Title: PSI 2019/2020 - QFT III - Lecture 1

Speakers:

Collection: PSI 2019/2020 - QFT III

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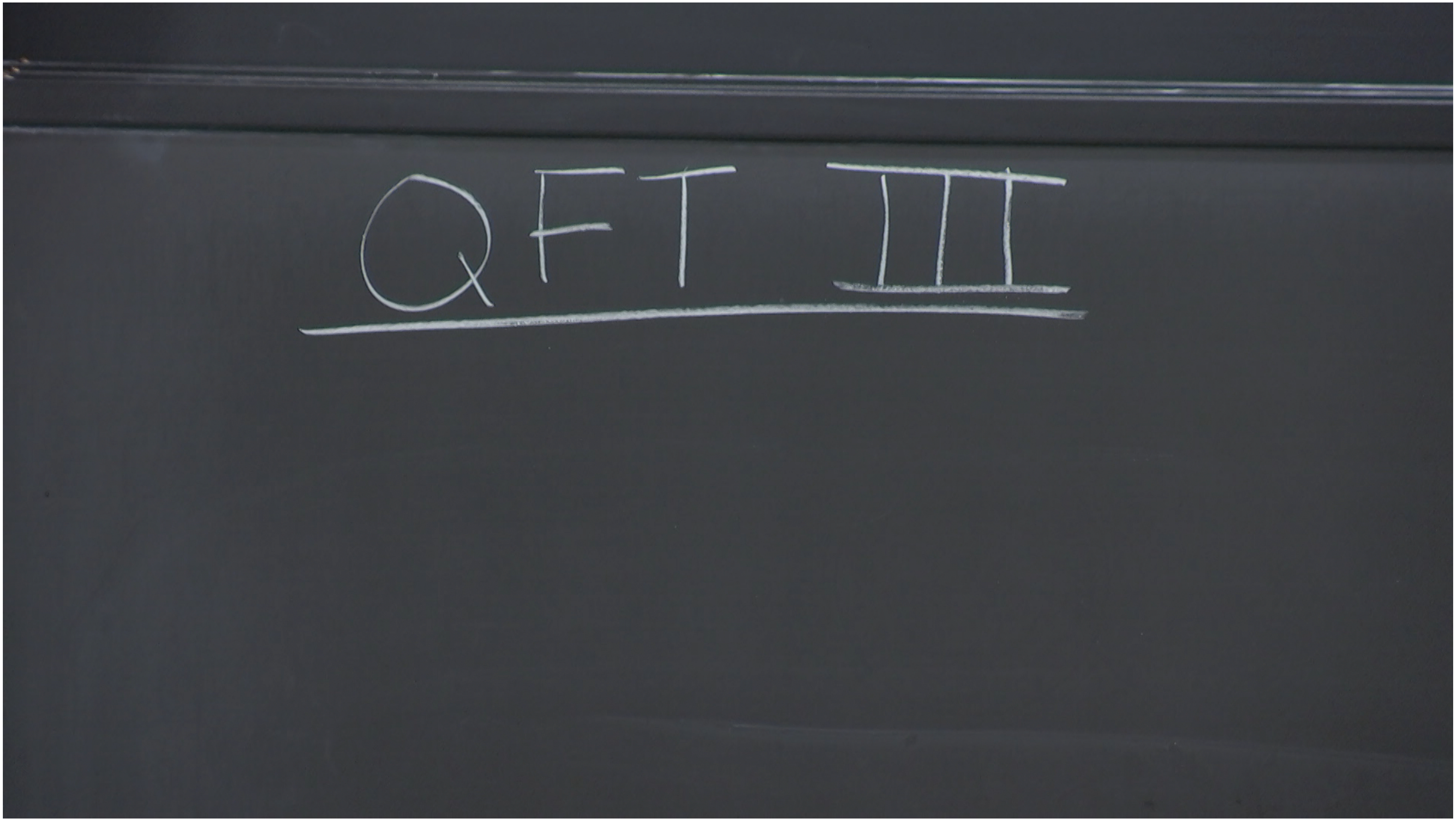

QFT III


QFT III
Instantons + Anomalies
trivial saddles of
idean functional integral
quantum effects
that spoil classical
symmetries
onformal Field Theory
QFT I + II
small
parameter
sums + integrals

QFT III

Instantons + Anomalies

non-trivial saddles of
Euclidean functional integral

quantum effects
that spoil classical
symmetries

Conformal Field Theory

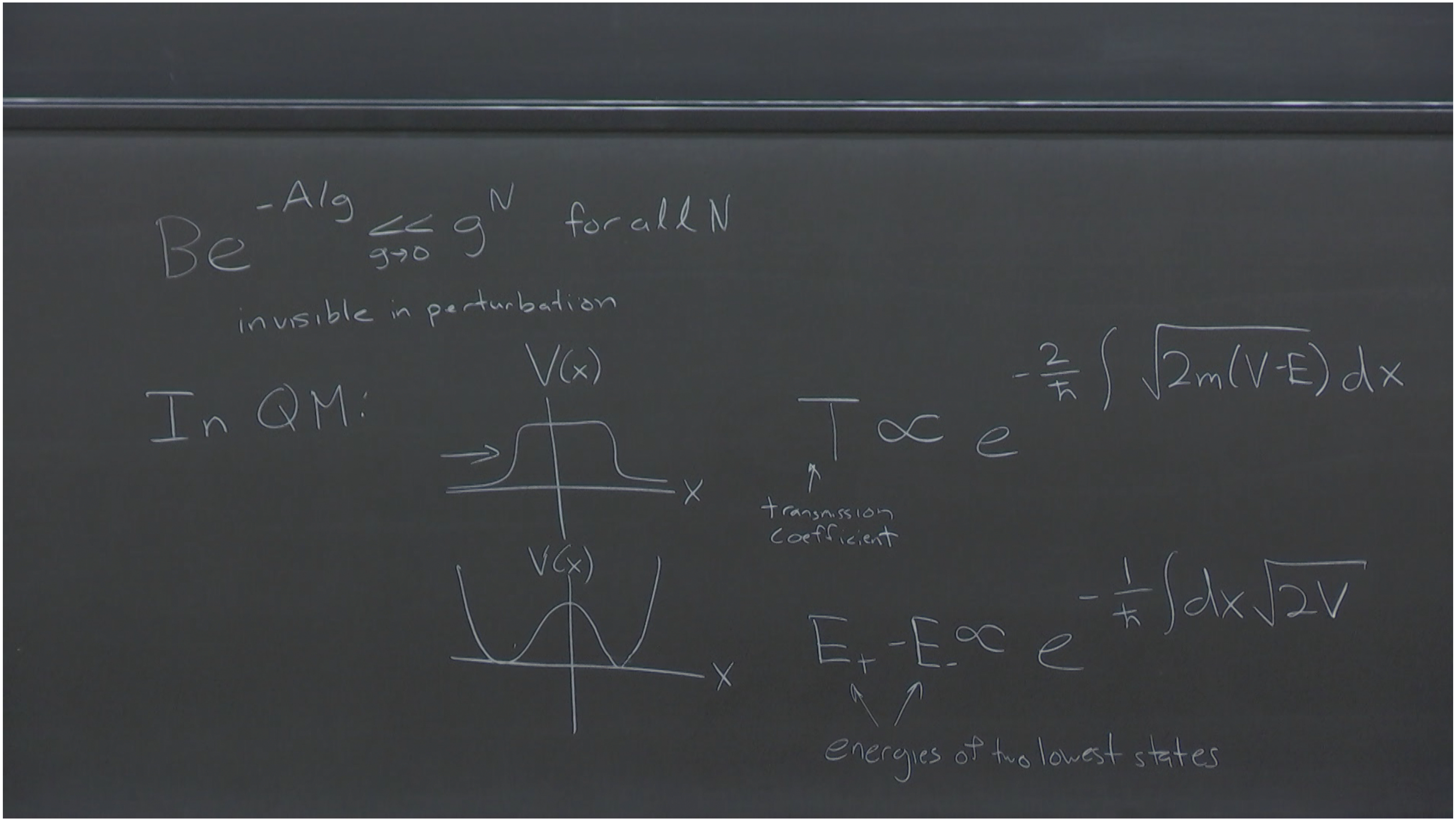
In QFT I + II

$$e^{-A/g} + f(g) \underset{g\to 0}{\sim} \sum_n a_n g^n$$

exact
quantity
of interest

small
parameter

swapping
sums + integrals


$Be^{-A/g} \ll_{g\to 0} g^N$ for all N
invisible in perturbation
In QM:
$V(x)$
x
$T \propto e^{-\frac{2}{\hbar}\int \sqrt{2m(V-E)}\,dx}$
transmission coefficient
$V(x)$
x
$E_+ - E_- \propto e^{-\frac{1}{\hbar}\int dx \sqrt{2V}}$
energies of two lowest states

can determine by finding $\psi(x)$

In QFT need $\underline{\Psi}[\varphi(x)]$

hard to find!

instantons easier for QFT

$$I(z) = \int_C g(w)\, e^{z f(w)}\, dw \underset{z\to\infty}{\sim} \sum_{\substack{\text{relevant}\\ \text{saddles}\\ w_0}} \sqrt{\frac{2\pi}{z}}\, \frac{e^{z f(w_0)}}{\sqrt{-f''(w_0)}}\, g(w_0)$$

generalization to QM + QFT?

For studying ground state can use instanton method

<u>Outline</u>

Instanton in QM - E_0 (ground state) from Euclidean functional integral

Instantons in QFT - Decay of a metastable state in QFT

- Θ vacua (ground state of YM theory)

From Math for QFT

$$\underset{\substack{\uparrow \\ \langle\ \rangle}}{I}\left(\underset{\substack{\Uparrow \\ \frac{1}{g}}}{z}\right) = \int_C g(w)\, e^{z \underset{\substack{\uparrow \\ S}}{f}(w)}\, dw \underset{z\to\infty}{\sim} \sum_{\substack{\text{relevant} \\ \text{saddles} \\ w_0}} \sqrt{\frac{2\pi}{z}}\, \frac{e^{z f(w_0)}}{\sqrt{-f''(w_0)}}\, g(w_0)$$

generalization to QM + QFT?

For studying ground state can use instanton method

<u>Outline</u>

Instanton in QM - E_0 (ground state) from Euclidean functional integral

Instantons in QFT - Decay of a metastable state in QFT

- Θ vacua (ground state of YM theory) of interest   small parameter   swapping sums + integrals

E_0 from Euclidean path integral

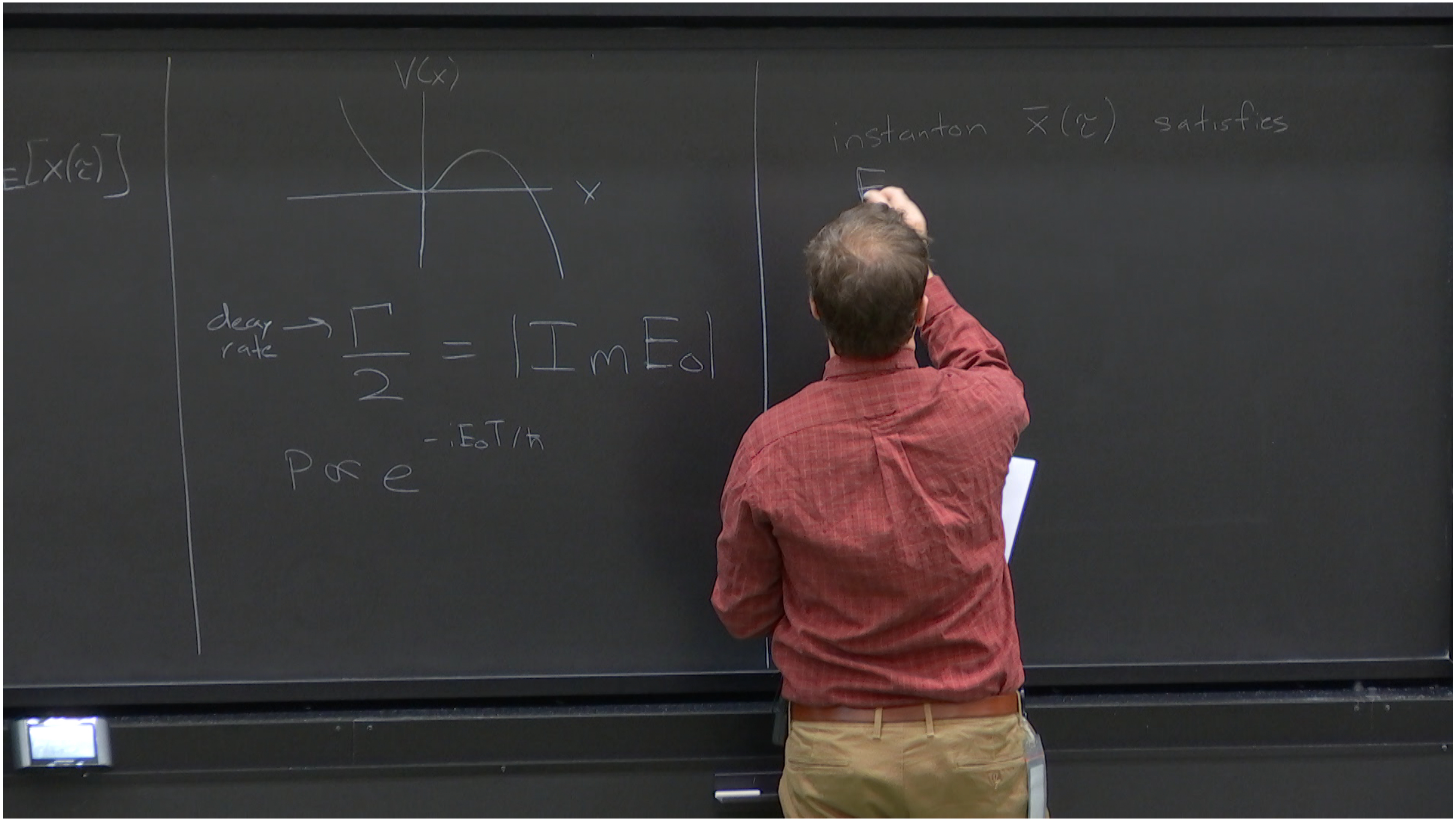
$$Z = \langle x_f | e^{-HT/\hbar} | x_i \rangle = \int_{x(-\frac{T}{2}) = x_i}^{x(\frac{T}{2}) = x_f} \mathcal{D}x(\tau)\, e^{-\frac{1}{\hbar} S_E[x(\tau)]}$$

$$Z = \sum_n e^{-E_n T/\hbar} \langle x_f | n \rangle \langle n | x_i \rangle$$

$$\uparrow$$

$$H|n\rangle = E_n |n\rangle$$

$$E_0 = -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$$


E[x(τ)]
V(x)
x
decay rate → Γ/2 = |Im E₀|
P ∝ e^{-iE₀T/ħ}
instanton x̄(τ) satisfies

E_0 from Euclidean path integral

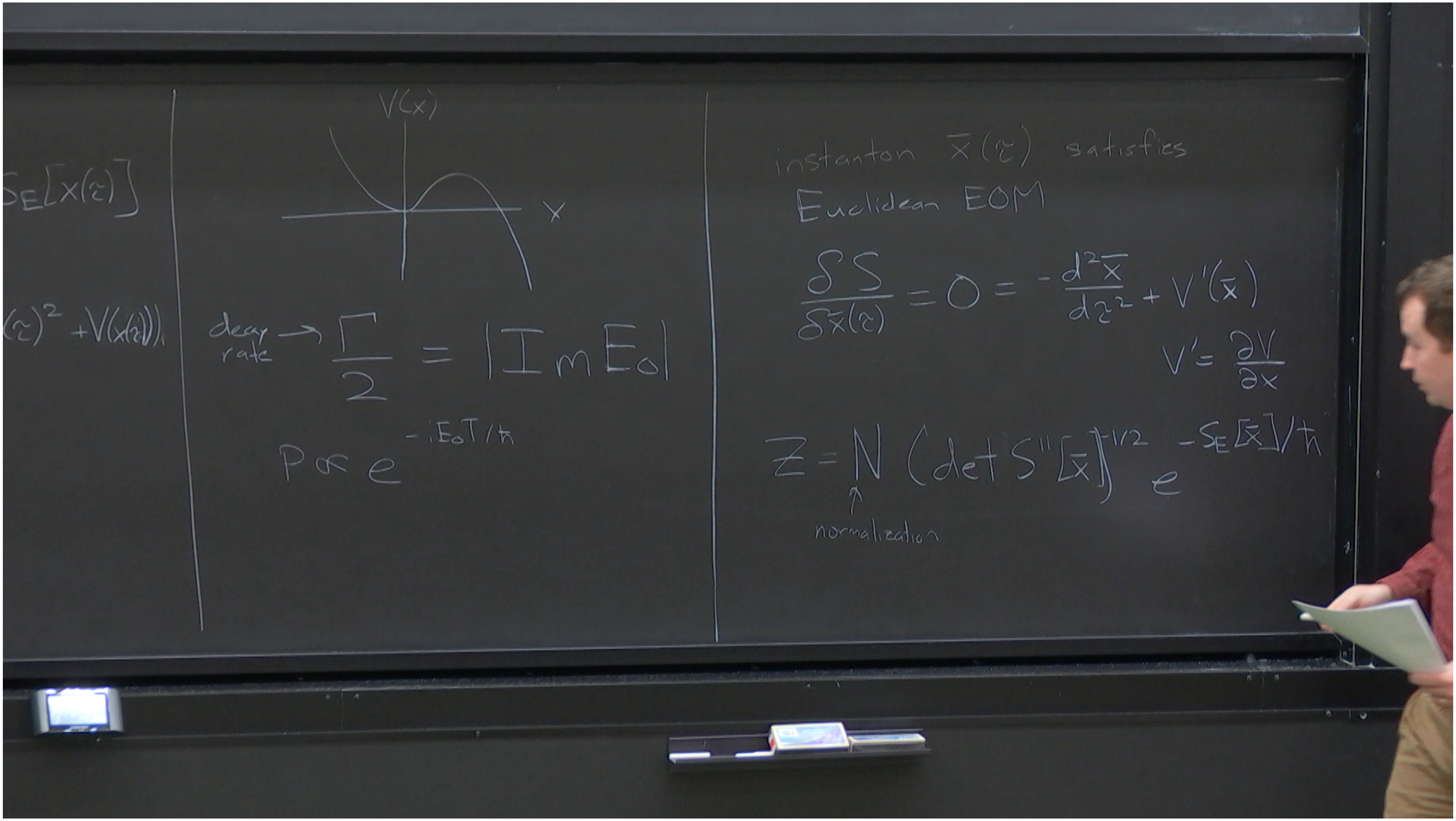
$$Z = \langle x_f | e^{-HT/\hbar} | x_i \rangle = \int_{x(-\frac{T}{2}) = x_i}^{x(\frac{T}{2}) = x_f} \mathcal{D}x(\tau)\; e^{-\frac{1}{\hbar} S_E[x(\tau)]}$$

$$S_E = \int d\tau \left(\dot{x}(\tau)^2 + V(x(\tau)) \right)$$

$$Z = \sum_n e^{-E_n T/\hbar} \langle x_f | n \rangle \langle n | x_i \rangle$$

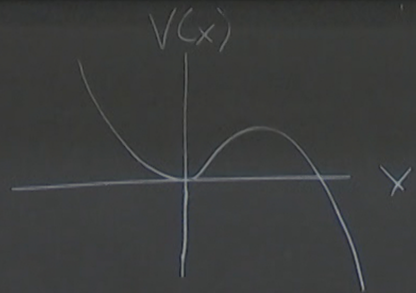
$$H|n\rangle = E_n |n\rangle$$

$$E_0 = -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$$


$S_E[x(\tau)]$
$(\tau))^2 + V(x(\tau))$
$V(x)$
x
decay rate
$\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$
$P \propto e^{-iE_0 T/\hbar}$
instanton $\bar{x}(\tau)$ satisfies
Euclidean EOM
$\frac{\delta S}{\delta \bar{x}(\tau)} = 0 = -\frac{d^2\bar{x}}{d\tau^2} + V'(\bar{x})$
$V' = \frac{\partial V}{\partial x}$
$Z = N\,(\det S''[\bar{x}])^{-1/2}\, e^{-S_E[\bar{x}]/\hbar}$
normalization

$S_E[x(\tau)]$

$(\tau)^2 + V(x(\tau))$



decay rate $\rightarrow \frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$

$$P \propto e^{-iE_0 T/\hbar}$$

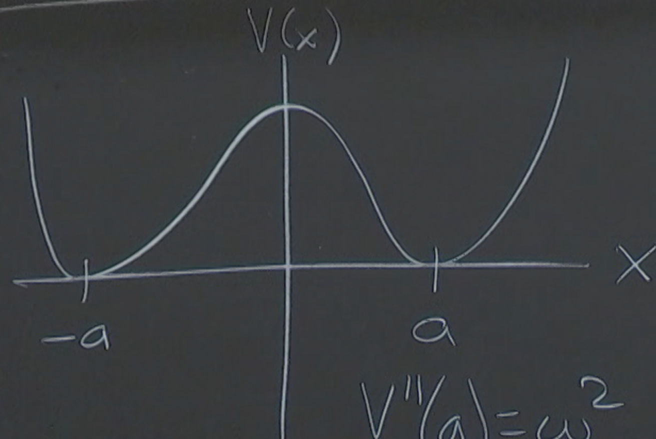
instanton $\bar{x}(\tau)$ satisfies Euclidean EOM

$$\frac{\delta S}{\delta \bar{x}(\tau)} = 0 = -\frac{d^2 \bar{x}}{d\tau^2} + V'(\bar{x})$$

$$V' = \frac{\partial V}{\partial x}$$

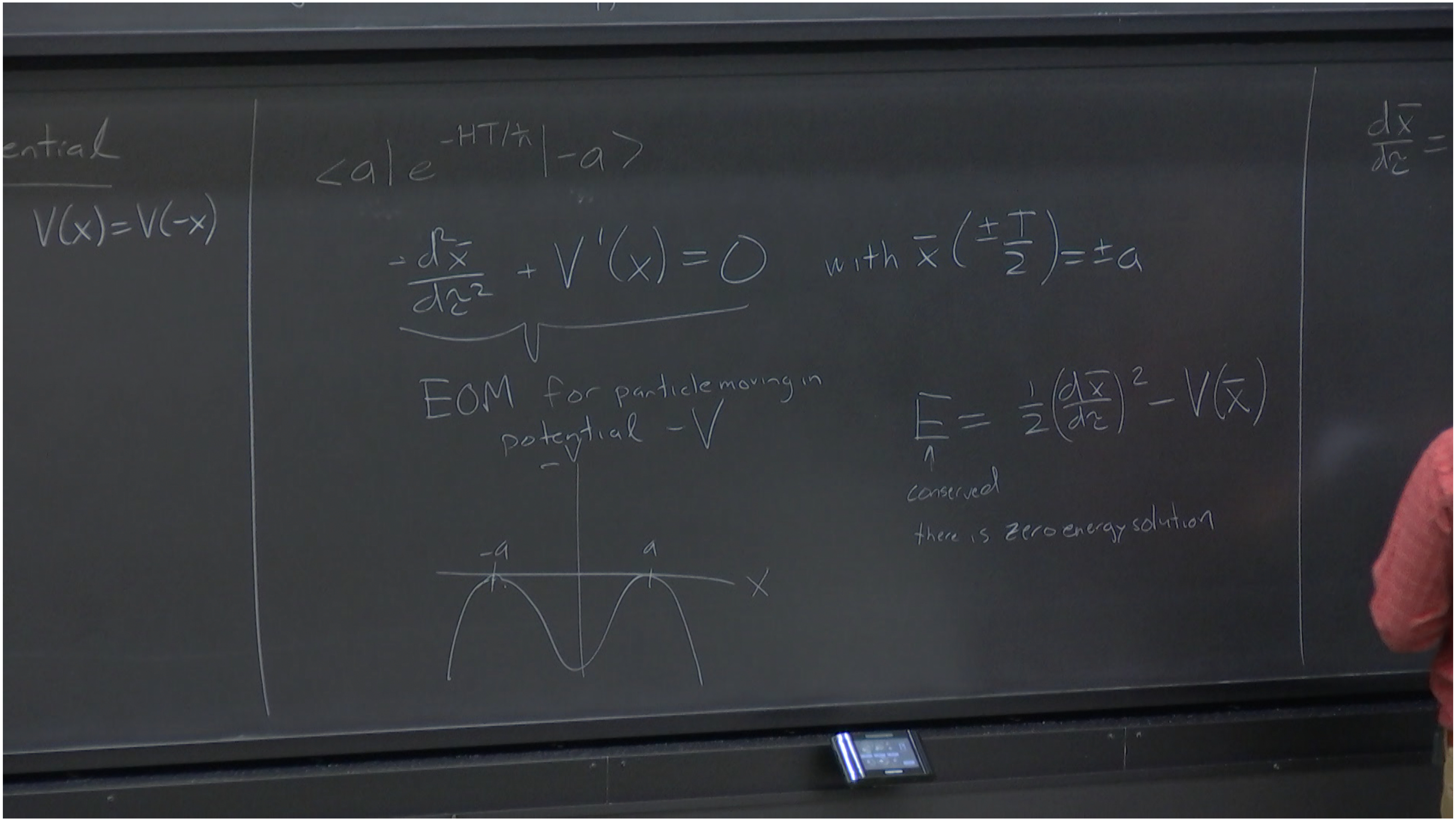
$$Z = \sum_{\bar{x}} \underset{\substack{\uparrow \\ \text{normalization}}}{N} \left(\det S''[\bar{x}]\right)^{-1/2} e^{-S_E[\bar{x}]/\hbar}$$

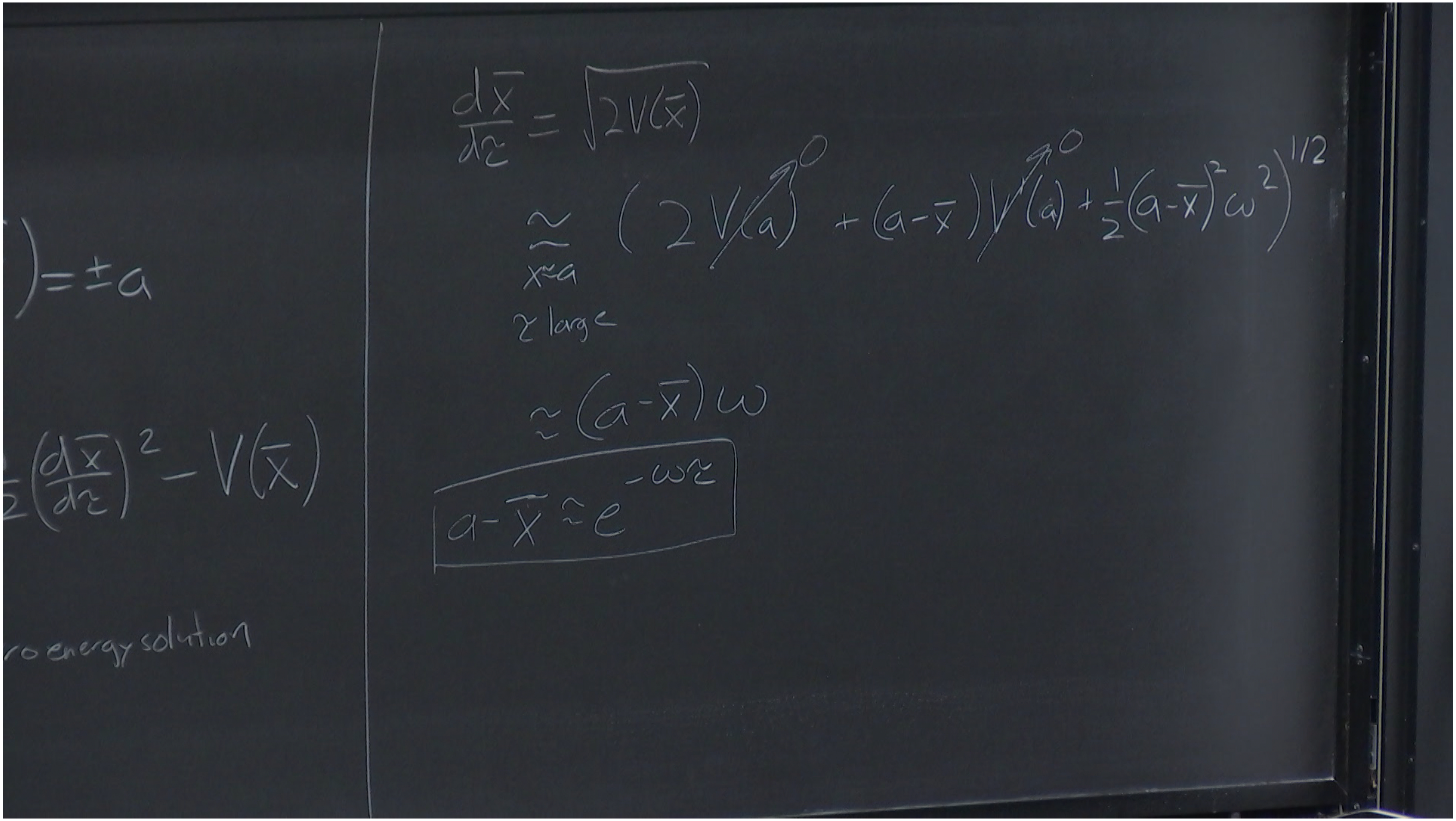
Double Well Potential

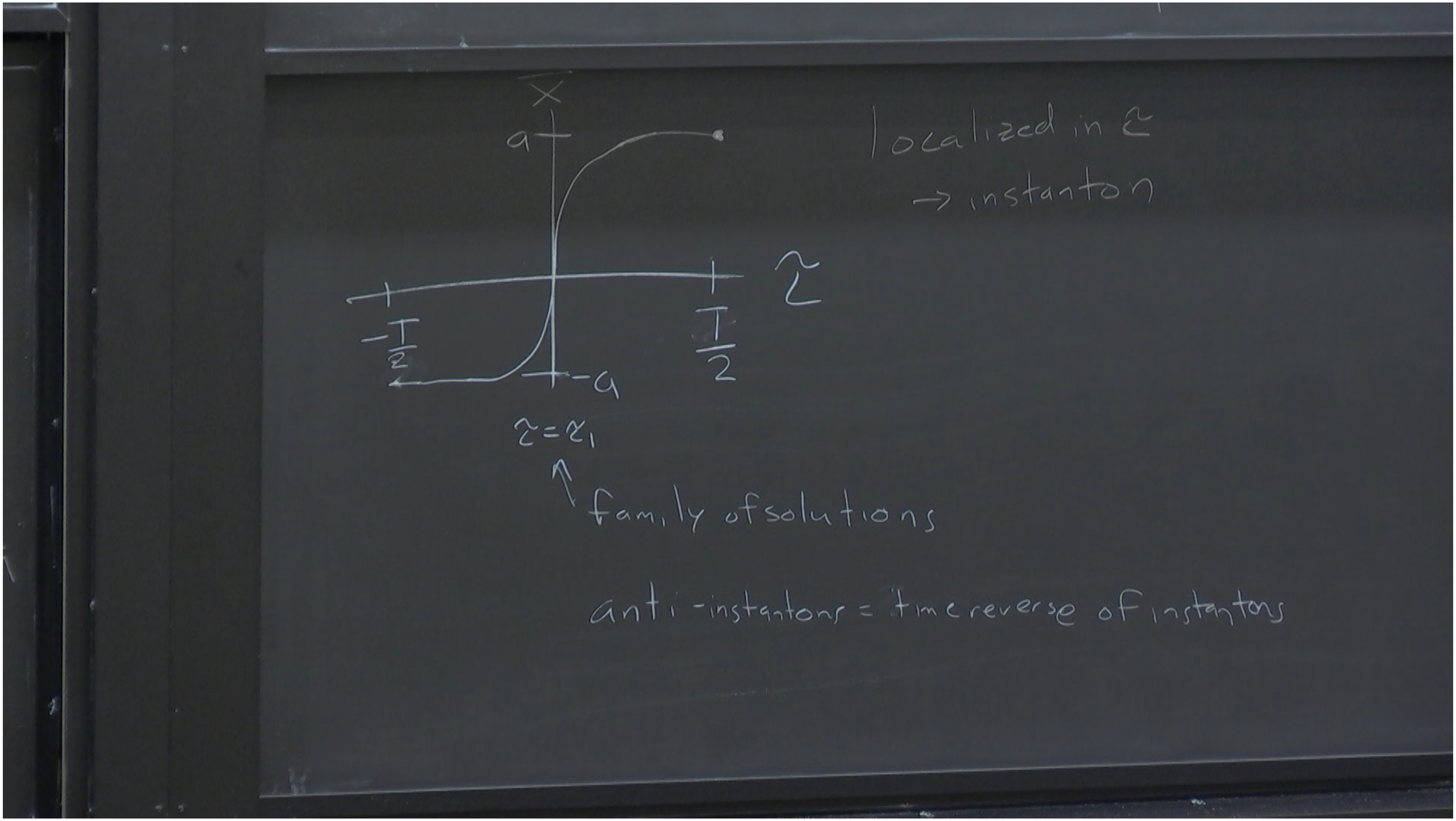


$$V(x) = V(-x)$$

$$V''(a) = \omega^2$$


ential
$V(x)=V(-x)$
$\langle a|e^{-HT/\hbar}|-a\rangle$
$-\frac{d^2\bar{x}}{d\tau^2}+V'(x)=0$ with $\bar{x}(\pm\frac{T}{2})=\pm a$
EOM for particle moving in potential $-V$
$-V$
$-a$
a
x
$E=\frac{1}{2}\left(\frac{d\bar{x}}{d\tau}\right)^2-V(\bar{x})$
conserved
there is zero energy solution
$\frac{d\bar{x}}{d\tau}=$


$) = \pm a$
$\frac{1}{2}\left(\frac{d\bar{x}}{d\tau}\right)^2 - V(\bar{x})$
ro energy solution
$\frac{d\bar{x}}{d\tau} = \sqrt{2V(\bar{x})}$
$\underset{\substack{x \simeq a \\ \tau \text{ large}}}{\approx} \left(\cancel{2V(a)}^{0} + (a-\bar{x})\cancel{V'(a)}^{0} + \frac{1}{2}(a-\bar{x})^2\omega^2 \right)^{1/2}$
$\simeq (a-\bar{x})\omega$
$\boxed{a - \bar{x} \simeq e^{-\omega\tau}}$


x
a
−a
τ
−T/2
T/2
τ = τ₁
family of solutions
localized in τ
→ instanton
anti-instantons = time reverse of instantons