Title: PSI 2019/2020 - QFT III - Lecture 5

Speakers:

Collection: PSI 2019/2020 - QFT III

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Anomalies

Noether's Theorem is a LIE (in QM)

- UV
- IR

Outline: Massless QED in $d=2$

Anomalies + Functional Integral Measure

Chiral Anomaly in massless D=2 QED

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi$$

$$\Psi \to e^{i\alpha}\Psi \quad \longleftrightarrow \quad J^\mu = J^\mu_V = \bar{\Psi}\gamma^\mu\Psi$$

vector curent

$$\Psi \to e^{i\alpha\gamma^5}\Psi \quad \longleftrightarrow \quad J^\mu_A = J^\mu_5 = \bar{\Psi}\gamma^\mu\gamma^5\Psi$$

In QED

$$\Psi \to e^{i\alpha}\Psi \qquad A_\mu \to A_\mu$$

same currents

$$\Psi \to e^{i\alpha\gamma^5}\Psi \qquad A_\mu \to A_\mu$$

In

$\{\gamma$

...less $D=2$ QED

$J^\mu = J_V^\mu = \bar{\Psi}\gamma^\mu\Psi$

vector curent

$\partial_\mu J^\mu = 0$

$\partial_\mu J_5^\mu = 0$

$J_A^\mu = J_5^\mu = \bar{\Psi}\gamma^\mu\gamma^5\Psi$

In 2D

same currents

$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ $\qquad \eta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$i\bar{\Psi}\not{\partial}\Psi$

$\psi \to e^{i\alpha}\psi \quad \longleftrightarrow \quad J^\mu = J^\mu_V = \bar{\Psi}\gamma^\mu\Psi \qquad \partial_\mu J^\mu = 0$

vector curent

$\psi \to e^{i\alpha\gamma^5}\psi \quad \longleftrightarrow \quad J^\mu_A = J^\mu_5 = \bar{\Psi}\gamma^\mu\gamma^5\Psi \qquad \partial_\mu J^\mu_5 =$

axial vector current

ED

$\psi \to e^{i\alpha}\psi \qquad A_\mu \to A_\mu$

same currents

$\psi \to e^{i\alpha\gamma^5}\psi \qquad A_\mu \to A_\mu$

In 2D

$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$

In chiral basis

$$\gamma^0 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^1 = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\gamma^5 = \sigma^3 = -\gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^5 P_\pm \psi = \pm P_\pm \psi$$

$$P_\pm = \frac{1 \pm \gamma^5}{2}$$

$$P_\pm \psi = \psi_\pm$$

$$\psi = \psi_+ + \psi_-$$

$$i\bar{\Psi} \not{D} \psi =$$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\gamma^0 \gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= P_\pm \Psi$

s

$= \Psi_\pm$

$\Psi_+ + \Psi_-$

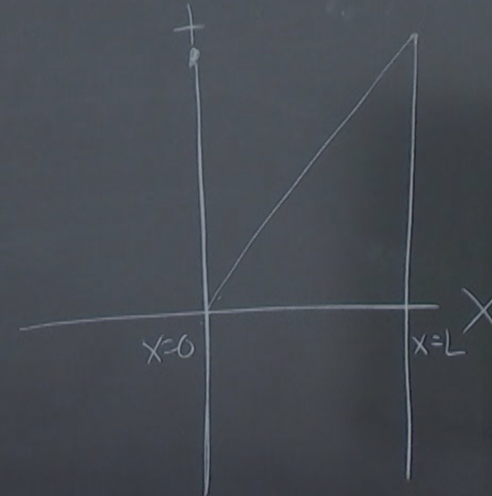
$$i \bar{\Psi} \not{D} \Psi = \Psi_+^\dagger \, i(D_0 + D_1)\Psi_+ + \Psi_-^\dagger \, i(D_0 - D_1)\Psi_-$$

$$\partial_\mu \underbrace{\bar{\Psi}_\pm \gamma^\mu \Psi_\pm}_{J^\mu_\pm} = 0 \quad \text{classically}$$

$$A_\mu = 0 \rightarrow \text{EOM: } i(\partial_0 \pm \partial_1)\Psi_\pm = 0$$

solns are waves moving to left or right with $v = 1$

Compactify space on a circle of radius L with $e_0 L \ll 1$



$$A_\mu = \sum a_\mu(t) \cos\left(\frac{2\pi n}{L} x\right)$$

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

$$\alpha = - \sum_{n \neq 0} \frac{L}{2\pi n} a_1(t) \sin\left(\frac{2\pi n}{L} x\right)$$

$$A_1(x,t) \to A_1(t)$$

$$\alpha = \frac{2\pi n}{L} x$$

$A_1(t) \to A_1(t) + \frac{2\pi n}{L} \to$ physical A_1 lives on circle of circumference $\frac{2\pi}{L}$

Quantum Mechanics of Ψ

A_1 is classical, slowly varying

$A_0 = 0$ (subleading effect)

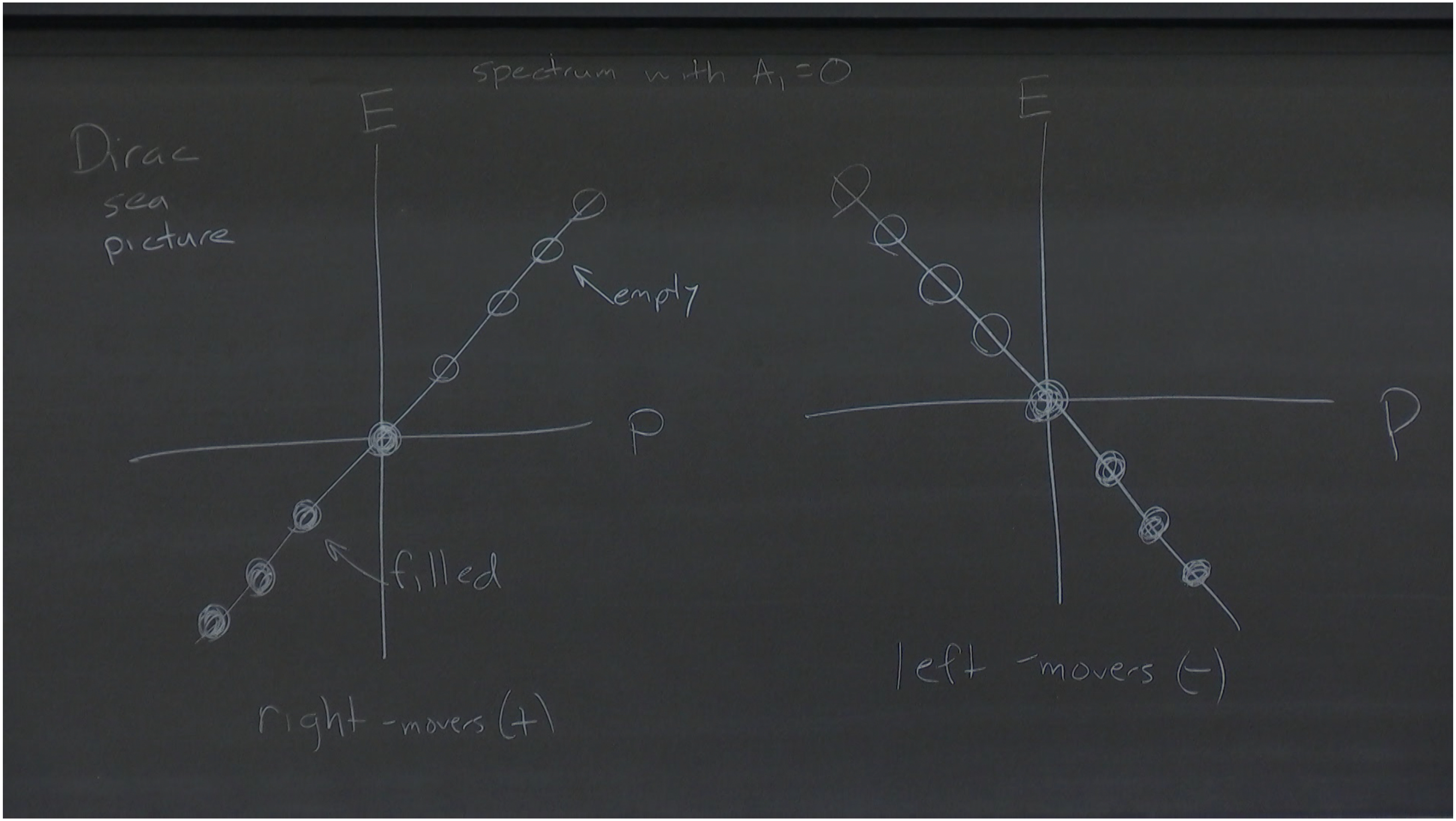
$$i \not{D} \Psi = 0$$

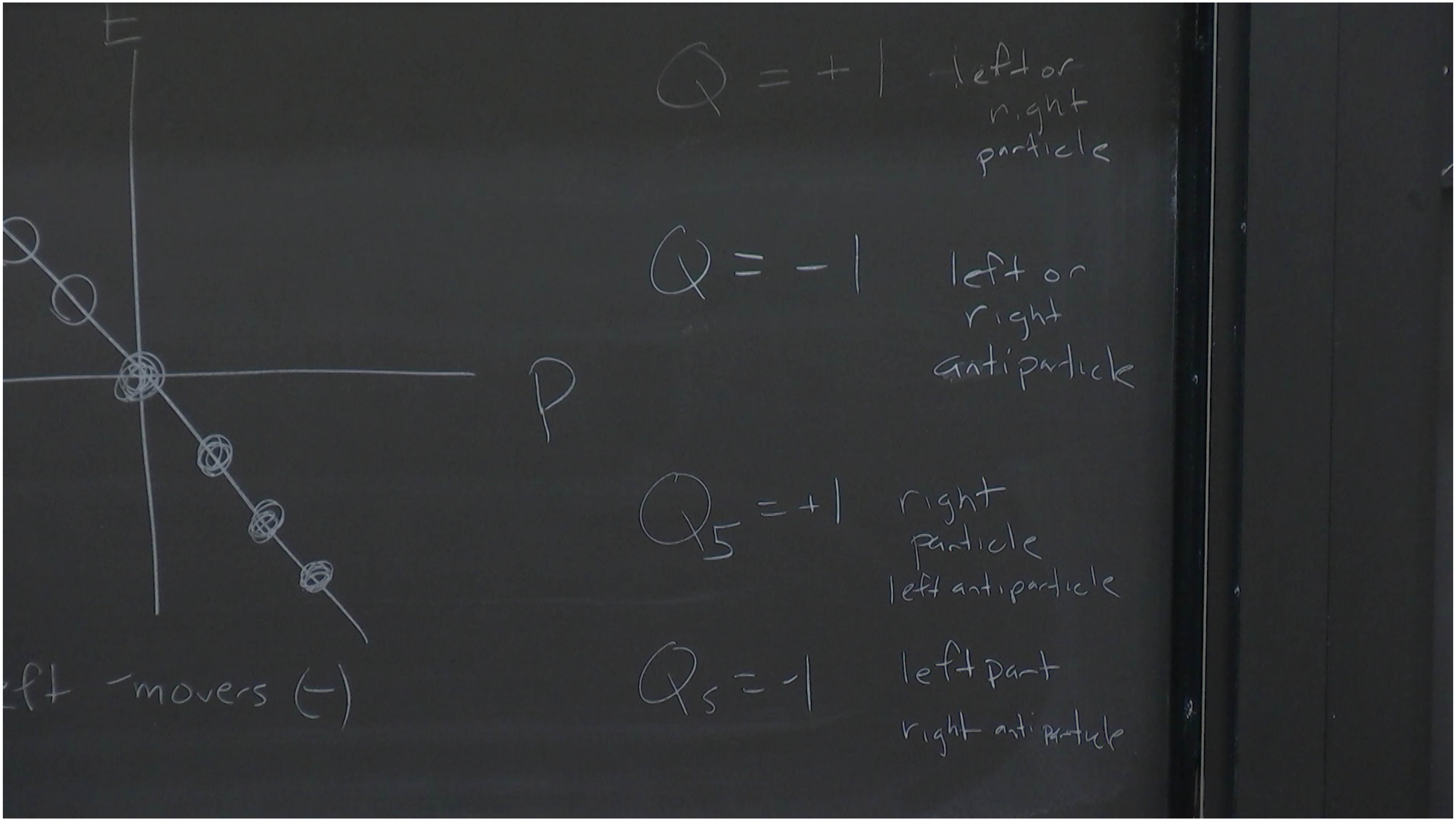
$$\partial_0 + \gamma^5(\partial_1 - iA_1)\Psi = 0$$

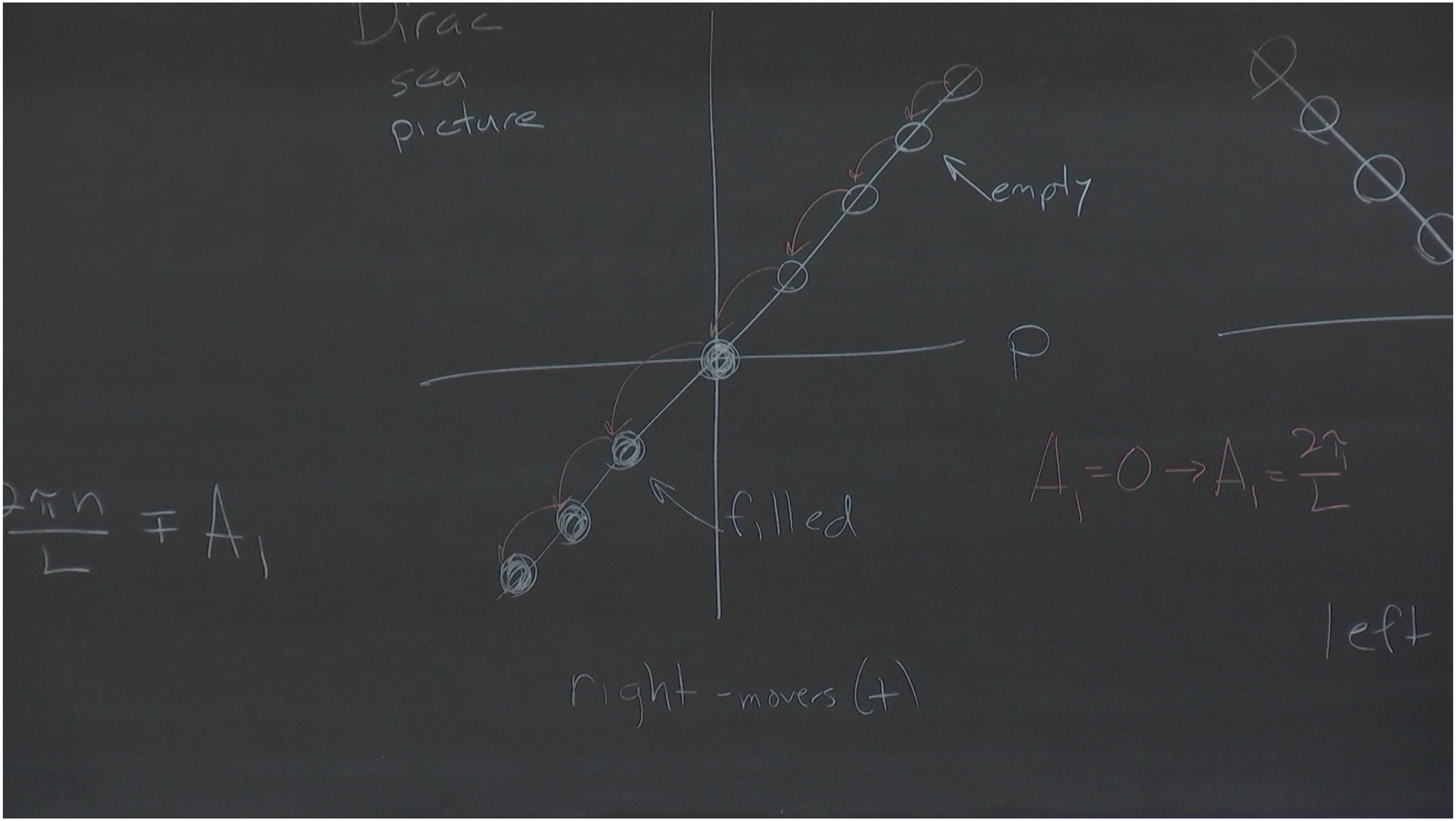
$$\Psi \propto e^{-iE_n t}\Psi_n(x)$$

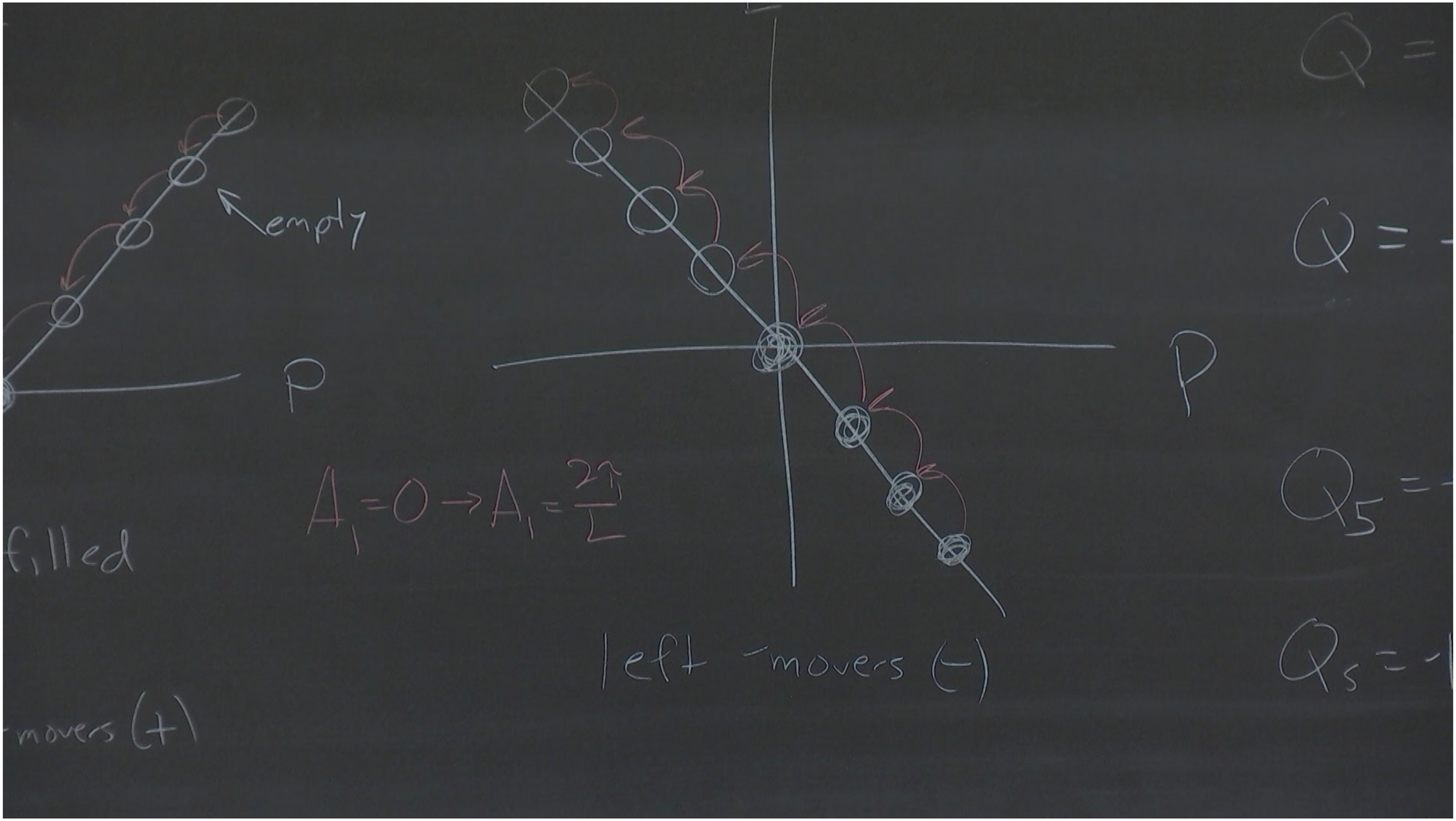
plug in $\rightarrow E_n^{\pm} = \pm \frac{2\pi n}{L} \mp A_1$

$$\Psi_n(x) \propto e^{2\pi i n x/L}$$


spectrum with $A_1 = 0$
Dirac sea picture
E
p
empty
filled
right-movers (+)
E
p
left-movers (−)


E
P
Q = +1 left or right particle
Q = -1 left or right antiparticle
Q5 = +1 right particle left antiparticle
Q5 = -1 left part right antiparticle
ft movers (-)


Dirac
sea
picture
empty
p
filled
$\frac{2\pi n}{L} \mp A_1$
$A_1 = 0 \to A_1 = \frac{2\pi}{L}$
left
right-movers (+)


empty
P
filled
movers (+)
A1=0 → A1 = 2π/L
P
left movers (−)
Q =
Q =
Q5 =
Q5 =

$$\Delta Q_5 = -2$$

$$\frac{\Delta Q_5}{\Delta t} = -\frac{L}{\tilde{\pi}} \frac{\Delta A_1}{\Delta t}$$

$$\dot{Q}_5 = -\frac{L}{\tilde{\pi}} \dot{A}_1$$

$$Q_5 = \int dx \, J_5^0$$

$$L A_1 = \int \epsilon^{0\mu} A_\mu dx$$

$$\boxed{\partial_\mu J_5^\mu = \frac{1}{\tilde{\pi}} \epsilon^{\mu\nu} F_{\mu\nu}}$$

anomalous non-conservation equation