Title: PSI 2019/2020 - Computational Physics - Lecture 5

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Collection: PSI 2019/2020 - Computational Physics

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URL: http://pirsa.org/20010082

Discretizing Functions

$[-1, +1]$

1. choose basis — "discretization method"
2. cut-off N — "resolution"

⋮

3. show result is indep. of N — "convergence"

+1]

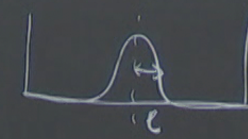
od"

Example: $P_\ell(x)$

$$f(x) = \sum_{i=0}^{N} c^i P_i(x)$$

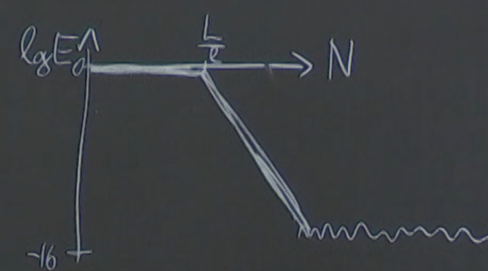
$$\| f \|_2 = \int_{-1}^{+1} |f(x)|^2 \, dx$$

$$h \sim \frac{L}{N}$$



$h \lesssim \ell$

$N \gtrsim \frac{L}{\ell}$



Derivatives:

$$f(x) = \sum_i c^i P_i(x)$$

$$f'(x) = \sum_i c^i P_i'(x)$$

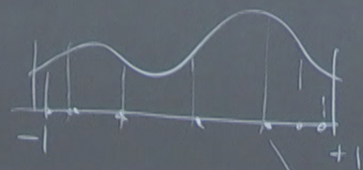
$$= \sum_i c^i \sum_j d_i^{\,j} P_j(x)$$

$$= \boxed{d_i^{\,j} c^i} P_j(x)$$

Qudrature:

Quadrature:

Gauß



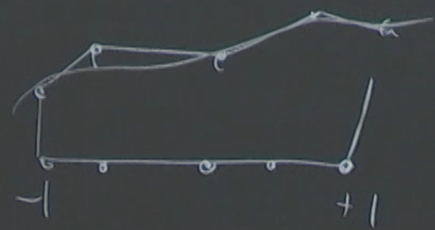
collocation points

Determine coefficients:

$$f(x) = c^i P_i(x)$$

$$\int_{-1}^{+1} P_k(x) f(x) = c^i \delta_i^k$$

PLC



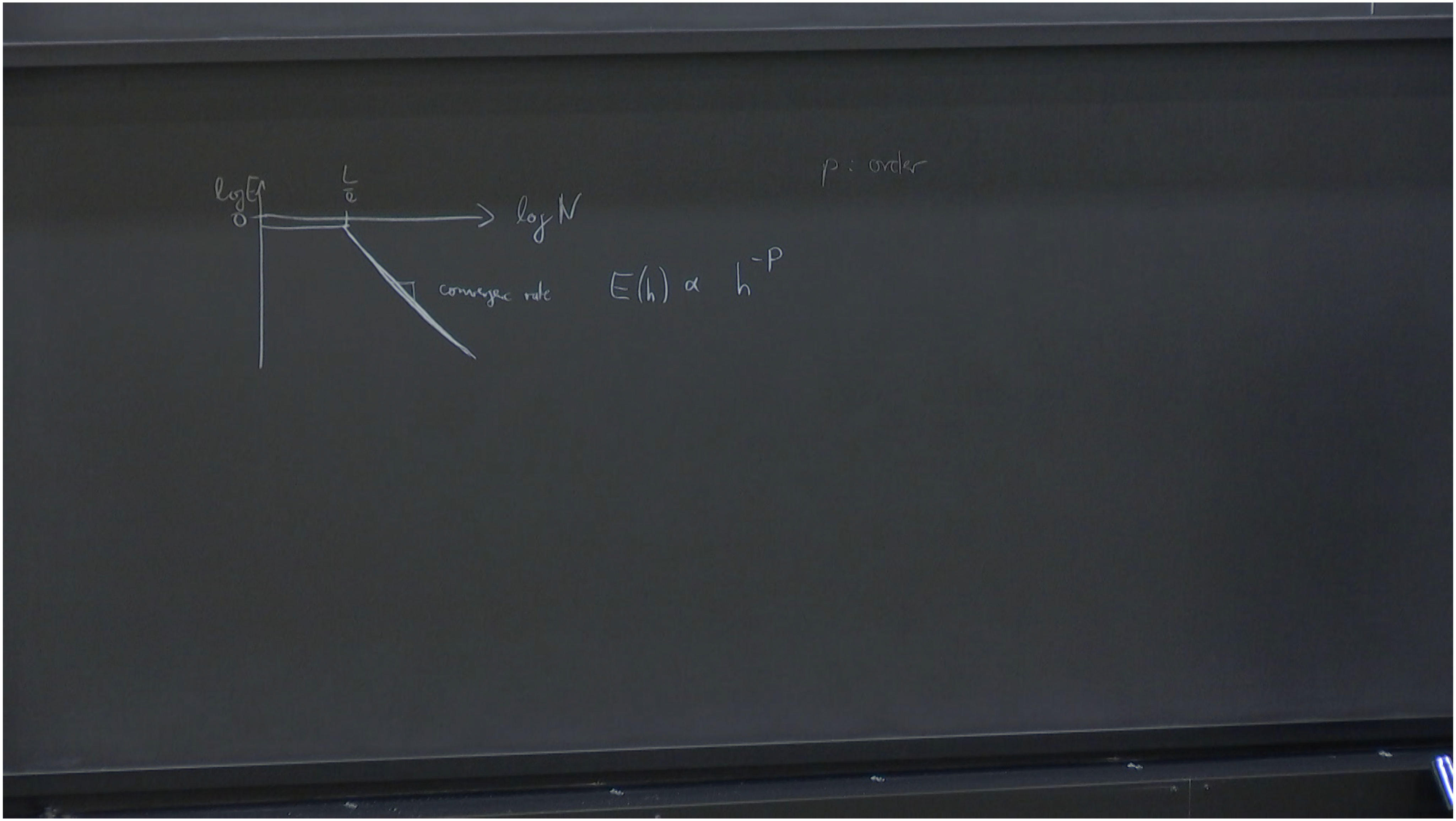
$$h = \frac{L}{N-1}$$

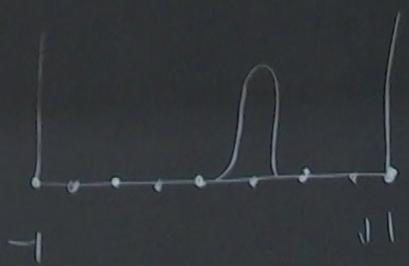
Finite Differences

basis for linear interpolation:

- PLC
- 1 at one coll point
- 0 at all other coll. points

"position basis"


p : order
log E
0
log N
convergence rate
E(h) ∝ h^-p



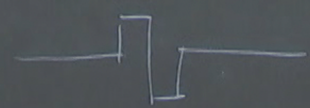
Richardson extrapolation

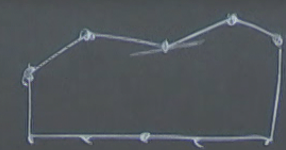
$$I^{N} = I + \alpha \cdot h_N^{2} + \cancel{\beta \cdot h_N^{3}} + \dots\dots$$

$$I^{2N} = I + \alpha \cdot h_{2N}^{2}$$

$$I^{4N} = \dots\dots$$

$$f(x) = \sum_i c^i \, b_i(x)$$

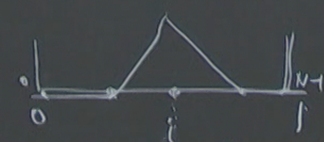




$$f'(x) = \sum_i c^i \, b'_i(x)$$

$$\approx d_i^{\,j} c^i \, b_j(x)$$

$$d_{FD} = \frac{1}{h} \begin{pmatrix} 0 & +\frac{1}{2} & & 0 \\ -\frac{1}{2} & \ddots & \ddots & \\ & \ddots & \ddots & +\frac{1}{2} \\ 0 & & -\frac{1}{2} & 0 \end{pmatrix}$$

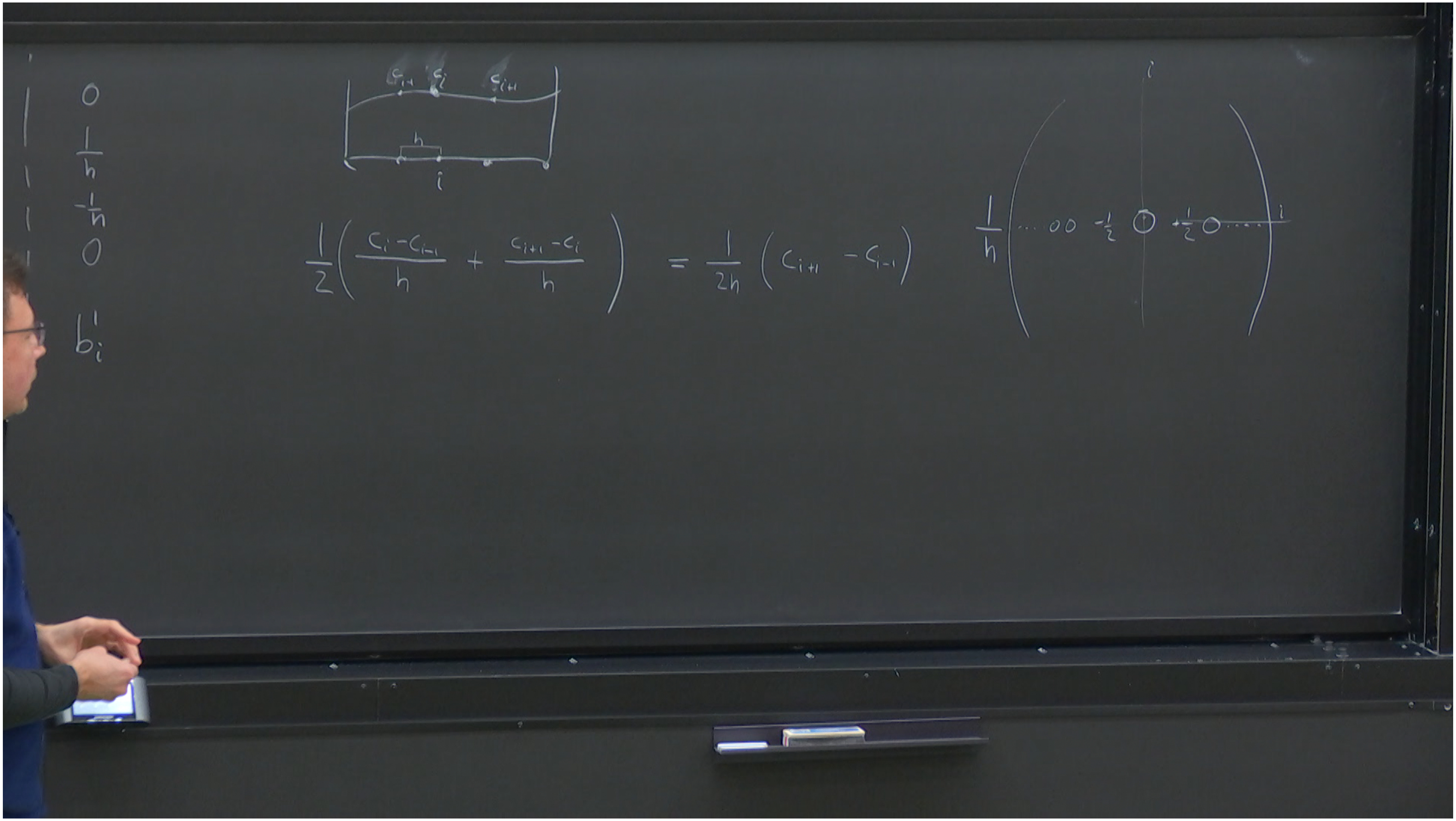


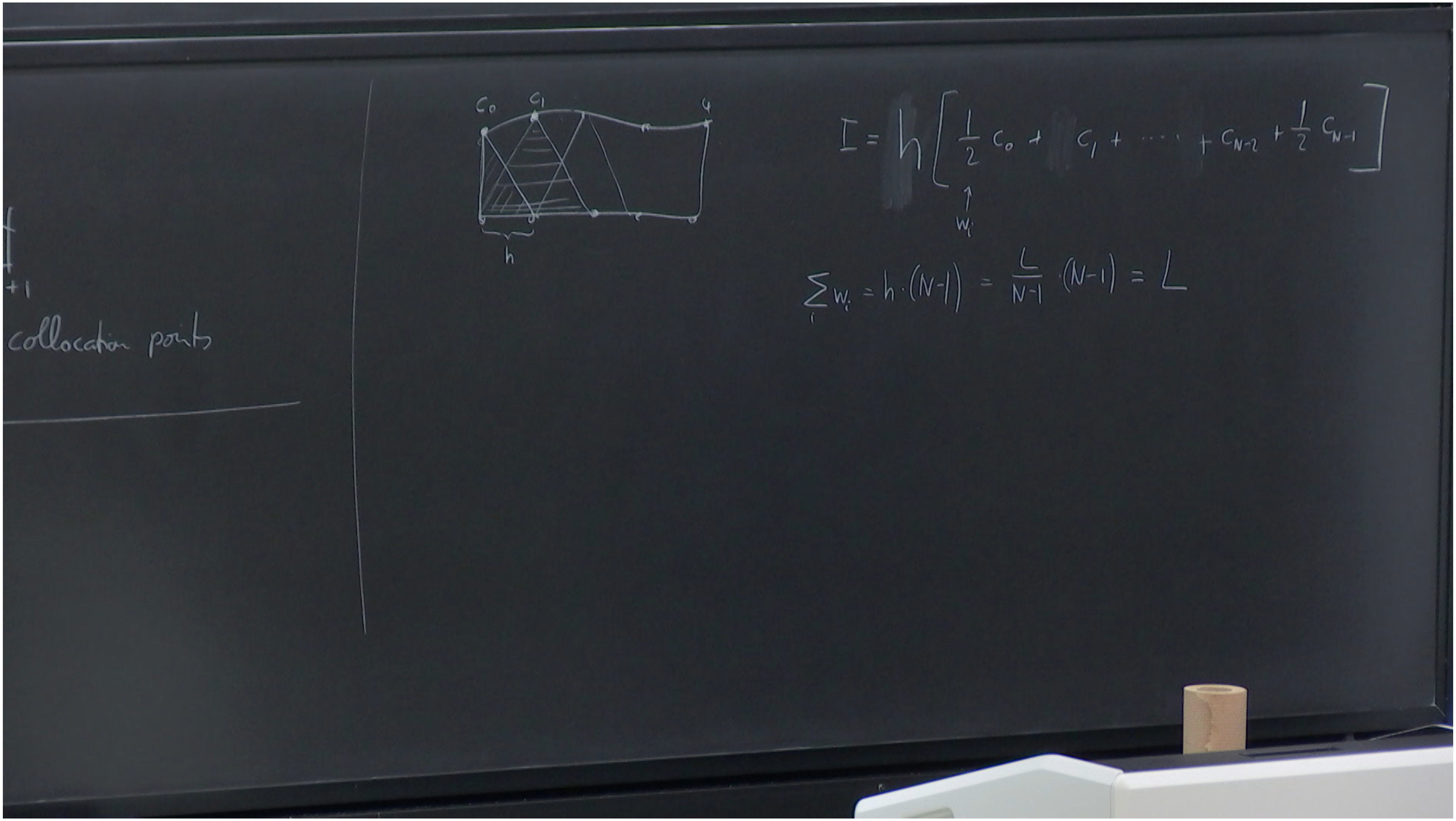
$$x_i = \frac{i}{N-1} = h \cdot i$$

$$h = \frac{1}{N-1}$$

$$b_i = \begin{cases} 0 & x \leq h\cdot(i-1) \\ \frac{1}{h}\cdot(x - h(i-1)) & h(i-1) \leq x \leq hi \\ -\frac{1}{h}\cdot(x - h(i+1)) & hi \leq x \leq h(i+1) \\ 0 & x \geq h\cdot(i+1) \end{cases}$$

$$b_i'$$


0
$\frac{1}{h}$
$-\frac{1}{h}$
0
b_i^1
c_{i-1} c_i c_{i+1}
h
i
$\frac{1}{2}\left(\frac{c_i - c_{i-1}}{h} + \frac{c_{i+1} - c_i}{h}\right) = \frac{1}{2h}\left(c_{i+1} - c_{i-1}\right)$
i
$\frac{1}{h}\left(\cdots\ 0\ 0\ -\frac{1}{2}\ 0\ +\frac{1}{2}\ 0\ \cdots\right)$ i


collocation points
c_0 c_1 c_4
h
$I = h\left[\frac{1}{2} c_0 + c_1 + \cdots + c_{N-2} + \frac{1}{2} c_{N-1}\right]$
w_i
$\sum_i w_i = h \cdot (N-1) = \frac{L}{N-1} \cdot (N-1) = L$