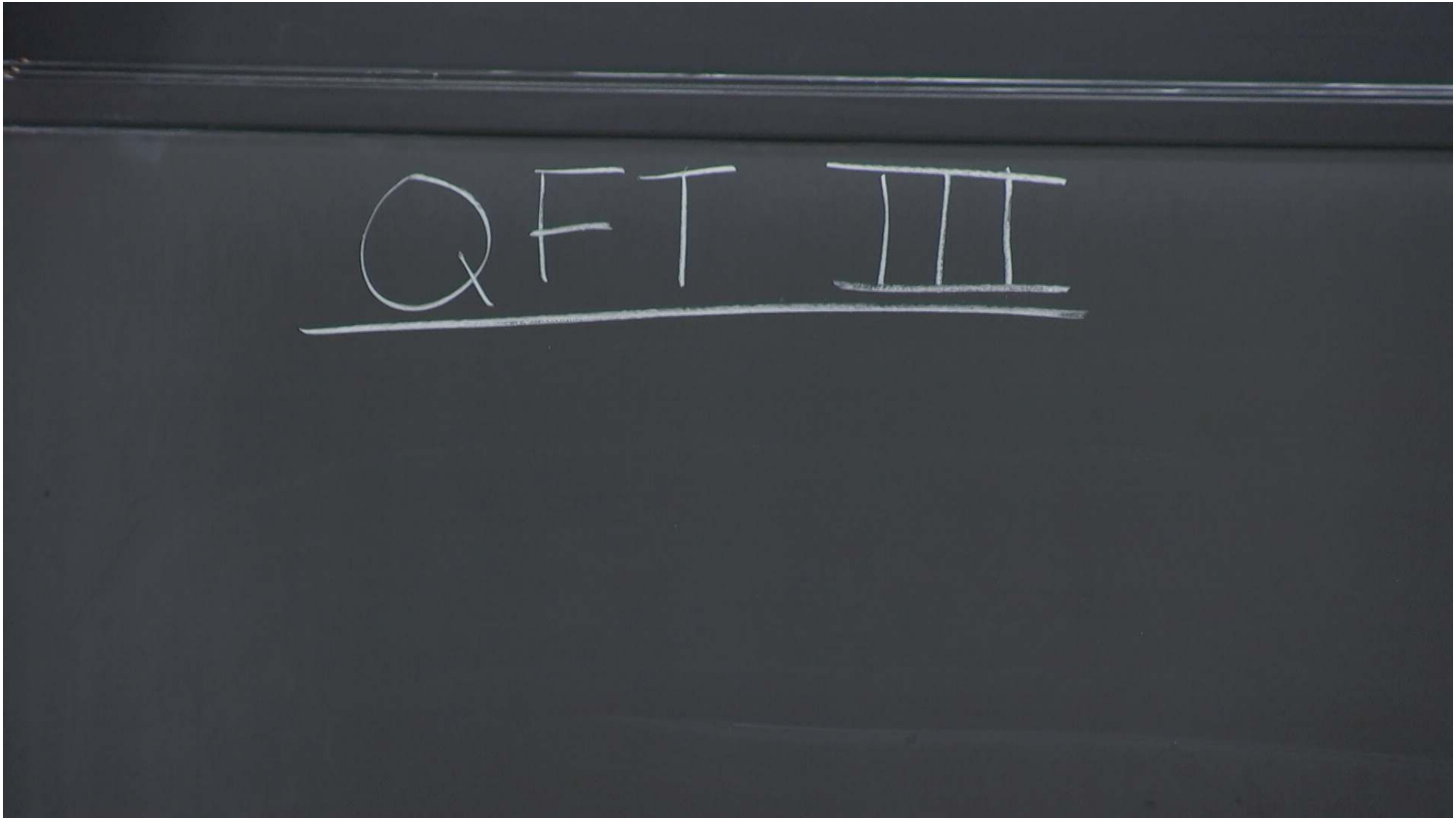
Title: PSI 2019/2020 - QFT III - Lecture 1

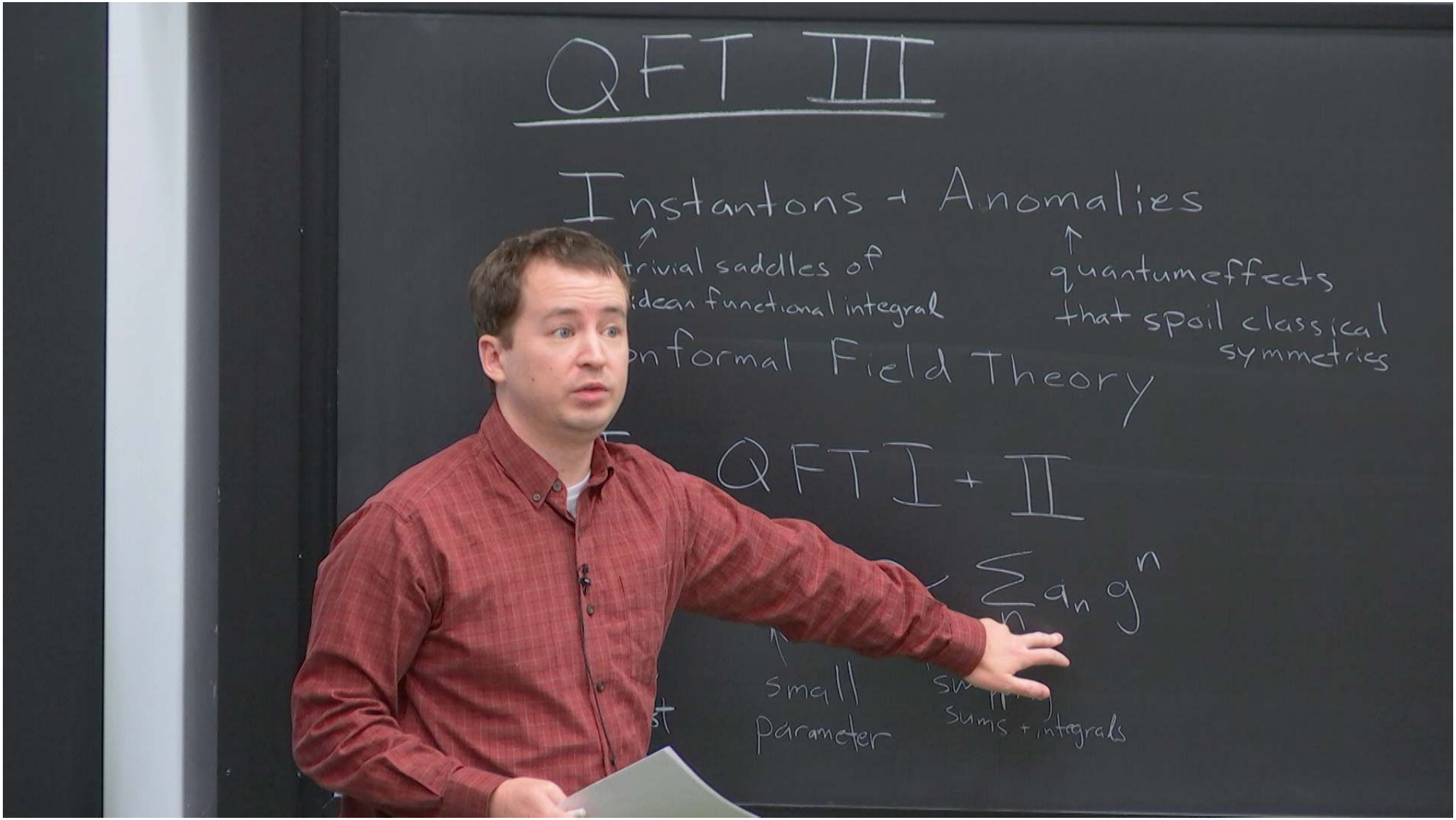
Speakers:

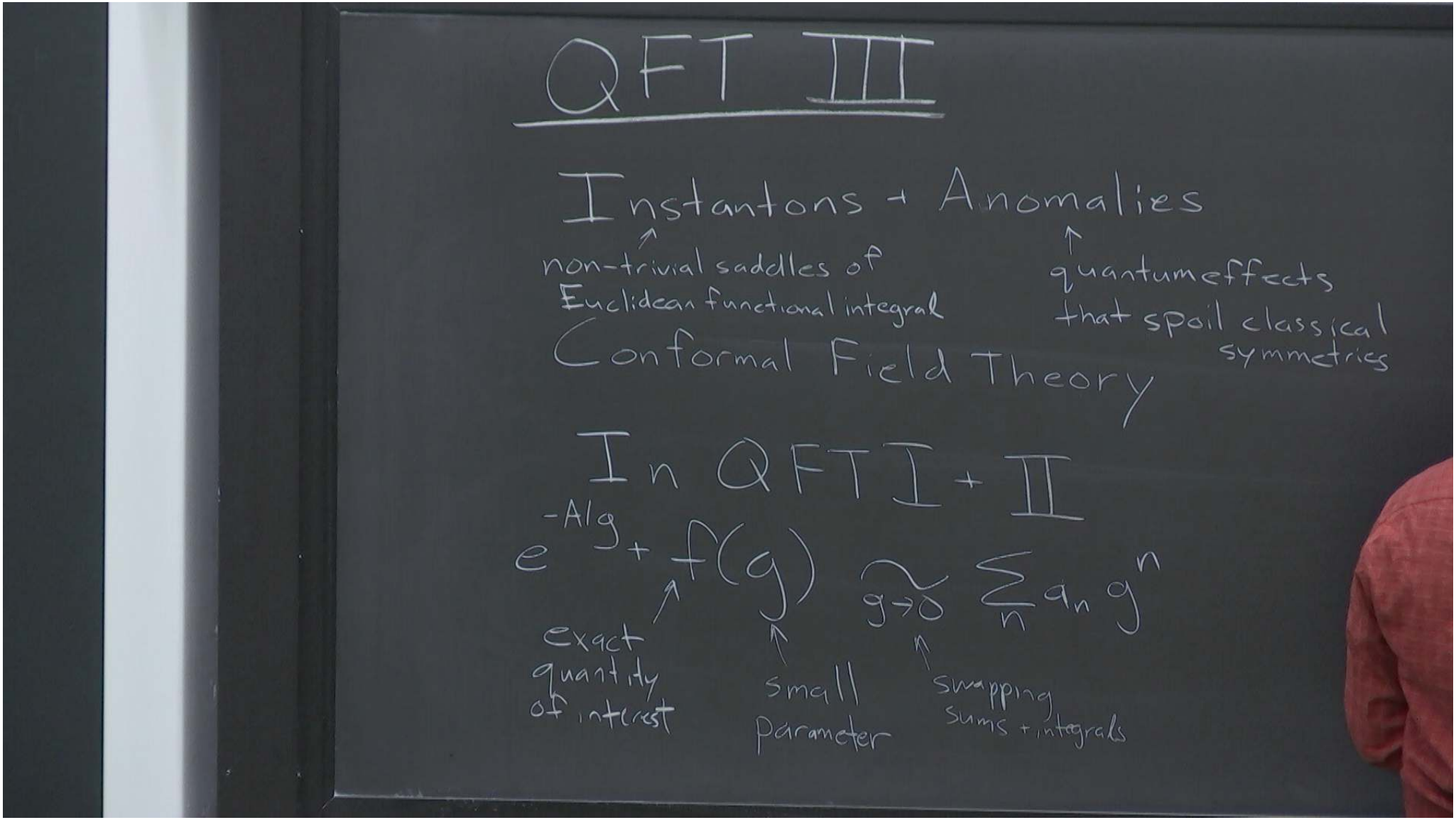
Collection: PSI 2019/2020 - QFT III

Date: January 08, 2020 - 11:30 AM

URL: http://pirsa.org/20010072


QFT III

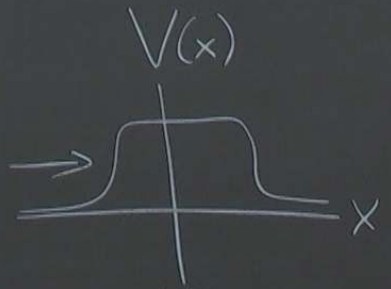

QFT III
Instantons + Anomalies
trivial saddles of
ideal functional integral
quantum effects
that spoil classical
symmetries
onformal Field Theory
QFT I + II
small
parameter
sums + integrals


QFT III
Instantons + Anomalies
non-trivial saddles of Euclidean functional integral
quantum effects that spoil classical symmetries
Conformal Field Theory
In QFT I + II
$e^{-A/g} + f(g) \underset{g\to 0}{\sim} \sum_n a_n g^n$
exact quantity of interest
small parameter
swapping sums + integrals

$$Be^{-A/g} \underset{g \to 0}{\ll} g^N \quad \text{for all } N$$

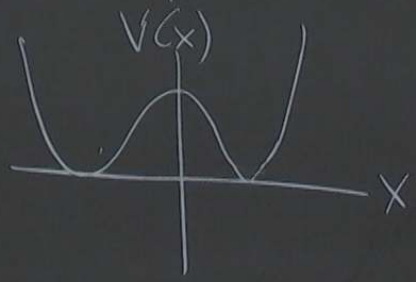
invisible in perturbation

In QM:



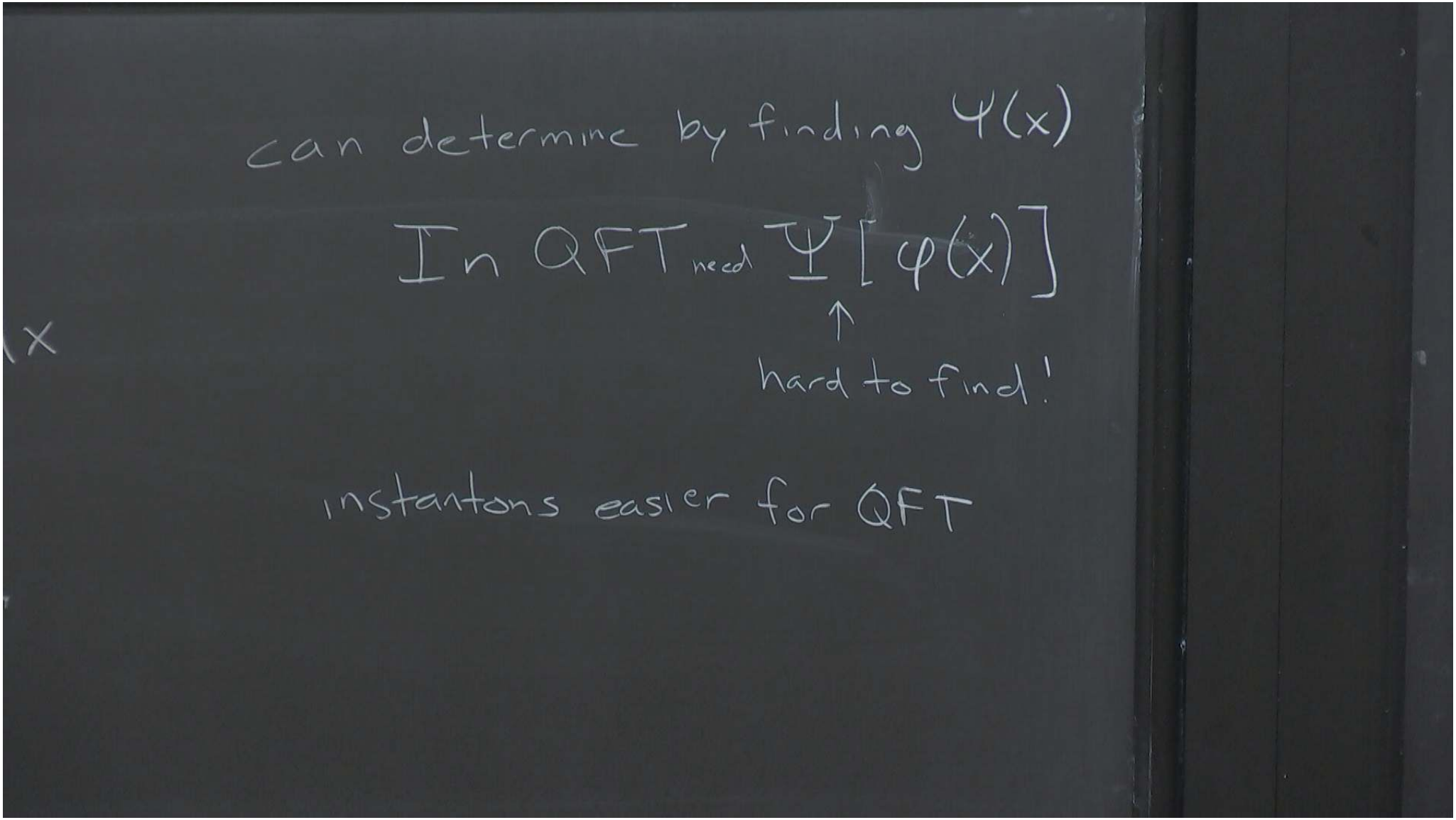
$$T \propto e^{-\frac{2}{\hbar}\int \sqrt{2m(V-E)}\,dx}$$

transmission coefficient



$$E_+ - E_- \propto e^{-\frac{1}{\hbar}\int dx \sqrt{2V}}$$

energies of two lowest states


can determine by finding $\Psi(x)$
In QFT need $\Psi[\varphi(x)]$
hard to find!
instantons easier for QFT

$$I(z) = \int_C g(w)\, e^{z f(w)}\, dw \underset{z\to\infty}{\sim} \sum_{\substack{\text{relevant} \\ \text{saddles} \\ w_0}} \sqrt{\frac{2\pi}{z}}\, \frac{e^{z f(w_0)}}{\sqrt{-f''(w_0)}}\, g(w_0)$$

generalization to QM + QFT?

For studying ground state can use instanton method

Outline

Instanton in QM - E_0 ← ground state from Euclidean functional integral

Instantons in QFT - Decay of a metastable state in QFT

- θ vacua (ground state of YM theory)

From Math for QFT

$$I(z) = \int_C g(w)\, e^{z f(w)}\, dw \underset{z\to\infty}{\sim} \sum_{\substack{\text{relevant} \\ \text{saddles} \\ w_0}} \sqrt{\frac{2\pi}{z}}\, \frac{e^{z f(w_0)}}{\sqrt{-f''(w_0)}}\, g(w_0)$$

I: $\uparrow$ $\langle \quad \rangle$

z: $\uparrow$ $\frac{1}{g}$

$f(w)$: $\uparrow$ S

generalization to QM + QFT?

For studying ground state can use instanton method

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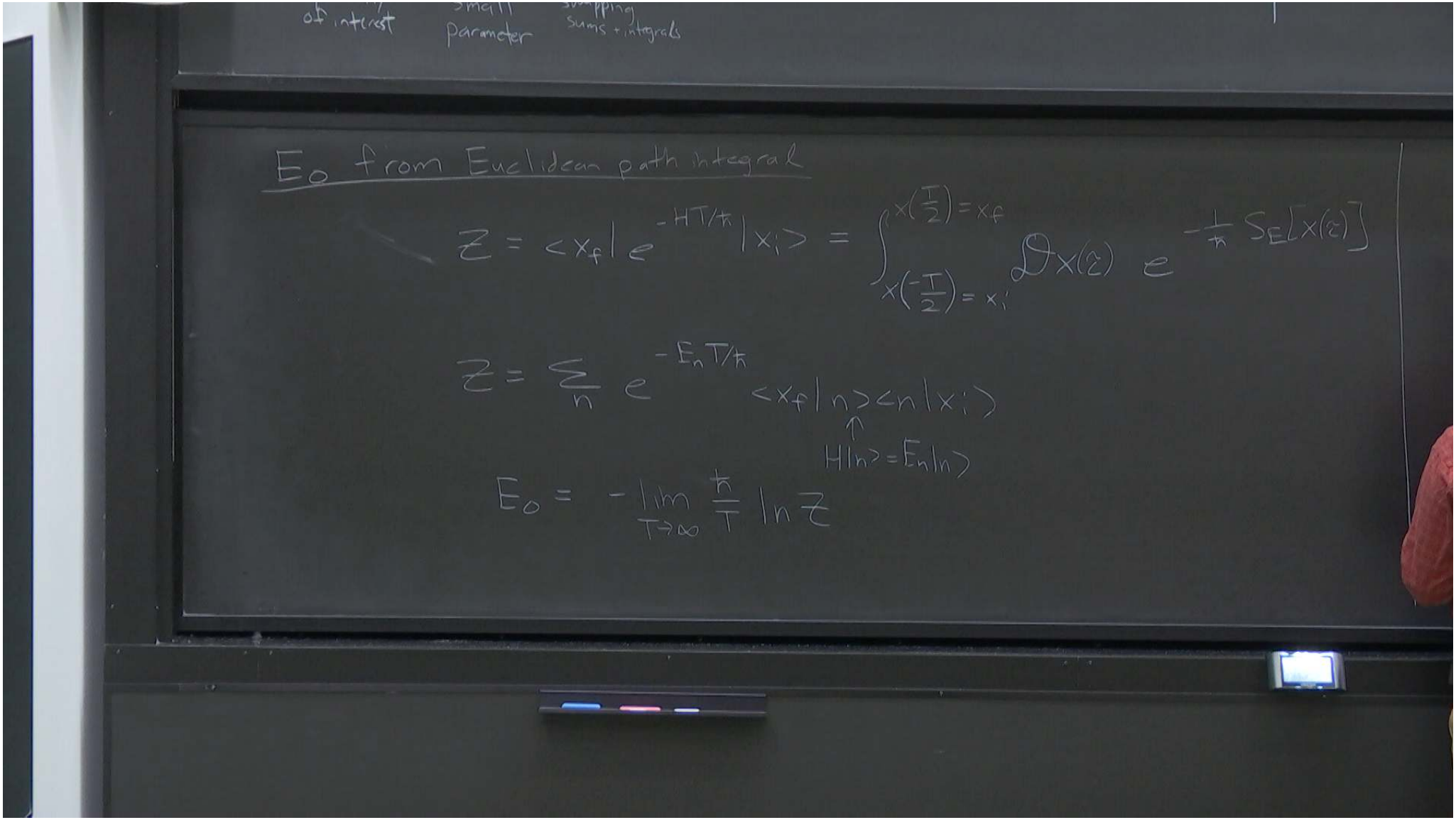
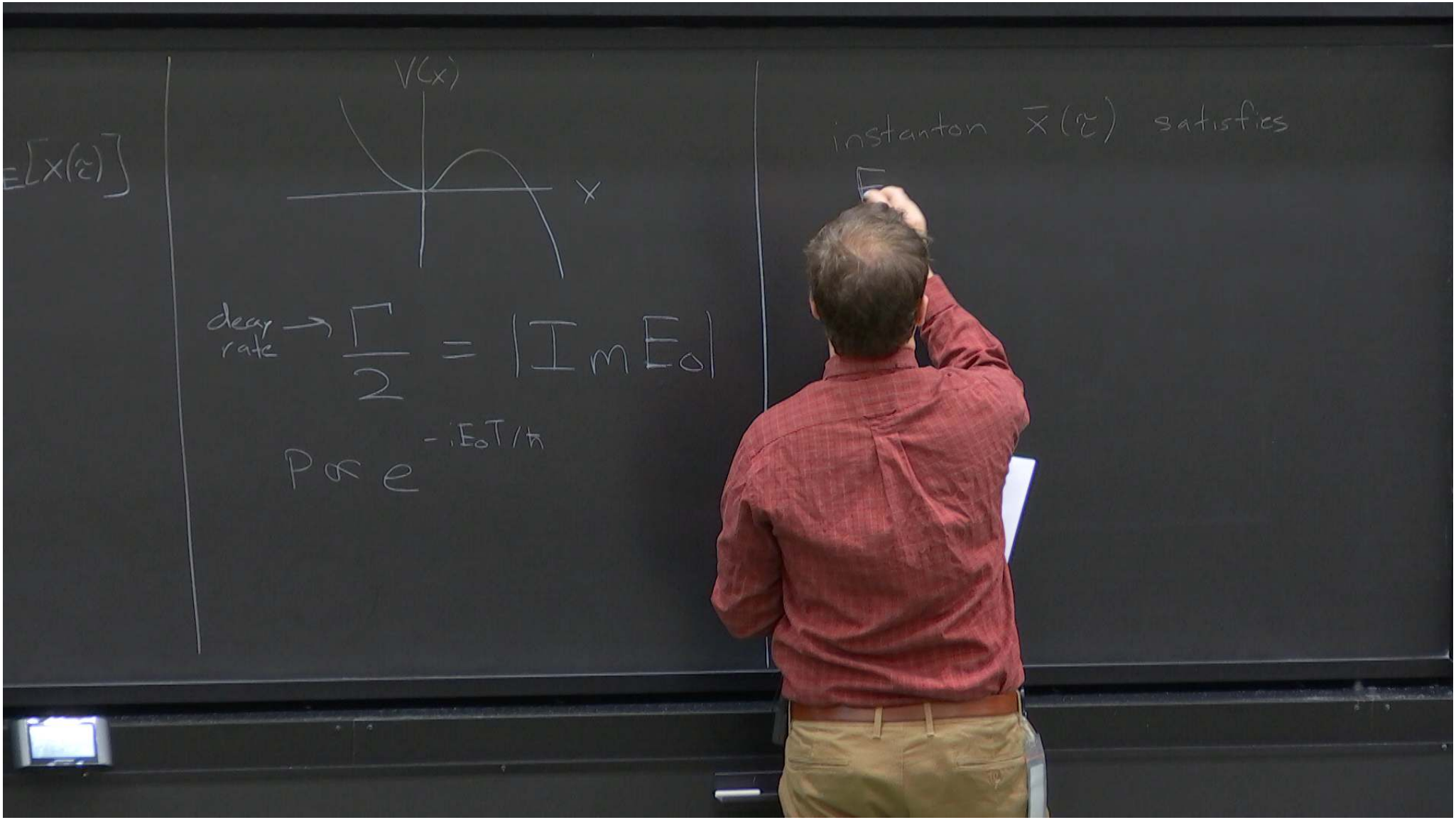
- θ vacua (ground state of YM theory)


of interest
small parameter
sums + integrals
E_0 from Euclidean path integral
$Z = \langle x_f | e^{-HT/\hbar} | x_i \rangle = \int_{x(-\frac{T}{2})=x_i}^{x(\frac{T}{2})=x_f} \mathcal{D}x(\tau)\, e^{-\frac{1}{\hbar} S_E[x(\tau)]}$
$Z = \sum_n e^{-E_n T/\hbar} \langle x_f | n \rangle \langle n | x_i \rangle$
$H|n\rangle = E_n|n\rangle$
$E_0 = -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$


V(x)
x
instanton $\bar{x}(\tau)$ satisfies
decay rate → $\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$
$\propto e^{-iE_0 T/\hbar}$

E_0 from Euclidean path integral

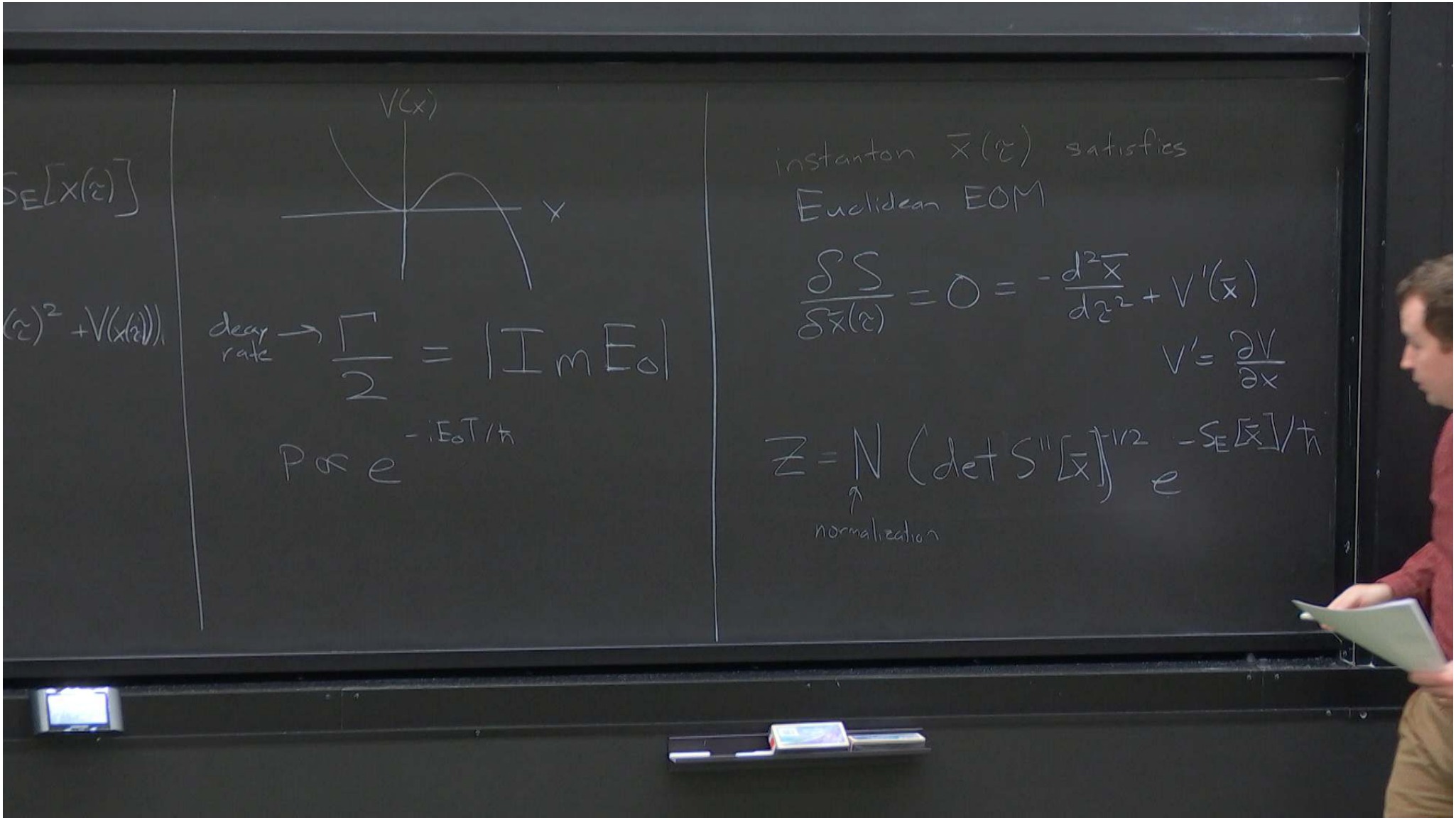
$$Z = \langle x_f | e^{-HT/\hbar} | x_i \rangle = \int_{x(-\frac{T}{2})=x_i}^{x(\frac{T}{2})=x_f} \mathcal{D}x(\tau)\, e^{-\frac{1}{\hbar} S_E[x(\tau)]}$$

$$S_E = \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\dot{x}(\tau)^2 + V(x(\tau)) \right) d\tau$$

$$Z = \sum_n e^{-E_n T/\hbar} \langle x_f | n \rangle \langle n | x_i \rangle$$

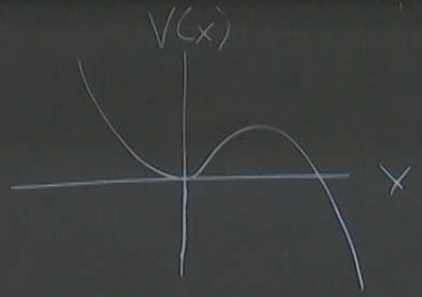
$$H|n\rangle = E_n |n\rangle$$

$$E_0 = -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$$


$S_E[x(\tau)]$
$(\dot{x}(\tau))^2 + V(x(\tau))$
V(x)
x
decay rate → $\frac{\Gamma}{2} = |\mathrm{Im}\, E_0|$
$P \propto e^{-iE_0 T/\hbar}$
instanton $\bar{x}(\tau)$ satisfies Euclidean EOM
$\frac{\delta S}{\delta \bar{x}(\tau)} = 0 = -\frac{d^2\bar{x}}{d\tau^2} + V'(\bar{x})$
$V' = \frac{\partial V}{\partial x}$
$Z = N \left(\det S''[\bar{x}]\right)^{-1/2} e^{-S_E[\bar{x}]/\hbar}$
normalization

$S_E[x(\tau)]$

$(\dot{x}(\tau))^2 + V(x(\tau))\Big]$



decay rate $\rightarrow \dfrac{\Gamma}{2} = |\mathrm{Im}\, E_0|$

$P \propto e^{-iE_0 T/\hbar}$

instanton $\bar{x}(\tau)$ satisfies Euclidean EOM

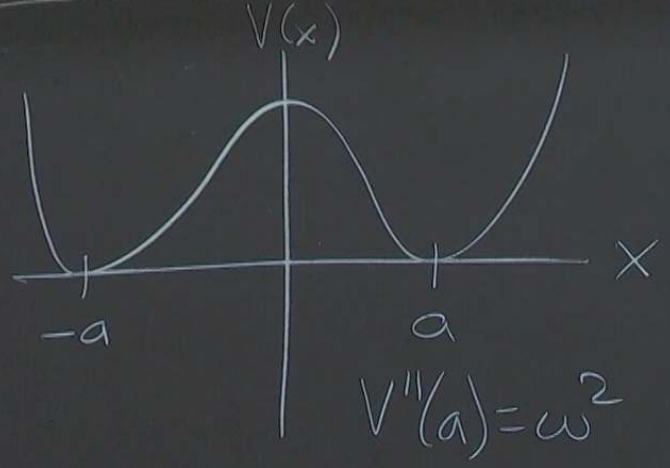
$$\frac{\delta S}{\delta x(\tau)} = 0 = -\frac{d^2\bar{x}}{d\tau^2} + V'(\bar{x})$$

$$V' = \frac{\partial V}{\partial x}$$

$$Z = \sum_{\bar{x}} N \left(\det S''[\bar{x}]\right)^{-1/2} e^{-S_E[\bar{x}]/\hbar}$$

N: normalization

Double Well Potential



$V(x) = V(-x)$

$V''(a) = \omega^2$

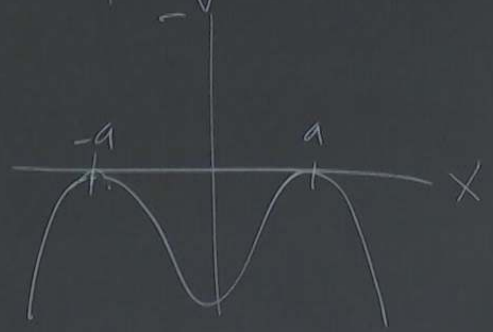
ential

$V(x) = V(-x)$

$\langle a | e^{-HT/\hbar} | -a \rangle$

$$-\frac{d^2\bar{x}}{d\tau^2} + V'(\bar{x}) = 0 \qquad \text{with } \bar{x}\left(\pm\frac{T}{2}\right) = \pm a$$

EOM for particle moving in potential $-V$



$$E = \frac{1}{2}\left(\frac{d\bar{x}}{d\tau}\right)^2 - V(\bar{x})$$

↑ conserved

there is zero energy solution

$\frac{d\bar{x}}{d\tau} =$

$\left(\frac{d\bar{x}}{d\tau}\right)^2 - V(\bar{x})$

ro energy solution

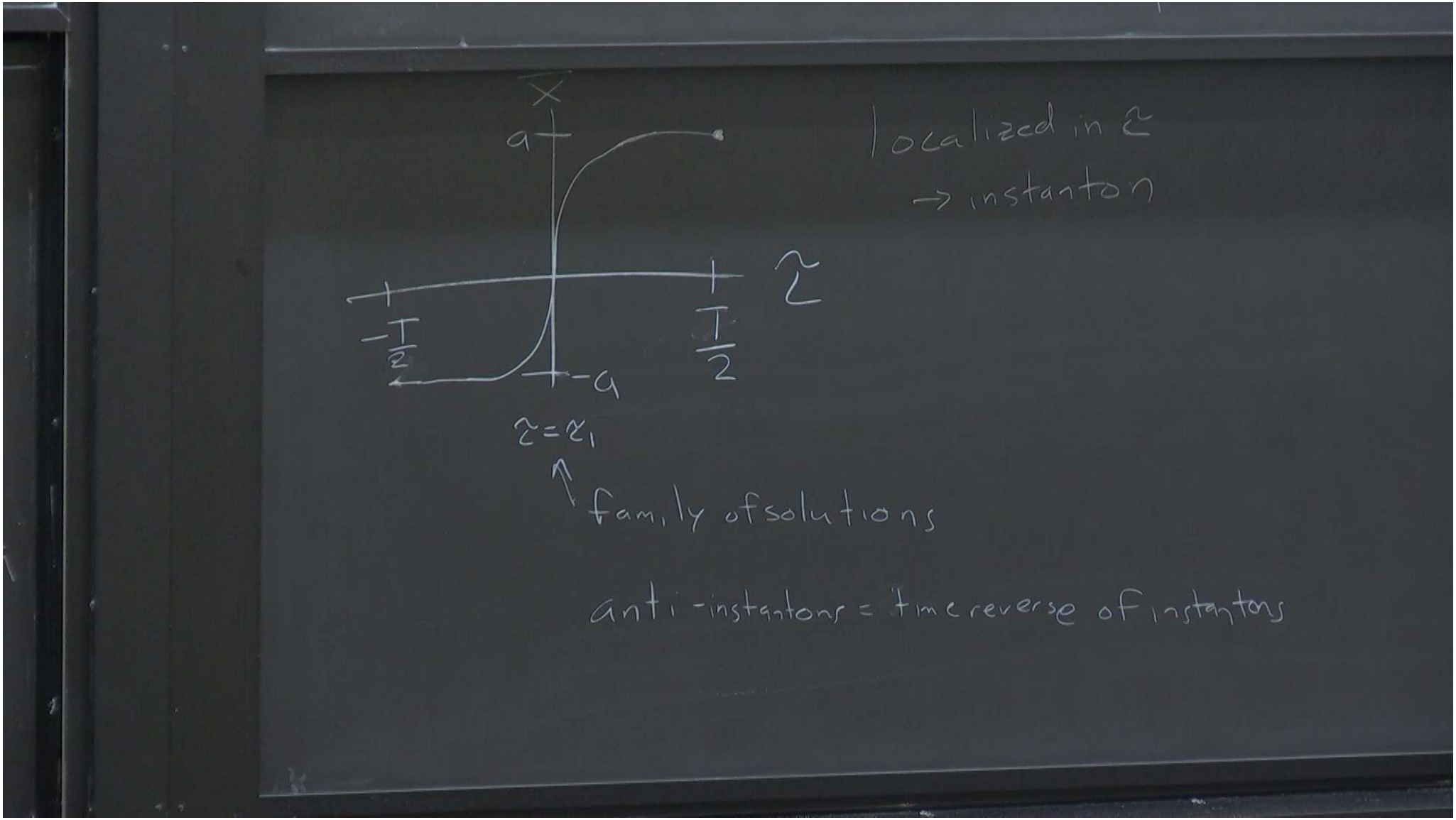
$\frac{d\bar{x}}{d\tau} = \sqrt{2V(\bar{x})}$

$\simeq_{\bar{x}\simeq a} \left(2\cancel{V(a)}^{0} + (a-\bar{x})\cancel{V'(a)}^{0} + \frac{1}{2}(a-\bar{x})^2\omega^2\right)^{1/2}$

τ large

$\simeq (a-\bar{x})\omega$

$\boxed{a-\bar{x} \simeq e^{-\omega\tau}}$


X
a
$\tilde{z}$
$-\frac{I}{2}$
$\frac{I}{2}$
$-a$
$\tilde{z}=\tilde{z}_1$
localized in $\tilde{z}$
→ instanton
↑ family of solutions
anti-instantons = time reverse of instantons