Title: Cosmic Structure Formation with Kinetic Field Theory - A New Approach to an Old Problem

Speakers: Elena Kozlikin

Series: Cosmology & Gravitation

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Abstract: Thus far, the non-linear regime of structure formation is only accessible through expensive numerical N-body simulations since the conventional ana-
lytic treatment of cosmic density fluctuations via the hydrodynamical equations runs into severe problems even in a mildly non-linear regime. I will give an overview of Kinetic Field Theory, a novel analytic approach to cosmic large-scale structure formation, which provides a density fluctuation power spectrum that agrees well with state of the art N-body simulations up to k = 10h Mpcâˆ’1. I will point out how Kinetic Field Theory avoids the difficulties of standard perturbation theory by construction and allows to proceed deeply into the non-linear regime of density fluctuations. I will give a summary of the most important results and recent developments for cosmic structure formation and briefly show applications of Kinetic Field Theory outside of cosmology, stressing the opportunities this method provides for the study of the physics behind structure formation. I will then focus my discussion on how the form of the interaction potential influences the formation of cosmic structures in the highly non-linear regime and the implications it bears on the NFW density profile of dark matter halos.

Cosmic structure formation with KFT – A new approach to an old problem

Elena Kozlikin

Cosmology Seminar
Perimeter Institute

January 7, 2020

Cosmic structure formation

2-Micron All-Sky Survey

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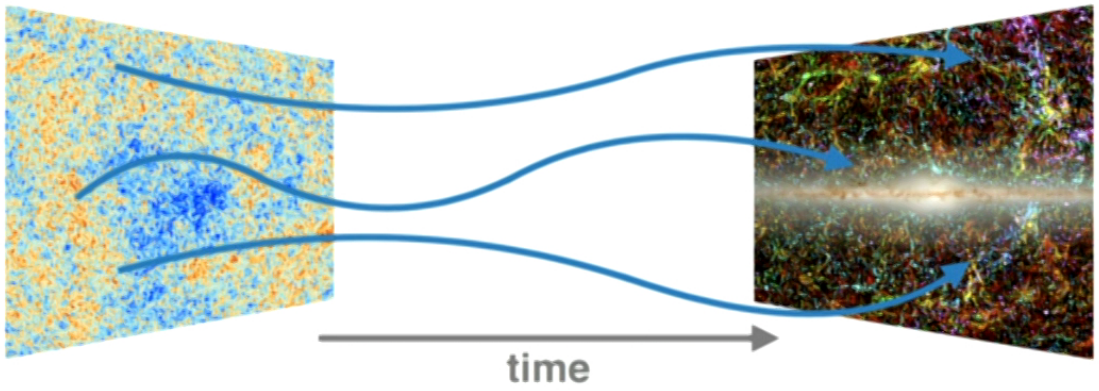
Cosmic structure formation

How can we understand cosmic structure formation from first principle?



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Theory of cosmic structure formation

Conventional:

- Hydrodynamical equations:

$$\dot{\delta} + \vec{\nabla} \cdot \vec{u} = 0$$
$$\dot{\vec{u}} + 2H\vec{u} = -\vec{\nabla}\phi$$
$$\vec{\nabla}^2\phi = 4\pi G\bar{\rho}\delta$$

- But: dark matter is not a fluid
- Multiple streams form where shocks would form in a fluid

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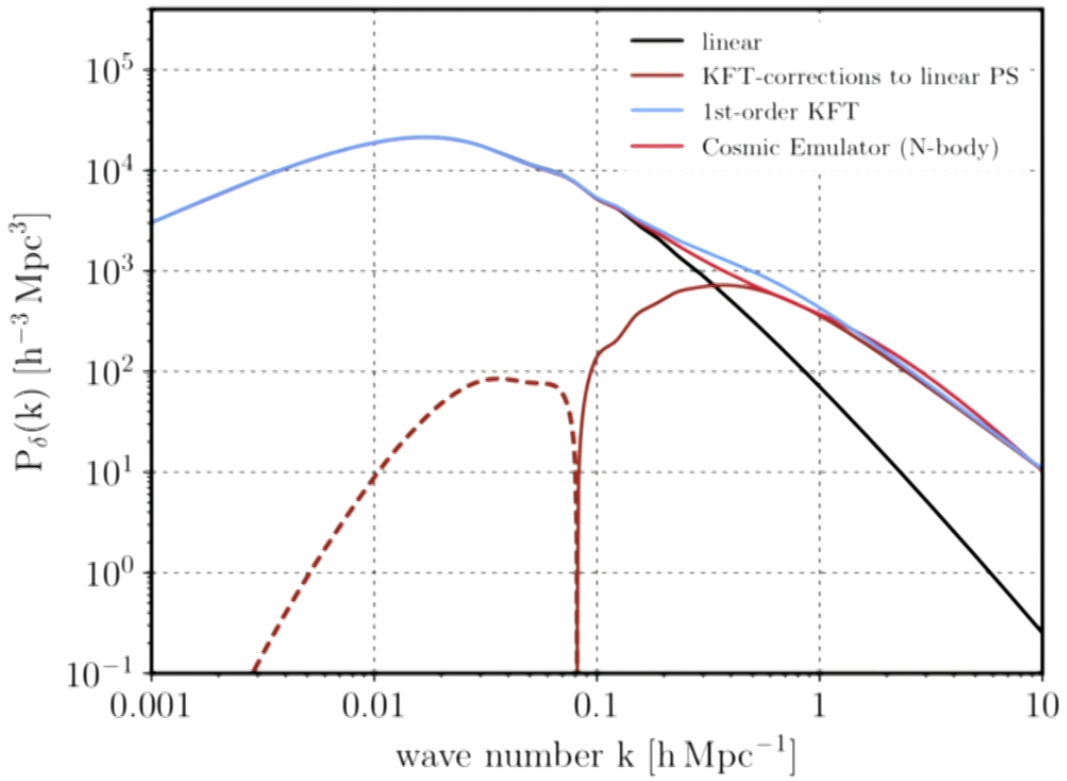
Kinetic Field Theory:

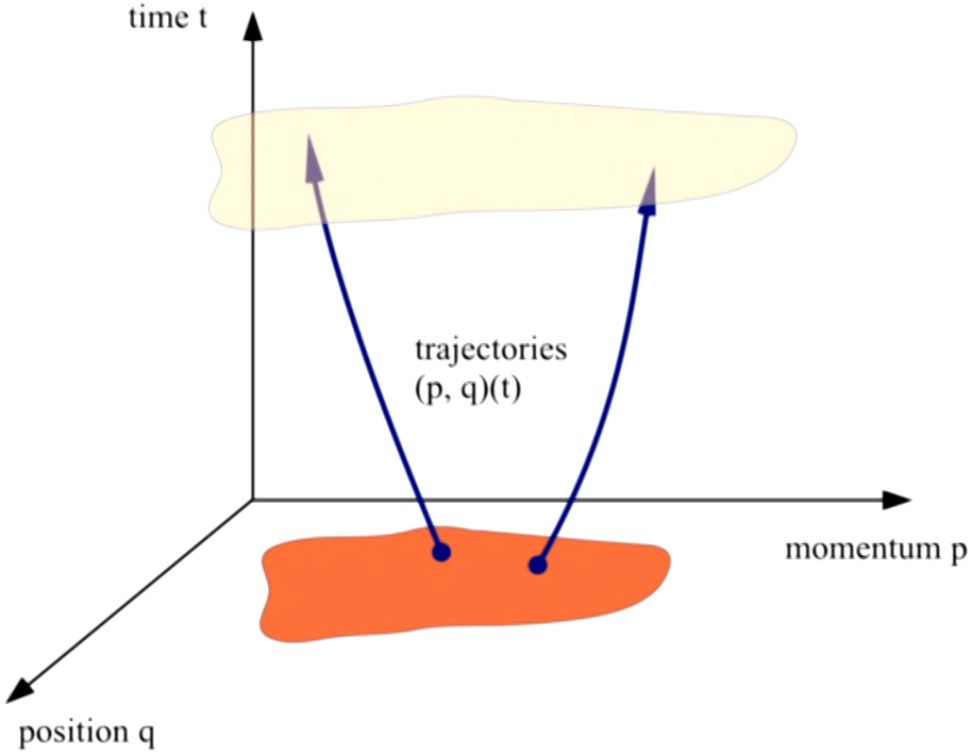
- Non-equilibrium statistics of N classical (initially correlated) particles
- Describe particle ensemble by generating functional Z
- Derive statistical properties by functional derivatives

Power spectrum with KFT



Bartelmann et al. 2016, NJP

Phase-space trajectories



Trajectories in phase space do not cross.

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Particle dynamics

- Hamiltonian dynamics

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + m\varphi \,, \qquad \vec{\nabla}^2\varphi = A_\varphi \delta$$

  (m and A_φ depend on time)

- Solution to e.o.m.

$$\vec{q}(t) = \vec{q}^{(i)} + g_{qp}(t,0)\,\vec{p}^{(i)} + \int_0^t dt'\, g_{qp}(t,t')\,\vec{f}(t')$$

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Particle dynamics

- Hamiltonian dynamics

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + m\varphi \,, \quad \vec{\nabla}^2\varphi = A_\varphi \delta$$

  (m and A_φ depend on time)

- Solution to e.o.m. for all N particles

$$\bar{\mathbf{x}}(t) = \underbrace{\mathcal{G}(t,0)\mathbf{x}^{(i)}}_{\text{inertial motion}} + \underbrace{\int_0^t dt'\, \mathcal{G}(t,t')\, \mathcal{F}(t')}_{\text{interactions}}$$

  ($\mathbf{x} = \vec{x}_i \otimes \vec{e}_i$ and $\vec{x}_i = (\vec{q}_i, \vec{p}_i)$)

- Free choice of propagator $g_{qp}(t,t')$ and force $\vec{f}(t')$ relative to it

Generating functional

- In general

$$Z = \int \mathcal{D}\varphi \, P[\varphi]$$

- KFT: Use point particles instead of fields; introduce source field $\mathbf{J}$

$$\begin{aligned} Z[\mathbf{J}] &= \int \mathcal{D}\mathbf{x} \, P(\mathbf{x}) e^{i \int dt' \langle \mathbf{J}, \mathbf{x} \rangle} \\ &= \int \mathcal{D}\mathbf{x} \int d\mathbf{x}^{(\mathbf{i})} \, P(\mathbf{x}^{(\mathbf{i})}) \, P(\mathbf{x}|\mathbf{x}^{(\mathbf{i})}) e^{i \int dt' \langle \mathbf{J}, \mathbf{x} \rangle} \\ &= \int \mathcal{D}\mathbf{x} \int d\Gamma \, \delta_D(\mathbf{x} - \bar{\mathbf{x}}) e^{i \int dt' \langle \mathbf{J}, \mathbf{x} \rangle} \\ &= \int d\Gamma \, e^{i \int dt' \langle \mathbf{J}, \bar{\mathbf{x}} \rangle} \end{aligned}$$

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Correlation functions and density operator

- n-point correlation function

$$\langle \rho(1)\rho(2)\dots\rho(n)\rangle = \hat{\rho}(1)\hat{\rho}(2)\dots\hat{\rho}(n)\, Z[J]\Big|_{J=0}$$

- number density operator:

$$\rho = \sum_{i=1}^{N} \delta_D\left(\vec{q} - \vec{q}_i\right) \quad \rightarrow \quad \sum_{i=1}^{N} \exp\left(-i\vec{k}\cdot\vec{q}_i\right)$$

$$\hat{\rho} = \sum_{i=1}^{N} \exp\left(-i\vec{k}\cdot\frac{\delta}{\delta J_{q_i}(t)}\right)$$

- Exponential of derivative: generates translation

$$\hat{\rho}(1)\cdots\hat{\rho}(n)\, Z[J]\Big|_{J=0} = Z[L]$$

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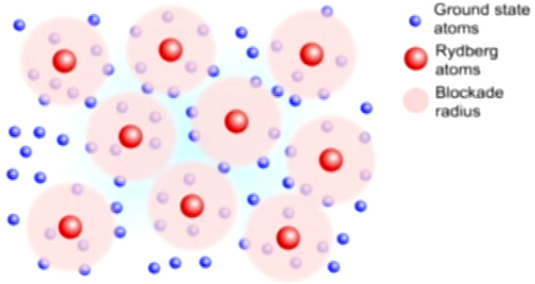
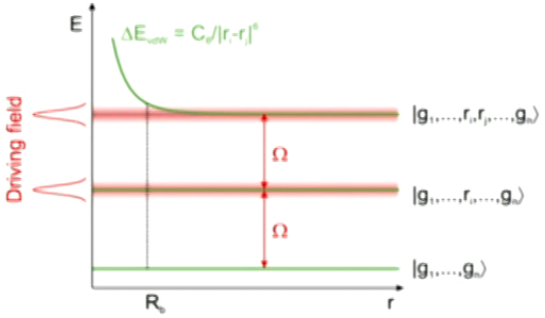
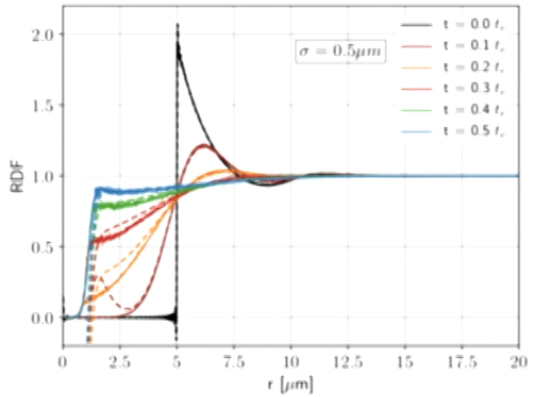
Applications beyond cosmology



Rydberg gas





E. Kozlikin et al. in prep

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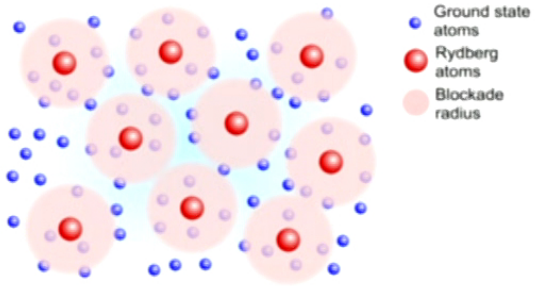
Applications beyond cosmology



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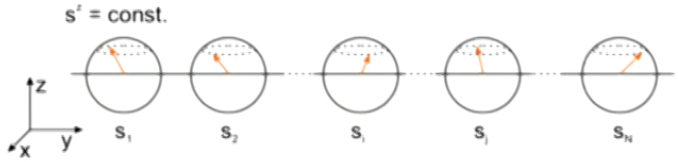
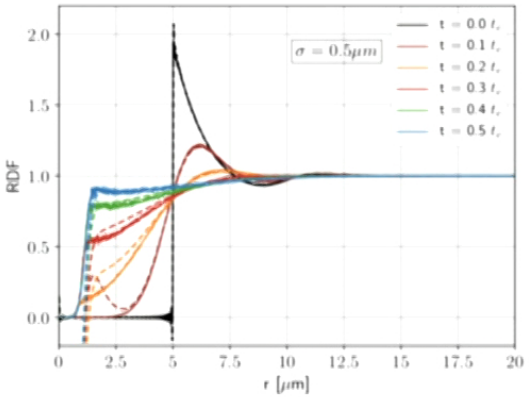


Spin(ning top)s on a chain



E. Kozlikin et al. in prep

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Choice of propagators

$$Z[\mathbf{J}] = \int d\Gamma \, e^{i \int dt' \langle \mathbf{J}, \bar{\mathbf{x}} \rangle}$$

- Choose $g_{qp}(t, t')$ to reproduce linear growth

$$g_{qp}(t, t') = D_+(t) - D_+(t') =: t - t'$$

(Zel'dovich trajectories)

- Or improved Zel'dovich trajectories to slow down re-expansion of structures
- Relative force

$$\vec{f} = -\vec{\nabla}\varphi + \frac{\dot{m}h}{m^2} \int_0^t dt' \, m \vec{\nabla}\varphi$$

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Initial conditions

$$Z[\mathbf{J}] = \int d\Gamma \, e^{i \int dt' \langle \mathbf{J}, \bar{\mathbf{x}} \rangle}$$

- Multivariate Gaussian

$$P(\mathbf{x}^{(i)}) = \mathcal{N} \hat{\mathcal{C}}(\mathbf{p}) \, e^{-\frac{1}{2} \mathbf{p}^\top C_{pp}^{-1} \mathbf{p}}$$

with

$$C_{pp} = \langle \vec{p}_i \otimes \vec{p}_j \rangle = \int d\vec{k} \, (\vec{k} \otimes \vec{k}) \, k^{-4} \, P_\delta^{(i)}(k) e^{i\vec{k} \cdot (\vec{q}_i - \vec{q}_j)}$$

- For cosmology: $\hat{\mathcal{C}}(\mathbf{p}) \approx 1$

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Free generating functional

- Free generating functional $Z_0[L]$ can be factorised such that full hierarchy of initial momentum-correlations can be included
- For 2-point function

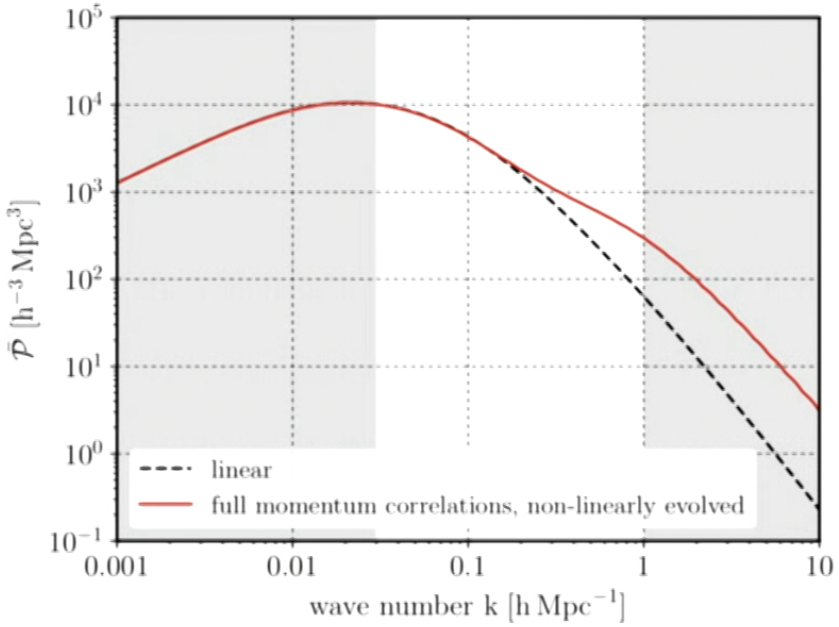
$$Z_0[L] = V^{-2}(2\pi)^3\delta_D\left(\vec{k}_1 + \vec{k}_2\right) e^{-Q_D}\,\mathcal{P}(k_1, t)$$

- Non-linear, free power spectrum

$$\mathcal{P}(k, t) = \int_q \left[e^{-g_{qp}^2(t,0)k^2 a_{\parallel}(q,\mu)} - 1\right] e^{i\vec{k}\cdot\vec{q}}$$

- Momentum correlation function $a_{\parallel}(q, \mu)$, damping term Q_D

Effect of initial correlations



Bartelmann et al. 2017, NJP

Shape of non-linear power spectrum dominated by initial momentum correlations on intermediate scales.

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Include particle interactions

Option 1: Perturbative

- Interaction operator

$$Z[\mathbf{J},\mathbf{K}] = e^{i\hat{S}_I} Z_0[\mathbf{J},\mathbf{K}]$$

$$\mathcal{P}_\delta(k,t) \propto \hat{\rho}\hat{\rho}\left[1 + i\hat{S}_I + ...\right] Z_0[\mathbf{J},\mathbf{K}]$$

Option 2: Mean-field

- Evaluated interaction term $\langle S_I \rangle$: force between particle pairs $\vec{\nabla}_1 v_2$ weighted by correlation function ξ_{12}

$$\bar{\mathcal{P}}_\delta(k,t) = e^{i\langle S_I\rangle - Q_D}\mathcal{P}(k)$$

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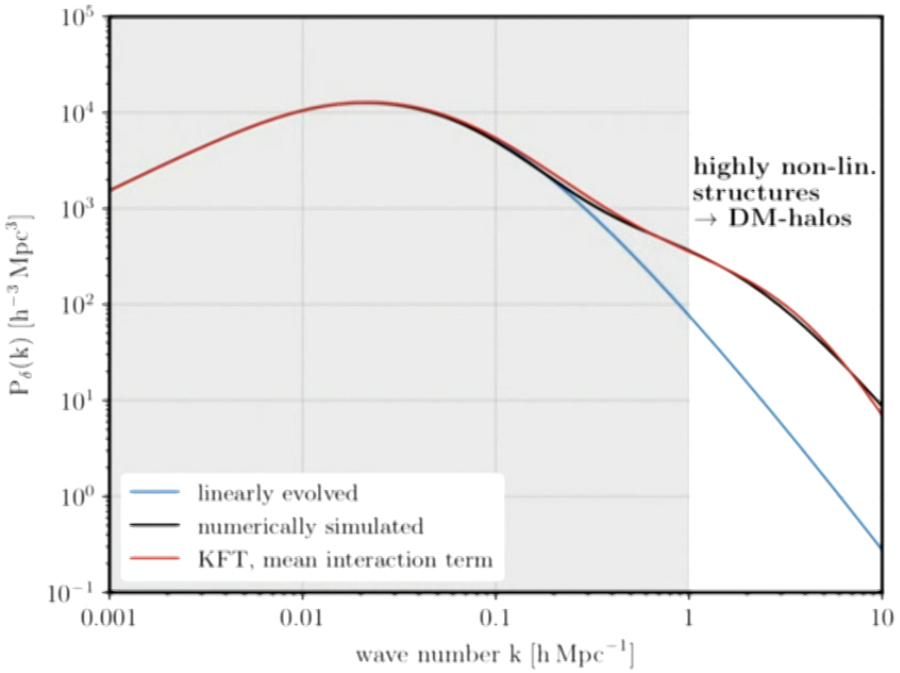
Effect of interaction potential

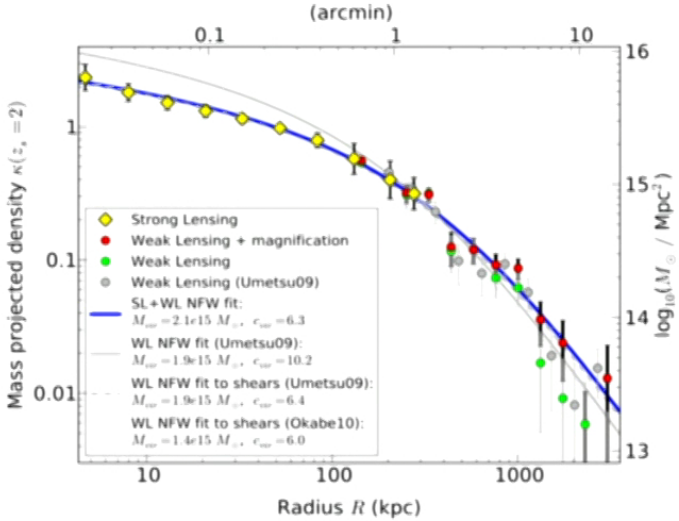


Bartelmann et al. 2019, Ann. Phys.

Halo profiles

How can we understand the self-similarity of cosmic structure?

Abell 2261, CLASH Project



Coe et al. 2012

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Halo profiles and interaction potential

- At $k > 1\,h\,\mathrm{Mpc}^{-1}$:

$$P_\delta(k) \propto \int dm \; \underbrace{n(m)}_{\substack{\text{mass} \\ \text{function}}} \; \underbrace{|\rho(k|m)|^2}_{\substack{\text{density} \\ \text{profile}}}$$

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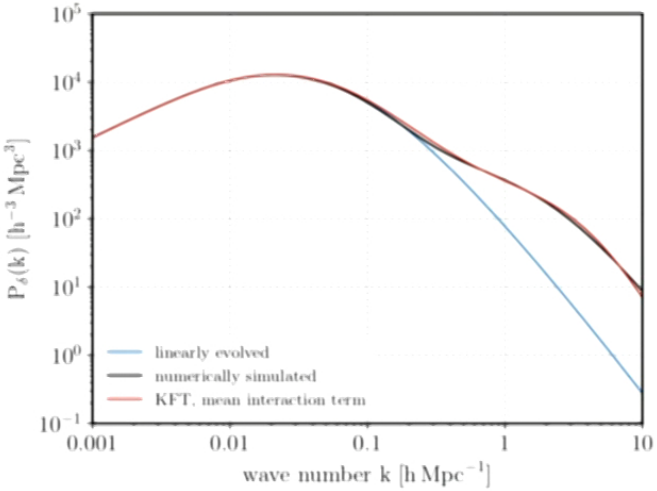
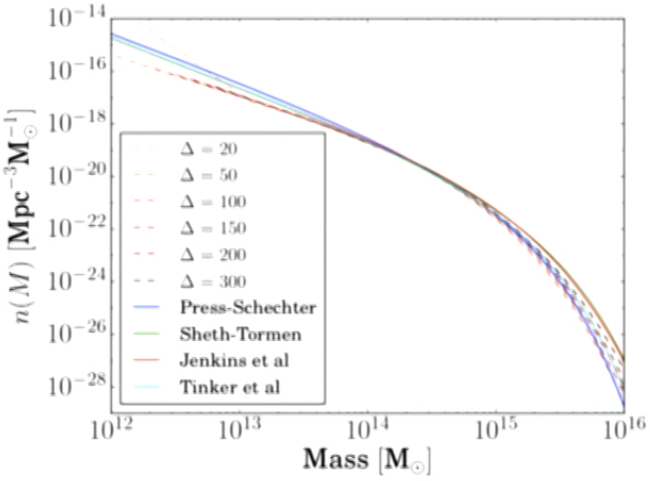
- $P_\delta(k)$ and $n(m)$ computable with KFT.

Results



Bartelmann et al. 2019, Ann.Phys.



Linke et al. arXiv:1712.04461

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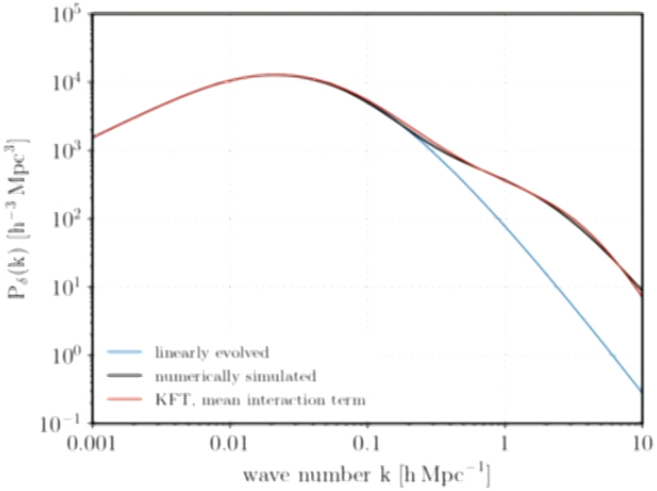
$\Rightarrow$ How does $v(r)$ affect the halo density profile?
Recovery of NFW? Self-similarity feature?

Results



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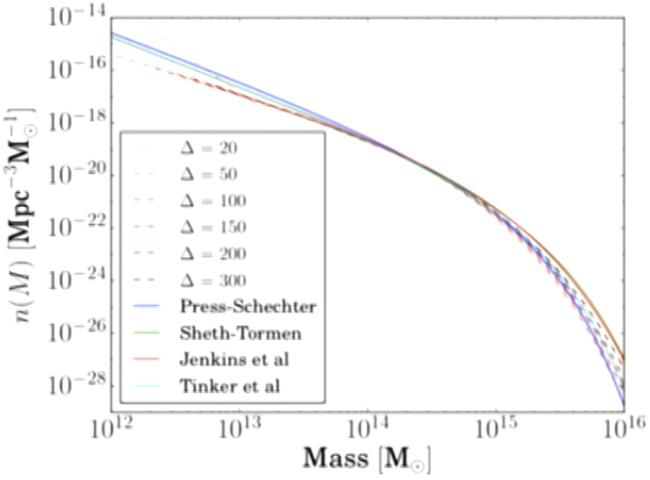

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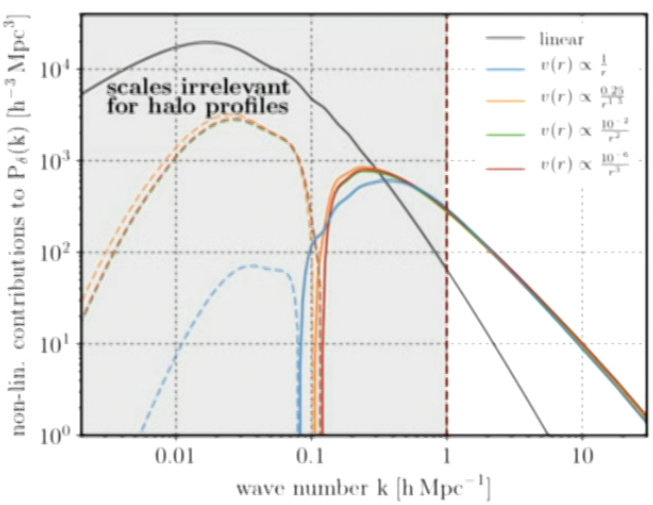
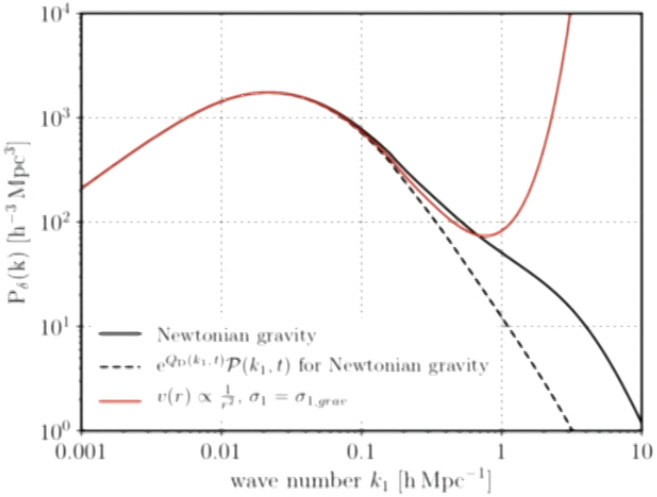
Preliminary results

Perturbative approach



slope at high k not
sensitive to potential

Mean-field



high sensitivity to potential

Kozlikin et al. in prep

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Preliminary results

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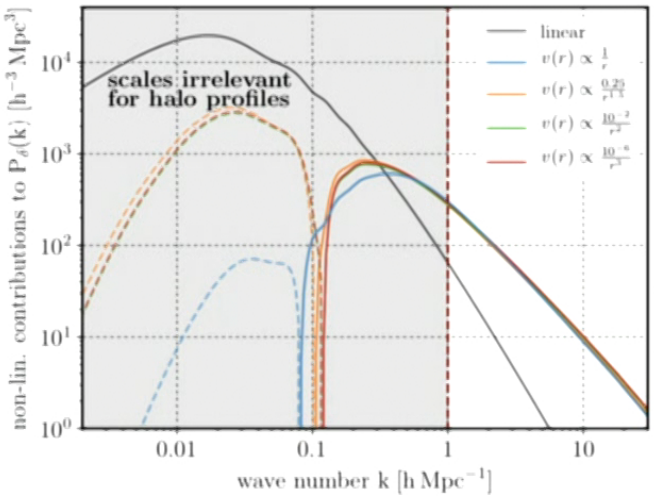
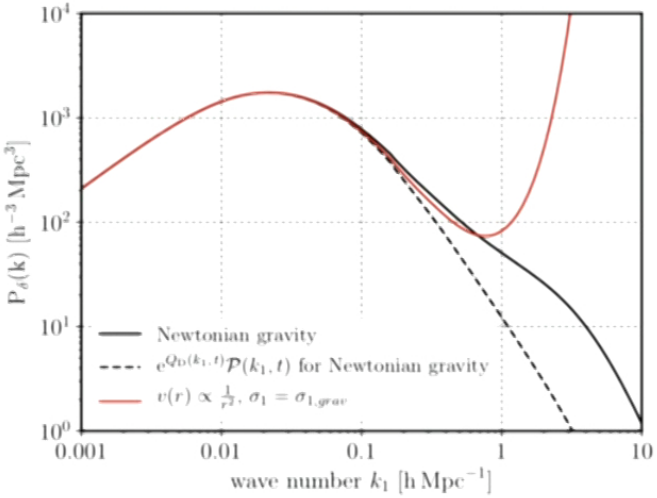


need higher order perturbations

Mean-field



too much structure produced

Kozlikin et al. in prep

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Recent developments for Newtonian gravity

- $\langle S_I \rangle$ contains force relative to inertial motion between particle pairs

$$\vec{f}_{12} = -\vec{\nabla}_1 v_2 + \frac{\dot{m}h}{m^2} \int_0^t dt' \, m \vec{\nabla}_1 v_2$$

- Inertial motion leads to linear growth for Zel'dovich trajectories
- Full particle interaction potential in $\vec{f}$ over-estimates structure growth
- Effective potential

$$\tilde{v} \propto -\frac{1}{k_0^2 + k^2}$$

- Time-dependent Yukawa scale k_0 determined self-consistently from KFT

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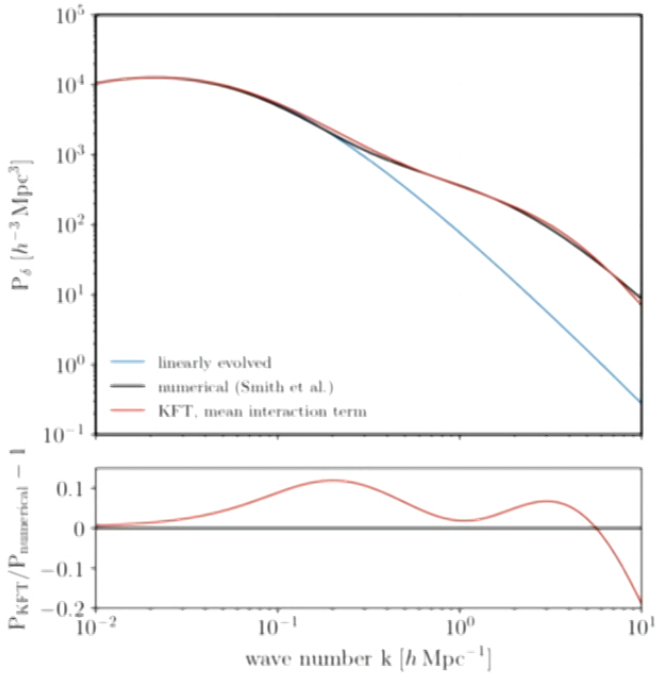
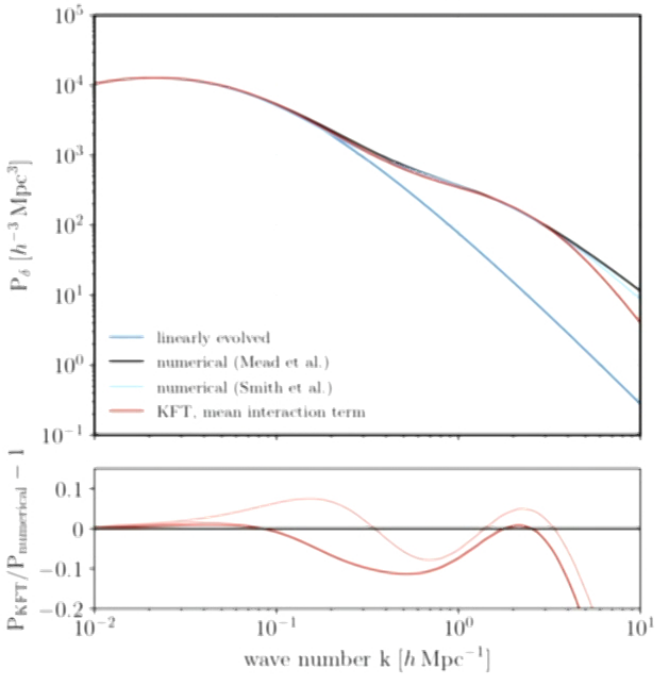
Recent developments for Newtonian gravity

Full interaction potential



Bartelmann et al. 2019, Ann.Phys.

Effective interaction potential



PRELIMINARY, Bartelmann et al. in prep

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Effective modified interaction potential

- Same procedure to find the effective form of the potential

$$\tilde{v} \propto -\frac{1}{g_n(k)} f_v(k_0; k)$$

  with Fourier transform of the potential $g_n(k)$
- Better consistency between damping and interaction term
- Expect less structure on small scales

In summary

- Parameter free, analytic description of non-linear structure formation.
- Investigate effects of physical processes on DM structure formation.
- Closed expression for the non-linear density power spectrum.
- Systematically construct effective interaction potentials.
- Extract information on halo profiles from non-linear power spectrum.

Outlook

- Compute $P_\delta(k, t)$ with effective interaction potential for different power-law potentials $\rightarrow$ halo profiles.
- Compute halo mass function directly from KFT for different potential slopes $\rightarrow$ halo profiles.
- Systematically construct (corrections to) mean-field interaction term.
- Compute higher-order spectra from mean-field approach.