

Title: PSI 2019/2020 - QFT III - Lecture 3

Speakers:

Collection: PSI 2019/2020 - QFT III

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URL: <http://pirsa.org/20010074>

(x)



action
to Euclidean
SM

$$S_E = \int d\tau d^3x \left[\frac{1}{2} \left(\frac{\partial \bar{\phi}}{\partial \tau} \right)^2 + \frac{1}{2} \left(\nabla \bar{\phi} \right)^2 + U(\bar{\phi}) \right]$$

like V in QM problem

Boundary conditions:

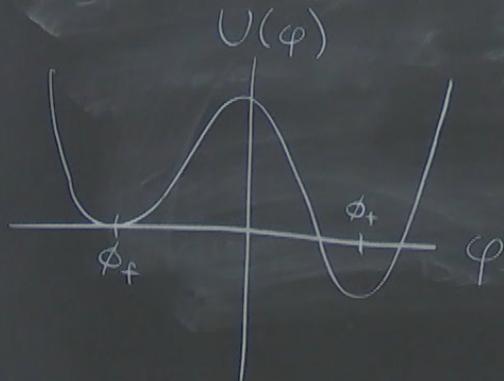
$$\left. \begin{aligned} \lim_{\tau \rightarrow \pm\infty} \bar{\phi} &= \phi_f \\ \lim_{|\vec{x}| \rightarrow \infty} \bar{\phi} &= \phi_f \end{aligned} \right\} \lim_{\rho \rightarrow \infty} \bar{\phi}(\rho) = \phi_f$$

$\rho^2 = \tau^2 + \vec{x}^2$

Can be proven that lowest action solution to Euclidean EOM is $O(4)$ symmetric

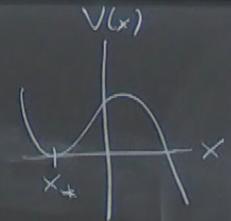
Decay of False Vacua in QFT

⊙ Vacua



$$\langle \varphi = \phi_f | e^{-HT/\hbar} | \varphi = \phi_f \rangle = Z$$

$$E_0 = -\lim_{T \rightarrow \infty} \frac{\hbar}{T} \ln Z$$



$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - U(\varphi)$$

QFT as ∞ dim QM

$$\frac{\Gamma}{\text{vol}} = A e^{-S_E[\bar{\varphi}]/\hbar}$$

\uparrow finite because decay is localized
 \uparrow $\propto \det' S''$
 \uparrow finite action soln to Euclidean EOM

$$S_E = \int d\tau d^3x \left[\frac{1}{2} \left(\frac{\partial \bar{\phi}}{\partial \tau} \right)^2 + \frac{1}{2} \left(\nabla \bar{\phi} \right)^2 + U(\bar{\phi}) \right]$$

like V in QM problem

Boundary conditions:

$$\left. \begin{array}{l} \lim_{\tau \rightarrow \pm\infty} \bar{\phi} = \phi_f \\ \lim_{|\vec{x}| \rightarrow \infty} \bar{\phi} = \phi_f \end{array} \right\} \lim_{\rho \rightarrow \infty} \bar{\phi}(\rho) = \phi_f$$

$\rho^2 = \tau^2 + \vec{x}^2$

Can be proven that lowest action solution to Euclidean EOM is $O(4)$ symmetric

$$\left. \frac{d\bar{\phi}}{d\rho} \right|_{\rho=0} = 0 \quad \text{non-singular}$$

$$\text{EOM: } \frac{d^2 \bar{\phi}}{d\rho^2} + \frac{3}{\rho} \frac{d\bar{\phi}}{d\rho} = U'(\bar{\phi})$$

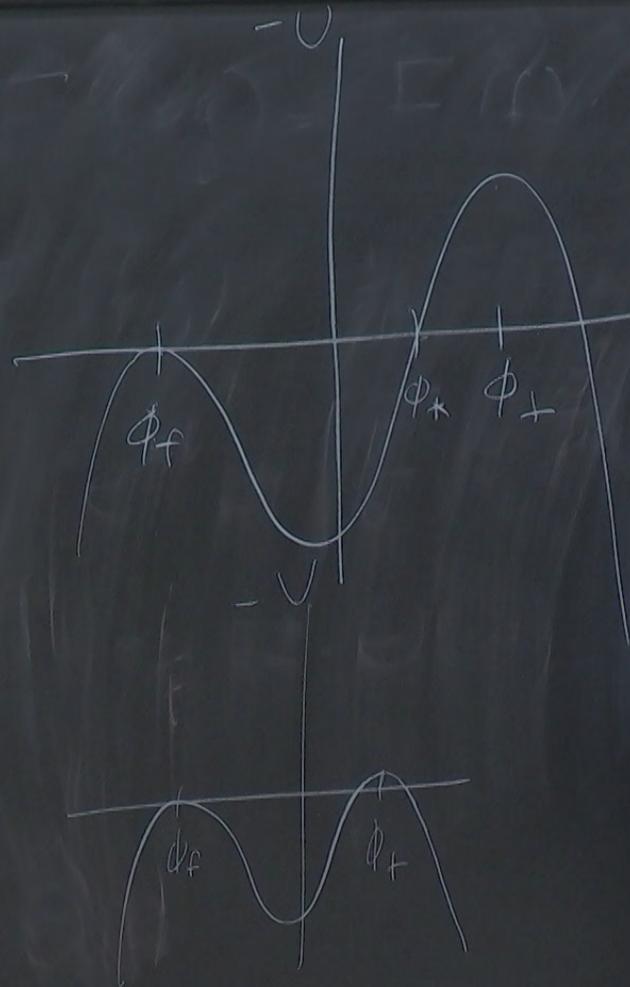
$$U' = \frac{dU}{d\phi}$$

like a particle moving in potential $-U$
with "time" or ρ dependent
damping force

$$+ \frac{3}{\rho} \frac{d\bar{\phi}}{d\rho} = U'(\bar{\phi})$$

$$U' = \frac{dU}{d\phi}$$

particle moving in potential $-U$
with "time" or ρ dependent
damping force



Strategy: Argue that there exists $\bar{\phi}(0)$ with $\dot{\bar{\phi}}(0) = 0$ so that $\bar{\phi}(\infty) = \phi_f$

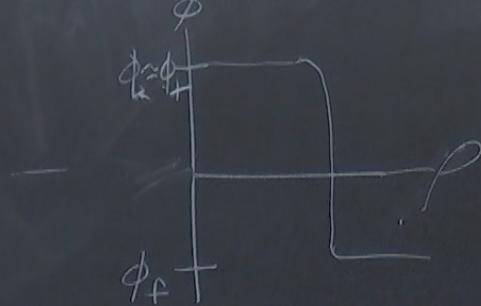
undershoot: $\bar{\phi}(0) = \phi_+$

overshoot: $\bar{\phi}(0) \approx \phi_+$

stay near ϕ_+ until $\frac{3}{\rho} \approx 0$

Thin-wall limit

$$U(\phi_+) \approx U(\phi_f)$$



$$S_E = -\frac{1}{2} \pi^2 R_0^4 \epsilon + 2 \pi^2 R_0^3 \sigma$$

↑ volume term
 ↑ $U(\phi_-) - U(\phi_+)$
 surface tension = $\int_{\phi_-}^{\phi_+} d\phi \sqrt{2U}$

$$\frac{dS_E}{dR_0} = 0 = -2\pi^2 R_0^3 \epsilon + 6\pi^2 R_0^2 \sigma$$

$$R_0 = \frac{3\sigma}{\epsilon}$$

$$S_E = \frac{27\pi^2 \sigma^4}{2\epsilon^3}$$

$$\frac{27\pi^2}{2} \approx 100$$

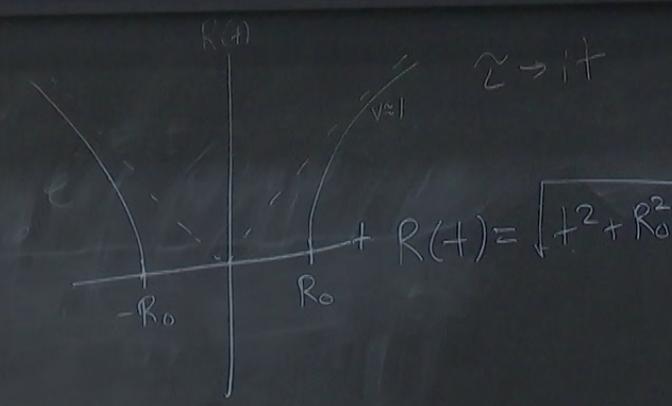
$$2\pi^2 R_0^3 \sigma$$

$1(\phi_+)$

$$+ 6\pi^2 R_0^2 \sigma$$

$$\frac{27\pi^2}{2} \approx 100$$

surface tension = $\int_{\phi_-}^{\phi_+} d\phi \sqrt{2U}$



$z \rightarrow t$

$$R(z) = \sqrt{z^2 + R_0^2}$$

$$E_0 = -\lim_{T \rightarrow \infty} \frac{1}{T} \ln Z$$

because decay is localized

$\propto \det S''$

EOM

Θ Vacua

Classical vacua of YM?

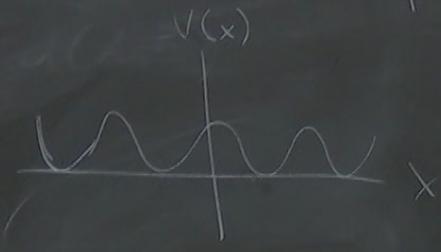
Expect: $A_\mu = 0 + iU \partial_\mu U^{-1}$

pure gauge

↑
not deformable into one another
restricting to pure gauge

$$A_{1\mu} = iU_1 \partial_\mu U_1^{-1}$$

$$A_{2\mu} = iU_2 \partial_\mu U_2^{-1}$$



dp $\rho=0$ non-singular

$$S_E = \frac{1}{2g} \int d^4x \text{tr} F_{\mu\nu} F^{\mu\nu}$$

$$iU_1 \partial_M U_1^{-1}$$

$$iU_2 \partial_M U_2^{-1}$$