Title: PSI 2019/2020 - QFT III - Lecture 6

Speakers:

Collection: PSI 2019/2020 - QFT III

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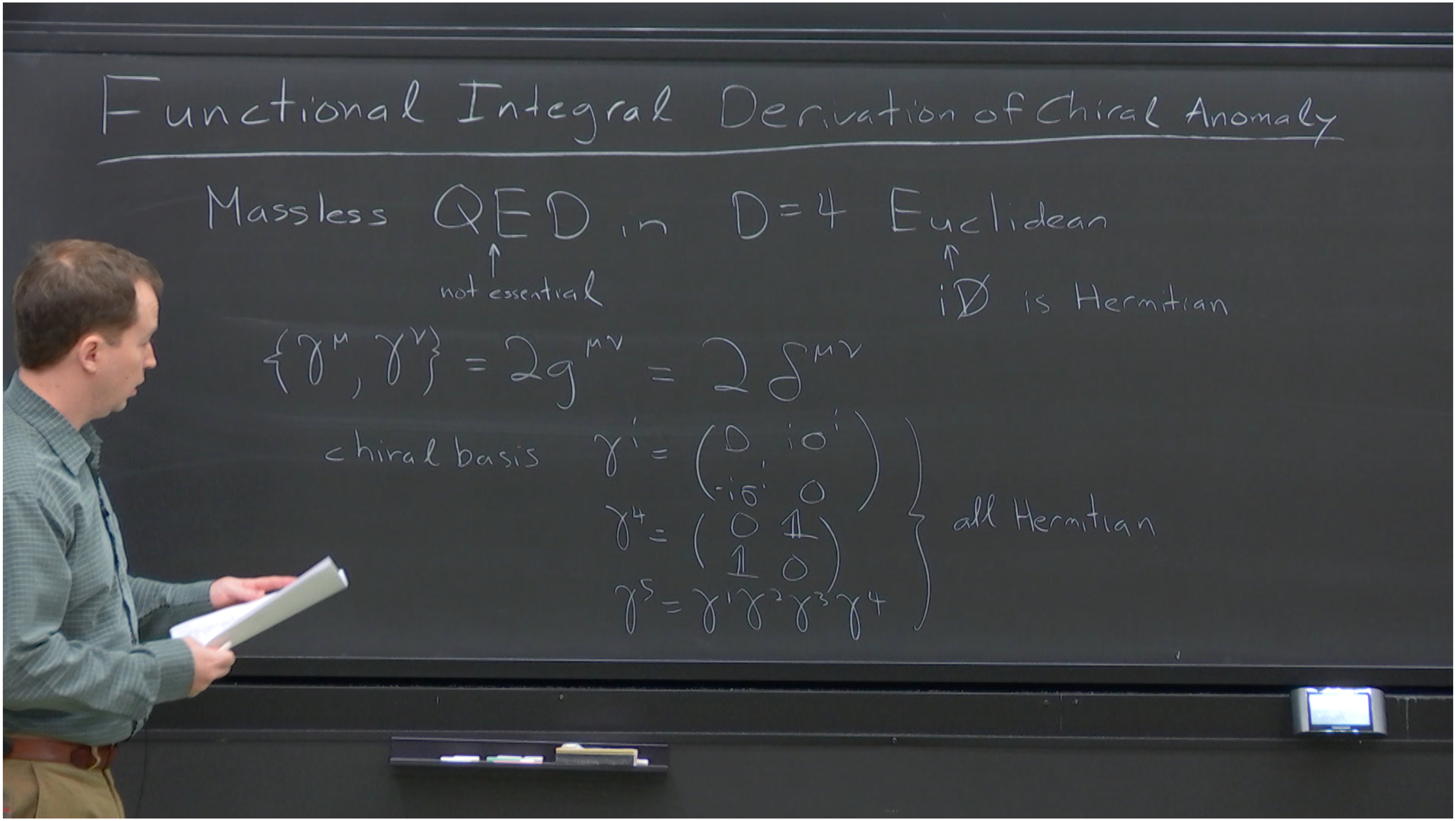
Functional Integral Derivation of Chiral Anomaly


Functional Integral Derivation of Chiral Anomaly
Massless QED in D=4 Euclidean
not essential
$i\not{D}$ is Hermitian
$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} = 2\delta^{\mu\nu}$
chiral basis
$\gamma^i = \begin{pmatrix} 0 & i\sigma^i \\ -i\sigma^i & 0 \end{pmatrix}$
$\gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\gamma^5 = \gamma^1\gamma^2\gamma^3\gamma^4$
all Hermitian

maly

$$Z[A] = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \, e^{-S[\Psi, \bar{\Psi}, A]}$$

no A integral

...maly

$$Z[A] = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi}\, e^{-S[\Psi,\bar{\Psi},A]}$$

no A integral

true classically

$\partial_\mu J^\mu_5 = 0$

$\langle \partial_\mu J^\mu_5 \rangle = \ ? \neq 0$ QFT

goal

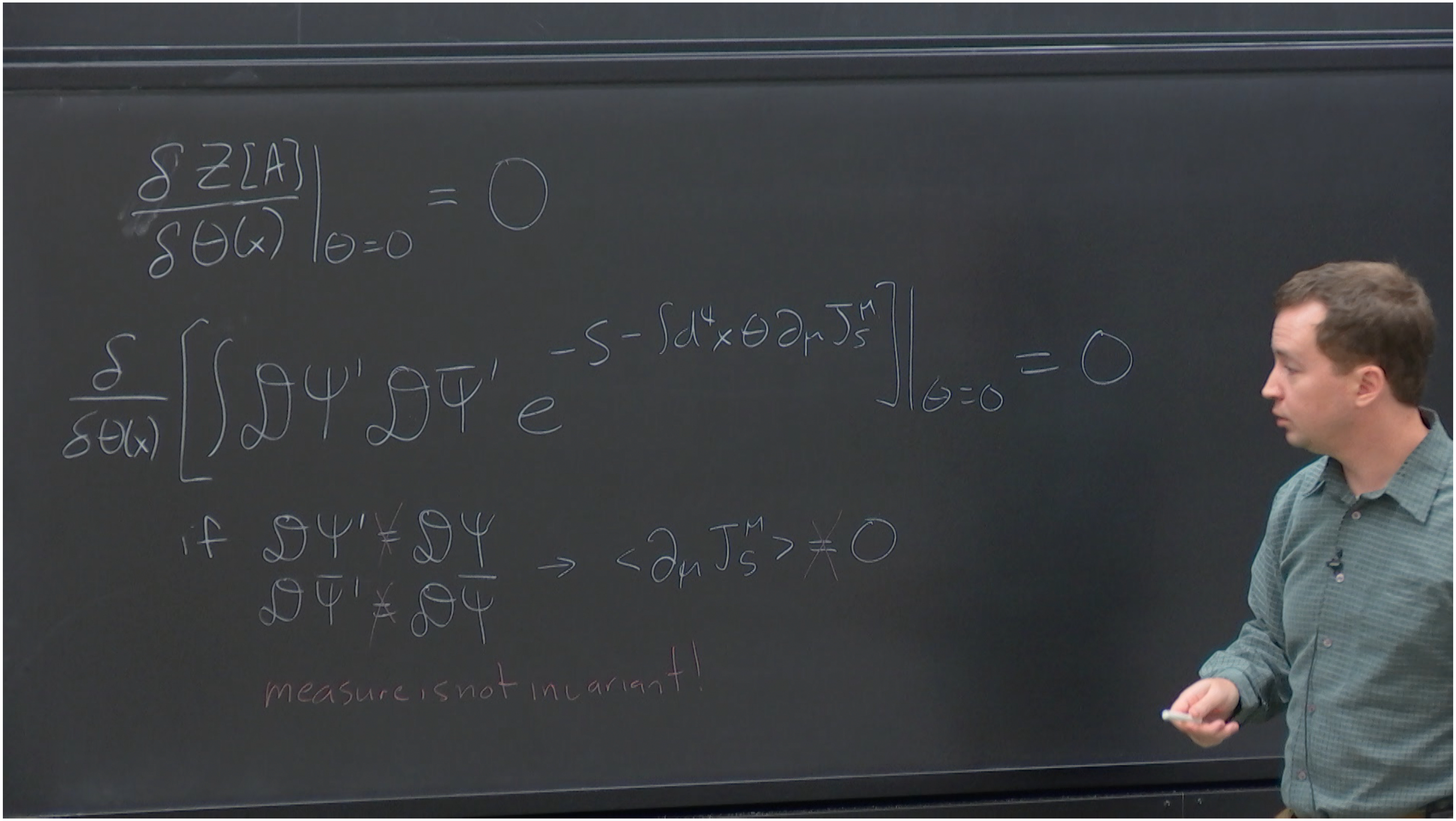
$$\Psi \to \Psi' = e^{i\theta(x)\gamma^5}\Psi$$

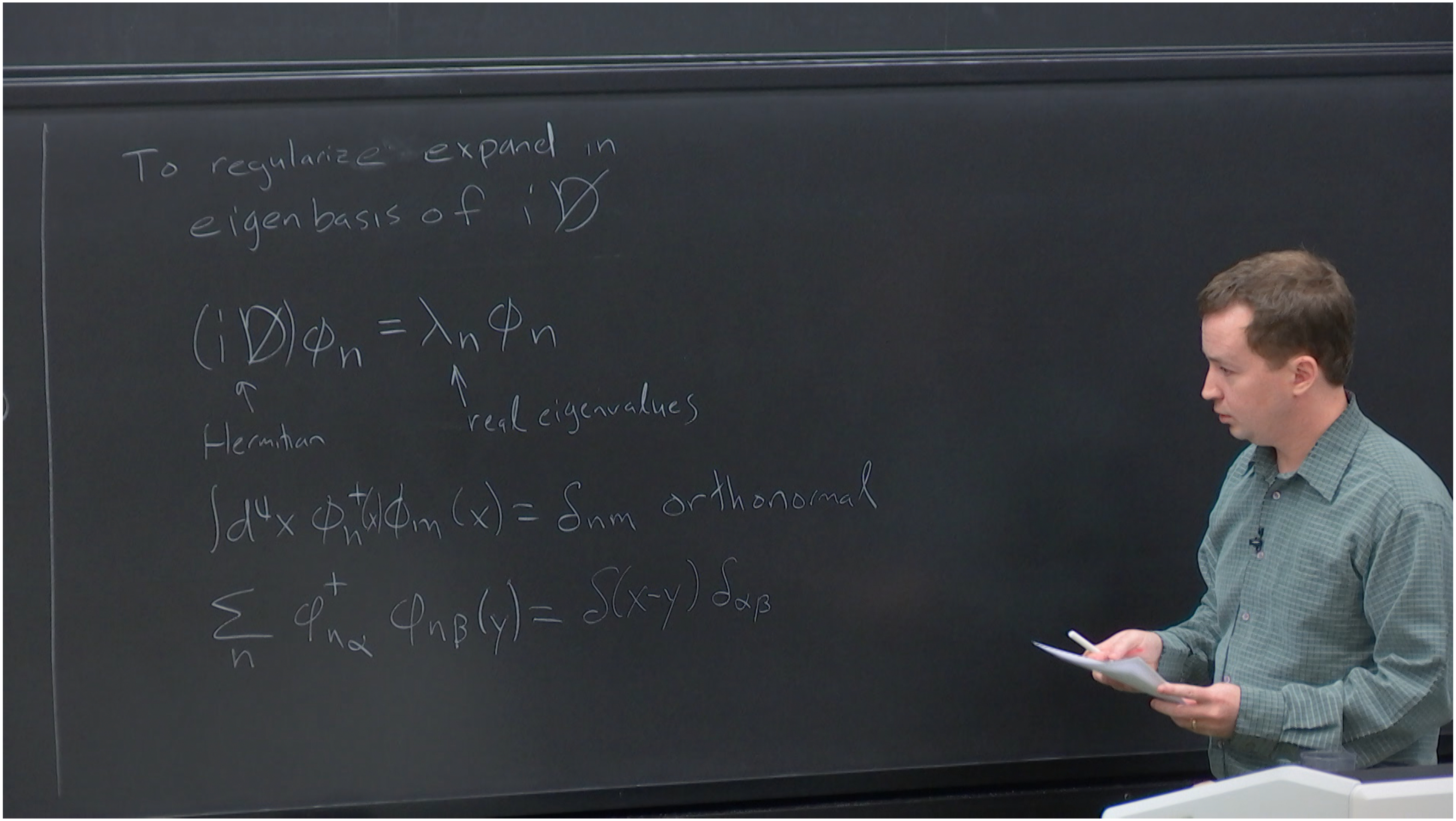
$$\bar{\Psi} \to \bar{\Psi}' = \bar{\Psi} e^{i\theta(x)\gamma^5}$$

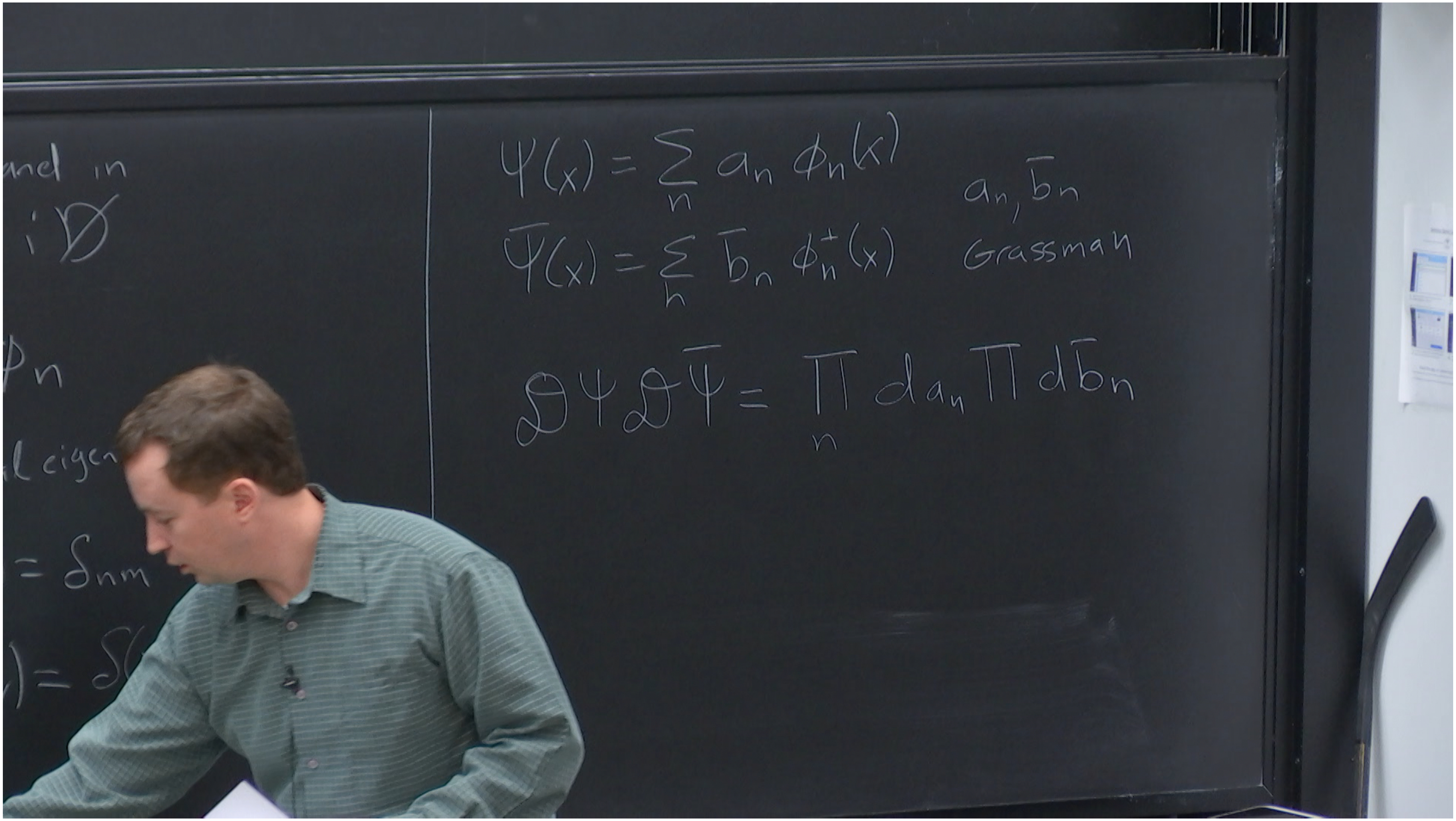
$$A_\mu \to A'_\mu = A_\mu$$

$$S \to S' = S + \int d^4x\, \theta\, \partial_\mu J^\mu_5$$

$$Z[A] \to Z[A]$$


$\frac{\delta Z[A]}{\delta \theta(x)}\Big|_{\theta=0} = 0$
$\frac{\delta}{\delta\theta(x)}\left[\int \mathcal{D}\psi' \mathcal{D}\bar{\psi}' e^{-S-\int d^4x\, \theta\, \partial_\mu J_5^\mu}\right]\Big|_{\theta=0} = 0$
if $\mathcal{D}\psi' \cancel{\neq} \mathcal{D}\psi$
$\mathcal{D}\bar{\psi}' \cancel{\neq} \mathcal{D}\bar{\psi}$
$\rightarrow \langle \partial_\mu J_5^\mu \rangle \cancel{\neq} 0$
measure is not invariant!


To regularize expand in
eigenbasis of $i\not{D}$
$(i\not{D})\phi_n = \lambda_n \phi_n$
Hermitian
real eigenvalues
$\int d^4x\, \phi_n^\dagger(x)\phi_m(x) = \delta_{nm}$ orthonormal
$\sum_n \varphi^\dagger_{n\alpha}\, \varphi_{n\beta}(y) = \delta(x-y)\,\delta_{\alpha\beta}$


$\Psi(x) = \sum_n a_n \phi_n(x)$
$a_n, \bar{b}_n$
$\bar{\Psi}(x) = \sum_n \bar{b}_n \phi_n^+(x)$
Grassman
$\mathcal{D}\Psi \mathcal{D}\bar{\Psi} = \prod_n da_n \prod d\bar{b}_n$
$= \delta_{nm}$

and in

$i\not{D}$

ϕ_n

al eigenvalues

$= \delta_{nm}$ orthonormal

$) = \delta(x-y)\,\delta_{\alpha\beta}$

$$\Psi(x) = \sum_n a_n \phi_n(x)$$

$$\bar{\Psi}(x) = \sum_n \bar{b}_n \phi_n^{+}(x)$$

$a_n, \bar{b}_n$ Grassman

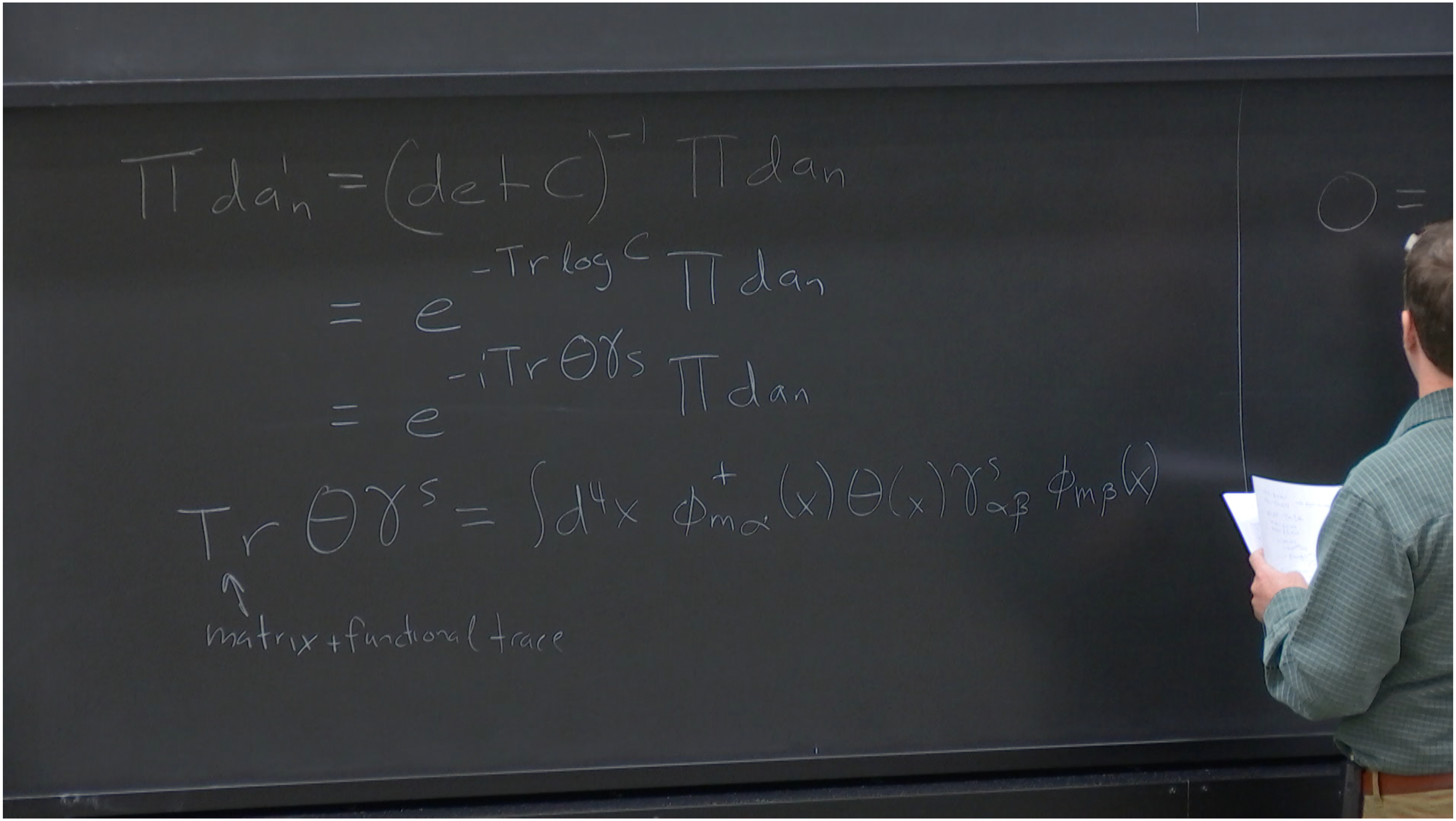
$$\mathcal{D}\Psi\,\mathcal{D}\bar{\Psi} = \prod_n da_n \prod d\bar{b}_n$$

$$\Psi'(x) = \sum_n a_n' \phi_n(x)$$

$$a_n' = \langle \phi_n | \Psi' \rangle$$

$$= \langle \phi_n | e^{i\theta\gamma^5} | \Psi \rangle$$

$$= \sum_m a_m \underbrace{\langle \phi_n | e^{i\theta\gamma^5} | \phi_m \rangle}_{C_{nm}}$$


$\prod da'_n = (\det C)^{-1} \prod da_n$
$= e^{-\mathrm{Tr}\log C} \prod da_n$
$= e^{-i\,\mathrm{Tr}\,\theta\gamma_5} \prod da_n$
$\mathrm{Tr}\,\theta\gamma^5 = \int d^4x\, \phi^\dagger_{m\alpha}(x)\,\theta(x)\,\gamma^5_{\alpha\beta}\,\phi_{m\beta}(x)$
matrix + functional trace
$\mathcal{O} =$

anomaly

$$0 = \frac{\delta Z[A]}{\delta \theta(x)}\Big|_{\theta=0}$$

$$0 = \frac{\delta}{\delta \theta(x)}\left[\int \mathcal{D}\psi \mathcal{D}\bar{\psi}\; e^{-S-\theta\partial_\mu J_5^\mu - 2i\,\mathrm{Tr}\,\theta\gamma^5}\right]\Big|_{\theta=0}$$

$$\mathrm{Tr}\,\Theta\gamma^5 = \int d^4x \, \underbrace{\delta^4(x-x)}_{\infty}\, \Theta(x)\, \underbrace{\mathrm{tr}\,\gamma^5}_{0}$$

$$\mathrm{Tr}\,\Theta\gamma^5 = \lim_{\Lambda\to\infty}\int d^4x \sum_n \Theta\, \phi_n^\dagger\, \gamma^5 f\left(\frac{-\lambda_n^2}{\Lambda^2}\right) \phi_n$$

FT + $i\not{D}\phi_n = \lambda_n \phi_n$

$f(0) = 1$

$f(\infty) = f'(\infty) = \dots = 0$

$$= \lim_{\Lambda\to\infty}\int d^4x \, \frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4} \sum_n \Theta\, e^{-ik'x}\, \tilde{\phi}_n^\dagger(k')\gamma^5 f\left(\frac{-(i\not{D})^2}{\Lambda^2}\right)\tilde{\phi}_n(k)\, e^{ik}$$

$$\sum \tilde{\Phi}_n^\dagger(k')\,\tilde{\Phi}_n(k) = 2\pi^4\,\delta(k-k')\,\mathbb{1}$$

spinor identity

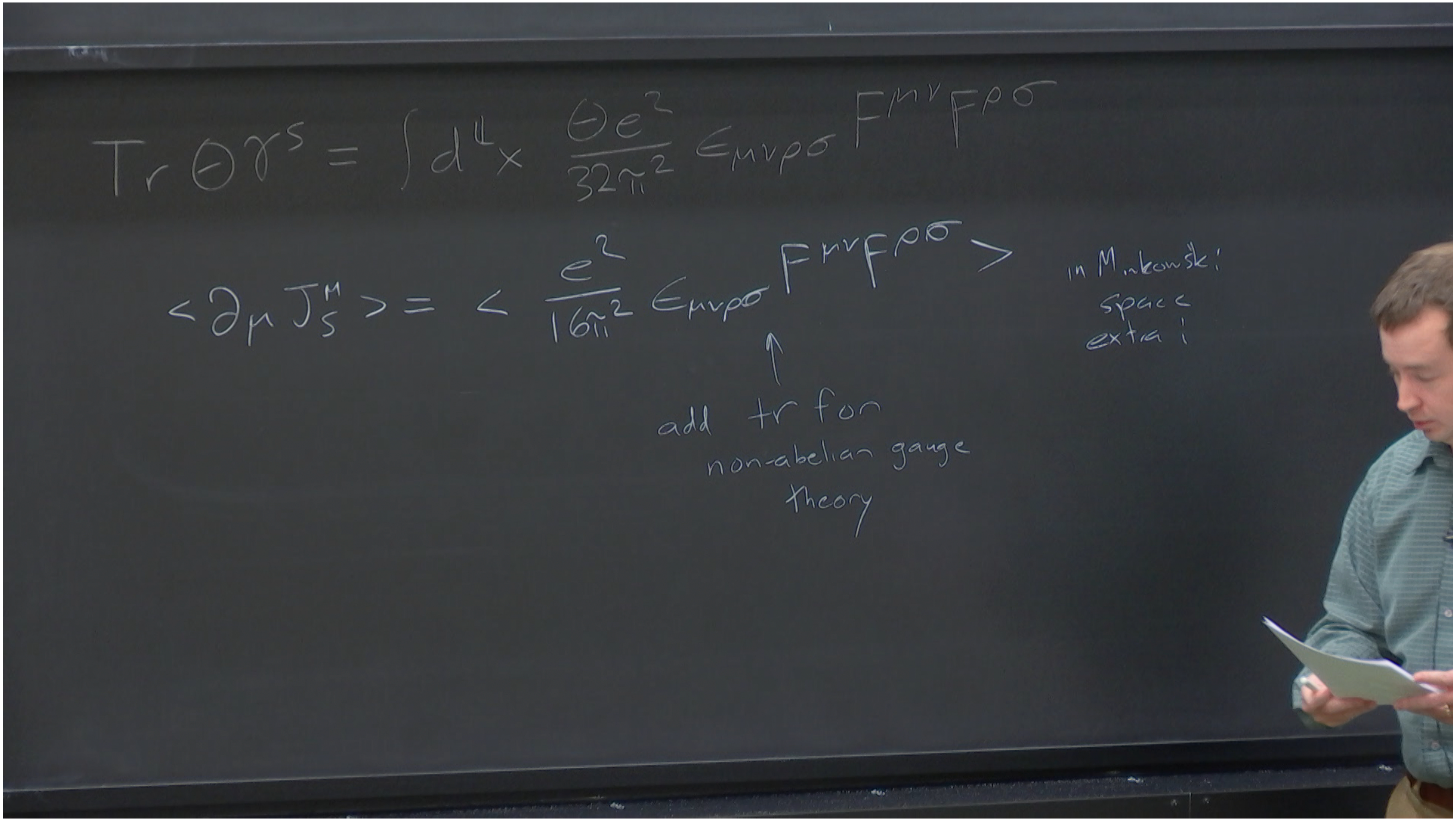
$$\mathrm{Tr}\,\Theta\gamma^5 = \lim_{\Lambda\to\infty}\int d^4x\,\frac{d^4k}{(2\pi)^4}\,\Theta\,\mathrm{tr}\,\gamma^5\sum_{N=0}^{\infty}\frac{1}{N!}\,\frac{(i\not{D})^{2N}}{\Lambda^{2N}}\,f^{(N)}\!\left(\frac{k^2}{\Lambda^2}\right)$$

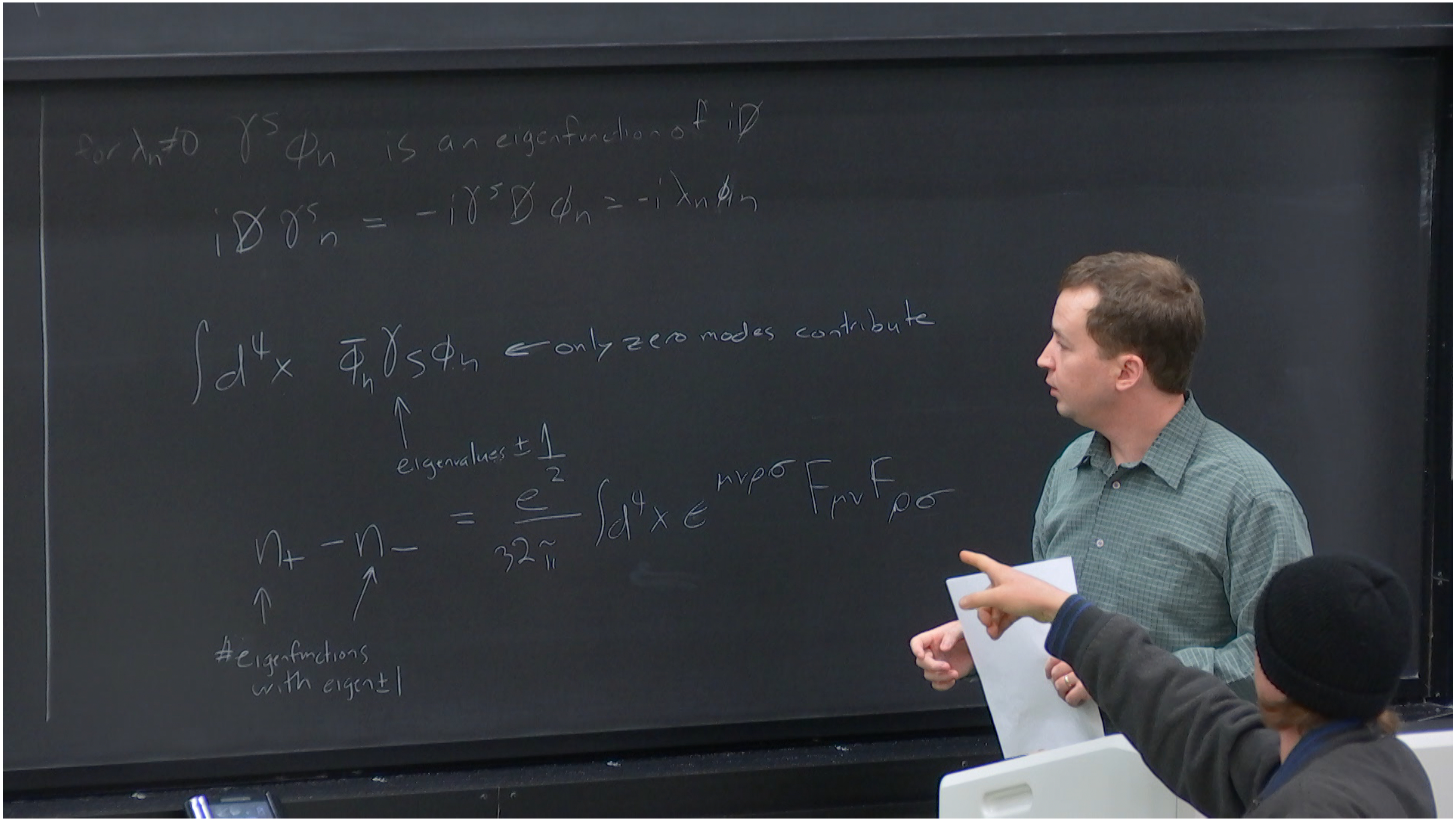
$$\mathrm{tr}\,\gamma^5 = \mathrm{tr}\,\gamma^5\gamma^\mu\gamma^\nu = 0$$

$$\lim_{\Lambda\to\infty}\int^{\Lambda}\frac{d^4k}{2\pi^4}\,\frac{1}{\Lambda^{2N}} = 0 \quad \text{for } N>2$$

$$\mathrm{tr}(\gamma^5\,\not{D}^4) \xrightarrow{\text{algebra}} e^2\,\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$\Phi_n(k)\,e^{ik\cdot x}$$


$\mathrm{Tr}\, \Theta \gamma^5 = \int d^4x \, \frac{\Theta e^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$
$\langle \partial_\mu J_5^\mu \rangle = \langle \frac{e^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \rangle$
in Minkowski space extra i
add tr for non-abelian gauge theory


for $\lambda_n \neq 0$ $\gamma^5 \phi_n$ is an eigenfunction of $i\not{D}$
$i\not{D}\gamma^5_n = -i\gamma^5 \not{D}\phi_n = -i\lambda_n \phi_n$
$\int d^4x \ \bar{\Phi}_n \gamma_5 \phi_n$ ← only zero modes contribute
eigenvalues $\pm \frac{1}{2}$
$n_+ - n_- = \frac{e^2}{32\pi^2} \int d^4x \, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$
#eigenfunctions
with eigen ±1