

Title: PSI 2019/2020 - Standard Model and Beyond - Part 1 - Lecture 8

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Lecture 8 SSB $\rightarrow$ SM

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4} \text{Tr}[G_{\mu\nu}^2] - \frac{1}{4} \text{Tr}[W_{\mu\nu}^2] - \frac{1}{4} F_{\mu\nu}^2 \\
 & - (D_\mu h)^2 + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots
 \end{aligned}$$

$$\begin{aligned}
 L = & -\frac{1}{4} \text{Tr}[G_{\mu\nu}^2] - \frac{1}{4} \text{Tr}[W_{\mu\nu}^2] - \frac{1}{4} B_{\mu\nu}^2 & G_{\mu\nu} \\
 & - (D_\mu h)^2 + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots
 \end{aligned}$$

$$\mathcal{L} = -\frac{1}{4} \text{Tr}[G_{\mu\nu}^2] - \frac{1}{4} \text{Tr}[W_{\mu\nu}^2] - \frac{1}{4} B_{\mu\nu}^2$$

$$G_{\mu\nu} = G_{\mu\nu}^a \lambda^a \quad \text{Tr}[\lambda^a \lambda^b] = \frac{1}{2} \delta^{ab}$$

$$- (D_\mu h)^2 + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

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$$G_{\mu\nu} = G_{\mu\nu}^a \lambda^a \quad \text{Tr}[\lambda^a \lambda^b] = \frac{1}{2} \delta^{ab}$$

$$- (D_\mu h)^2 + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

$$= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2$$

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$$- (D_\mu h)^2 + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

$$= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2$$

$$- (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

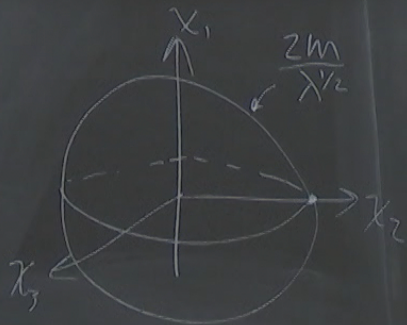
$$= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2 \\ - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

$$(h^\dagger h) = X$$

$$m^2 - \frac{\lambda}{2} X = 0$$

$$(h^\dagger h) = \frac{2m^2}{\lambda}$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} x_1 + i x_2 \\ x_3 + i x_4 \end{pmatrix}$$



$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = \frac{4m^2}{\lambda}$$

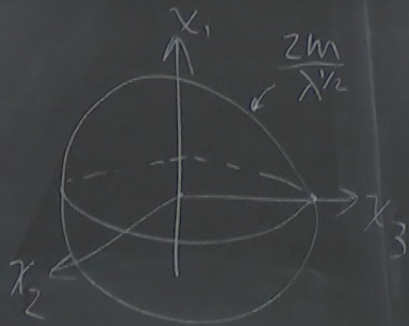
$$= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2 \\ - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

$$(h^\dagger h) = X$$

$$m^2 - \frac{\lambda}{2} X = 0$$

$$(h^\dagger h) = \frac{2m^2}{\lambda}$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \chi_3 + i\chi_4 \end{pmatrix}$$



$$\chi_1^2 + \chi_2^2 + \chi_3^2 + \chi_4^2 = \frac{4m^2}{\lambda} = v^2$$

expect 3 Goldstones.

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$V = su(3) \oplus su(2) \oplus u(1)$$

8

3

1

λ^a

T_i

Y

"hypercharge"

1)
1
Y
"hypercharge"

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

	$SU(3)$	$SU(2)$	$U(1)$
h	1	2	$1/2$

g

$$h = e^{ix} h$$

1



$$= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2 - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

	SU(3)	SU(2)	U(1)
h	1	2	$\frac{1}{2}$

$$g h = e^{i\vec{x} \cdot \vec{\tau}} h$$

$$X \langle h \rangle = 0$$

$$\lambda^a \in W$$

$$\text{Tr}[T^i T^j] = \frac{1}{2} \delta^{ij}$$

$$T^i = \frac{1}{2} \sigma^i$$

$$h \rightarrow e^{i\alpha(\frac{1}{2})} h \quad Y = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2 - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

	SU(3)	SU(2)	U(1)
h	1	2	$\frac{1}{2}$

$$g h = e^{i\alpha} h$$

$$X \langle h \rangle = 0$$

$$X^a \in W$$

$$\text{Tr}[T^i T^j] = \frac{1}{2} \delta^{ij}$$

$$T^i = \frac{1}{2} \sigma^i$$

$$h \rightarrow e^{i\alpha(\frac{1}{2})} h \quad Y = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\alpha Y + \beta_i T^i = \frac{1}{2} \begin{pmatrix} \alpha + \beta_3 & \beta_1 - i\beta_2 \\ \beta_1 + i\beta_2 & \alpha - \beta_3 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix} = 0$$

$$= \frac{\sqrt{2}}{2\sqrt{2}} \begin{pmatrix} \beta_1 - i\beta_2 \\ \alpha - \beta_3 \end{pmatrix} \quad \begin{matrix} \beta_1 = \beta_2 = 0 \\ \alpha = \beta_3 \end{matrix}$$

$$Y + T^3 = Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

"electric charge"

$$W: \boxed{\lambda^9 + Q}$$

$$8 + 1 = 9$$

$$V/W = \dim(12 - 9) = \underline{\underline{3}} = \text{Massive vectors in SM}$$

$$h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \quad \text{"unitary gauge"}$$

$$D_\mu h = \left(\partial_\mu + ig_2 W_\mu + ig_1 B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left(\partial_\mu + \frac{ig_2}{2} W_\mu^i \sigma^i + \frac{i}{2} g_1 B_\mu \right) \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\partial_\mu H + \frac{i}{2} \begin{pmatrix} g_1 B_\mu + g_2 W_\mu^3 & g_2 (W_\mu^1 - iW_\mu^2) \\ g_2 (W_\mu^1 + iW_\mu^2) & g_1 B_\mu - g_2 W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right]$$

gauge"

$$N_m + ig_1 B_m \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix} + \frac{i}{2} g_1 B_m \begin{pmatrix} 0 \\ v+H \end{pmatrix} \right)$$

$$\left[\begin{array}{cc} W_m^3 & g_2(W_m^1 - iW_m^2) \\ W_m^2 & g_1 B_m - g_2 W_m^3 \end{array} \right] \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$\rightarrow \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ \partial_\mu H \end{pmatrix} + \frac{i v}{2} \begin{pmatrix} g_2(W_m^1 - iW_m^2) \\ g_1 B_m - g_2 W_m^3 \end{pmatrix} + \dots \right]$$

$$-(D_\mu h)^\dagger (D^\mu h) = -\frac{1}{2} \left[\frac{v^2}{4} g_2^2 ((W_m^1)^2 + (W_m^2)^2) + (\partial_\mu H)^2 + \frac{v^2}{4} (g_1 B_m - g_2 W_m^3)^2 \right]$$

SB $\rightarrow$ SM

$$\begin{aligned}
 \mathcal{L} &= -\frac{1}{2} \text{Tr}[G_{\mu\nu}^2] - \frac{1}{2} \text{Tr}[W_{\mu\nu}^2] - \frac{1}{4} B_{\mu\nu}^2 \\
 &\quad - (D_\mu h)^2 + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots \\
 &= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu}^2 \\
 &\quad - (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \frac{\lambda}{4} (h^\dagger h)^2 + \dots \rightarrow \frac{m^4}{\lambda} - m^2 H^2 + \dots
 \end{aligned}$$

$$G_{\mu\nu} = G_{\mu\nu}^a \lambda^a \quad \text{Tr}[\lambda^a \lambda^b] = \frac{1}{2} \delta^{ab}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned}
 & -\frac{1}{4}G_{\mu\nu}^a{}^2 - \frac{1}{4}W_{\mu\nu}^a{}^2 - \frac{1}{4}B_{\mu\nu}^2 - \frac{1}{2}\left[\left(\frac{Vg_2}{2}\right)^2(W_\mu^1{}^2 + W_\mu^2{}^2) + \frac{V^2}{4}(g_1 B_\mu - g_2 W_\mu^3)^2\right] \\
 & -\frac{1}{2}(\partial_\mu H)^2 - m^2 H^2 + \frac{M^4}{\lambda}
 \end{aligned}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$$

$$[W_m^3]^2]$$

$$-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\mu^2 A_\mu^2$$

8 G_μ^a 's are massless (8 "gluons")

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2) = W_\mu^1, W_\mu^2 \text{ are massive}$$

$$m_W = \frac{g_2 V}{2}$$

$$= \frac{1}{\sqrt{2}} \left(\partial_\mu + \frac{1}{2} W_\mu^3 + 2 g_1 B_\mu \right) (v + H)$$

$$= \frac{1}{\sqrt{2}} \left[\partial_\mu H + \frac{i}{2} \begin{pmatrix} g_1 B_\mu + g_2 W_\mu^3 & g_2 (W_\mu^1 - i W_\mu^2) \\ g_2 (W_\mu^1 + i W_\mu^2) & g_1 B_\mu - g_2 W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \right]$$

$$-\frac{1}{4} W_{\mu\nu}^3{}^2 - \frac{1}{4} B_{\mu\nu}^2 - \frac{1}{2} (W_\mu^3 B_\nu - B_\mu^3 W_\nu) \hat{M}^2 \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$-\frac{1}{4} Z_{\mu\nu}^2 - \frac{1}{4} A_{\mu\nu}^2 - \frac{1}{2} (Z_\mu A_\nu - A_\mu Z_\nu) \hat{M}^2 \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \Theta \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$A_{\mu\nu} = F_{\mu\nu}$$

$$\hat{M}^2 = \frac{V^2}{4} \begin{pmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix} = \Theta^T \hat{M}^2 \Theta$$

$$\sin \Theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$\cos \Theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$$

$$\hat{M}^2 = \begin{pmatrix} \frac{V^2}{4} (g_1^2 + g_2^2) & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Theta = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix}$$

$$m_Z = \frac{V}{2} \sqrt{g_1^2 + g_2^2}$$

$$m_A = 0$$

$$\frac{m_W}{m_Z} = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{82}{91}$$