Title: PSI 2019/2020 - Standard Model and Beyond - Part 1 - Lecture 3

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Collection: PSI 2019/2020 - Standard Model and Beyond - Part 1

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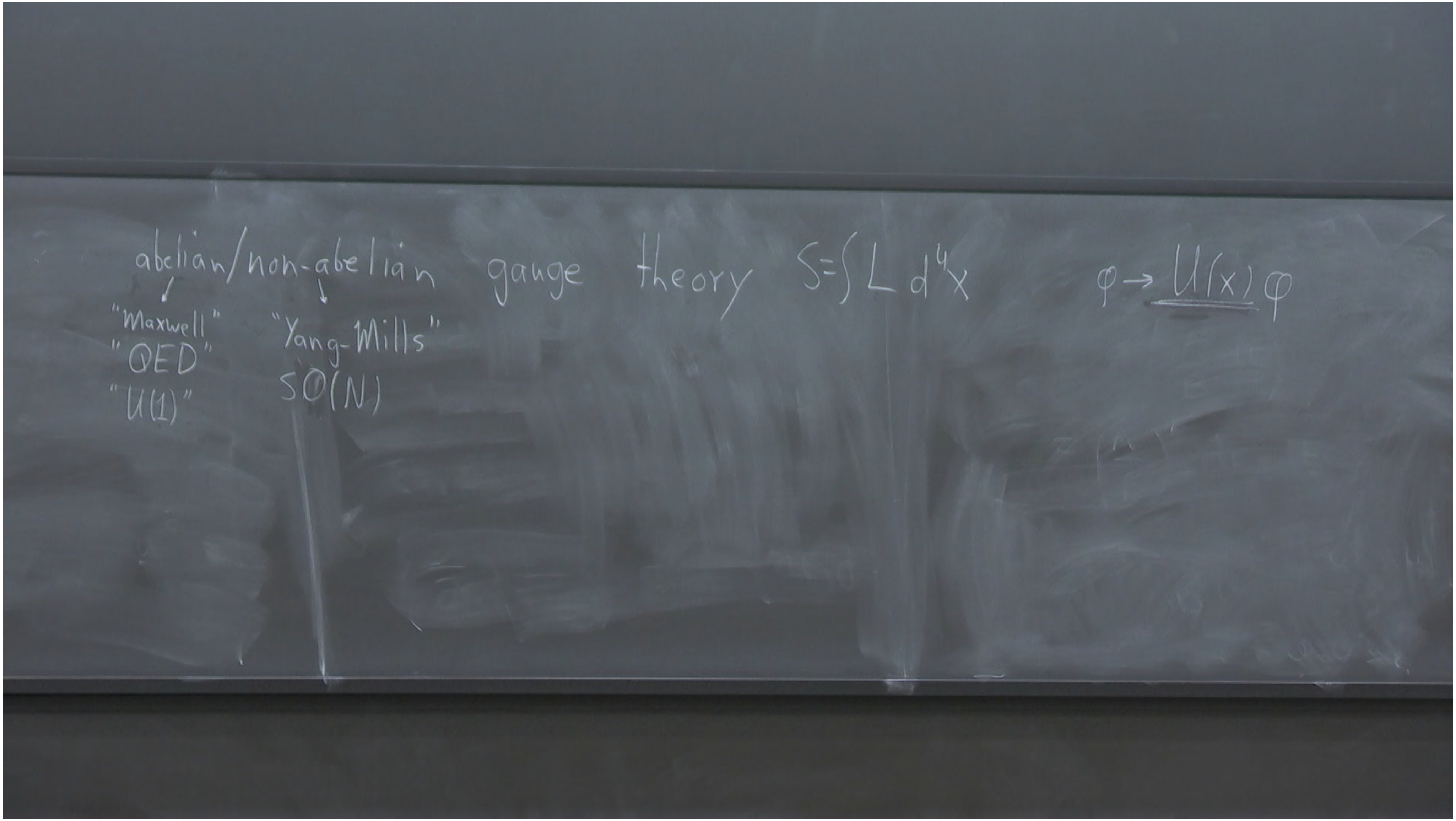

abelian/non-abelian gauge theory $S=\int L\, d^4x$
$\varphi \to U(x)\,\varphi$
"Maxwell"
"QED"
"U(1)"
"Yang-Mills"
SO(N)

$$L_S = -(\partial_\mu \varphi)^\dagger (\partial^\mu \varphi) - m^2 \varphi^\dagger \varphi$$

$$L_F = \bar{\psi}\, i \not{\partial}\, \psi - m \bar{\psi} \psi$$

$$\varphi \longrightarrow U(x)\varphi$$

$$\partial_\mu \longrightarrow \boxed{D_\mu = \partial_\mu + i g A_\mu}$$

$$D_\mu \varphi \longrightarrow U D_\mu \varphi = D'_\mu \varphi' = D'_\mu (U \varphi)$$

$$\boxed{D_\mu \longrightarrow D'_\mu = U D_\mu U^{-1}}$$

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U) U^{-1}$$

$$[D_\mu, D_\nu]\varphi = i g F_{\mu\nu} \varphi$$

$$(U\varphi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g [A_\mu, A_\nu]$$

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$\boxed{g A_\mu}$$

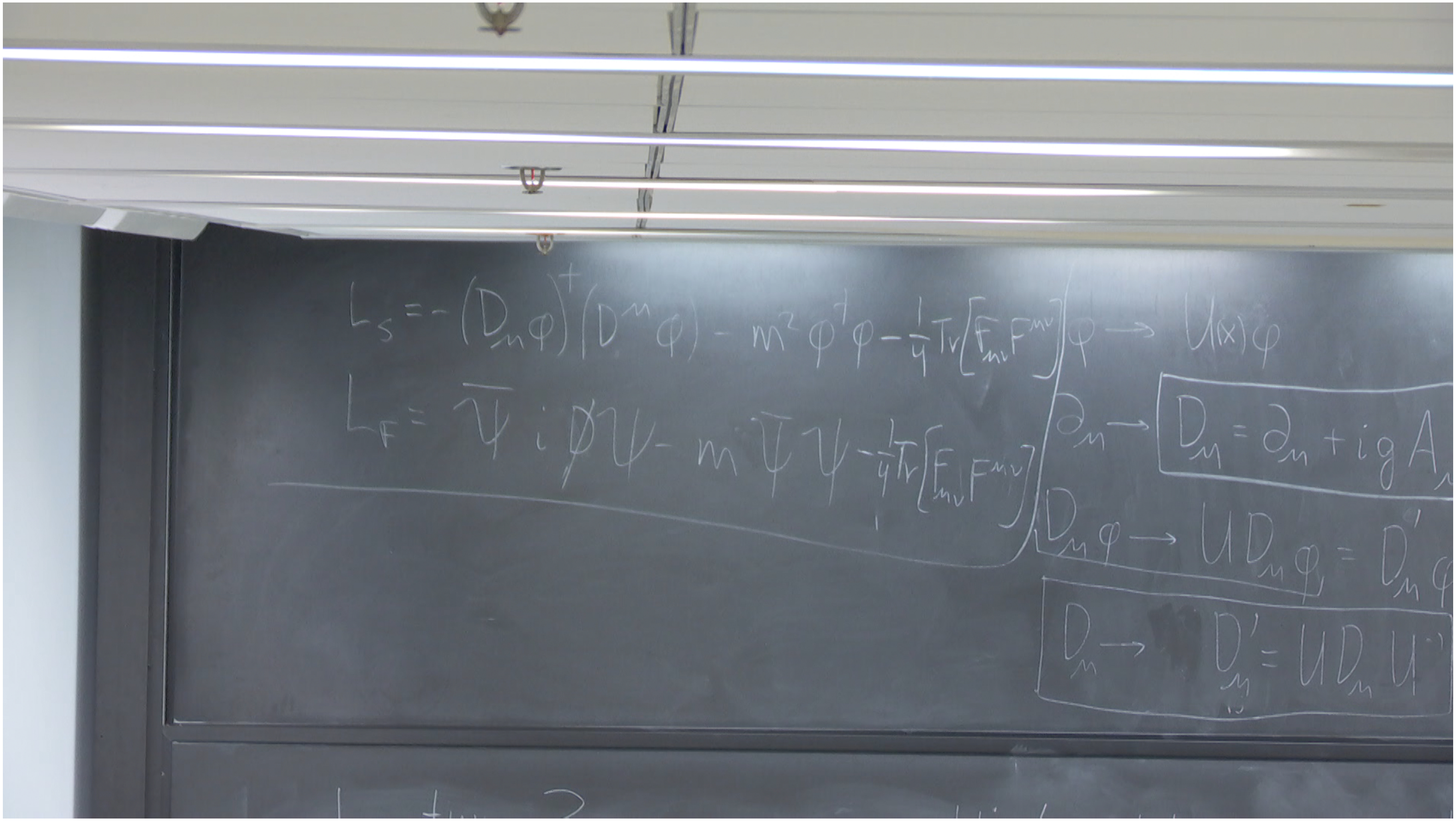
$$U\,[D_\mu, D_\nu]\,\varphi = i g F_{\mu\nu}\,\varphi$$

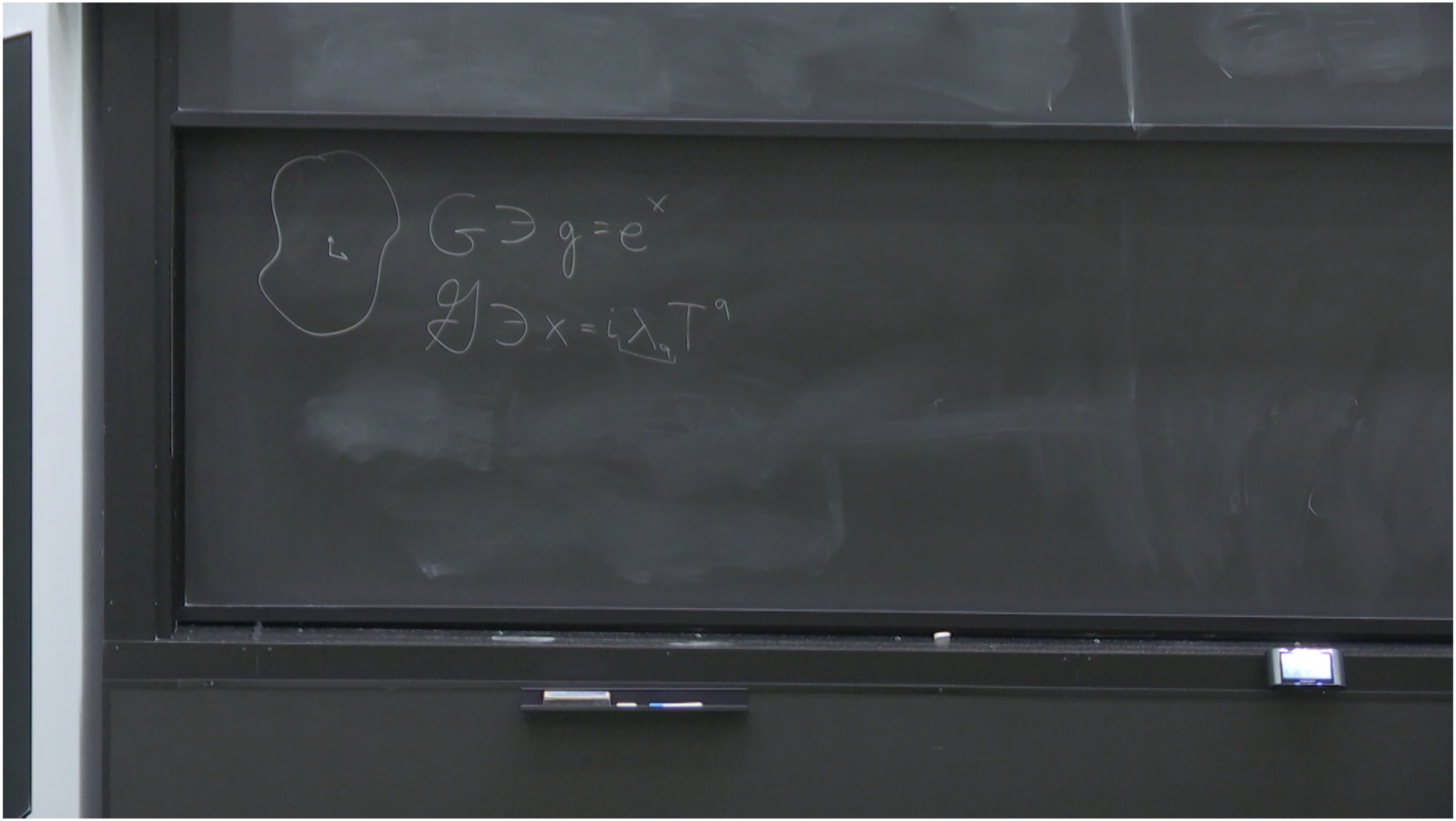
$$D'_\mu \varphi' = D'_\mu (U\varphi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i g [A_\mu, A_\nu]$$

$$\boxed{U^{-1}}$$

$$F_{\mu\nu} \to U F_{\mu\nu} U^{-1}$$


$L_S = -(D_\mu \varphi)^\dagger (D^\mu \varphi) - m^2 \varphi^\dagger \varphi - \frac{1}{4} Tr[F_{\mu\nu} F^{\mu\nu}]$
$L_F = \overline{\psi}\, i \not{D} \psi - m \overline{\psi} \psi - \frac{1}{4} Tr[F_{\mu\nu} F^{\mu\nu}]$
$\varphi \longrightarrow U(x) \varphi$
$\partial_\mu \longrightarrow \boxed{D_\mu = \partial_\mu + i g A_\mu}$
$D_\mu \varphi \longrightarrow U D_\mu \varphi = D'_\mu \varphi'$
$\boxed{D_\mu \longrightarrow D'_\mu = U D_\mu U^{-1}}$


$G \ni g = e^{x}$
$\mathfrak{g} \ni x = i\lambda_a T^a$

Step 1: choose G

Step 2: choose $R(G)$

$G \ni g = e^{x} = e^{i\lambda_m T^m}$

$a, b = 1, \dots, r$

$\mathfrak{g} \ni x = i\lambda_m T^m \qquad m = 1, \dots, d = \dim(\mathfrak{g})$

e.g.

$G = SU(2) \qquad d = 3$

r-dim $R(G): \; g \longrightarrow \underset{(r \times r)}{U(g)}{}^{a}{}_{b} = e^{i\lambda_m T^{m\,a}_{(r\times r)\,b}}$

$V_r \ni \varphi^a$

(2) $\quad d = 3$

$$\varphi^a \longrightarrow U^a{}_b \, \varphi^b$$

$$D_\mu \varphi^a = U^a{}_b \, D_\mu \varphi^b \qquad D_\mu = \partial_\mu + igA_\mu$$

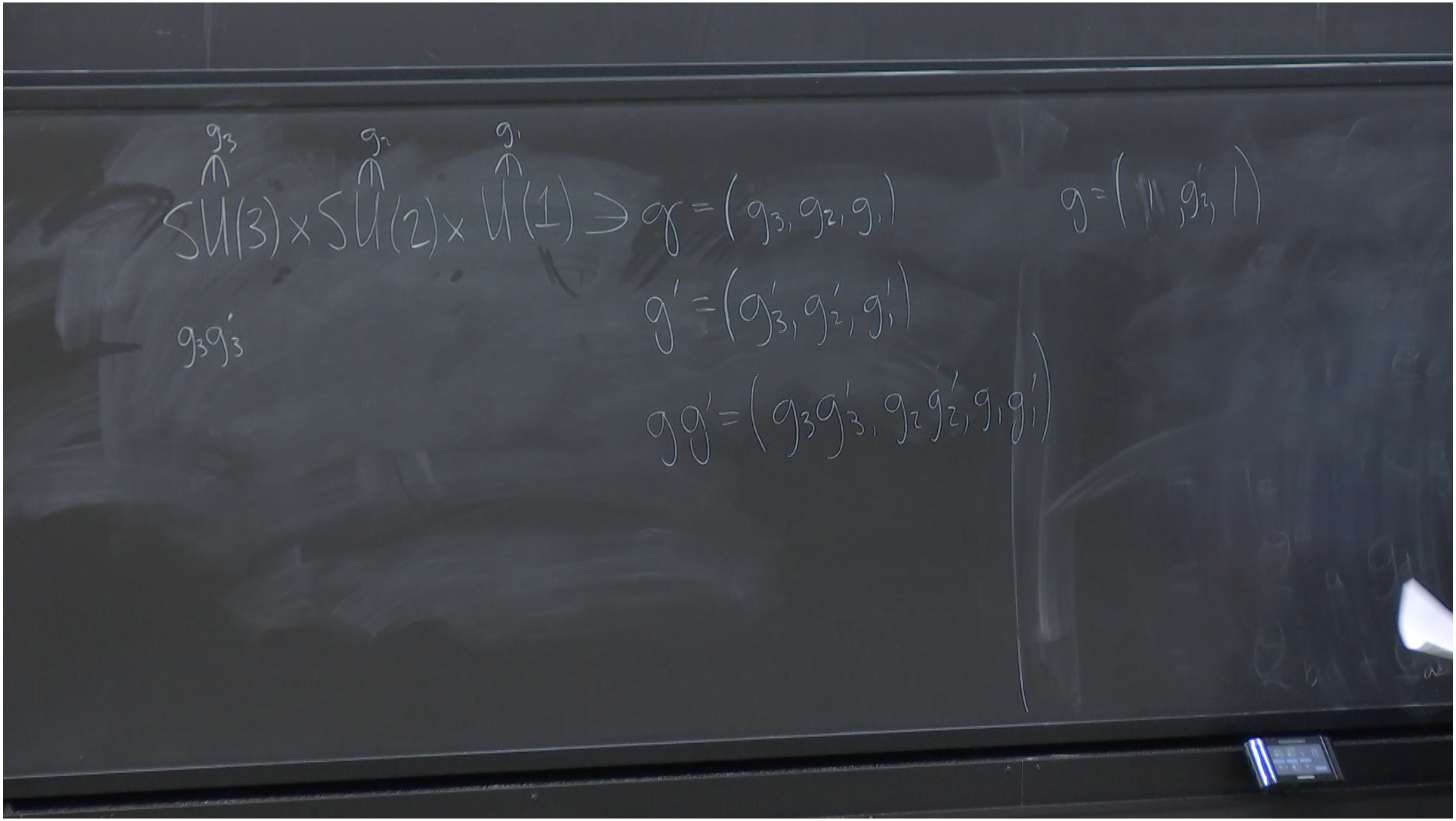
$$A_\mu(x) = A^m_\mu(x) \, T^m$$

$U(2)$ $\quad d = 3$

$$\varphi^a \longrightarrow U^a{}_b \, \varphi^b$$

$$D_\mu \varphi^a = U^a{}_b \, D_\mu \varphi^b \qquad D_\mu = \partial_\mu + igA_\mu$$

$$A_\mu(x) = A^m_\mu(x) \, T^m_{(r\times r)}$$


$SU(3) \times SU(2) \times U(1) \ni g = (g_3, g_2, g_1)$
$g_3 \quad g_2 \quad g_1$
$g' = (g_3', g_2', g_1')$
$gg' = (g_3 g_3', g_2 g_2', g_1 g_1')$
$g_3 g_3'$
$g = (1, g_2', 1)$

	$SU(3)$ (g_3)	$SU(2)$ (g_2)	$U(1)$ (g_1)
$\begin{pmatrix} u_L \\ d_L \end{pmatrix} = q_L^i$	3	2	1/6
u_R^i	3	1	2/3
d_R^i	3	1	−1/3
$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = l_L^i$	1	2	−1/2
e_R^i	1	1	−1
h	1	2	1/2

$$SU(3)\,SU(2)\,U(1) \ni g = (g_3, g_2, g_1)$$

$$g' = (g_3', g_2', g_1')$$

$$gg' = (g_3 g_3', g_2 g_2', g_1 g_1')$$

$$e^X = e^{i\lambda_m T^m}$$

$a, b = 1, \dots, r$

$\lambda_m T^m \qquad m = 1, \dots, d = \dim(\mathfrak{g})$

$$\to \underset{(r\times r)}{U(g)}{}^a{}_b = e^{i\lambda_m T^m_{(r\times r)}{}^a{}_b}$$

$$D_\mu \to U D_\mu U$$

e.g.

$G = SU(2) \qquad d = 3$

ϕ^a

D_μ