Title: PSI 2019/2020 - Standard Model and Beyond - Part 1 - Lecture 7

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Collection: PSI 2019/2020 - Standard Model and Beyond - Part 1

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Lecture 7

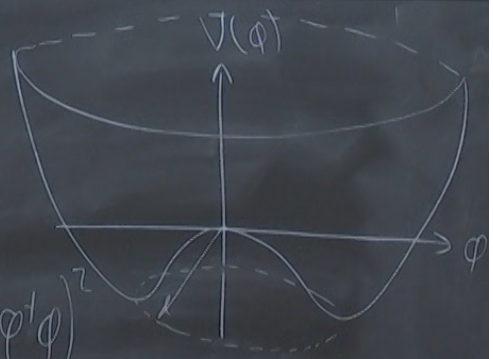
SSB → Global (Goldstone's Thm)
→ Gauge (Higgs Mechanism)

Global U(1)

$\varphi \to e^{i\alpha}\varphi$

$$S = \int d^4x \left[-(\partial_\mu \varphi)^\dagger (\partial^\mu \varphi) - V(\varphi) \right]$$

$$V(\varphi) = -m^2(\varphi^\dagger\varphi) + \frac{\lambda}{4}(\varphi^\dagger\varphi)^2$$



$\delta S = 0$

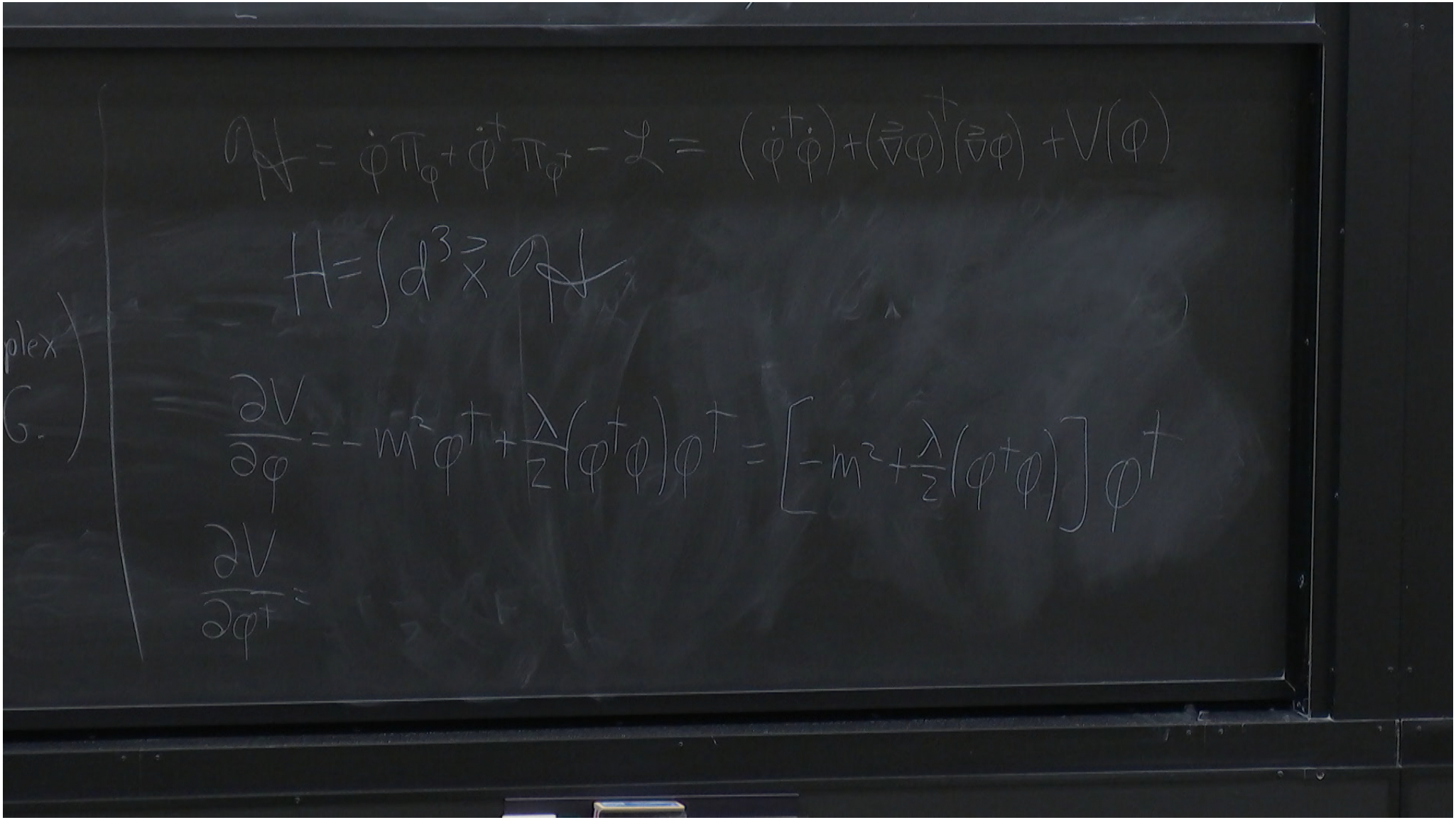
$$\delta_{\varphi^\dagger} S = \int d^4x \left[-\partial_\mu \delta\varphi^\dagger (\partial^\mu \varphi) - \frac{\partial V}{\partial \varphi^\dagger} \delta\varphi^\dagger \right] =$$

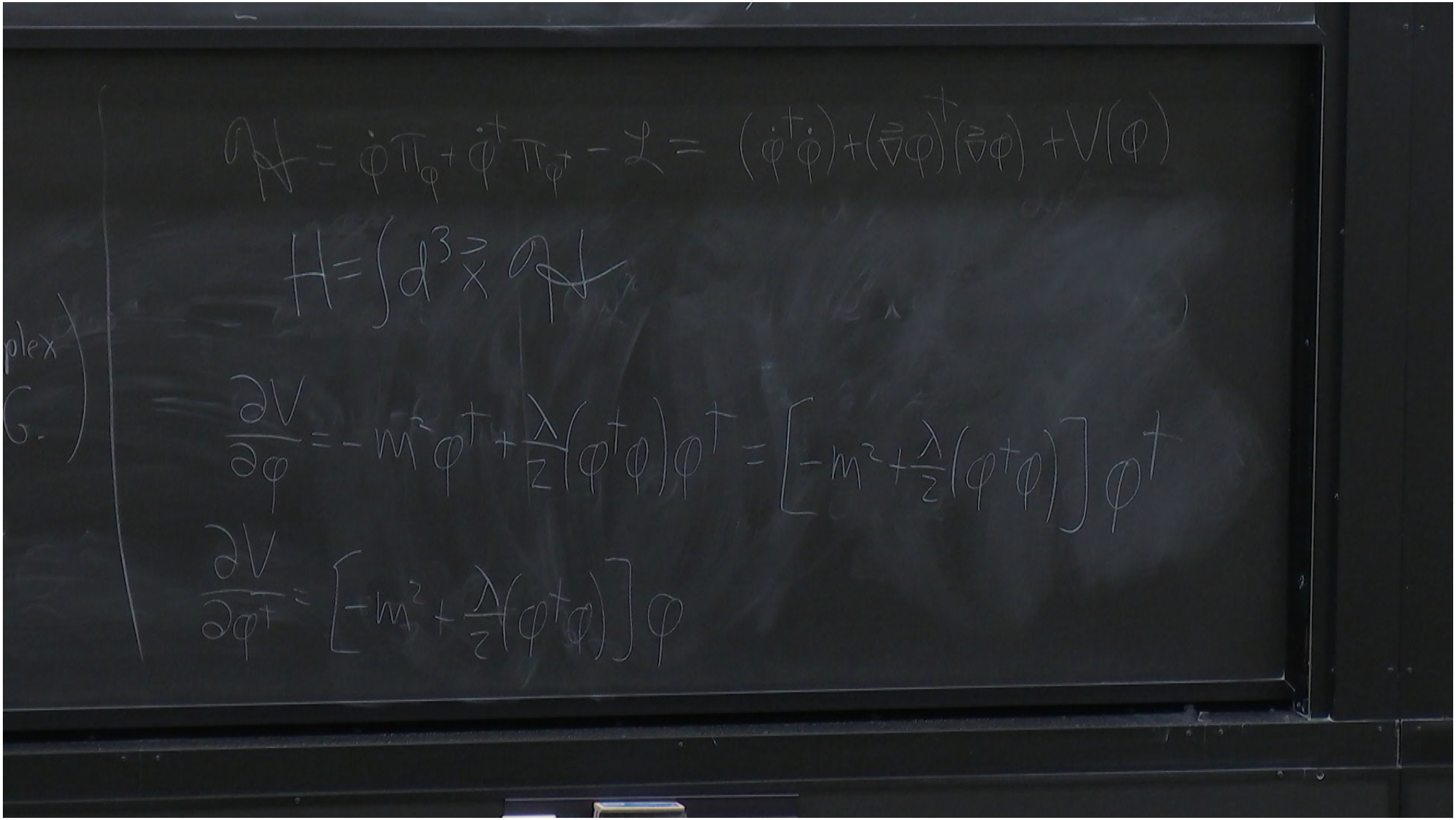
$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$$

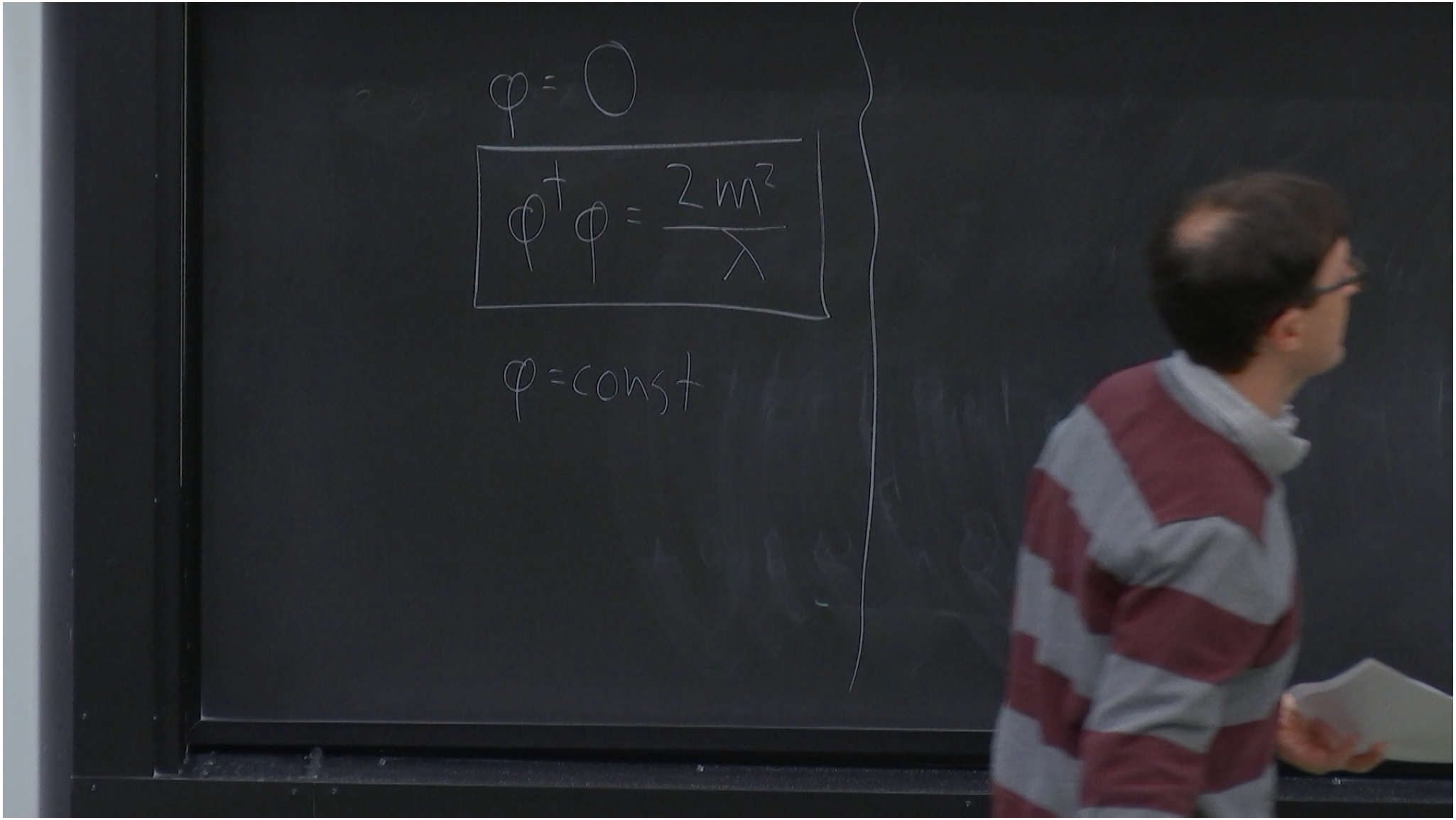
$$\bar{\varphi} = \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$$

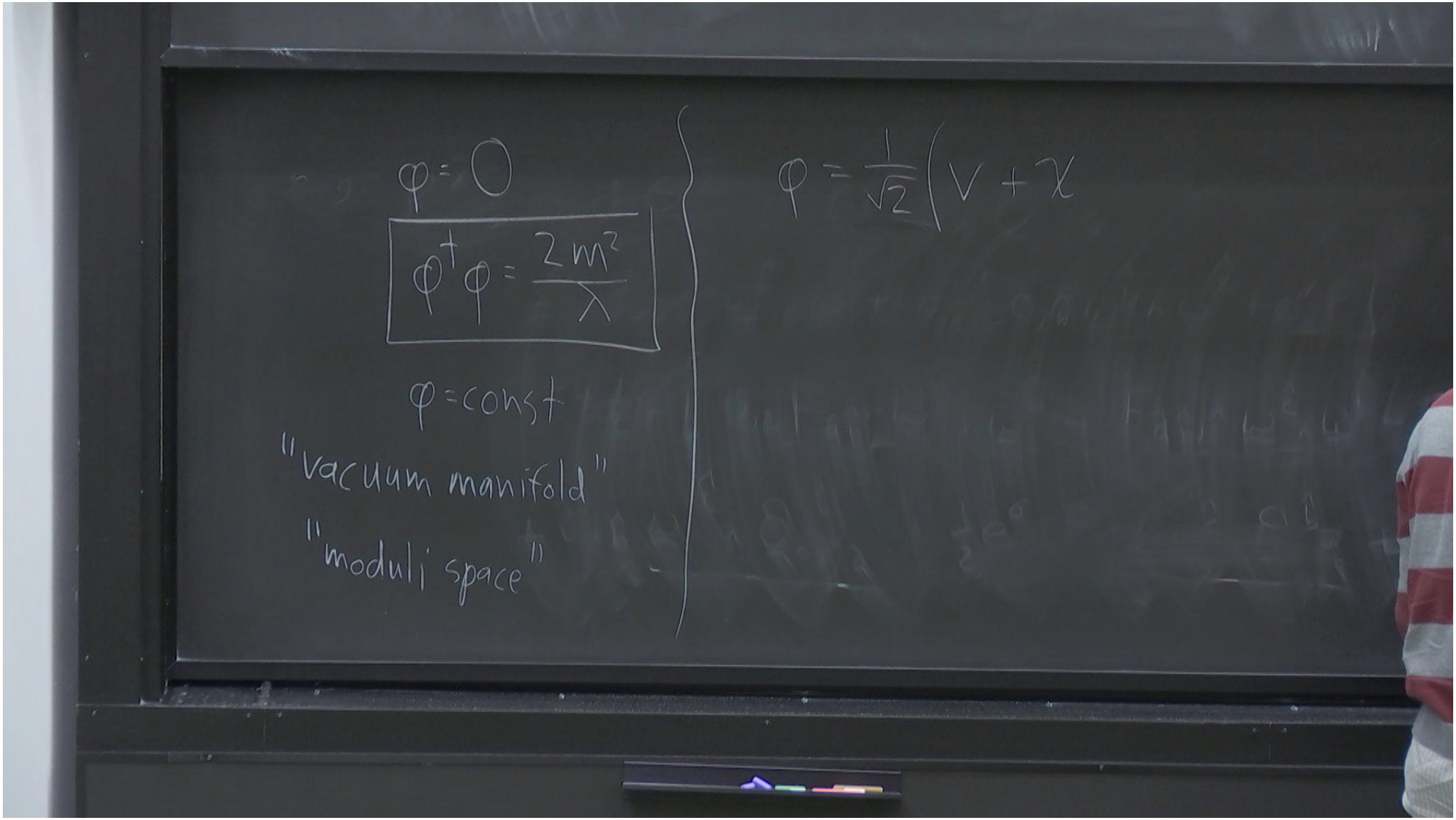
$$= \int d^4x \left[\underline{\partial_\mu \partial^\mu \varphi - \frac{\partial V}{\partial \varphi^\dagger}} \right] \delta\varphi^\dagger \qquad \partial_\mu \partial^\mu \varphi = \frac{\partial V}{\partial \varphi^\dagger} \quad \left(\begin{matrix} \text{complex} \\ \text{K.G.} \end{matrix} \right)$$

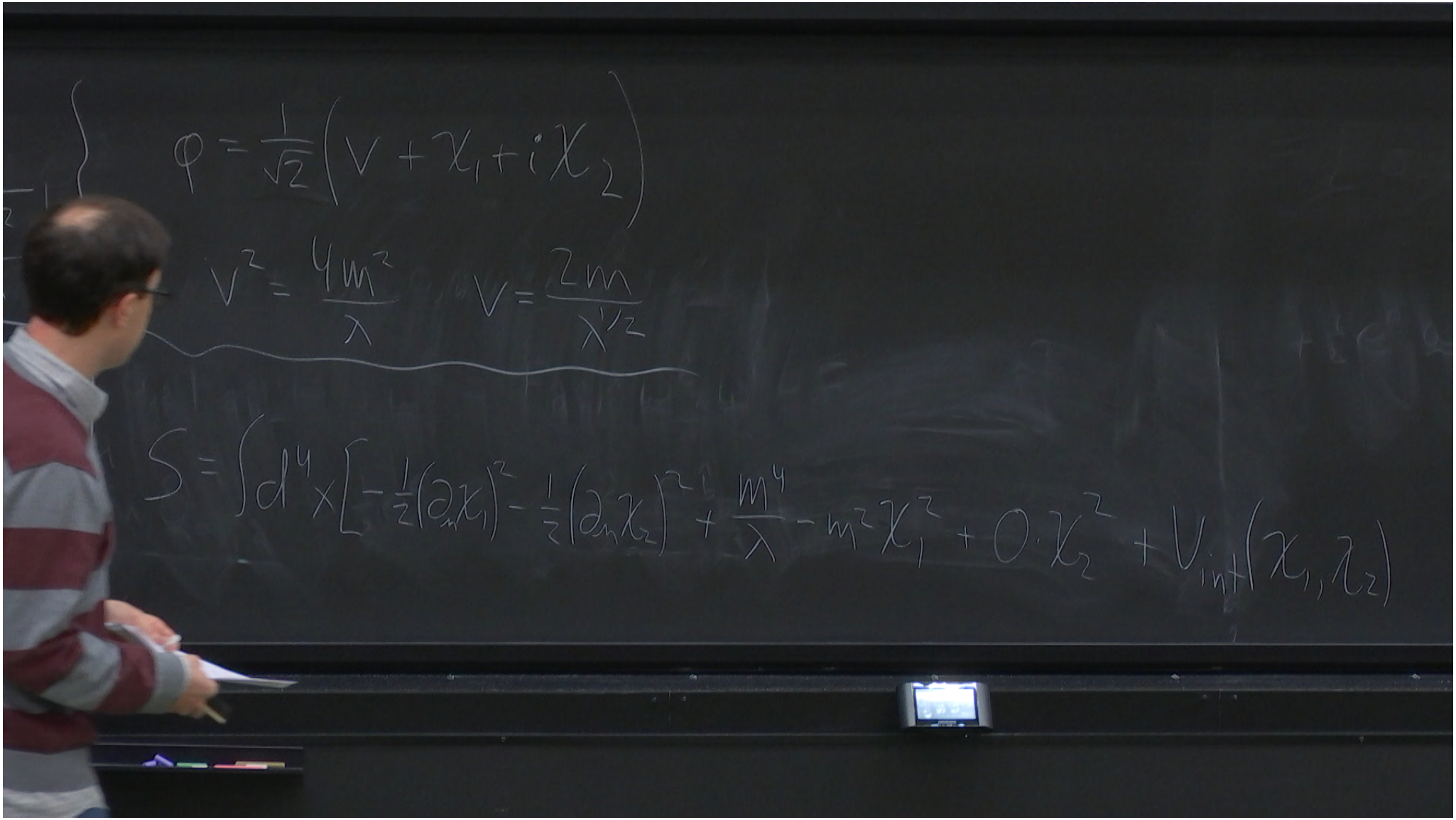
$$\Pi_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \dot{\varphi}^\dagger \qquad \Pi_{\varphi^\dagger} = \dot{\varphi}$$

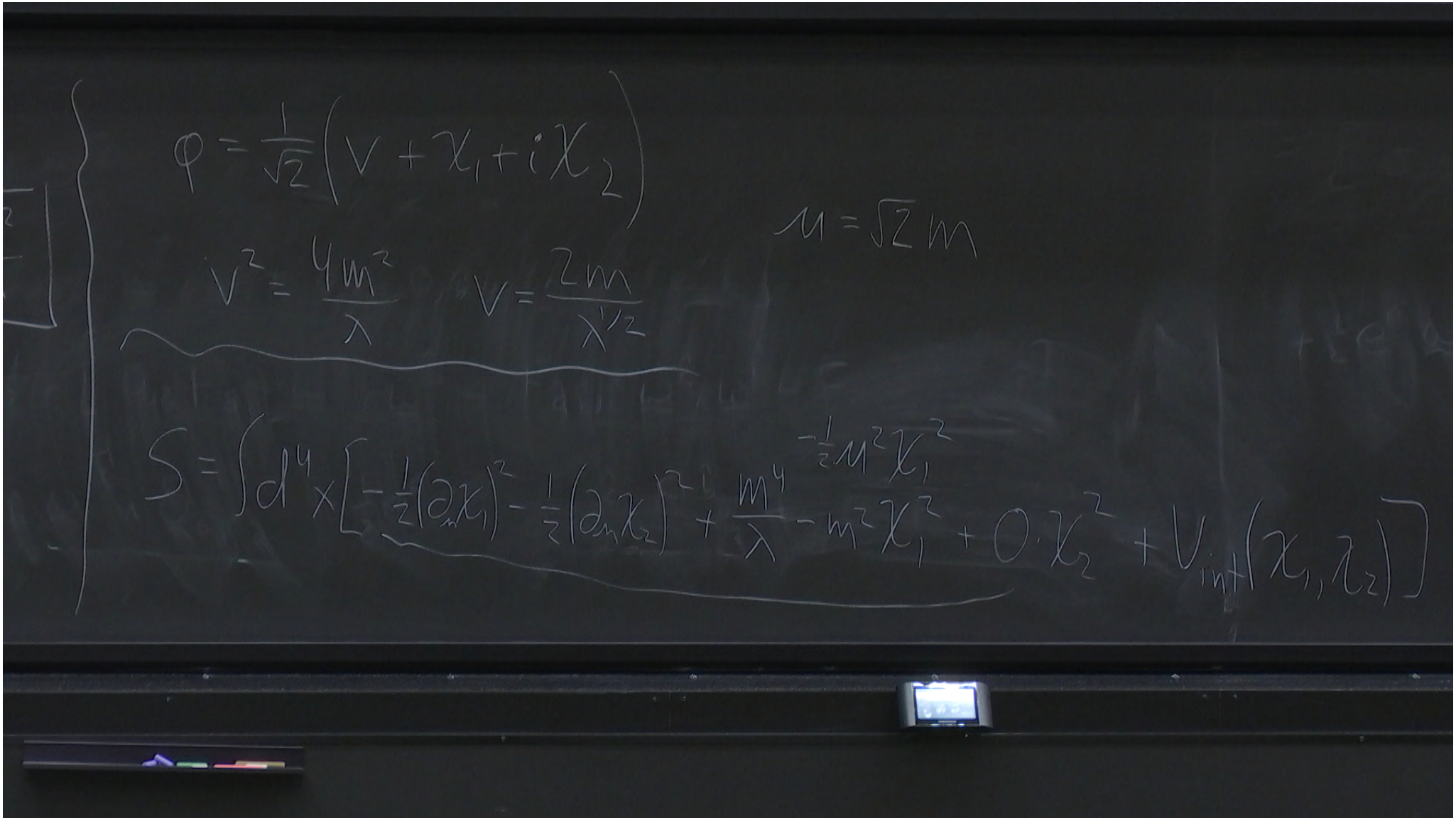

$\mathcal{H} = \dot{\phi}\pi_\phi + \dot{\phi}^\dagger \pi_{\phi^\dagger} - \mathcal{L} = (\dot{\phi}^\dagger\dot{\phi}) + (\vec{\nabla}\phi)^\dagger(\vec{\nabla}\phi) + V(\phi)$
$H = \int d^3\vec{x}\,\mathcal{H}$
$\frac{\partial V}{\partial \phi} = -m^2\phi^\dagger + \frac{\lambda}{2}(\phi^\dagger\phi)\phi^\dagger = \left[-m^2 + \frac{\lambda}{2}(\phi^\dagger\phi)\right]\phi^\dagger$
$\frac{\partial V}{\partial \phi^\dagger} =$

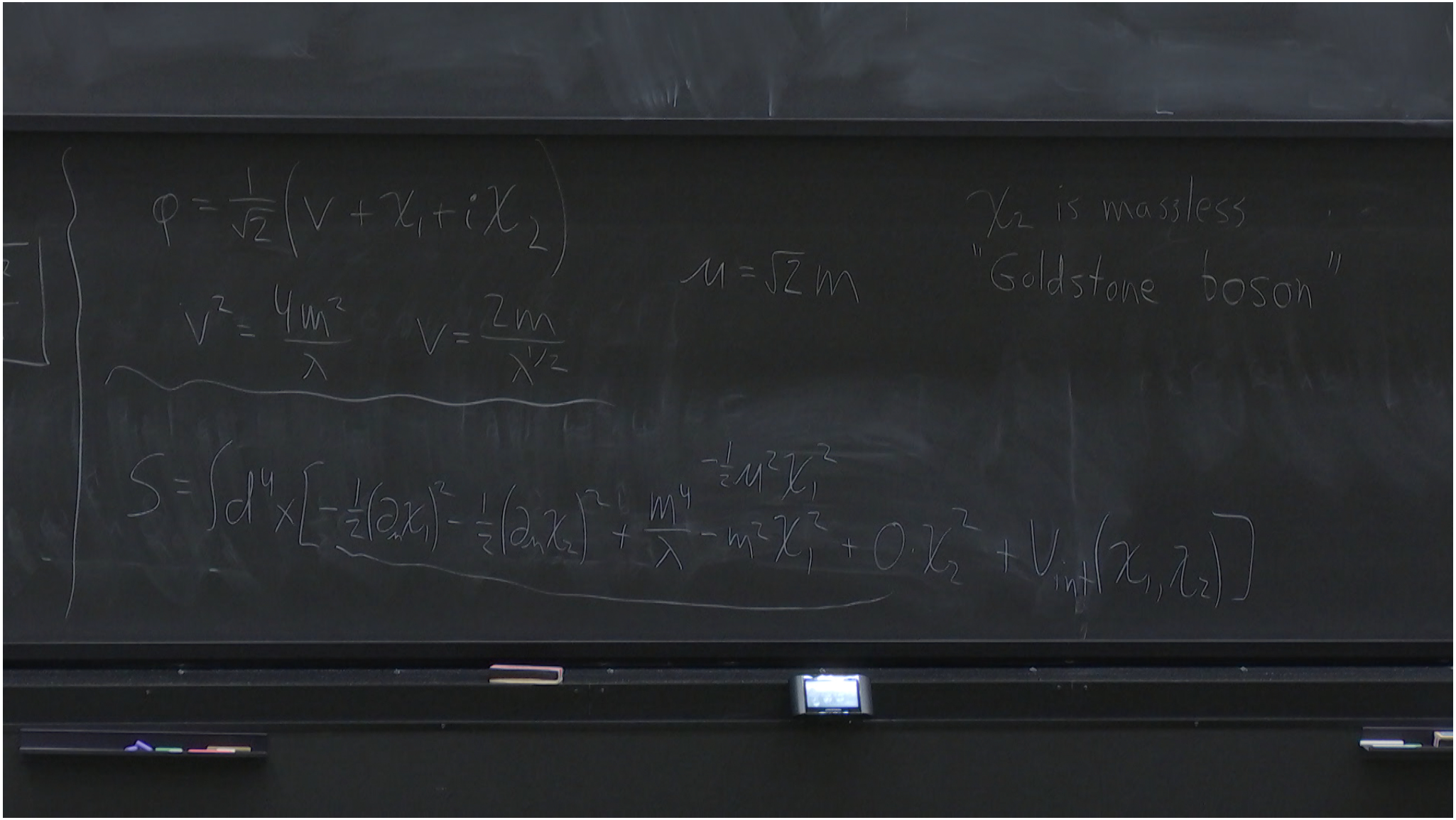

$\mathcal{H} = \dot{\varphi}\pi_\varphi + \dot{\varphi}^\dagger \pi_{\varphi^\dagger} - \mathcal{L} = (\dot{\varphi}^\dagger\dot{\varphi}) + (\vec{\nabla}\varphi)^\dagger(\vec{\nabla}\varphi) + V(\varphi)$
$H = \int d^3\vec{x}\, \mathcal{H}$
$\frac{\partial V}{\partial \varphi} = -m^2\varphi^\dagger + \frac{\lambda}{2}(\varphi^\dagger\varphi)\varphi^\dagger = \left[-m^2 + \frac{\lambda}{2}(\varphi^\dagger\varphi)\right]\varphi^\dagger$
$\frac{\partial V}{\partial \varphi^\dagger} = \left[-m^2 + \frac{\lambda}{2}(\varphi^\dagger\varphi)\right]\varphi$

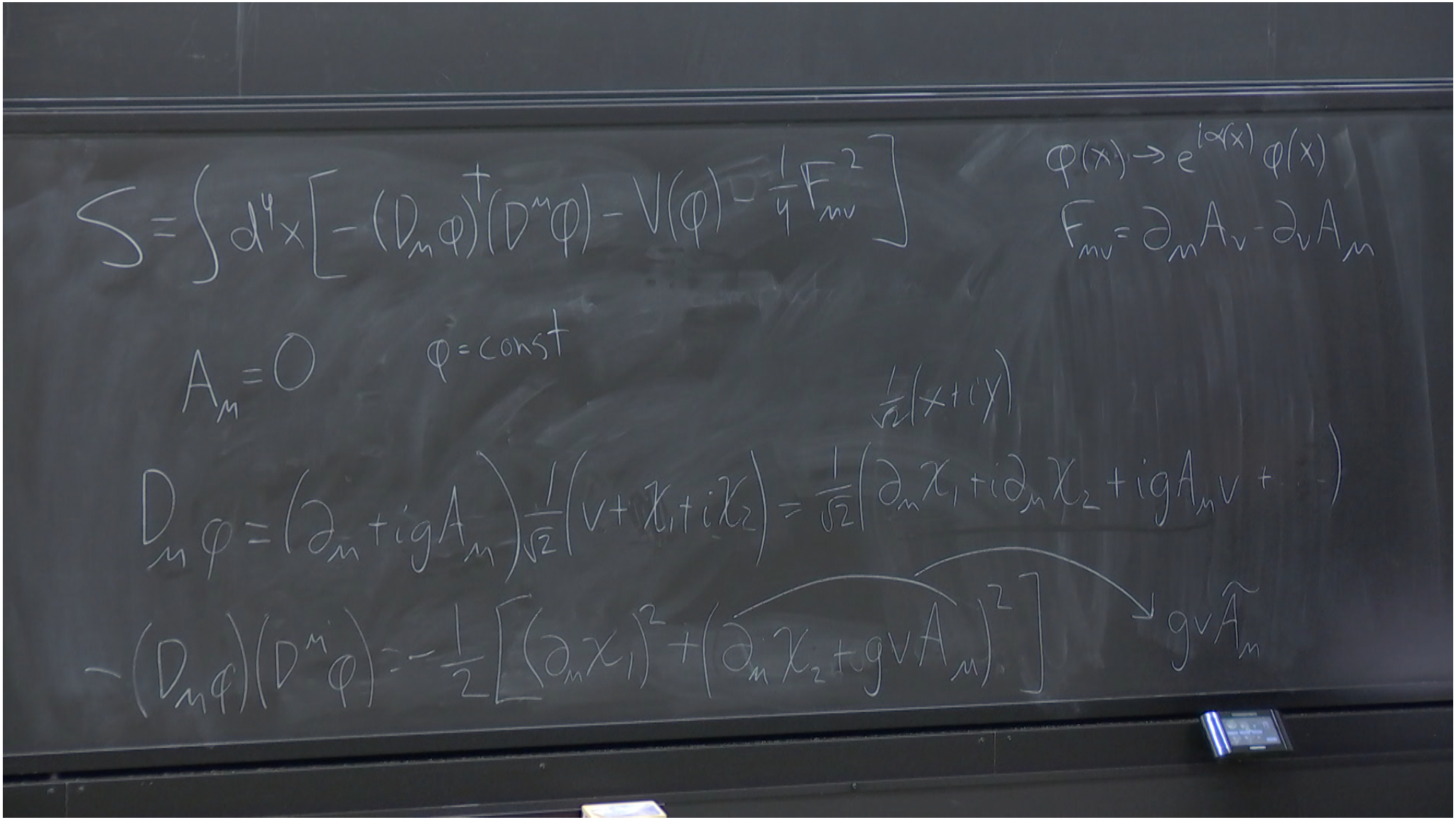

φ = 0
φ† φ = 2m² / λ
φ = const

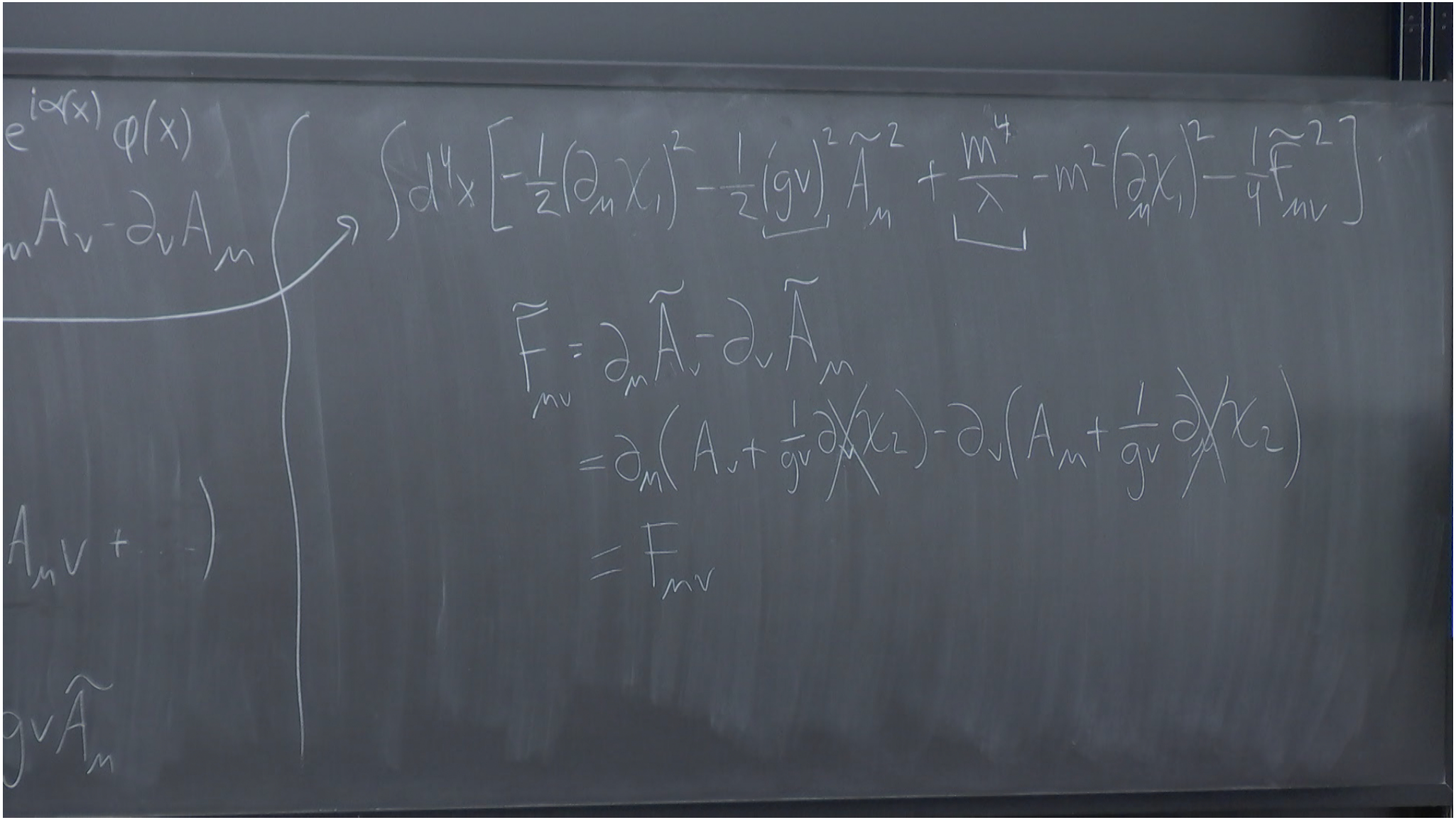

$\varphi = 0$
$\varphi^\dagger \varphi = \frac{2m^2}{\lambda}$
$\varphi = \text{const}$
"vacuum manifold"
"moduli space"
$\varphi = \frac{1}{\sqrt{2}}(v + \chi$


$\varphi = \frac{1}{\sqrt{2}}(v + \chi_1 + i\chi_2)$
$v^2 = \frac{4m^2}{\lambda} \quad v = \frac{2m}{\lambda^{1/2}}$
$S = \int d^4x \left[-\frac{1}{2}(\partial_\mu \chi_1)^2 - \frac{1}{2}(\partial_\mu \chi_2)^2 + \frac{m^4}{\lambda} - m^2 \chi_1^2 + 0 \cdot \chi_2^2 + V_{int}(\chi_1, \chi_2) \right]$


$\varphi = \frac{1}{\sqrt{2}}(v + \chi_1 + i\chi_2)$
$v^2 = \frac{4m^2}{\lambda} \quad v = \frac{2m}{\lambda^{1/2}}$
$u = \sqrt{2}m$
$S = \int d^4x \left[-\frac{1}{2}(\partial_\mu \chi_1)^2 - \frac{1}{2}(\partial_\mu \chi_2)^2 + \frac{m^4}{\lambda} - \frac{1}{2}u^2\chi_1^2 - m^2\chi_1^2 + 0\cdot\chi_2^2 + V_{int}(\chi_1, \chi_2) \right]$


$\varphi = \frac{1}{\sqrt{2}}(v + \chi_1 + i\chi_2)$
χ_2 is massless
"Goldstone boson"
$v^2 = \frac{4m^2}{\lambda}$ $v = \frac{2m}{\lambda^{1/2}}$
$\mu = \sqrt{2}m$
$S = \int d^4x \left[-\frac{1}{2}(\partial\chi_1)^2 - \frac{1}{2}(\partial\chi_2)^2 + \frac{m^4}{\lambda} - m^2\chi_1^2 + 0\cdot\chi_2^2 + V_{int}(\chi_1,\chi_2) \right]$
$-\frac{1}{2}\mu^2\chi_1^2$


$S = \int d^4x \left[-(D_\mu \varphi)^\dagger (D^\mu \varphi) - V(\varphi) - \frac{1}{4} F_{\mu\nu}^2 \right]$
$\varphi(x) \to e^{i\alpha(x)} \varphi(x)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$A_\mu = 0$
$\varphi = \text{const}$
$\frac{1}{\sqrt{2}}(x + iy)$
$D_\mu \varphi = (\partial_\mu + igA_\mu) \frac{1}{\sqrt{2}} (v + \chi_1 + i\chi_2) = \frac{1}{\sqrt{2}} (\partial_\mu \chi_1 + i\partial_\mu \chi_2 + igA_\mu v + \dots)$
$-(D_\mu \varphi)(D^\mu \varphi) \supset -\frac{1}{2} \left[(\partial_\mu \chi_1)^2 + (\partial_\mu \chi_2 + gvA_\mu)^2 \right]$
$gv\widetilde{A}_\mu$

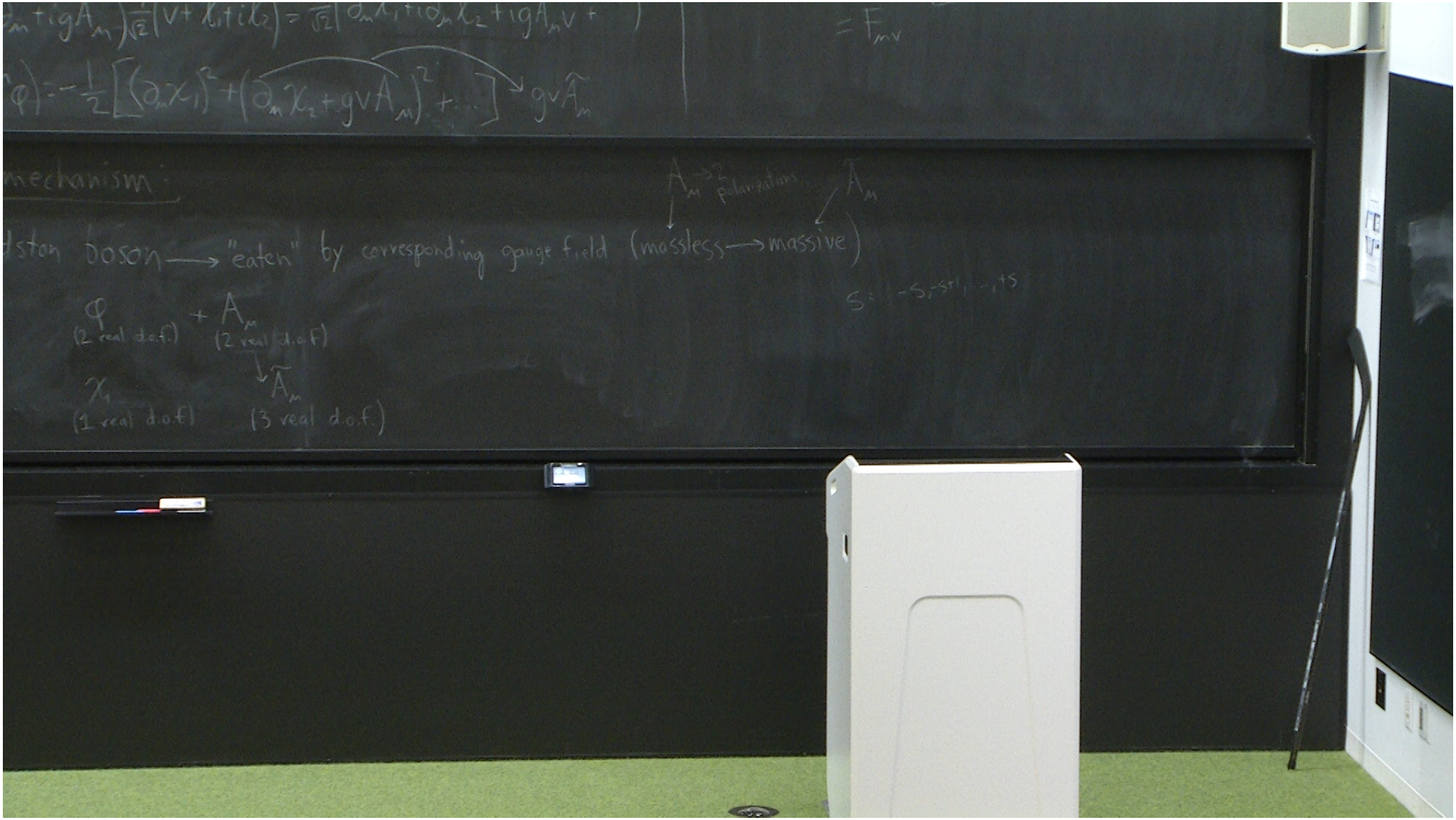

$e^{i\alpha(x)}\varphi(x)$
$\partial_\mu A_\nu - \partial_\nu A_\mu$
$A_\mu v + \ldots)$
$gv\tilde{A}_\mu$
$\int d^4x \left[-\frac{1}{2}(\partial_\mu \chi_1)^2 - \frac{1}{2}(gv)^2 \tilde{A}_\mu^2 + \frac{m^4}{\lambda} - m^2(\partial_\mu \chi_1)^2 - \frac{1}{4}\tilde{F}_{\mu\nu}^2 \right]$
$\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$
$= \partial_\mu \left(A_\nu + \frac{1}{gv}\partial_\nu \chi_2 \right) - \partial_\nu \left(A_\mu + \frac{1}{gv}\partial_\mu \chi_2 \right)$
$= F_{\mu\nu}$

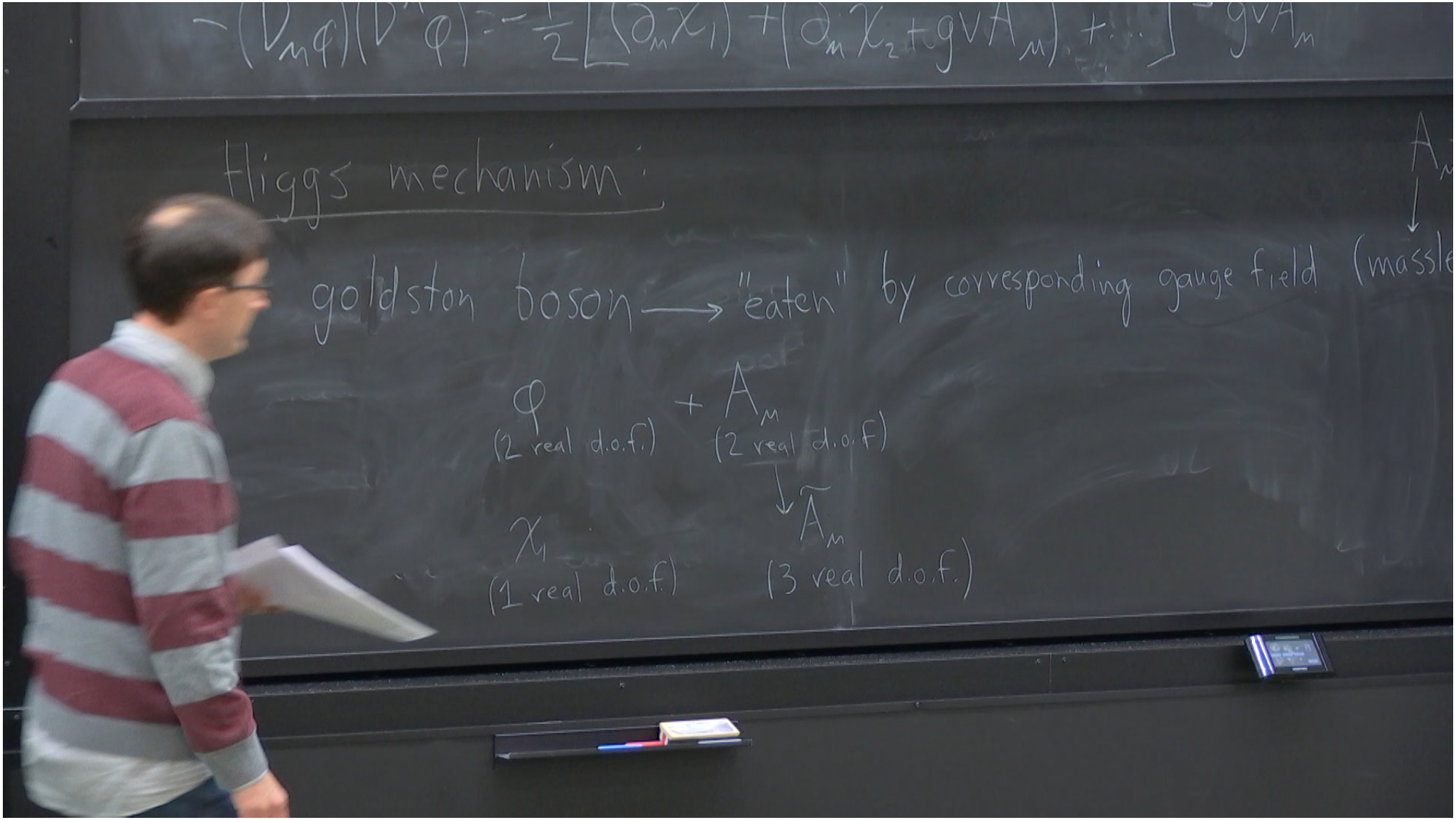
$-(D_\mu\varphi)(D^\mu\varphi)^* = -\frac{1}{2}\left[(\partial_\mu\chi_1)^2 + (\partial_\mu\chi_2 + gvA_\mu)^2 + \dots\right] \to gv\widehat{A}_\mu$

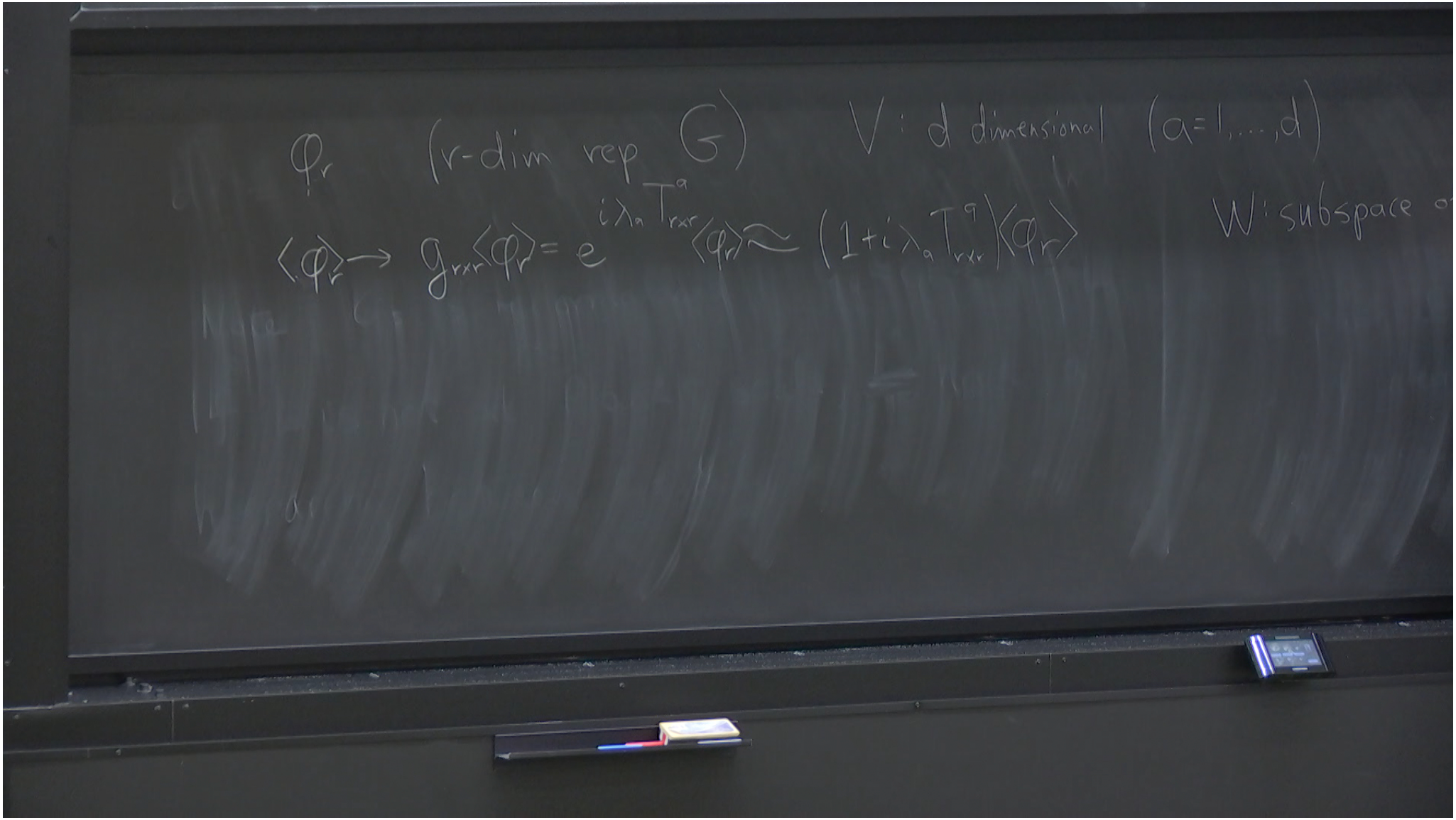
Higgs mechanism:

$A_\mu \to$ 2 polarizations

goldston boson $\longrightarrow$ "eaten" by corresponding gauge field (massless $\longrightarrow$ massive)

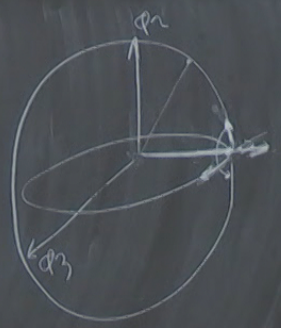

$= F_{\mu\nu}$
mechanism
A_μ → 3 polarizations
A_μ
ston boson → "eaten" by corresponding gauge field (massless → massive)
φ (2 real d.o.f.) + A_μ (2 real d.o.f.)
$\tilde{A}_\mu$
χ_1 (1 real d.o.f.)
(3 real d.o.f.)


Higgs mechanism:
goldston boson ⟶ "eaten" by corresponding gauge field (massle
φ
(2 real d.o.f.)
+ A_μ
(2 real d.o.f)
χ_1
(1 real d.o.f)
Ã_μ
(3 real d.o.f.)


φ_r (r-dim rep G) V: d dimensional (a=1,...,d)
$\langle\varphi_r\rangle \to g_{r\times r}\langle\varphi_r\rangle = e^{i\lambda_a T^a_{r\times r}}\langle\varphi_r\rangle \simeq (1+i\lambda_a T^a_{r\times r})|\varphi_r\rangle$
W: subspace

φ_r (r-dim rep G) V: d dimensional $(a = 1, \ldots, d)$

$\langle \varphi_r \rangle \to g_{r\times r} \langle \varphi_r \rangle = e^{i\lambda_a T^a_{r\times r}} \langle \varphi_r \rangle \approx (1 + i\lambda_a T^a_{r\times r}) \langle \varphi_r \rangle$



1 massive ← 1 massless

2 massless scalars ⟷ 2 massive vectors

W: subspace of dim $k \leq d$

$\lambda_a T^a_{r\times r} \langle \varphi \rangle = 0$

V/W = vec space of dim $d-k$