

Title: PSI 2019/2020 - QFT III - Lecture 4

Speakers:

Collection: PSI 2019/2020 - QFT III

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Θ Vacua

Introduction to Anomalies

Thermodynamics

Q Vacua

Introduction to Anomalies

$$S_E = \frac{1}{2g^2} \int d^4x \operatorname{tr} F_{\mu\nu} F^{\mu\nu}$$

finite action $\rightarrow F_{\mu\nu} \xrightarrow{p \rightarrow \infty} 0$ faster than $\frac{1}{p^2}$

$$F_{\mu\nu} = O\left(\frac{1}{p^3}\right) \rightarrow A_\mu = O\left(\frac{1}{p^2}\right) + iU \partial_\mu U^{-1}$$

$$U = O(p^0) \rightarrow U(x) = U(\hat{x})$$

↑
function of angles
only

$$A_\mu \rightarrow h A_\mu h^{-1} + i h \partial_\mu h^{-1}$$

$$= i(hU)A_\mu(hU)^{-1} + \mathcal{O}(\frac{1}{p^2})$$

If $hU=1$ as $p \rightarrow \infty$
generally not possible

Simpler case: $U(1)$ in $D=2$

$$F_{\mu\nu} =$$

Q Vacua

Introduction to Anomalies

$$S_E = \frac{1}{2g^2} \int d^4x \text{tr} F_{\mu\nu} F^{\mu\nu}$$

finite action $\rightarrow F_{\mu\nu} \xrightarrow{\rho \rightarrow \infty} 0$ faster than $\frac{1}{\rho^2}$

$$F_{\mu\nu} = O\left(\frac{1}{\rho^3}\right) \rightarrow A_\mu = O\left(\frac{1}{\rho^2}\right) + iU \partial_\mu U^{-1}$$

$$U = O(\rho^0) \rightarrow U(x) = U(\hat{x})$$

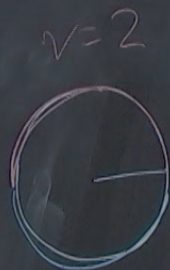
↑
function of angles only

trivial
 identity
 winds ν
 times

$$U^{(0)}(\theta) = 1$$

$$U^{(1)}(\theta) = e^{i\theta}$$

$$U^{(\nu)}(\theta) = e^{i\nu\theta}$$



trivial $U^{(0)}(\theta) = 1$

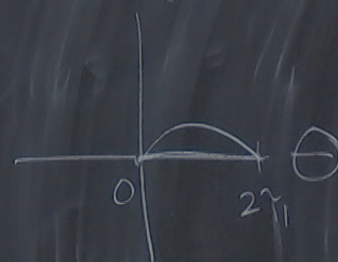
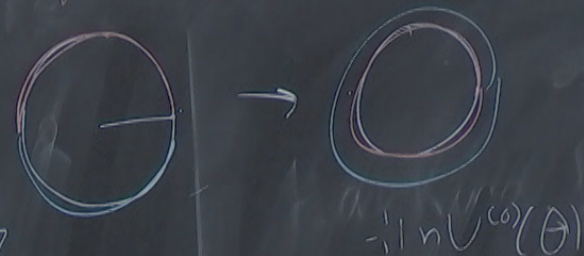
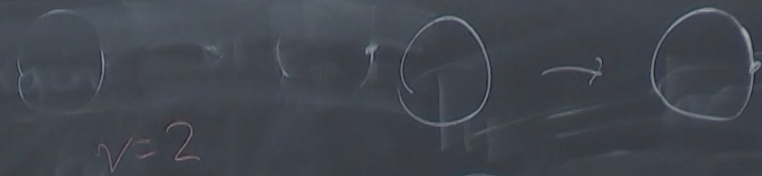
identity $U^{(1)}(\theta) = e^{i\theta}$

winds v times $U^{(v)}(\theta) = e^{iv\theta}$ $v \in \mathbb{Z}$

any map $f: S^1 \rightarrow S^1$ continuously deformable to exactly one of these maps

$$v = \frac{i}{2\pi} \int_0^{2\pi} d\theta \left(U \frac{d}{d\theta} U^{-1} \right)$$

$$= \frac{i}{2\pi} \int_0^{2\pi} d\theta \left(e^{iv\theta} (-iv) e^{-iv\theta} \right)$$



$$= \frac{1}{2\pi} \int_0^{2\pi} d\theta (e^{i\nu\theta} (-i\nu) e^{-i\nu\theta})$$

$$U \rightarrow U + \delta U$$

$$U \frac{d}{d\theta} U^{-1} \rightarrow U \frac{d}{d\theta} U^{-1} + \delta U \left(\frac{d}{d\theta} U^{-1} \right) + U \frac{d}{d\theta} (U^{-1} \delta U U^{-1})$$

$$= U \frac{d}{d\theta} U^{-1} + \cancel{(1-1)} \delta U \left(\frac{d}{d\theta} U^{-1} \right) + U \left(\frac{d}{d\theta} U^{-1} \right) \delta U U^{-1} - \left(\frac{d}{d\theta} \delta U \right) U^{-1}$$

$$= U \frac{d}{d\theta} U^{-1} - \frac{d}{d\theta} (U^{-1} \delta U)$$

↑
vanishes upon integration

$$U^{(\gamma)}(\theta) = U^{(\gamma_1)}(\theta) U^{(\gamma_2)}(\theta)$$

$\gamma = \gamma_1 + \gamma_2 \rightarrow$ instantons change winding number

$$\gamma = \frac{i}{4\pi} \int d^2x \epsilon_{\mu\nu} F^{\mu\nu}$$

chiral anomaly

$D=4$ for $SU(2)$

$$U = e^{i\lambda \frac{\sigma^i}{2}} = a \mathbb{1} + ib_i \sigma^i \quad a_i, b_i \in \mathbb{R}$$

$$U^\dagger U = \mathbb{1} \rightarrow a^2 + b_i b_i = 1$$

$\uparrow$
3-sphere

$$\text{maps: } \begin{array}{ccc} \uparrow & & \uparrow \\ S^3 & \rightarrow & S^3 \\ \uparrow & & \uparrow \\ \text{at } p=\infty & & SU(2) \end{array}$$

$$U^{(v)}(\hat{x}) = (\hat{x} + i\hat{x} \cdot \vec{\sigma})^v \quad \text{standard maps}$$

$$\nu = -\frac{1}{24\pi^2} \int d\Omega \operatorname{Tr} [e^{ijk} (U \partial_i U^{-1}) (U \partial_j U^{-1}) (U \partial_k U^{-1})]$$

$$= \frac{1}{16\pi^2} \int d^4x \underbrace{\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \operatorname{Tr} F^{\mu\nu} F^{\rho\sigma}}_{\text{chiral anomaly in } D=4}$$

$U(1)$ no analog of winding number

f. $S^3 \rightarrow S^1$ all deformable to trivial map

Thm: $f: S^3 \rightarrow G$ deformable to $f: S^3 \rightarrow S^3$

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta} |n\rangle$$

$\uparrow$ energy eigenstate $\uparrow$ definite ν states

$$\lim_{T \rightarrow \infty} \langle \theta' | e^{-HT} | \theta \rangle = \lim_{T \rightarrow \infty} \sum_{n,m} \langle n | e^{-HT} | m \rangle e^{in\theta' - im\theta}$$

$$= \lim_{T \rightarrow \infty} \sum_n \frac{e^{-in(\theta' - \theta)}}{S(\theta' - \theta)} \sum_{\nu} [DA]_{\nu} e^{i\nu\theta} e^{-S[A]}$$

$\uparrow$ extra term