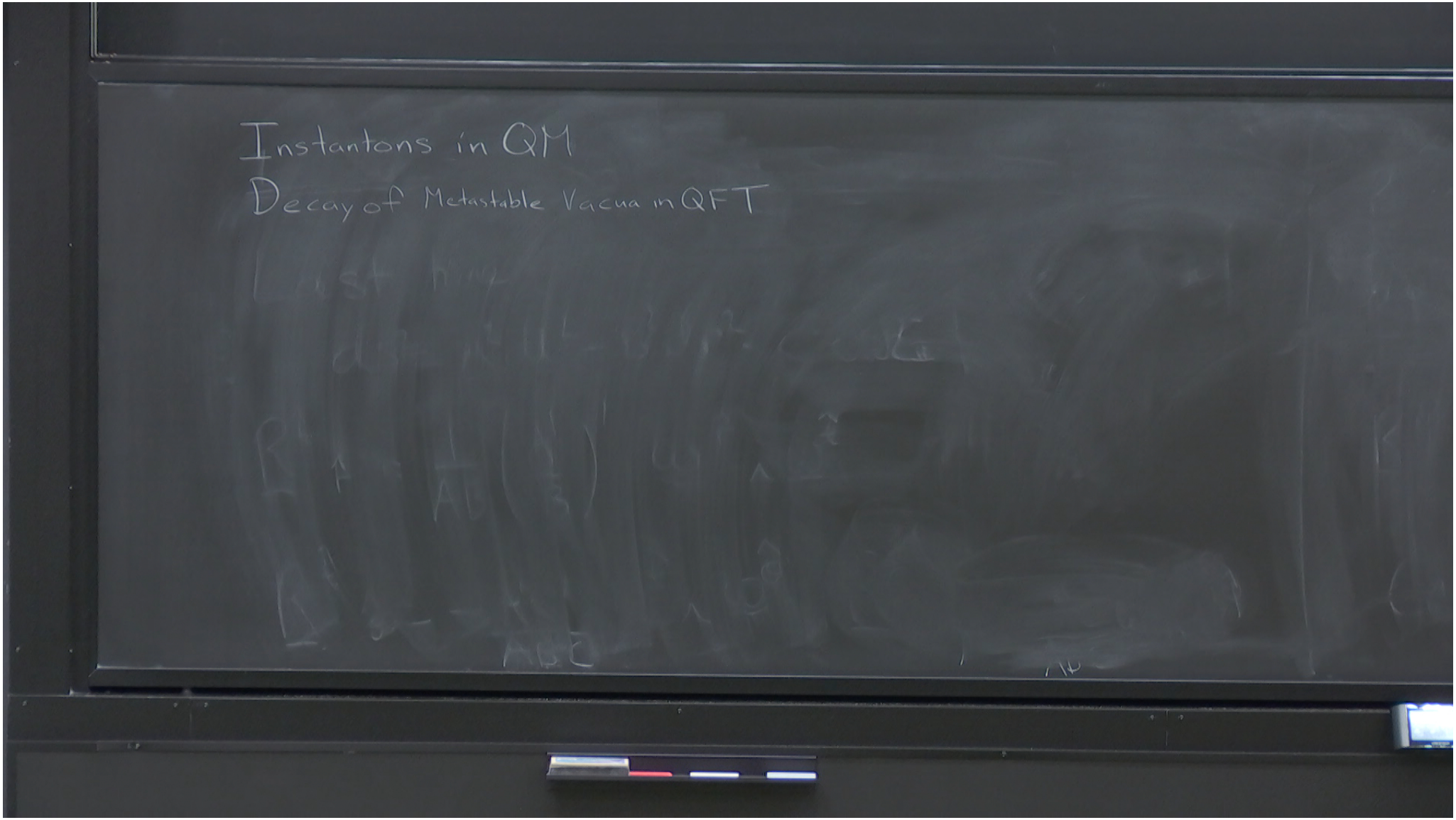
Title: PSI 2019/2020 - QFT III - Lecture 2

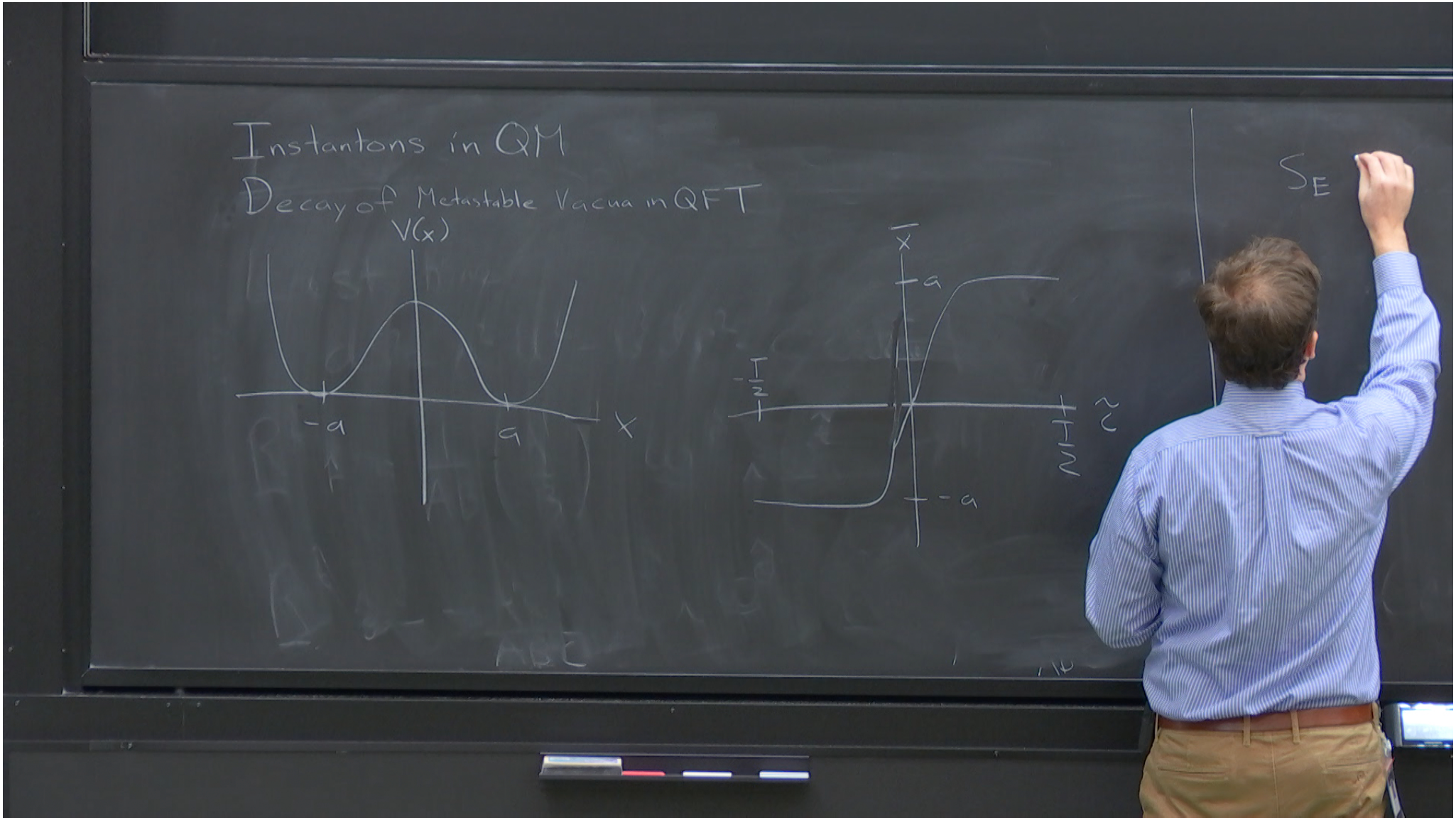
Speakers:

Collection: PSI 2019/2020 - QFT III

Date: January 10, 2020 - 11:30 AM

URL: http://pirsa.org/20010073

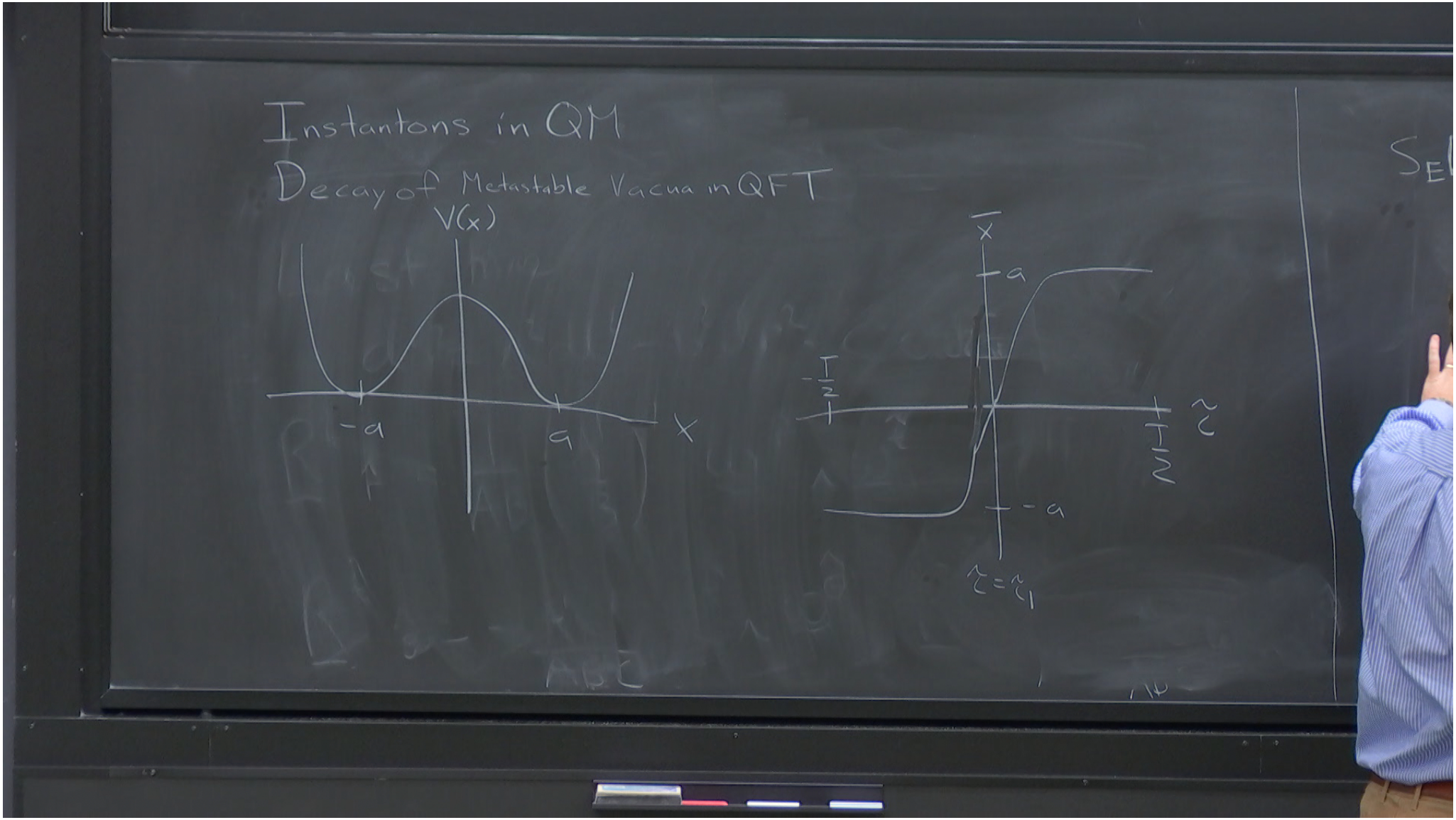

Instantons in QM
Decay of Metastable Vacua in QFT

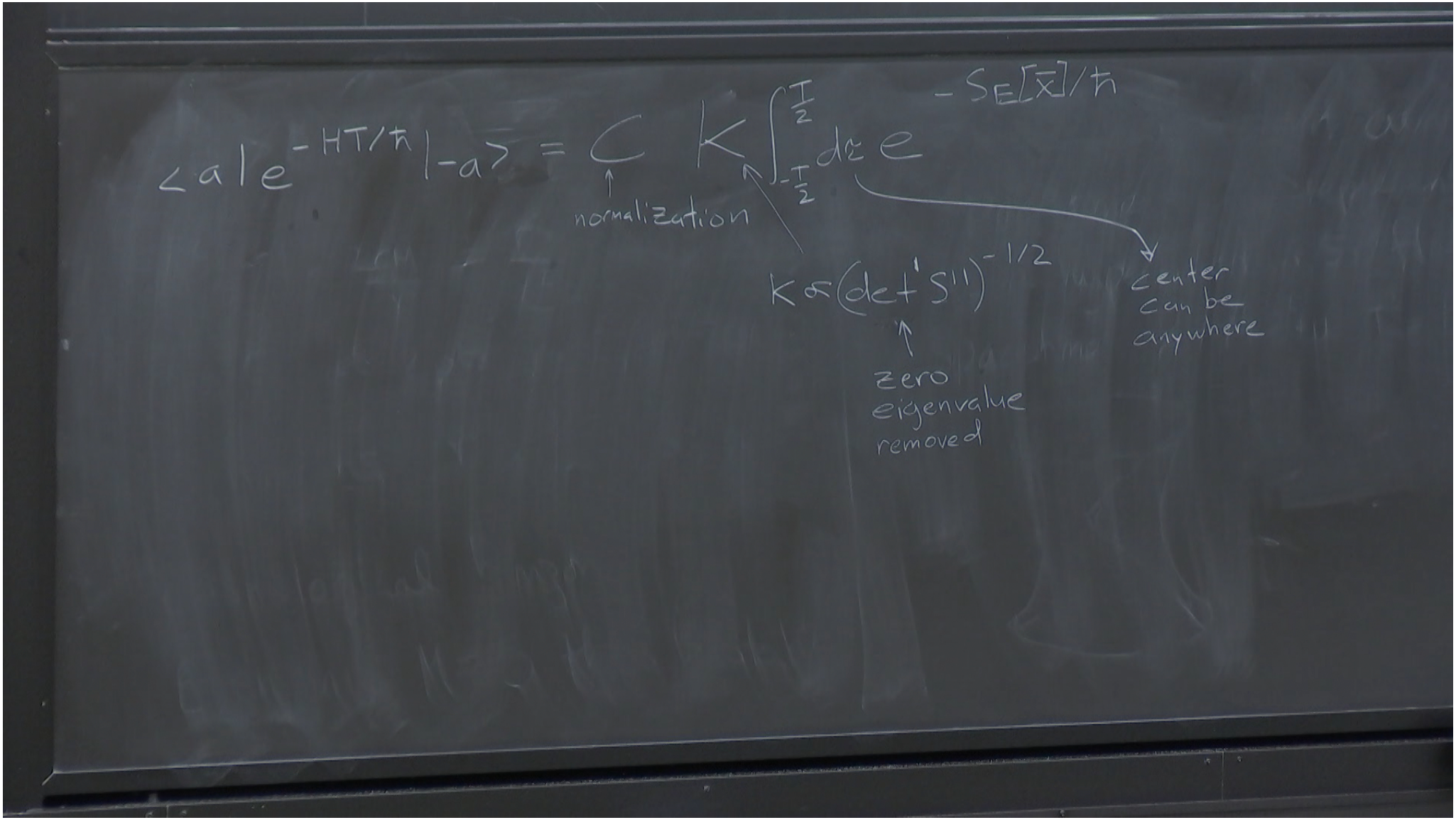

Instantons in QM
Decay of Metastable Vacua in QFT
V(x)
-a
a
x
S_E
x
a
-a
τ

$$S_E[\bar{x}] \equiv S_0 = \int_{-\frac{I}{2}}^{\frac{I}{2}} d\bar{\tau} \left(\frac{1}{2}\left(\frac{d\bar{x}}{d\bar{\tau}}\right)^2 + V(\bar{x}) \right)$$

$$= \int_{-a}^{a} d\bar{x} \, \frac{d\bar{\tau}}{d\bar{x}} \left(\frac{1}{2}\dot{\bar{x}}^2 + V \right) \qquad \text{using } \frac{d\bar{x}}{d\bar{\tau}} = \sqrt{2V} \leftrightarrow E = 0$$

$$= \int_{-a}^{a} d\bar{x} \sqrt{2V(\bar{x})}$$


Instantons in QM
Decay of Metastable Vacua in QFT
V(x)
-a
a
x
x
a
-a
τ
τ=τ1

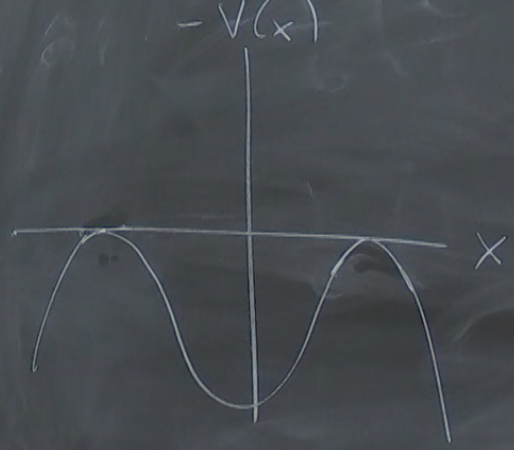

$\langle a | e^{-HT/\hbar} | -a \rangle = C \; K \int_{-\frac{T}{2}}^{\frac{T}{2}} d\bar{x} \; e^{-S_E[\bar{x}]/\hbar}$
normalization
$K \propto (\det' S'')^{-1/2}$
zero eigenvalue removed
center can be anywhere

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} d\bar{\tau} \left(\frac{1}{2} \left(\frac{d\bar{x}}{d\bar{\tau}} \right)^2 + V(\bar{x}) \right)$$

$$= \int_{-a}^{a} d\bar{x} \, \frac{d\bar{\tau}}{d\bar{x}} \left(\frac{1}{2} \dot{\bar{x}}^2 + V \right)$$

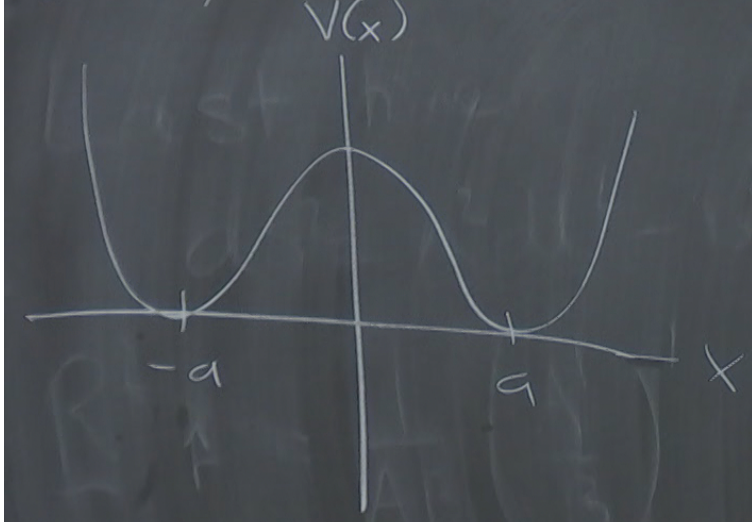
$$= \int_{-a}^{a} d\bar{x} \sqrt{2 V(\bar{x})}$$

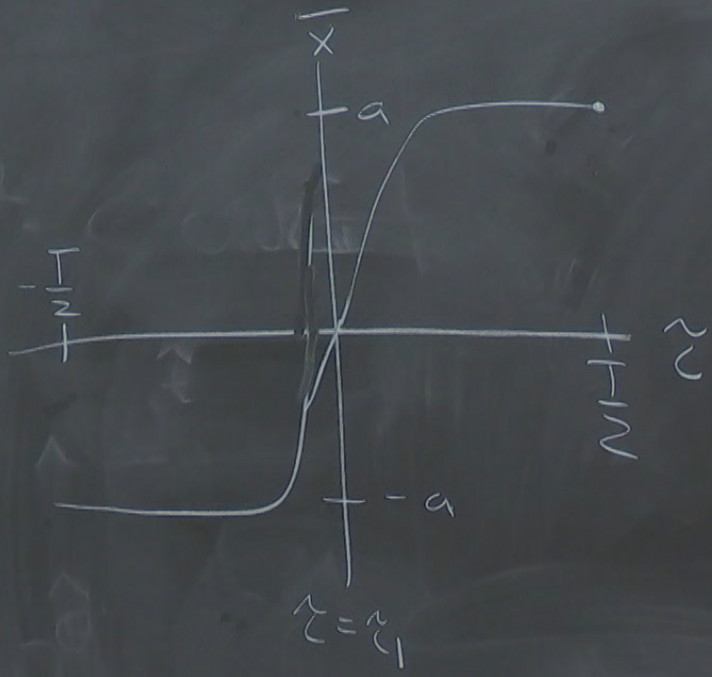
using $\frac{d\bar{x}}{d\bar{\tau}} = \sqrt{2V} \leftrightarrow E = 0$



Instantons in QM

Decay of Metastable Vacua in QFT





$S_E[\bar{x}$


$\bar{x}$
+a
$\tilde{z}$
-a
multi-instanton sdutions
exact solution in $T \to \infty$ limit
provided density of instantons is low

$$\langle a | e^{-HT/\hbar} | -a \rangle_{\text{one-instanton}} = C \, K \int_{-\frac{I}{2}}^{\frac{I}{2}} d\tilde{z} \, e^{-S_E[\bar{x}]/\hbar}$$

normalization

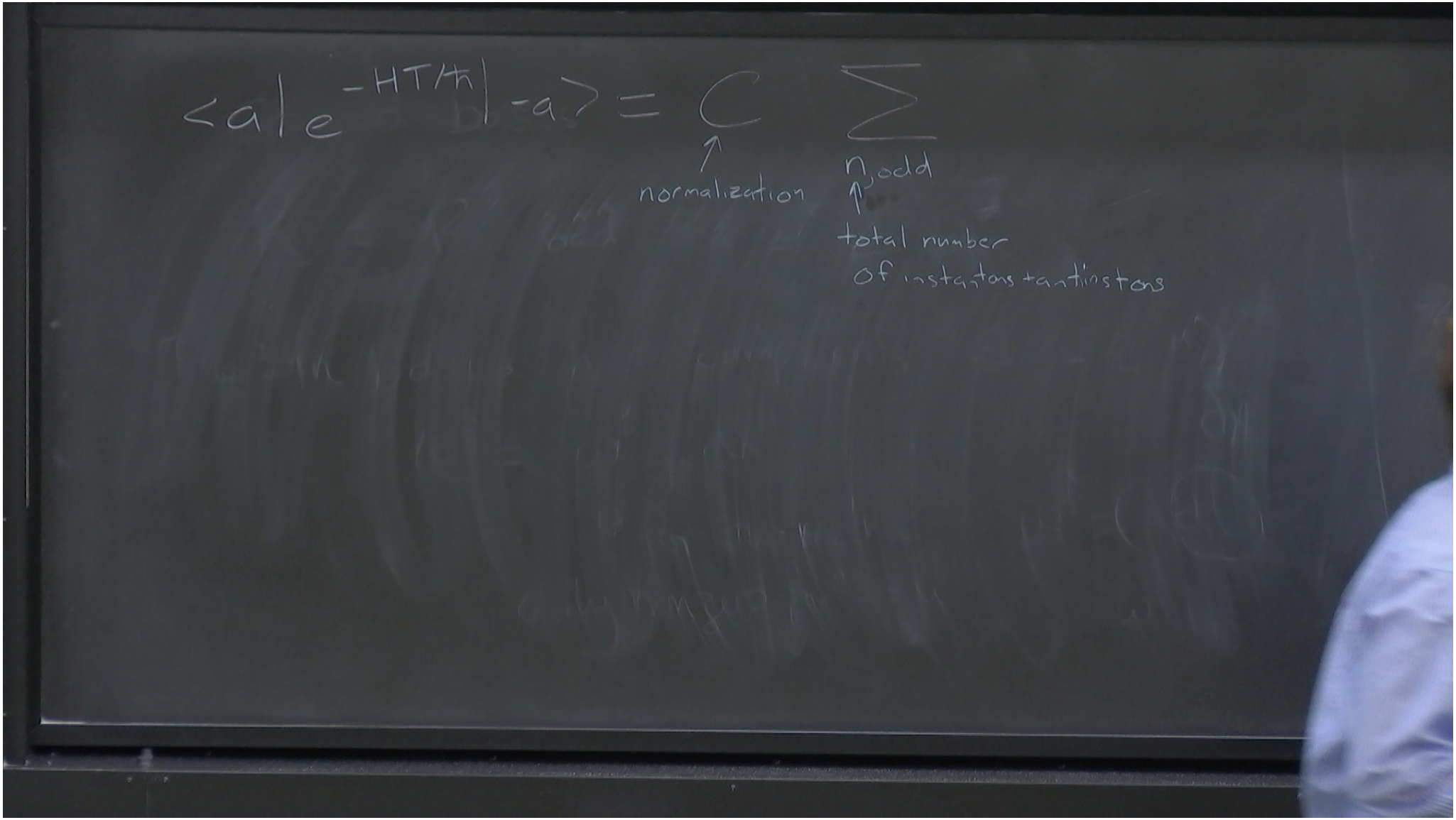
$$K \propto (\det{}' S'')^{-1/2}$$

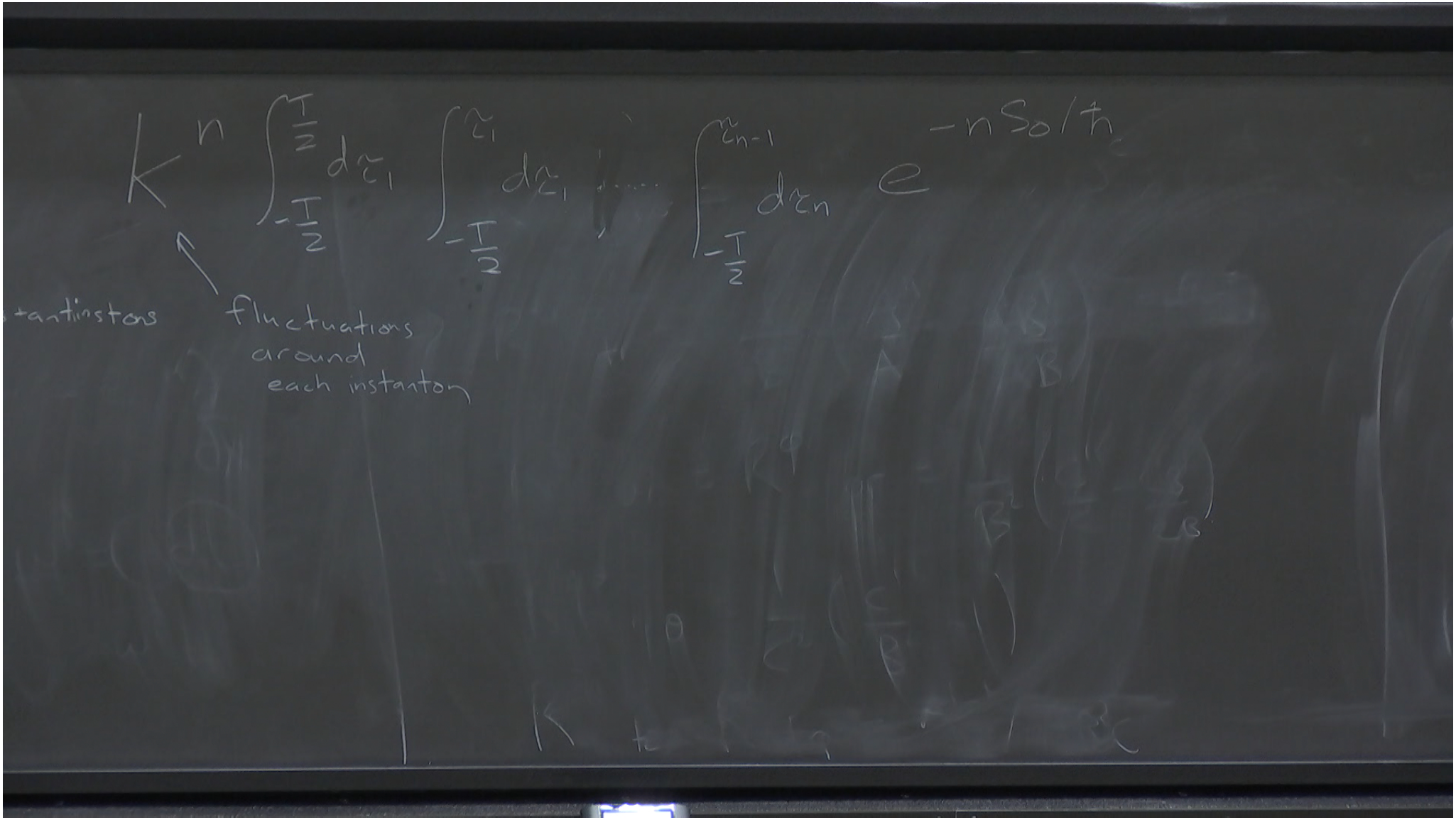
zero eigenvalue removed

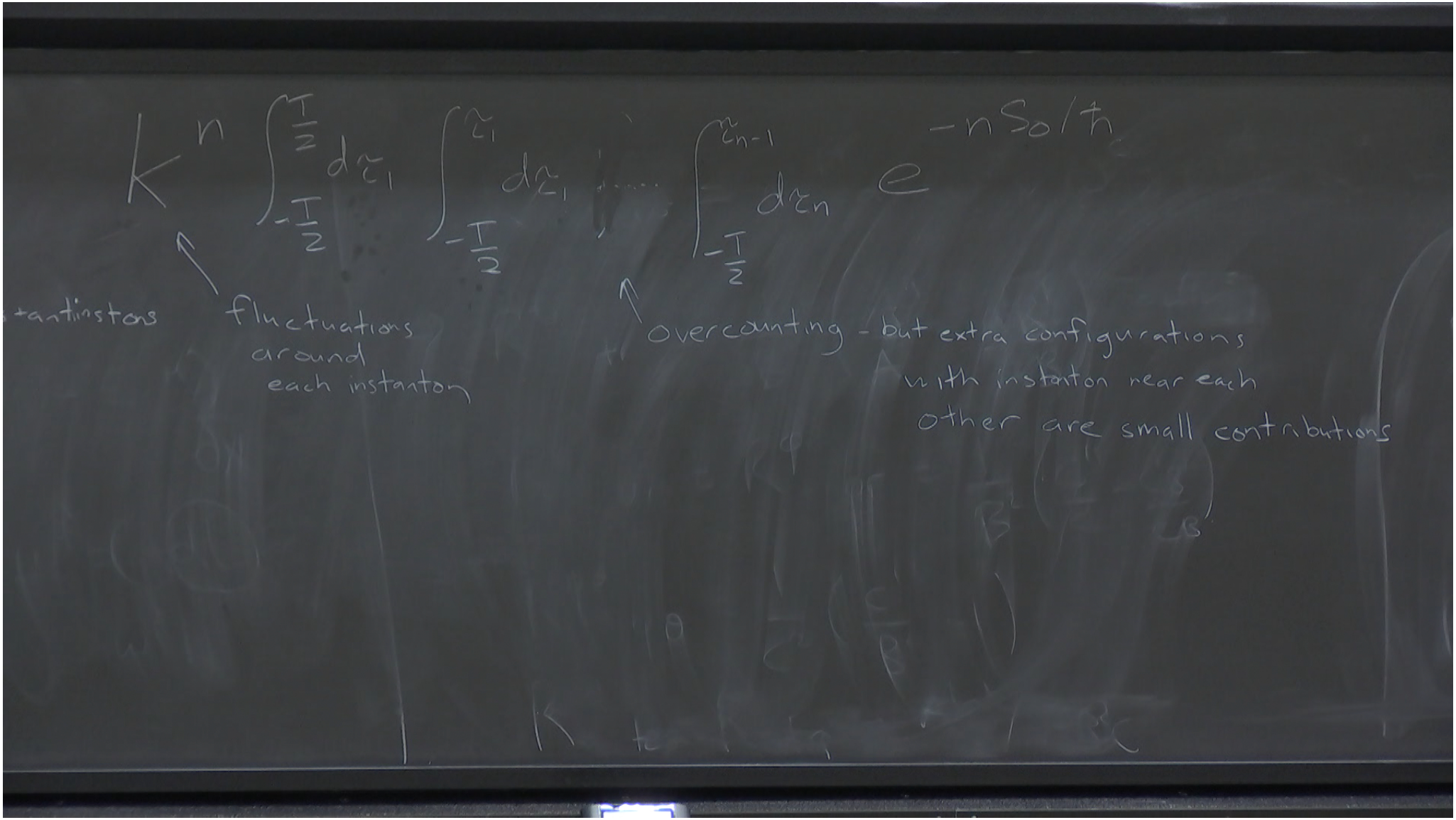
center can be anywhere

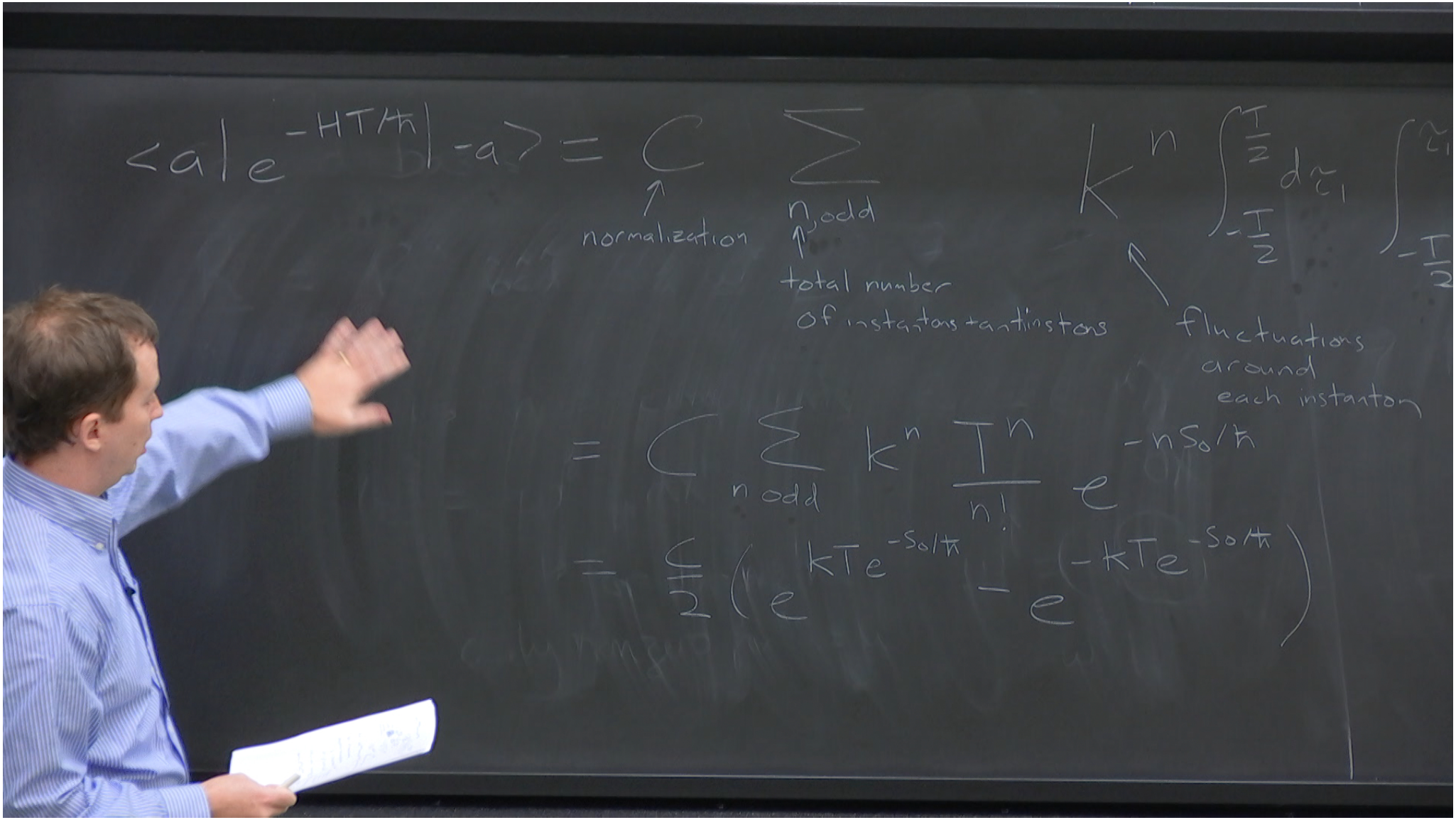
$$\langle a | e^{-HT/\hbar} | -a \rangle = \int_{x(-\frac{I}{2})=-a}^{x(\frac{I}{2})=a} \mathcal{D}x \, e^{-S[x]/\hbar}$$

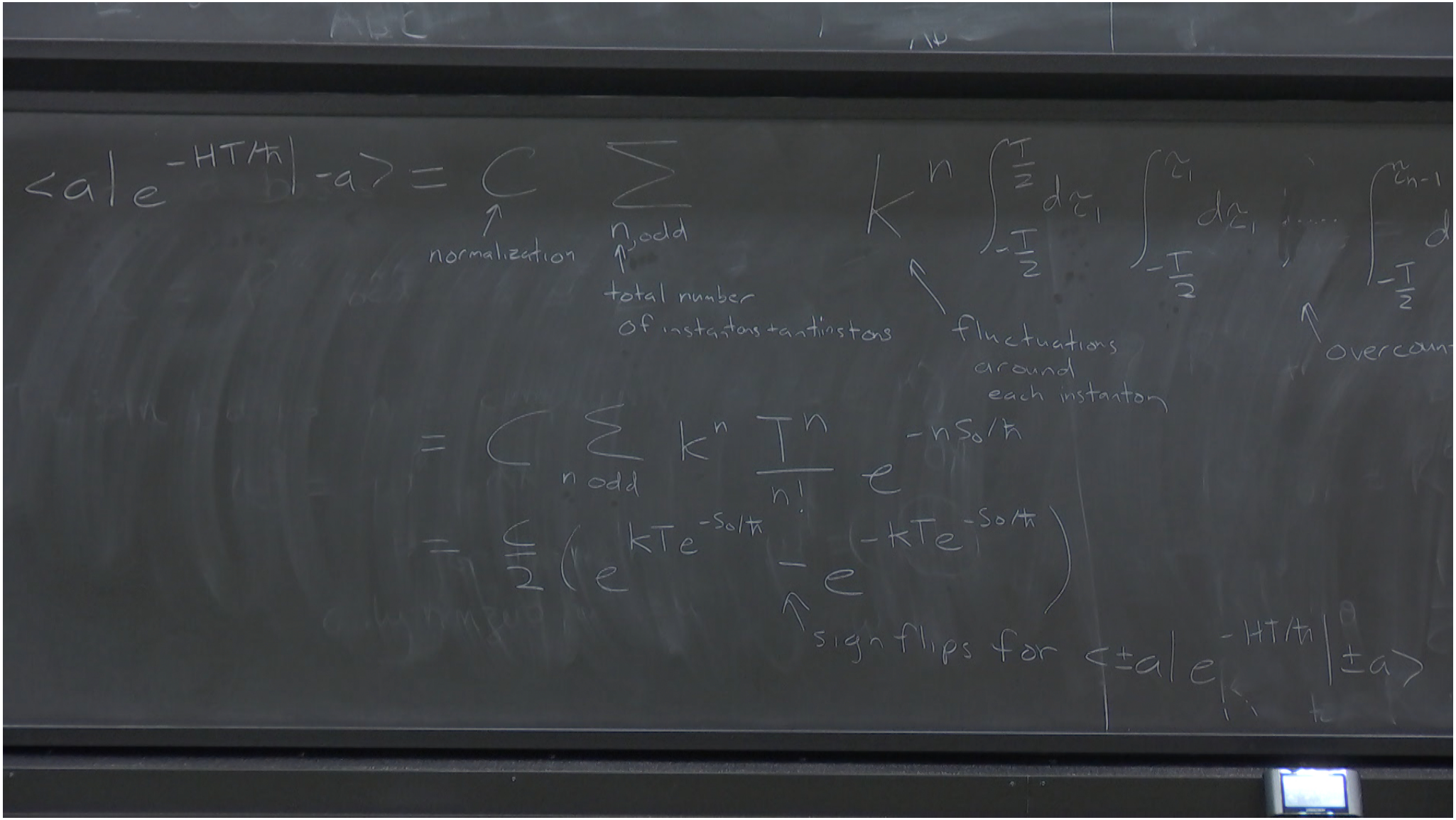
$$S[x] = S[\bar{x}] + \tilde{x} S'' \hat{x} + \cdots$$

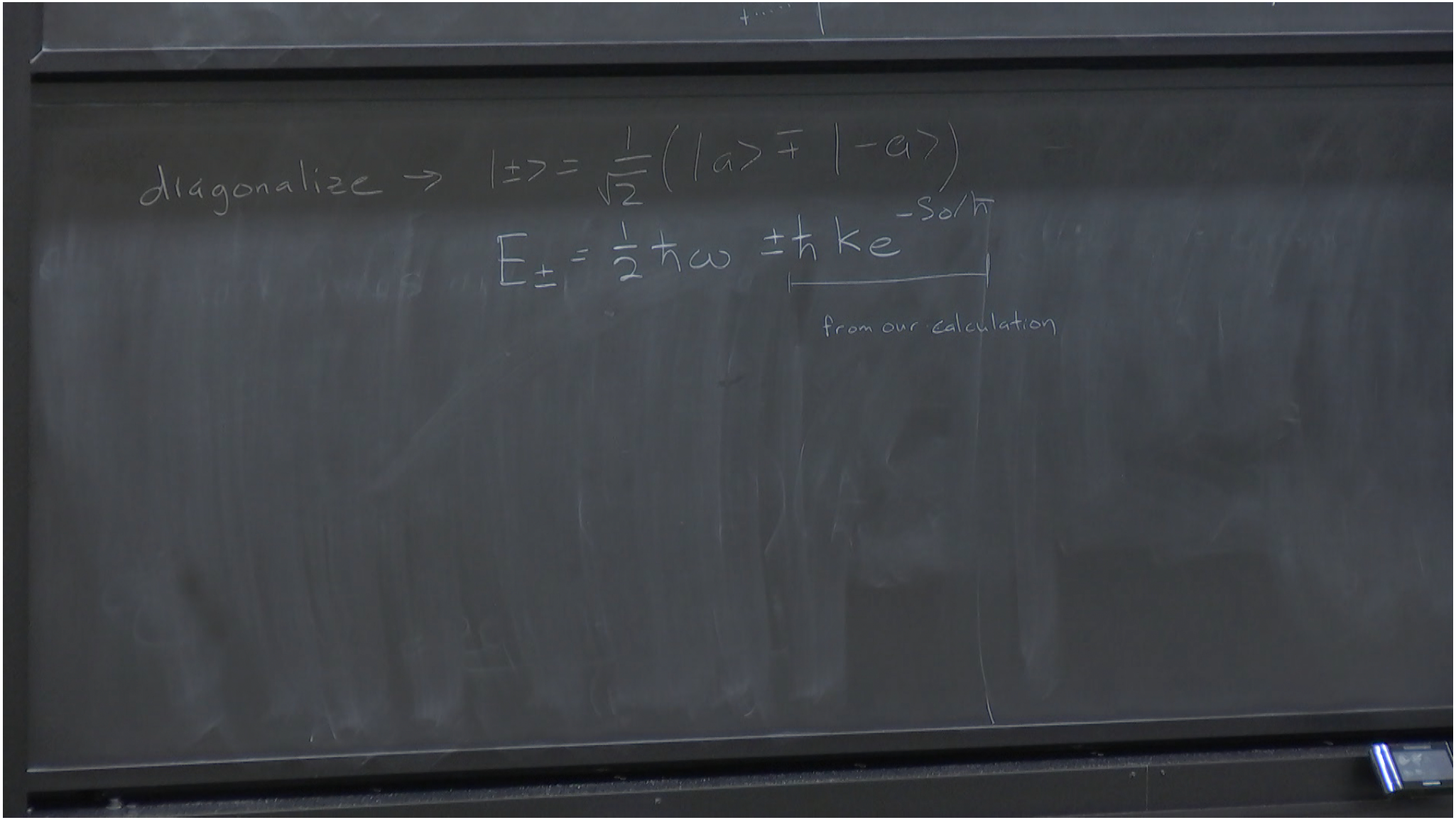

$\langle a | e^{-HT/\hbar} | -a \rangle = C \sum_{n,\text{odd}}$
normalization
total number
of instantons + anti-instantons


$K^n \int_{-\frac{I}{2}}^{\frac{I}{2}} d\tau_1 \int_{-\frac{I}{2}}^{\tau_1} d\tau_2 \cdots \int_{-\frac{I}{2}}^{\tau_{n-1}} d\tau_n \; e^{-nS_0/\hbar}$
tantinstons
fluctuations around each instanton


$K^n \int_{-\frac{I}{2}}^{\frac{I}{2}} d\tau_1 \int_{-\frac{I}{2}}^{\tau_1} d\tau_2 \cdots \int_{-\frac{I}{2}}^{\tau_{n-1}} d\tau_n \; e^{-nS_0/\hbar}$
tantinstons
fluctuations around each instanton
overcounting - but extra configurations with instanton near each other are small contributions


$\langle a | e^{-HT/\hbar} | -a \rangle = C \sum_{n\,\text{odd}} k^n \int_{-\frac{I}{2}}^{\frac{I}{2}} d\tau_1 \int_{-\frac{I}{2}}^{\tau_1}$
normalization
total number of instantons + anti-instantons
fluctuations around each instanton
$= C \sum_{n\,\text{odd}} k^n \frac{I^n}{n!} e^{-nS_0/\hbar}$
$= \frac{C}{2}\left(e^{kTe^{-S_0/\hbar}} - e^{-kTe^{-S_0/\hbar}} \right)$


$\langle a | e^{-HT/\hbar} | -a \rangle = C \sum_{n,\text{odd}} k^n \int_{-\frac{T}{2}}^{\frac{T}{2}} d\tau_1 \int_{-\frac{T}{2}}^{\tau_1} d\tau_2 \cdots \int_{-\frac{T}{2}}^{\tau_{n-1}}$
normalization
total number of instantons + anti-instantons
fluctuations around each instanton
$= C \sum_{n\ \text{odd}} k^n \frac{T^n}{n!} e^{-nS_0/\hbar}$
$= \frac{C}{2}\left(e^{kTe^{-S_0/\hbar}} - e^{-kTe^{-S_0/\hbar}}\right)$
sign flips for $\langle \pm a | e^{-HT/\hbar} | \pm a \rangle$


diagonalize → $|\pm\rangle = \frac{1}{\sqrt{2}}(|a\rangle \mp |-a\rangle)$
$E_\pm = \frac{1}{2}\hbar\omega \pm \hbar k e^{-S_0/\hbar}$
from our calculation

$-nS_0/\hbar$

e

$d\xi_n$

counting - but extra configurations with instanton near each other are small contributions

$$E_0 = -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$$

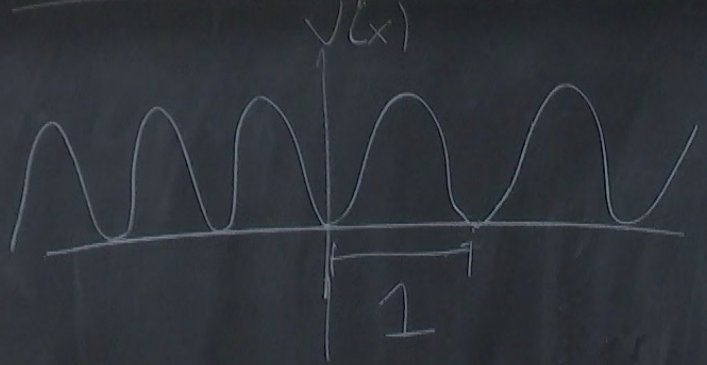
$$\sum \frac{x^n}{n!} \text{ grow until } x \approx n$$

$$kTe^{-S_0/\hbar} \geq n$$

$$ke^{-S_0/\hbar} \geq \frac{n}{T}$$

density

Periodic potential

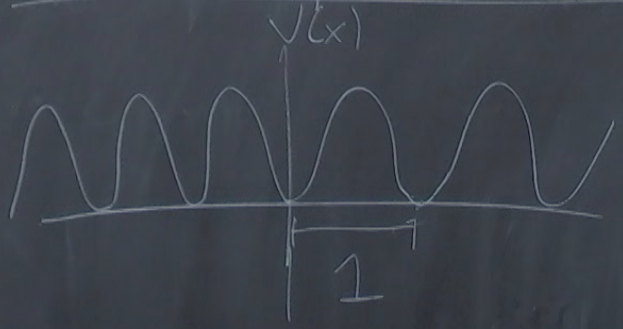


relevant to YM

$\langle x=2 | e^{-HT/\hbar} | x=5 \rangle$

$\langle x=\ell | e^{-HT/\hbar} | x=k \rangle = C\Sigma$

Periodic potential



$\times$

relevant to YM

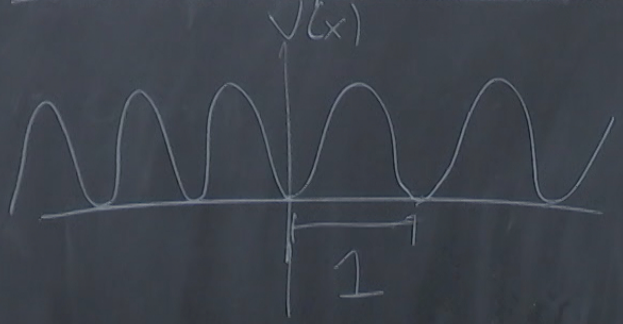
$\langle x=2 | e^{-HT/\hbar} | x=5 \rangle$

$\langle x=\ell | e^{-HT/\hbar} | x=k \rangle = C \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \delta_{n-\bar{n},\ell-k}$

instanton anti-instanton

$\cdot \frac{1}{n!\bar{n}!} \left(kTe^{-S_0/\hbar} \right)^{n+\bar{n}}$

Periodic potential



$\times$

$\langle r=2 | e^{-HT/\hbar} | x=5 \rangle$

relevant to YM

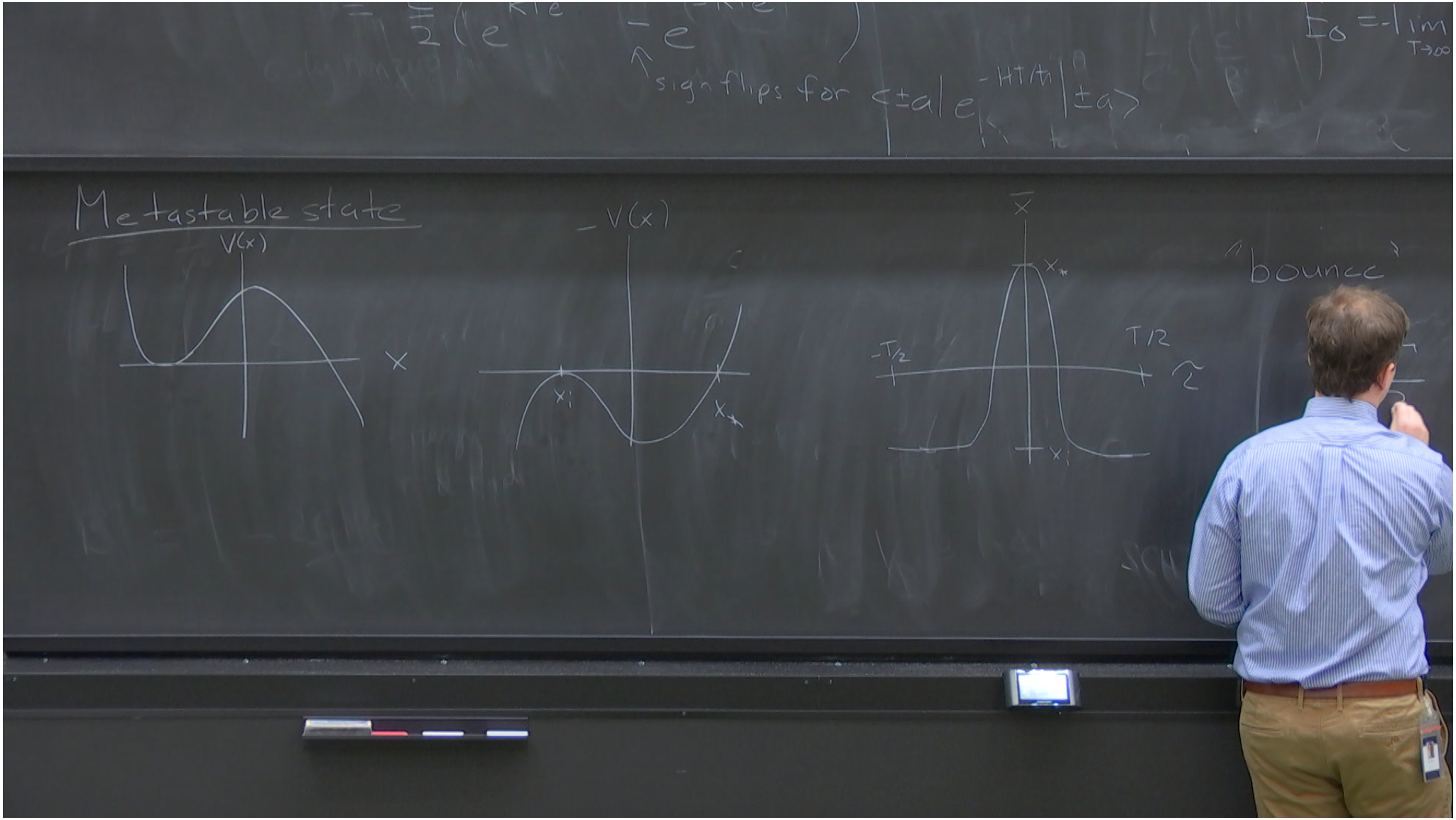
$$\langle x=l | e^{-HT/\hbar} | x=k \rangle = C \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \delta_{n-\bar{n},\, l-k}$$

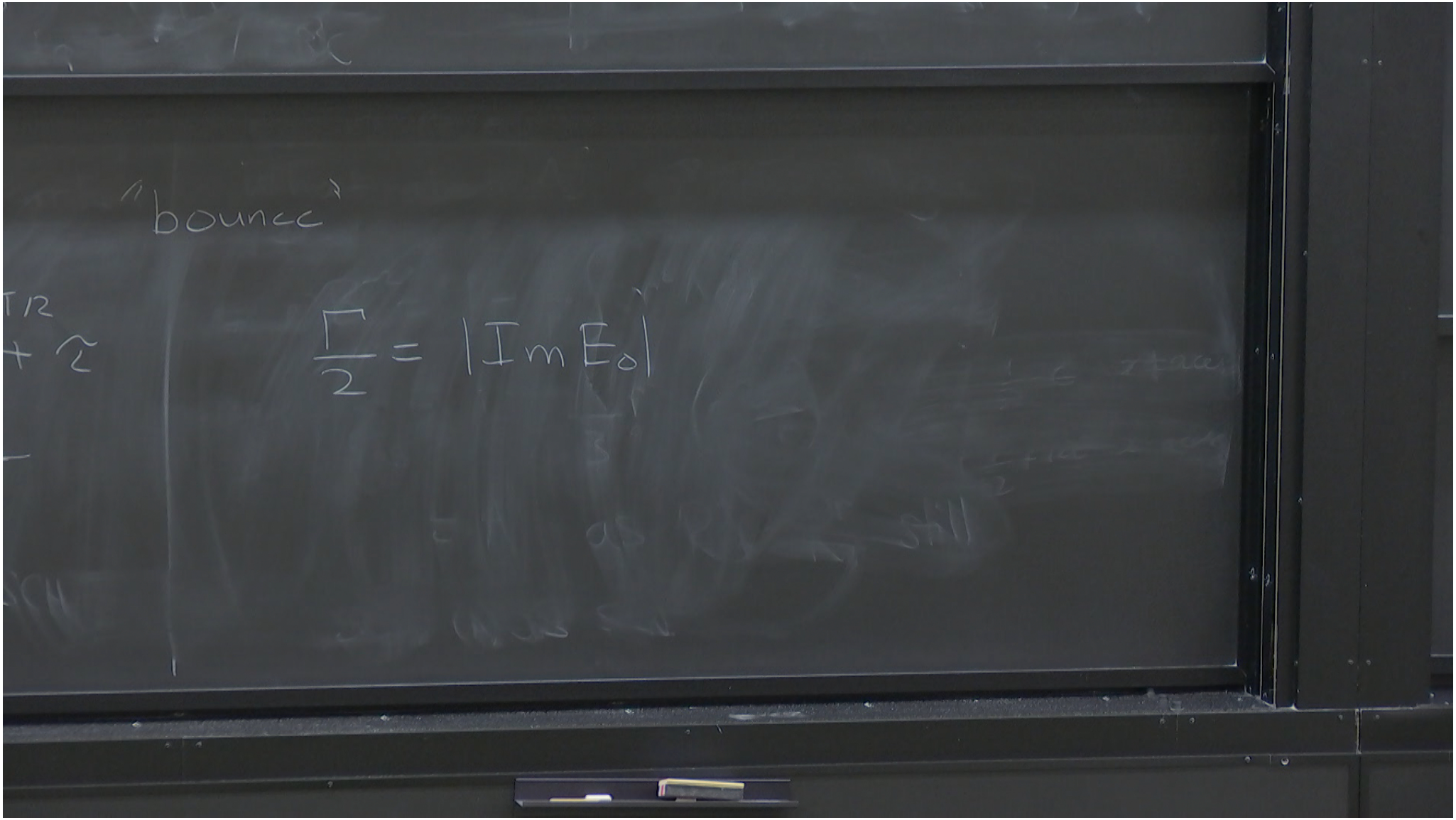
instanton (↑) anti-instanton (↑)

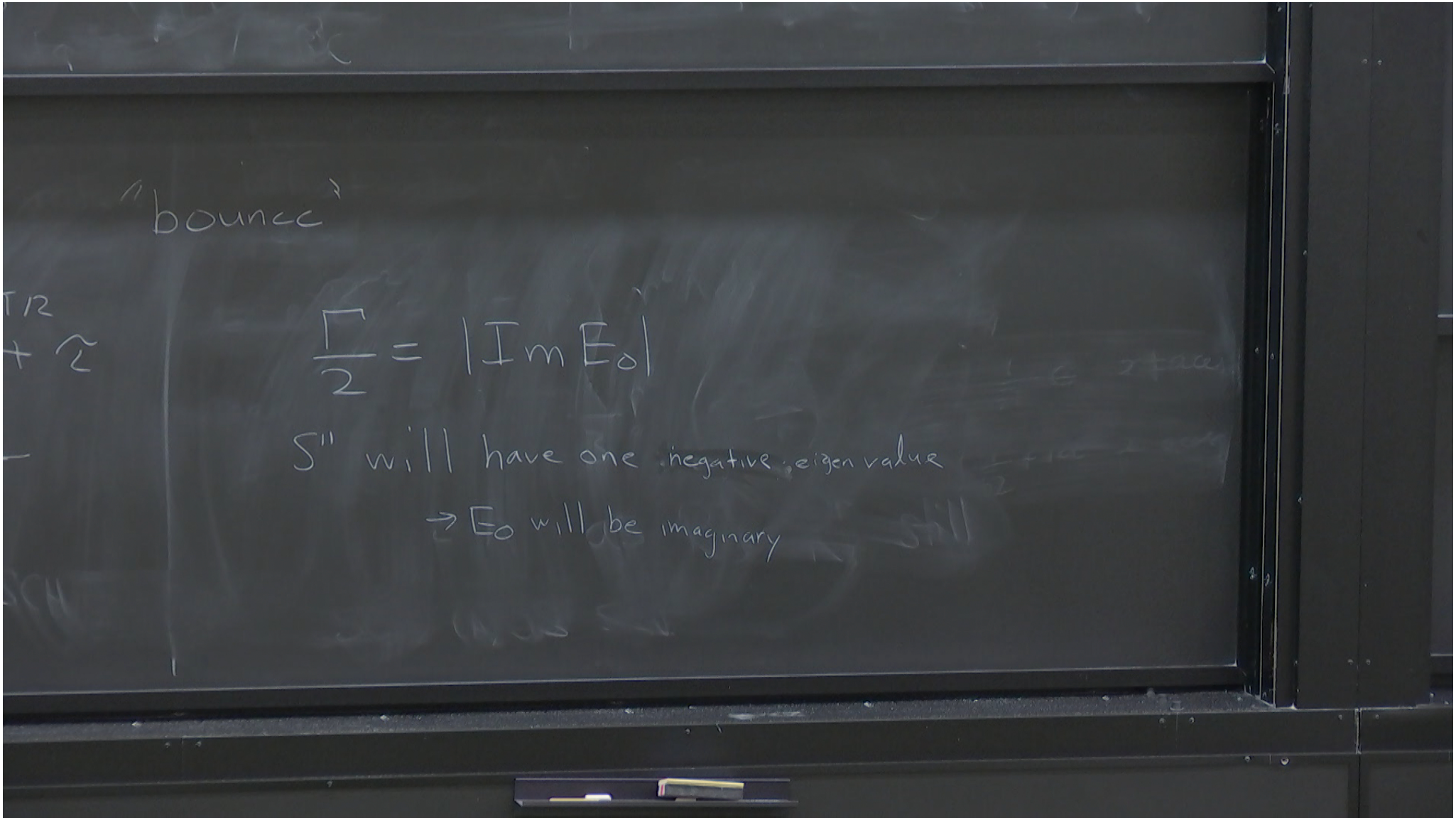
$$\cdot \frac{1}{n!\bar{n}!} \left(kT e^{-S_0/\hbar} \right)^{n+\bar{n}}$$

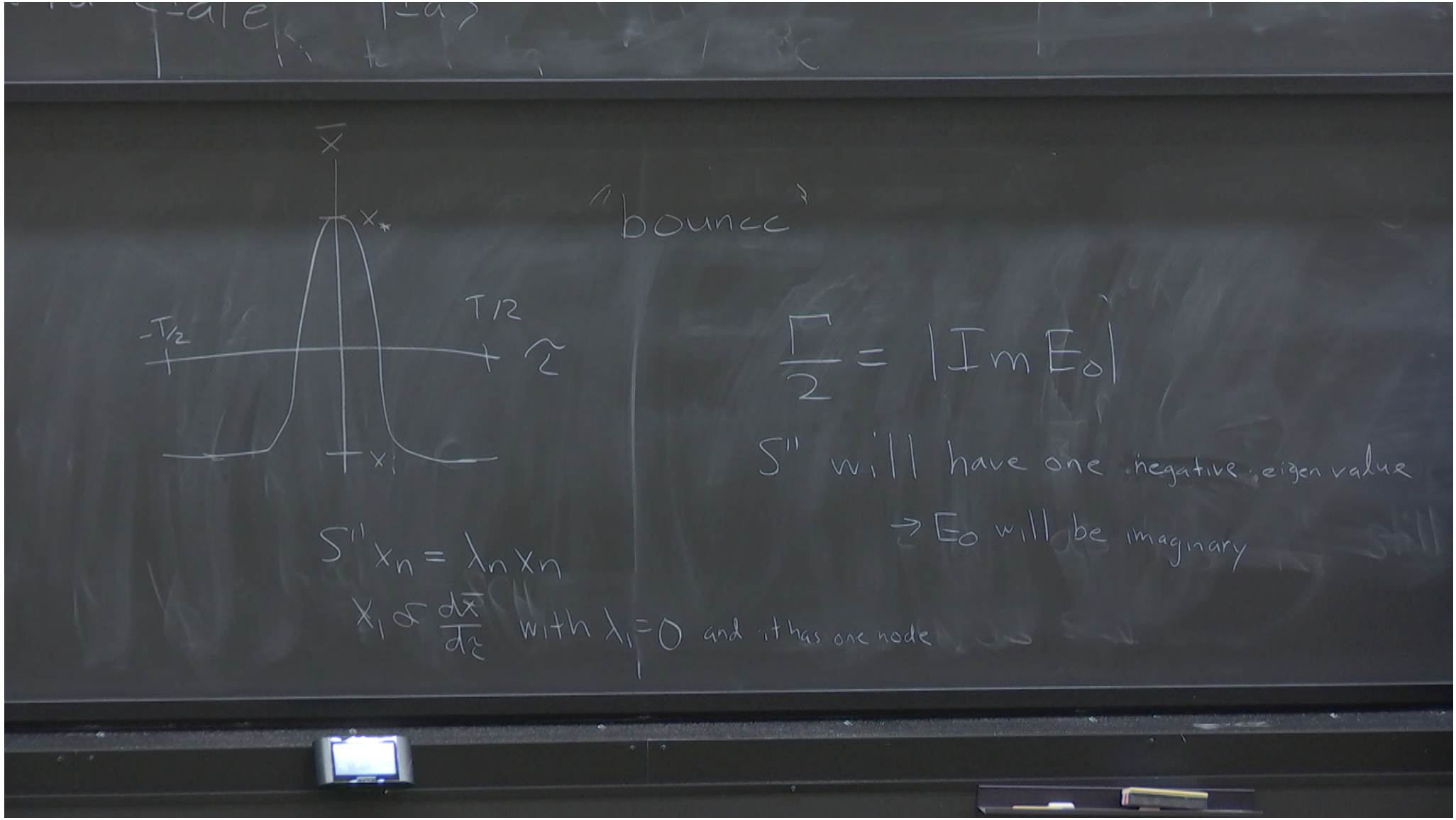
→ leads to continuum of energy eigenstates (tutorial)

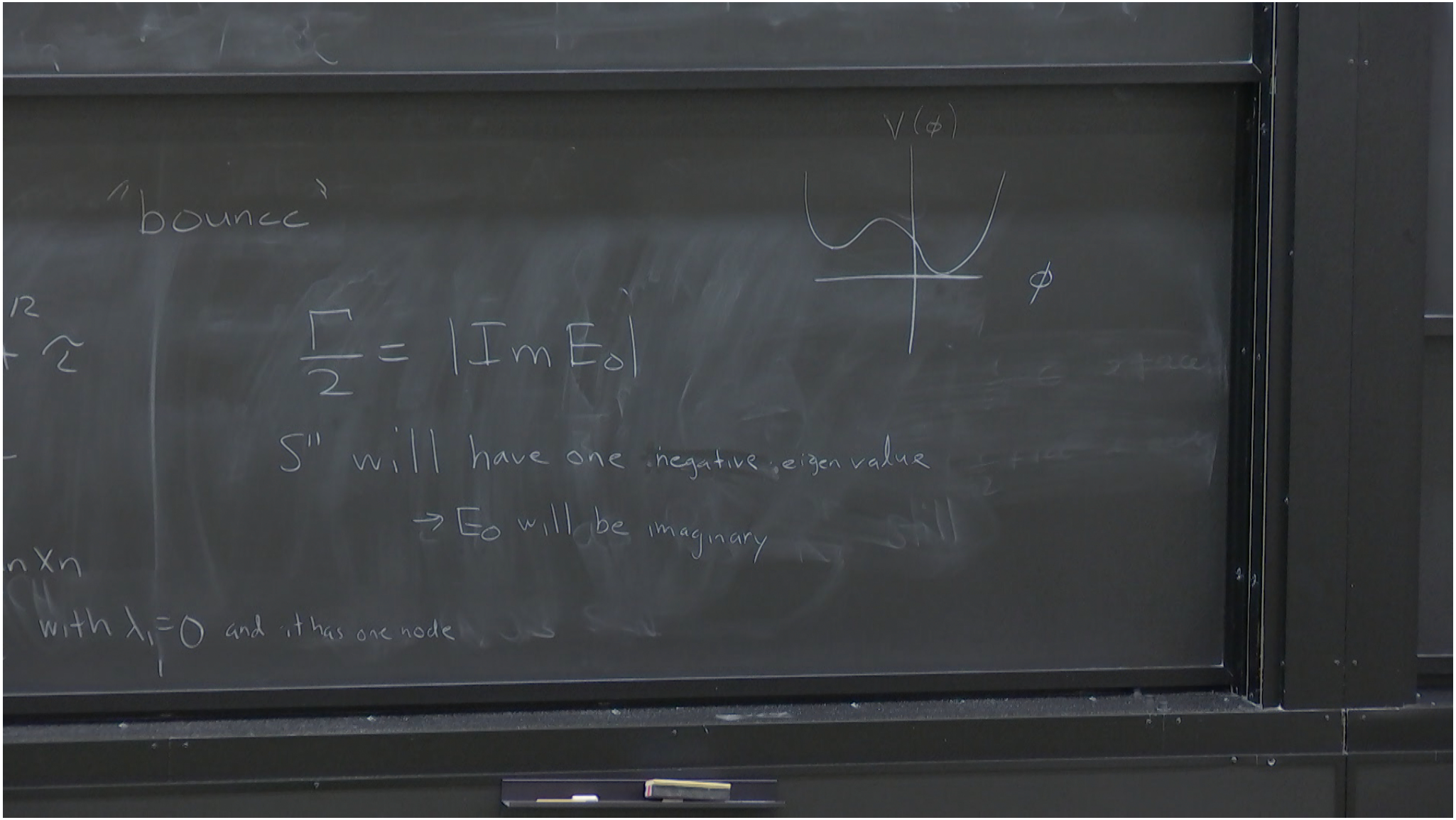

1
$\langle x=2 | e^{-HT/\hbar} | x=5 \rangle$
$\langle x=l | e^{-HT/\hbar} | x=k \rangle = C \sum$
$n=0$
instanton
$\frac{1}{n! \bar{n}!}$
→ leads to continu
(tu


Metastable state
V(x)
x
-V(x)
x_i
x_*
$\bar{x}$
x_*
x_i
-T/2
T/2
"bounce"


"bounce"
$\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$


"bounce"
$\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$
S'' will have one negative eigenvalue
→ E_0 will be imaginary


$\bar{x}$
x_*
$-T/2$
$T/2$
$\tilde{\tau}$
x_1
"bounce"
$\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$
S'' will have one negative eigenvalue
$\rightarrow E_0$ will be imaginary
$S'' x_n = \lambda_n x_n$
$x_1 \propto \frac{d\bar{x}}{d\tilde{\tau}}$ with $\lambda_1 = 0$ and it has one node


"bounce"
V(φ)
φ
Γ/2 = |Im E₀|
S'' will have one negative eigenvalue
→ E₀ will be imaginary
with λ₁=0 and it has one node