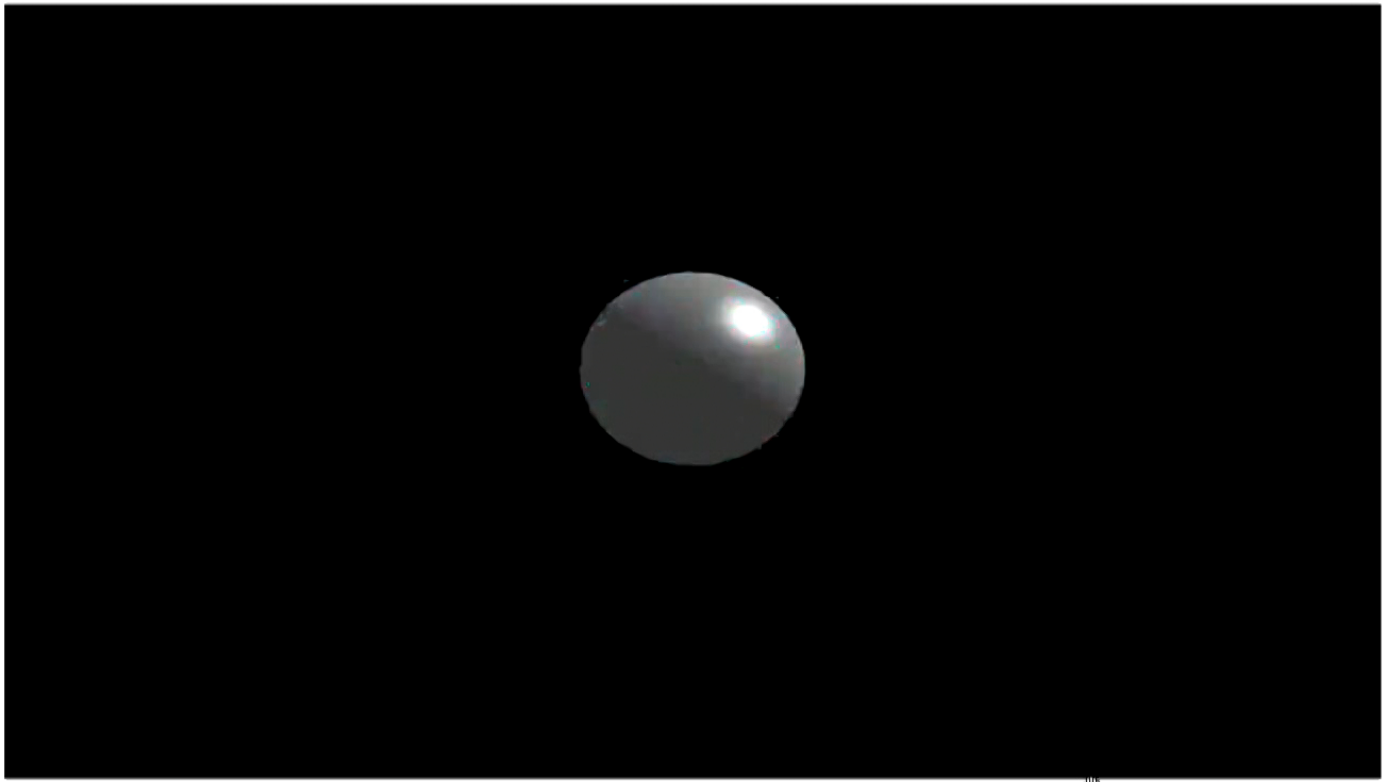

[animation courtesy of Steve Drasco]

Outline

1. Intro to EMRIs
2. EMRI model requirements
3. Self-force theory: the local problem
4. Self-force theory: the global problem
   - First order
   - Second order

Gravitational self-force theory

- m perturbs the spacetime of M:

$$\mathsf{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \dots$$

  where $\epsilon \sim m/M$

- this deformation of the geometry affects m's motion
  $\Rightarrow$ exerts a *self-force*

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F_1^\mu + \epsilon^2 F_2^\mu + \dots$$

How high order?

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F_1^\mu + \epsilon^2 F_2^\mu + \dots$$

- force is small; inspiral occurs very slowly, on time scale $\tau \sim 1/\epsilon$
- suppose we neglect F_2^μ; leads to error $\delta\left(\frac{D^2 z^\mu}{d\tau^2}\right) \sim \epsilon^2$
  $\Rightarrow$ error in position $\delta z^\mu \sim \epsilon^2 \tau^2$
  $\Rightarrow$ after time $\tau \sim 1/\epsilon$, error $\delta z^\mu \sim 1$

$\therefore$ accurately describing orbital evolution requires second order

Hierarchy of self-force models [Hinderer and Flanagan]

- when self-force is accounted for, E, L_z, and Q evolve with time
- on an inspiral timescale $t \sim 1/\epsilon$, the phase of the gravitational wave has an expansion (excluding resonances)

$$\phi = \frac{1}{\epsilon}\phi_0 + \phi_1 + \mathcal{O}(\epsilon)$$

- a model that gets ϕ_0 right should (hopefully) be enough to detect most signals
- a model that gets both ϕ_0 and ϕ_1 should be enough for prescise parameter extraction

Hierarchy of self-force models [Hinderer and Flanagan]

Adiabatic order

determined by

- averaged dissipative piece of F_1^μ

- ..., E, L_z, and Q evolve with time
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Hierarchy of self-force models [Hinderer and Flanagan]

Adiabatic order

determined by
- averaged dissipative piece of F_1^μ

Post-adiabatic order

determined by
- averaged dissipative piece of F_2^μ
- conservative piece of F_1^μ
- oscillatory dissipative piece of F_1^μ

an expansion (excluding resonances)

$$\phi = \frac{1}{\epsilon}\phi_0 + \phi_1 + \mathcal{O}(\epsilon)$$

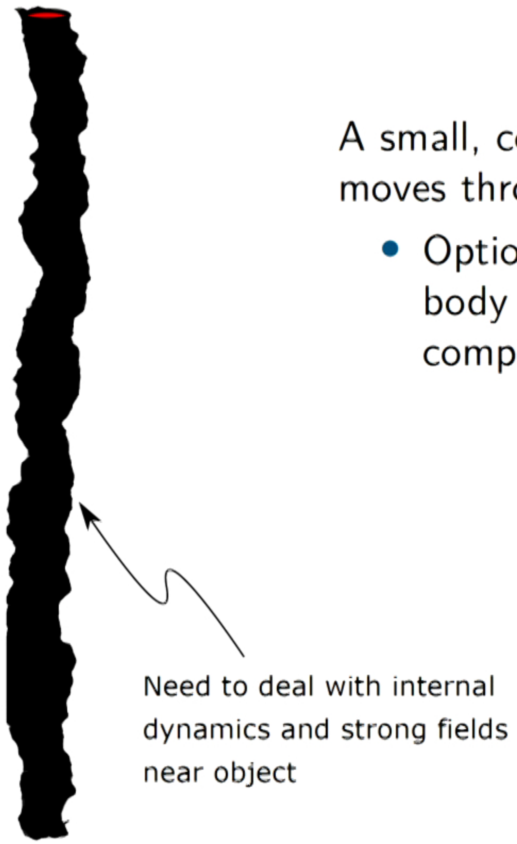
- a model that gets ϕ_0 right should (hopefully) be enough to detect most signals
- a model that gets both ϕ_0 and ϕ_1 should be enough for prescise parameter extraction

What is the status of these models?

- Efficient method of calculating adiabatic inspirals was developed ~ 15 years ago
- Most effort over last 23 years has been on calculating full F_1^μ
  —but this isn't an improvement over adiabatic approximation *if we don't also have averaged dissipative piece of* F_2^μ

Outline

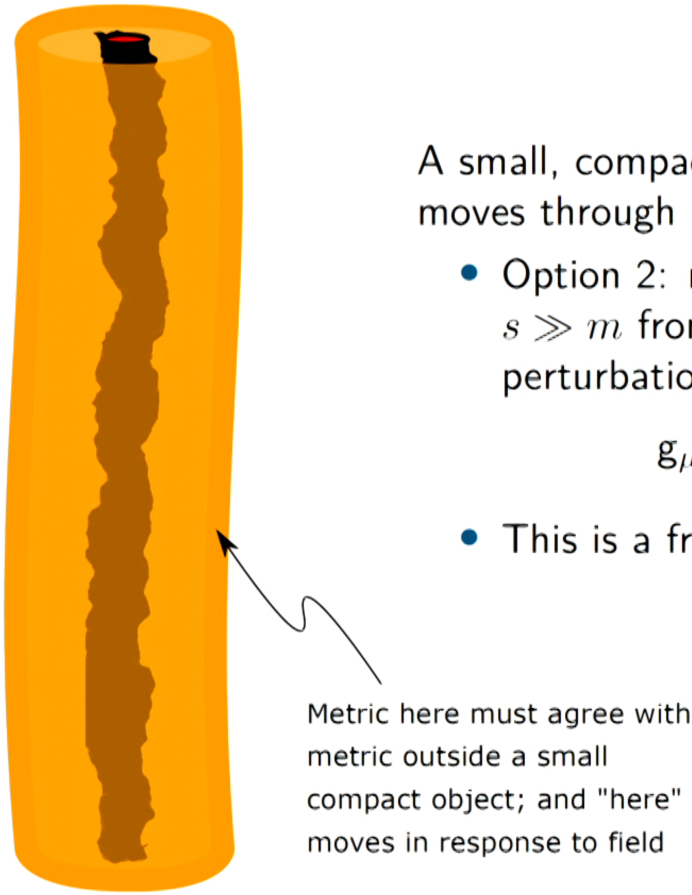
What is the problem we want to solve?



A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 1: tackle the problem directly, treat the body as finite sized, deal with its internal composition

What is the problem we want to solve?



A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 2: restrict the problem to distances $s \gg m$ from the object, treat m as source of perturbation of external background $g_{\mu\nu}$:

$$\mathrm{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \ldots$$

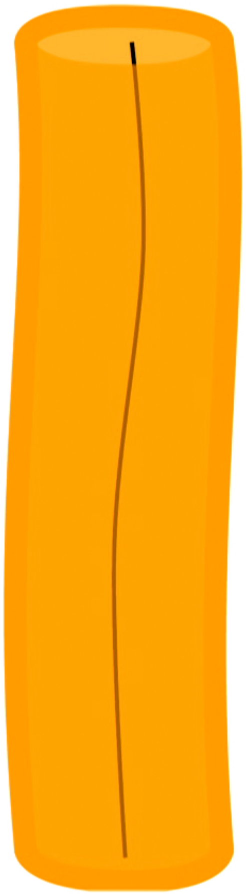
- This is a free boundary value problem

What is the problem we want to solve?

A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 3: treat the body as a point particle
  - takes behavior of fields outside object and extends it down to a fictitious worldline
  - so $h^1_{\mu\nu} \sim 1/s$ ($s =$distance from object)
  - second-order field equation $\delta G[h^2] \sim -\delta^2 G[h^1] \sim (\partial h^1)^2 \sim 1/s^4$ —no solution unless we restrict it to points off worldline, which is equivalent to FBVP

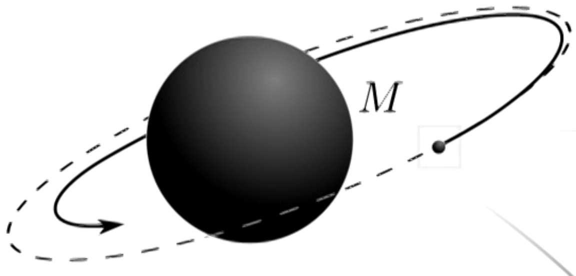
What is the problem we want to solve?



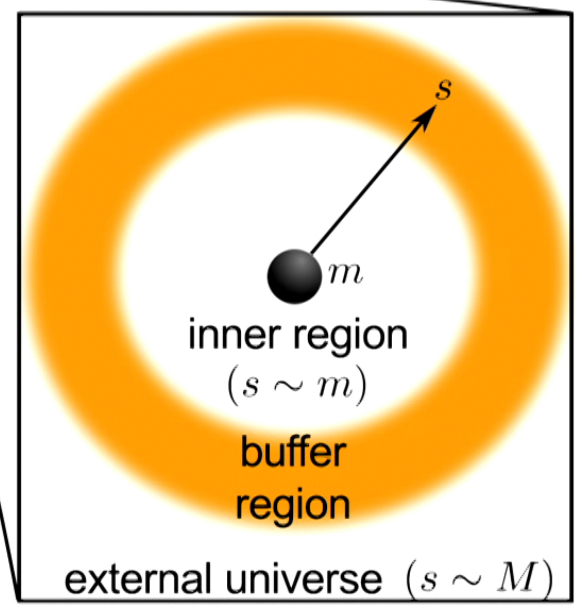
A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 4: transform the FBVP into an *effective* problem using a *puncture*, a local approximation to the field outside the object
- This will be the method emphasized here [Mino, Sasaki, Tanaka 1996; Quinn & Wald 1996; Detweiler & Whiting 2002-03; Gralla & Wald 2008-2012; Pound 2009-2017; Harte 2012]

Matched asymptotic expansions

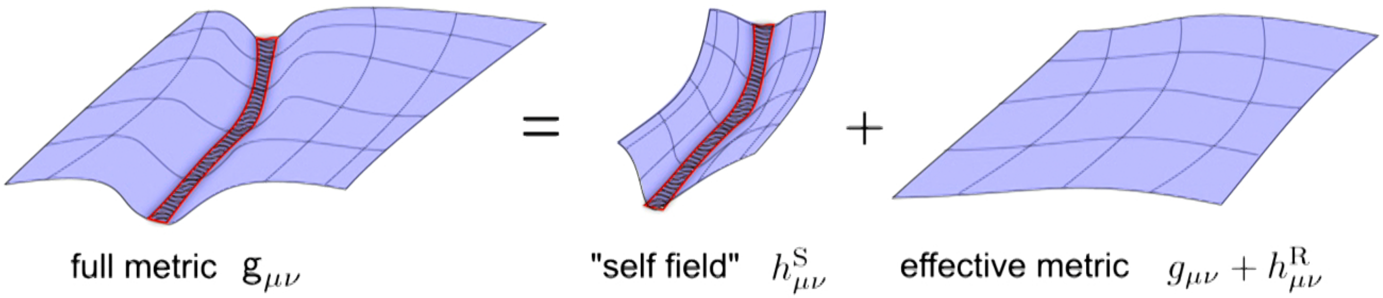


- *outer expansion*: in external universe, treat field of M as background
- *inner expansion*: in inner region, treat field of m as background
- in buffer region, feed information from inner expansion into outer expansion



Self-field and effective field

- based on local solution to EFE in buffer region, we split local metric into a "self-field" and an effective metric



- $h^{S}_{\mu\nu}$ directly determined by object's multipole moments
- $g_{\mu\nu} + h^{R}_{\mu\nu}$ is a *smooth vacuum metric* determined by global boundary conditions

Equation of motion

Solving EFE in buffer region yields equations of motion for object's effective center of mass

1st order, arbitrary compact object [MiSaTaQuWa 1996]:

$$\frac{D^2 z^\mu}{d\tau^2} = -\frac{1}{2}\left(g^{\alpha\delta} + u^\alpha u^\delta\right)\left(2h^{\rm R1}_{\delta\beta;\gamma} - h^{\rm R1}_{\beta\gamma;\delta}\right)u^\beta u^\gamma + \frac{1}{2m}R^\alpha{}_{\beta\gamma\delta}u^\beta S^{\gamma\delta} + O(m^2)$$

(motion of spinning test body in $g_{\mu\nu} + h^{\rm R1}_{\mu\nu}$)

2nd-order, nonspinning, spherical compact object [Pound 2012]:

$$\frac{D^2 z^\mu}{d\tau^2} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(g_\nu{}^\rho - h^{\rm R}_\nu{}^\rho\right)\left(2h^{\rm R}_{\rho\sigma;\lambda} - h^{\rm R}_{\sigma\lambda;\rho}\right)u^\sigma u^\lambda + O(m^3)$$

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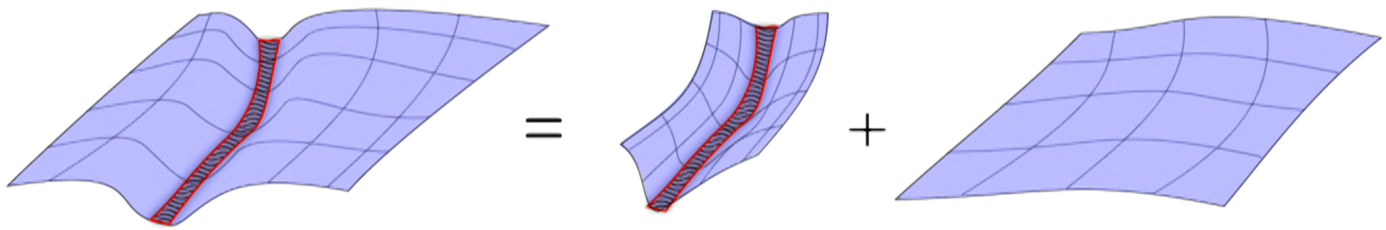
(geodesic motion in $g_{\mu\nu} + h^{\mathrm{R}}_{\mu\nu}$)

- these results are derived *directly from EFE outside the object*; there's no regularization of infinities, and no assumptions about $h^{\mathrm{R}}_{\mu\nu}$

Point particles and punctures

- replace "self-field" with "singular field"



- at 1st order, can use this to *replace object with a point particle*

$$T^{1}_{\mu\nu} := \frac{1}{8\pi}\delta G_{\mu\nu}[h^1] \sim m\delta(x - z)$$

- beyond 1st order, point particles not well defined—but can replace object with a *puncture*, a local singularity in the field, moving on z^{μ}, equipped with the object's multipole moments

Point particles and punctures

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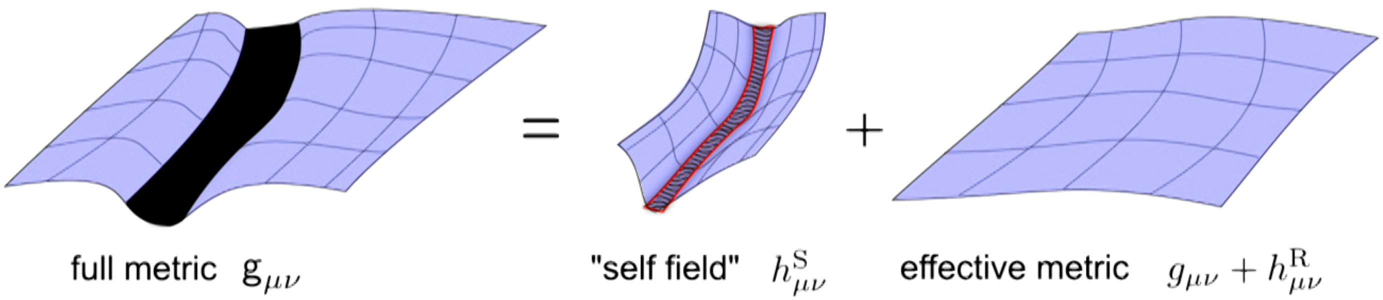

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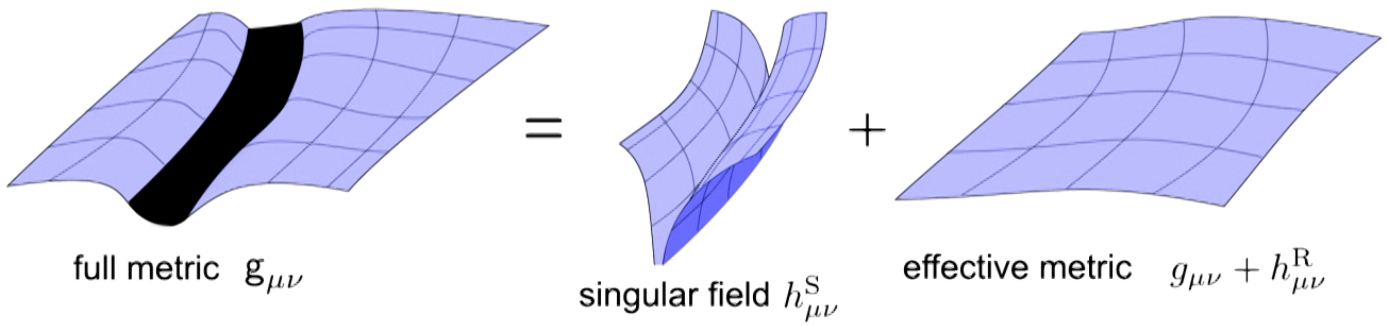

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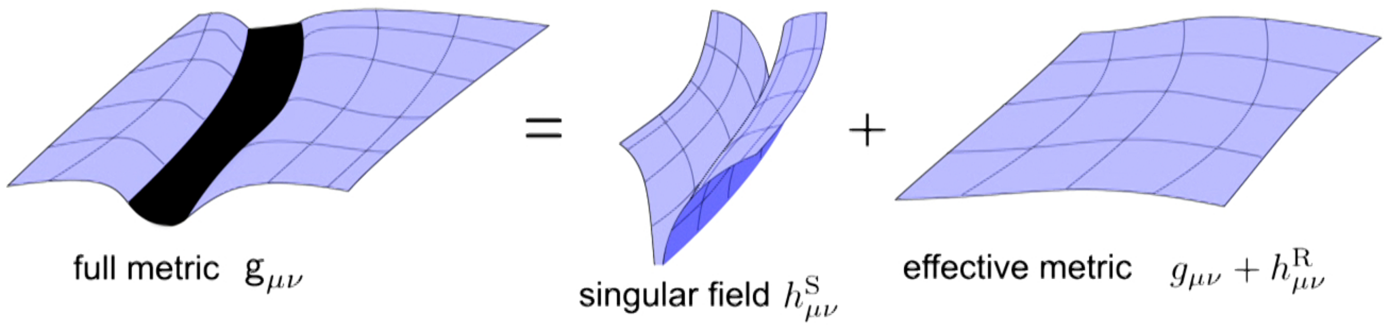

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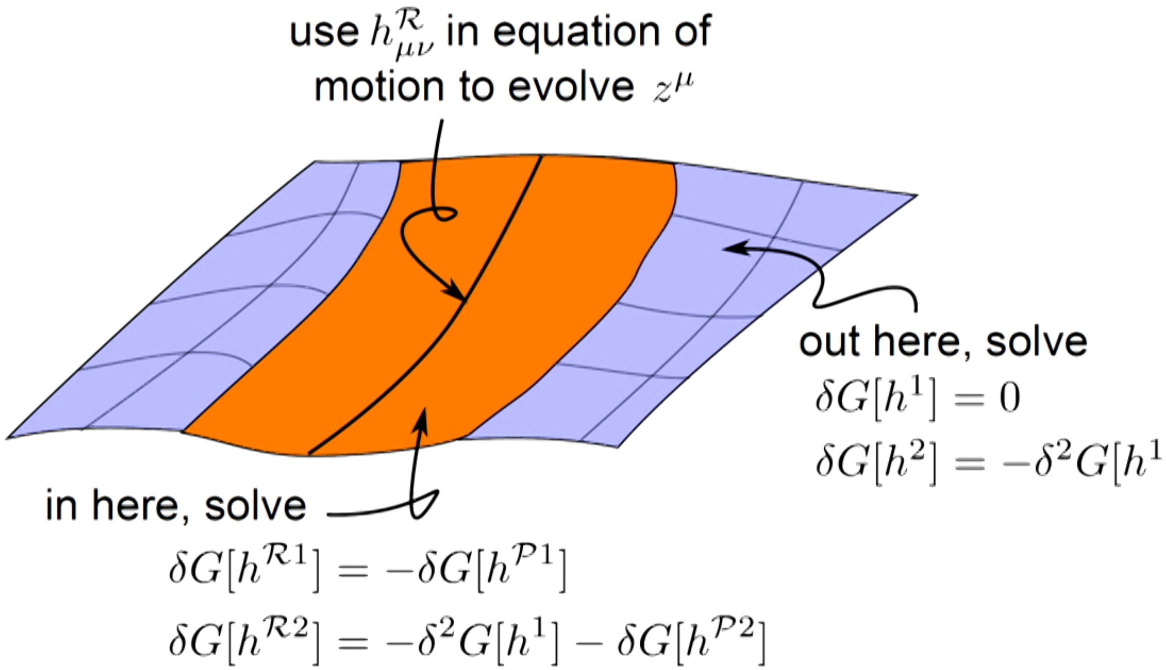
How you replace an object with a puncture



Solving the Einstein equations globally

- solving the local problem told us how to replace the small object with a moving puncture in the field equations:

$$\delta G_{\mu\nu}[h^{\mathcal{R}1}] = -\delta G_{\mu\nu}[h^{\mathcal{P}1}]$$
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Solving the Einstein equations globally

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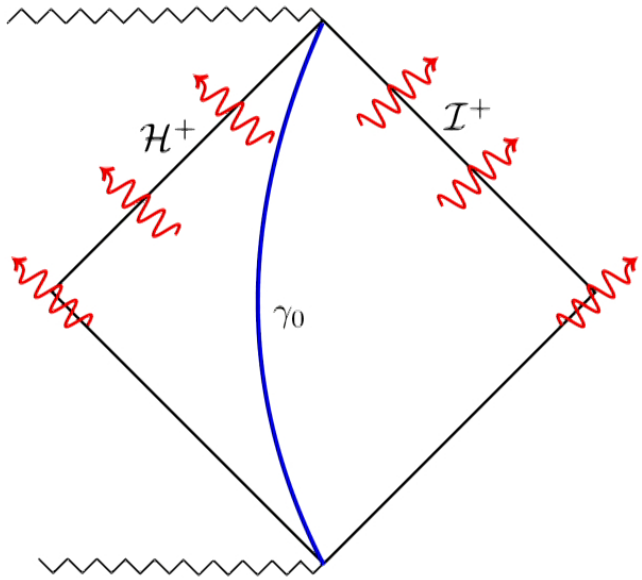
Typical calculation at first order

[Barack et al, Evans et al, van de Meent, many others]



- approximate the source orbit as a bound geodesic
- impose outgoing-wave BCs at $\mathcal{I}^+$ and $\mathcal{H}^+$
- solve field equation numerically, compute self-force from solution
- breaks down on *dephasing time* $\sim 1/\sqrt{\epsilon}$, when $|z^\mu - z_0^\mu| \sim M$

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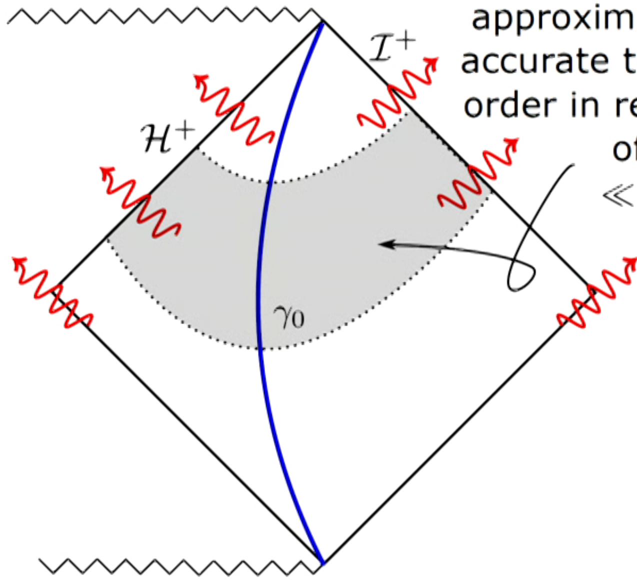

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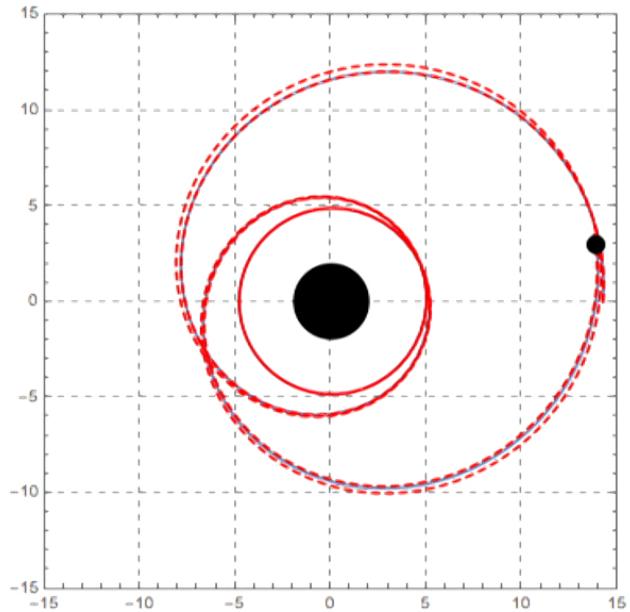
First-order force



[image courtesy of Warburton]

- complete inspirals simulated in Schwarzschild using full F_1^μ (including spin force) [Warburton et al]
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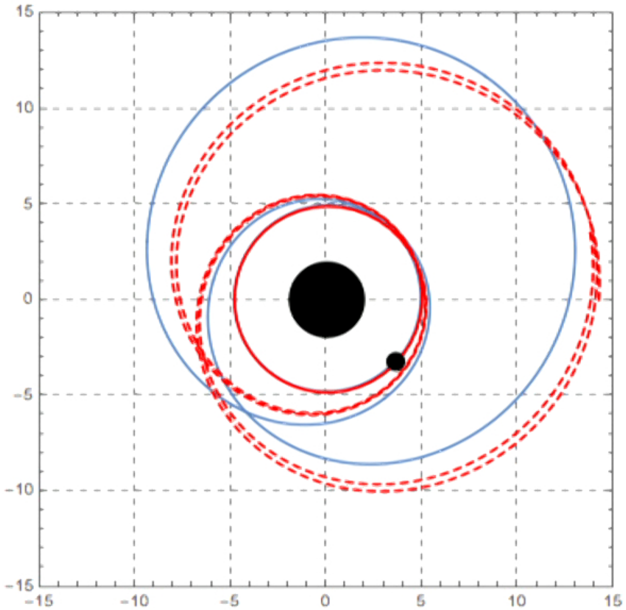

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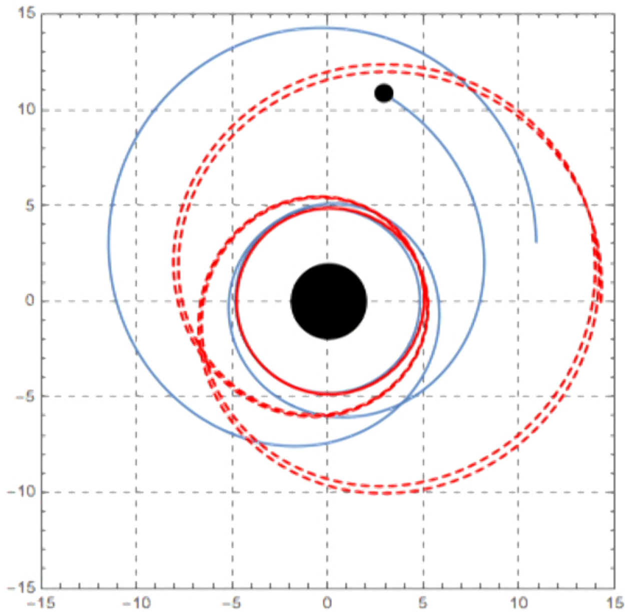

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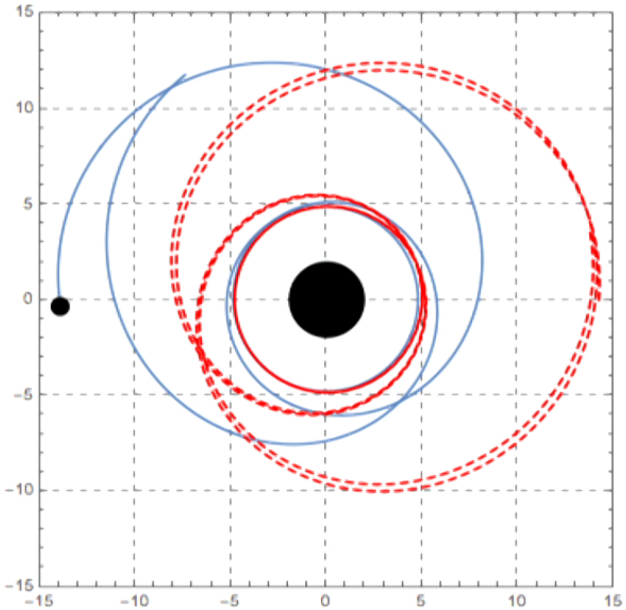

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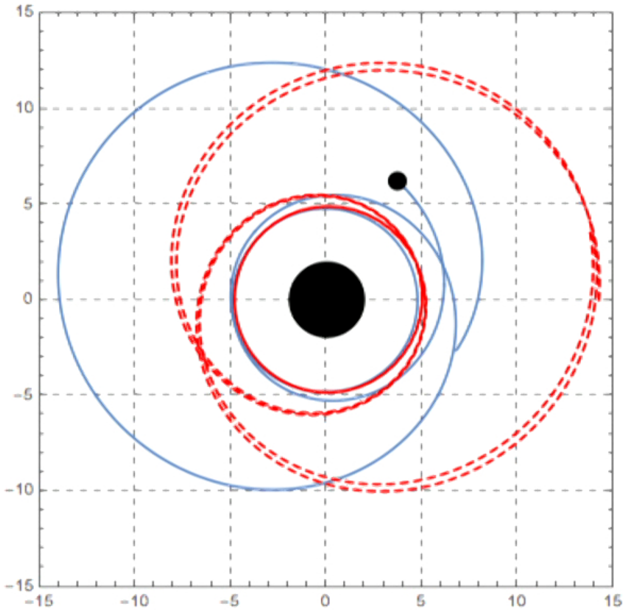

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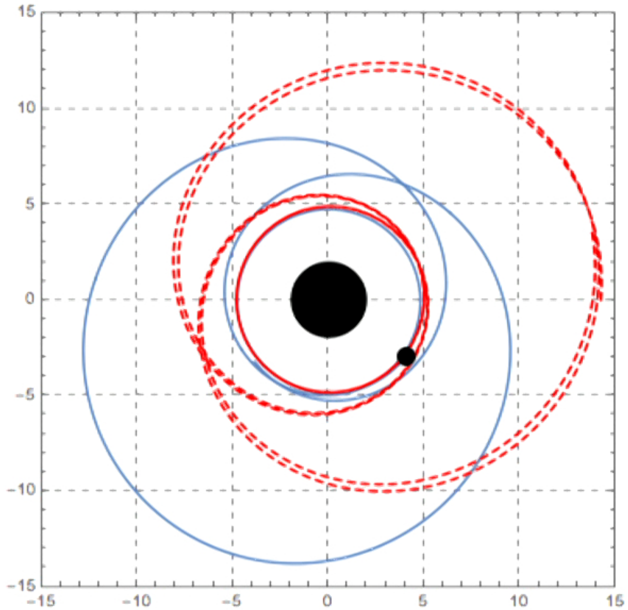

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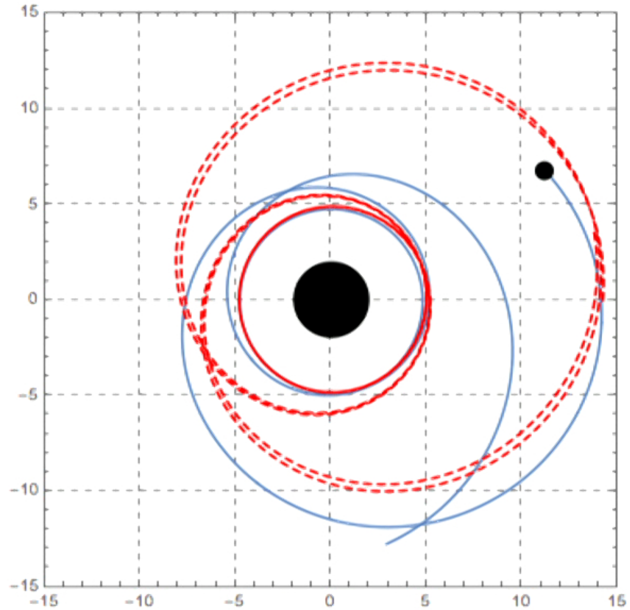

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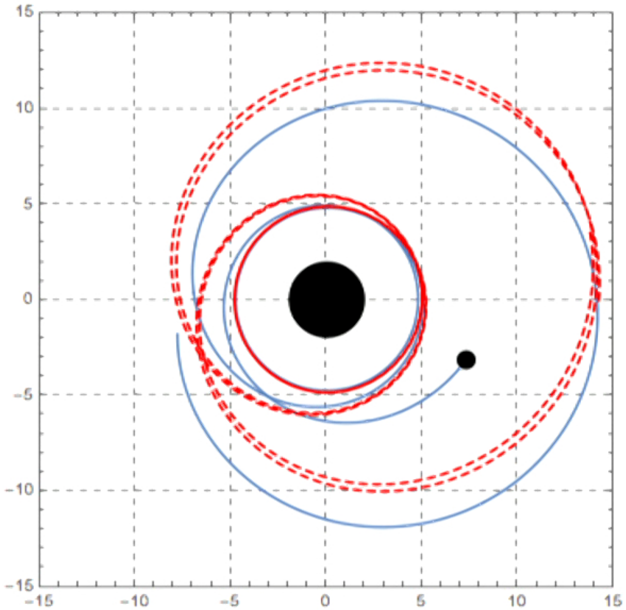

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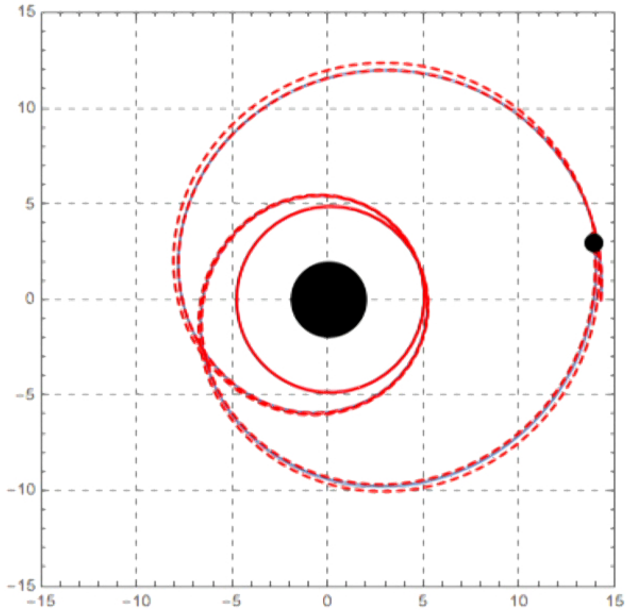

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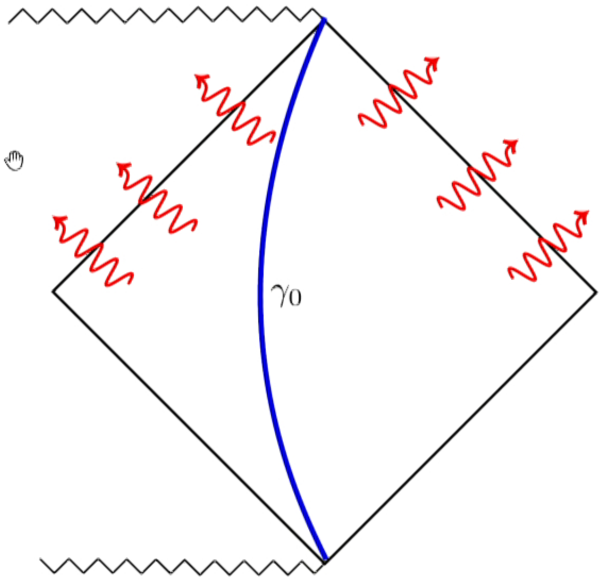
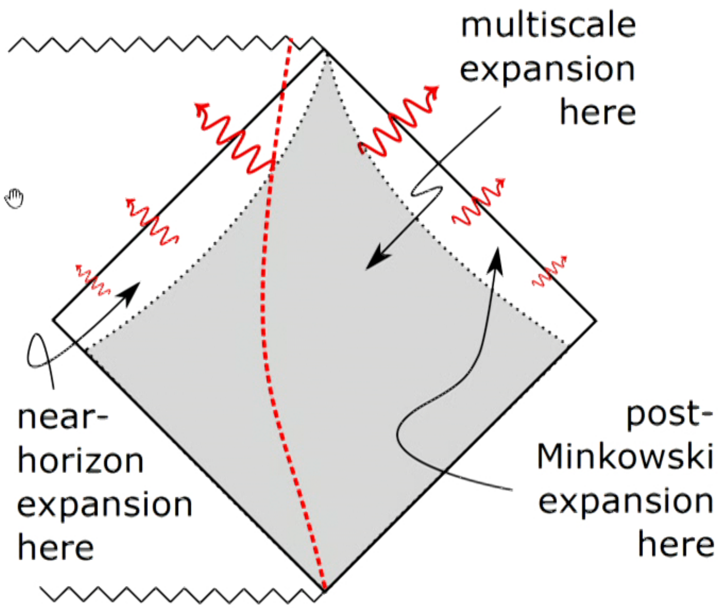
Infrared problems at second order [Pound 2015]



- suppose we try to use "typical" $h^1_{\mu\nu}$ to construct source for $h^2_{\mu\nu}$
- because $|z^\mu - z^\mu_0|$ blows up with time, $h^2_{\mu\nu}$ does likewise
- because $h^1_{\mu\nu}$ contains outgoing waves at all past times, the source $\delta^2 R_{\mu\nu}[h^1]$ decays too slowly, and *its retarded integral does not exist*
- instead, we must construct a uniform approximation
  - $h^1_{\mu\nu}$ must include evolution of orbit
  - radiation must decay to zero in infinite past

Matched expansions [Pound, Miller, Moxon, Flanagan, Hinderer, Yamada, Isoyama, Tanaka]



Multiscale expansion

$$J^\alpha = J_0^\alpha(\tilde{t}) + \epsilon J_1^\alpha(\tilde{t}) + \dots$$

$$h_{\mu\nu}^n \sim \sum_{k^\alpha} h_{k_\alpha}^n(\tilde{t}) e^{-ik^\alpha q_\alpha(\tilde{t})}$$

- (J^α, q_α) are action-angle variables for z^μ, and $\tilde{t} \sim \epsilon t$ is a "slow time"
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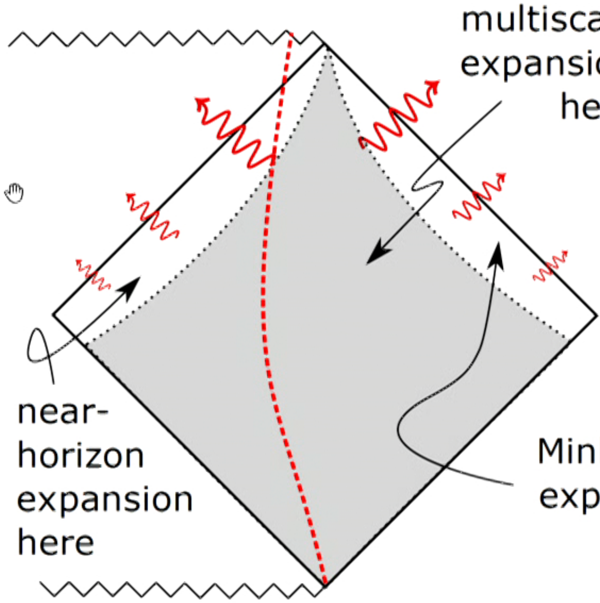

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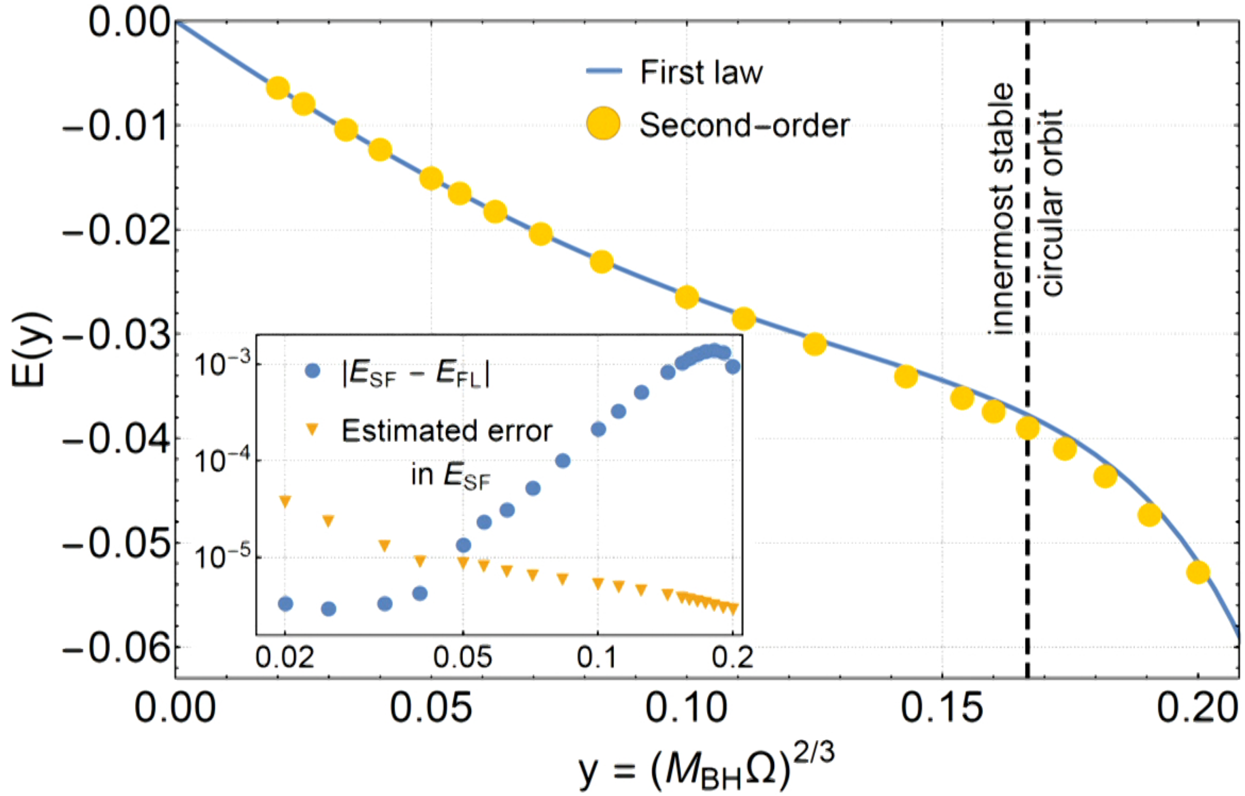
Binding energy [Pound, Wardell, Warburton, Miller]

Second-order piece of $E_{\text{bind}} = M_{\text{Bondi}} - m - M_{BH}$



Calculations performed as of 2005

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓		
	generic	✓		
Kerr	circular	✓		
	generic			**holy grail**

Calculations performed as of 2019

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	generic	✓	✓	
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	generic	✓	✓	**holy grail**

Conclusion

Challenge

- accurate (post-adiabatic) model requires second-order self-force calculations for generic bound orbits in Kerr

Prospects

- adiabatic inspirals should suffice for signal detection in most cases. Templates are on the way.
- post-adiabatic inspirals should enable high-precision parameter estimation. Lots of work to do.

For more information, see recent review by Barack and Pound (arXiv:1805.10385)

Equation of motion

Solving EFE in buffer region yields equations of motion for object's effective center of mass

1st order, arbitrary compact object [MiSaTaQuWa 1996]:

$$\frac{D^2 z^\mu}{d\tau^2} = -\tfrac{1}{2}\left(g^{\alpha\delta} + u^\alpha u^\delta\right)\left(2h^{\rm R1}_{\delta\beta;\gamma} - h^{\rm R1}_{\beta\gamma;\delta}\right) u^\beta u^\gamma + \tfrac{1}{2m} R^\alpha{}_{\beta\gamma\delta} u^\beta S^{\gamma\delta} + O(m^2)$$

(motion of spinning test body in $g_{\mu\nu} + h^{\rm R1}_{\mu\nu}$)

2nd-order, nonspinning, spherical compact object [Pound 2012]:

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(geodesic motion in $g_{\mu\nu} + h^{\rm R}_{\mu\nu}$)

- these results are derived *directly from EFE outside the object*; there's no regularization of infinities, and no assumptions about $h^{\rm R}_{\mu\nu}$