Title: PSI 2019/2020 - Gravitational Physics - Lecture 15

Speakers: Ruth Gregory

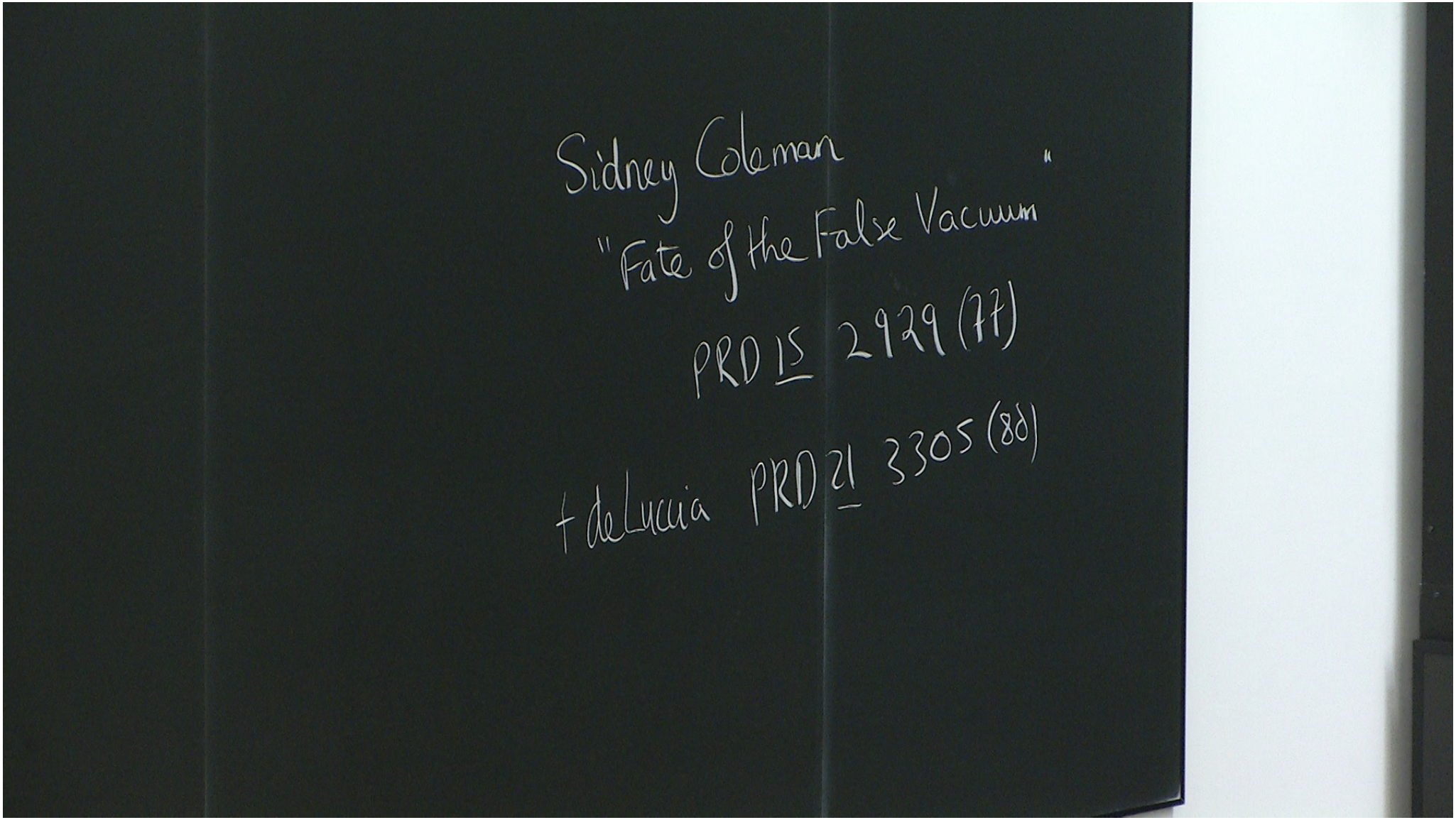
Collection: PSI 2019/2020 - Gravitational Physics

Date: January 24, 2020 - 10:15 AM

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Lecture 15 Gravitational Instantons

Tunnelling is one of the more startling implications of QM - a particle can literally traverse a forbidden region. We understand it via a damping of the wave fn.


Sidney Coleman
"Fate of the False Vacuum"
PRD 15 2929 (77)
+ deLuccia PRD 21 3305 (80)

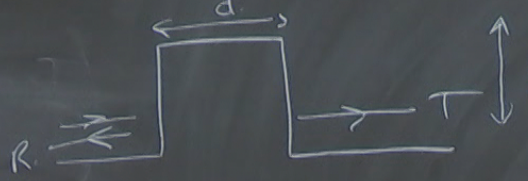
tan tons

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can literally

derstand it

Reminder : Schrödinger tunnelling:



$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E\Delta V.}} \sim e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}\Delta V \sim 2\Delta V.$$

Schrödinger tunnelling:

ΔV.

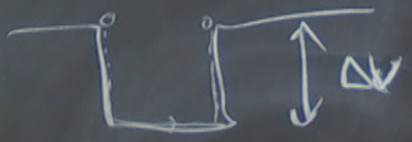
$\sim e^{-2\kappa d}$

$\sinh^2 \kappa d$

$4E\Delta V$

$\sim 2\Delta V$.

Consider a classical ptcle in potential well



$$\frac{1}{2}\dot{x}^2 = \Delta V$$

$$\Rightarrow \int_0^d \sqrt{2\Delta V}\, dx = \int \sqrt{2\Delta V}\, \dot{x}\, dt$$

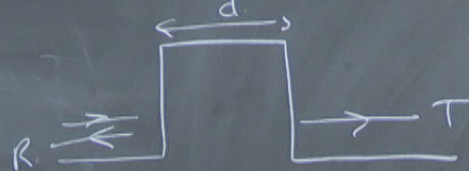
$$= \int 2\Delta V\, dt$$

$$= \int \left(\Delta V + \frac{1}{2}\dot{x}^2\right) dt$$

al Instantons

more startling
rticle can literally
we understand it
fn.

Reminder : Schrödinger tunnelling:



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$$\Omega^2 = \frac{2m}{\hbar^2}\Delta V \sim 2\Delta V.$$

$$2\int_0^d \Omega\, dx$$

$$2\int_0^{d''} \sqrt{2\Delta V}\, dx.$$

Consid
in poten

$$\frac{1}{2}\dot{x}^2$$

$$\Rightarrow \int_0^d \sqrt{2\Delta$$

al particle in

expnent of

lity

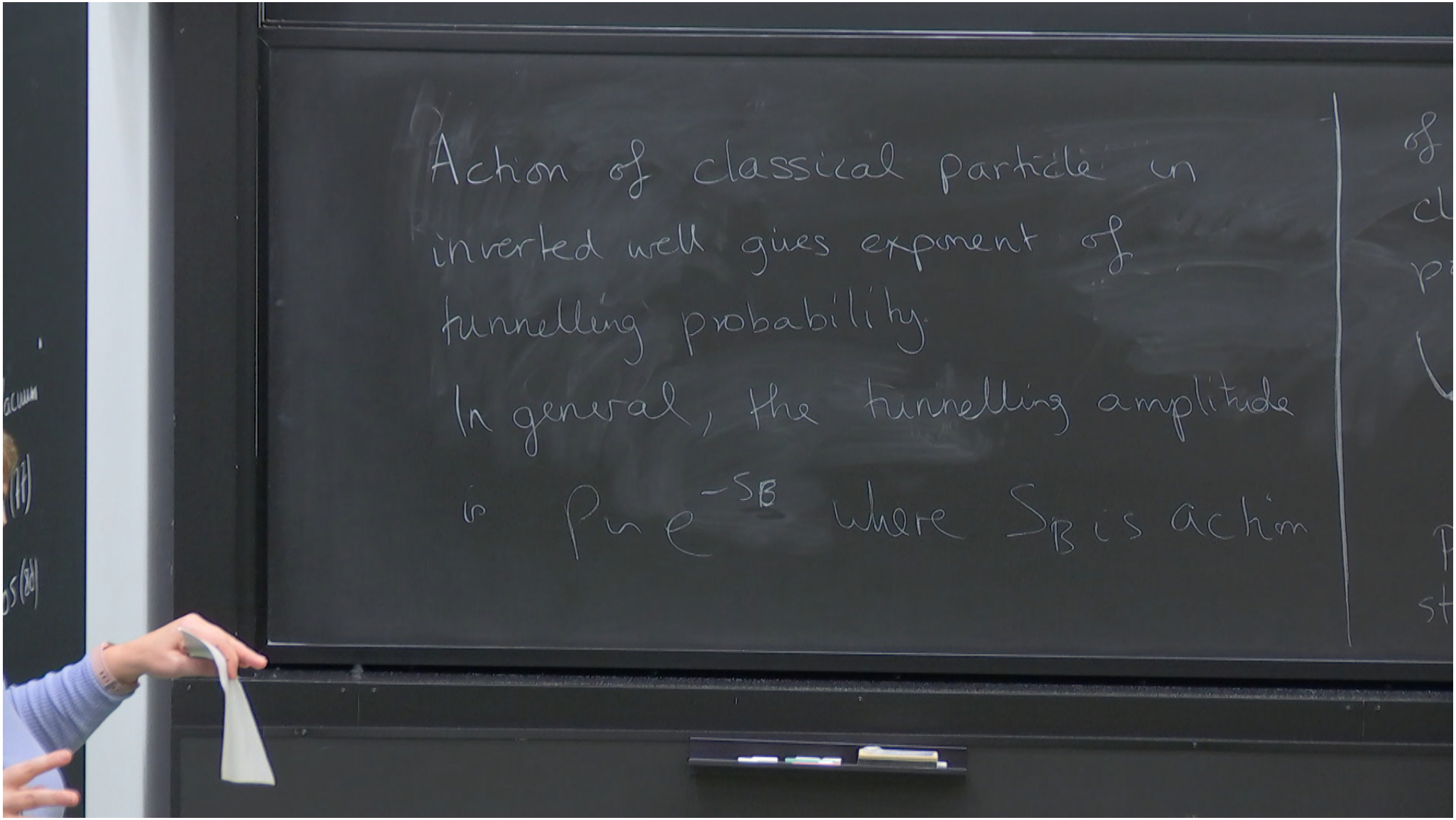
tunnelling amplitude

where S_B is actn

of a Euclidean soln of classical motion in inverted potential



particle rolls from metastable state to "exit" & back - the bounce

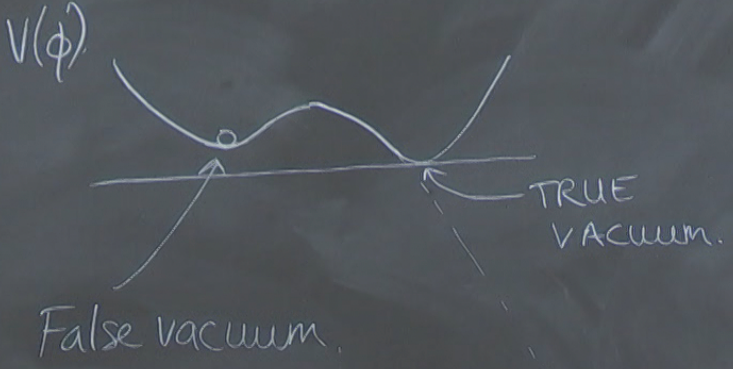

Action of classical particle in
inverted well gives exponent of
tunnelling probability
In general, the tunnelling amplitude
is Ae^{-S_B} where S_B is actin

...an soln of
...tion in inverted



...s from metastable
...' & back - the bounce

In QFT, we have interaction pot.



$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2 - \eta^2)^2 - \frac{\epsilon(\phi - \eta)}{2\eta}$$

$\phi \simeq \eta$, $V = 0$ TRUE.

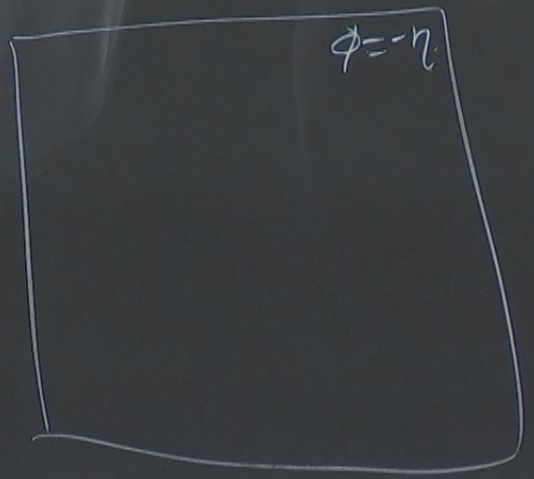
$\phi \simeq -\eta$, $V = \epsilon$ FALSE.

Look

Look for Euclidean soln:

$$\frac{d^2\phi}{d\tau^2} + \underbrace{\nabla^2\phi}_{\mathbb{R}^3 \text{ cart.}} = \frac{\partial V}{\partial \phi} = 2\lambda\phi(\phi^2 - \eta^2) - \frac{\epsilon}{2\eta}$$

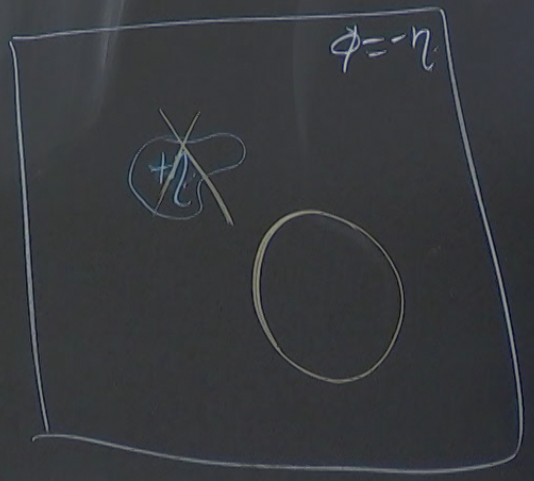
Imagine a false vac

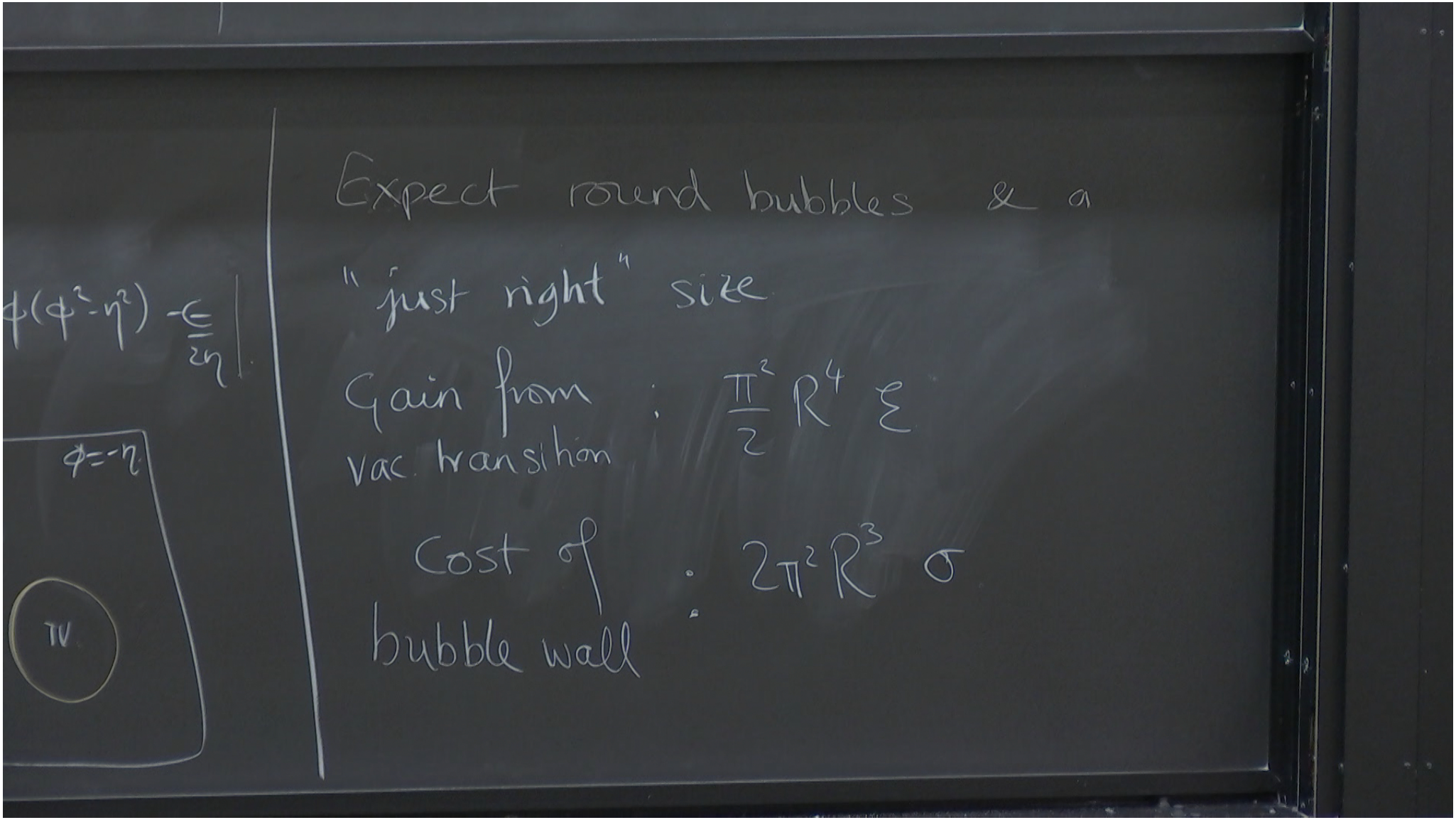


Look for Euclidean soln:

$$\frac{d^2\phi}{d\tau^2} + \underbrace{\nabla^2\phi}_{\mathbb{R}^3 \text{ cart.}} = \frac{\partial V}{\partial \phi} = 2\lambda\phi(\phi^2 - \eta^2) - \frac{\epsilon}{2\eta}$$

Imagine a false vac




$\phi(\phi^2-\eta^2)-\frac{\epsilon}{2\eta}$
$\phi=-\eta$
TV
Expect round bubbles & a
"just right" size
Gain from vac. transition : $\frac{\pi^2}{2}R^4\,\epsilon$
Cost of bubble wall : $2\pi^2R^3\,\sigma$

Balance: $\frac{\partial}{\partial R}$: $2\pi^2 R^3 \epsilon = 6\pi^2 R^2 \sigma$

$\Rightarrow$ $\underline{R = 3\sigma/\epsilon}$

Back to ϕ: take SO(4) symmetry.

$\rho^2 = \tau^2 + x^2$.

$\rightarrow$ $\phi'' + \boxed{\frac{3\phi'}{\rho}} = 2\lambda\phi(\phi^2 - \eta^2)$.

$\epsilon = 6\pi^2 R^2 \sigma$

$R = 3\sigma/\epsilon$

SO(4) symmetry.

$2\lambda\phi(\phi^2 - \eta^2)$.

Approx soln

$\phi = -\eta \tanh(\sqrt{\lambda}\eta(\rho - \rho_0)) + \delta\phi$

for $\rho_0 \gg 1/\sqrt{\lambda}\eta$.

$\sigma = \frac{4}{3}\sqrt{\lambda}\eta^3$.

Action:

wall: $\int \frac{1}{2}(\phi'^2 + (\partial\phi)^2) + V.$

$\simeq 2\pi^2 R^3 \sigma$.

Tu

N

$\sqrt{\lambda}\eta(\varphi-\rho_0)) + \delta\phi$

$\delta = \frac{4}{3}\sqrt{\lambda}\eta^3$

$\phi'^2 + (\nabla\phi)^2) + V.$

$\pi^2 R^3 \sigma$

Justifies Goldilocks argument.

Note $R = 3\sigma/\epsilon \gg 1/\sqrt{\lambda}\eta$

$$S_B = \frac{27\pi^2}{2}\frac{\sigma^4}{\epsilon^3}$$

$(\sqrt{\lambda}\eta\varphi - \rho_0)) + \delta\phi$

$\delta = \frac{4}{3}\sqrt{\lambda}\eta^3$.

η.

$(\phi'^2 + (\nabla\phi)^2) + V$.

$2\pi^2 R^3 \sigma$.

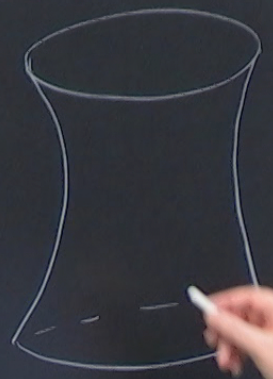
Justifies Goldilocks argument.

Note $R = 3\sigma/\epsilon \gg 1/\sqrt{\lambda}\eta$

$$S_\beta = \frac{27\pi^2}{2}\frac{\sigma^4}{\epsilon^3}$$

Analytically continue.

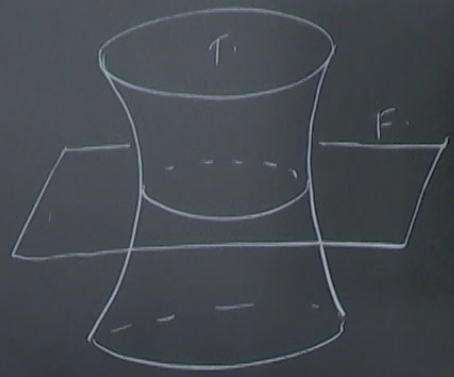
$\vec{t}^2 + \vec{x}^2 = \rho^2 R^2 \longrightarrow -t^2 + r^2 = R^2$



Justifies Goldilocks argument.

Note $R = 3\sigma/\epsilon \gg 1/\sqrt{\Lambda}\eta$

$$S_\beta = \frac{27\pi^2}{2}\frac{\sigma^4}{\epsilon^3}$$



Lorentzian soln.

Analytically continue.

$$-T^2 + X^2 = \rho^2 = R^2 \longrightarrow -t^2 + r^2 = R^2$$

⊕ Gravity.

$V(\phi_{FV}) > 0$

$\Rightarrow \Lambda_{FV} = 8\pi G \varepsilon$ ie dS!

Euclidean dS is S^4.

radius l : $\Lambda = 3/l^2$

Wall

dS!

Wall will look similar,

but now satisfies Israel eqns.

$$ds_+^2 = l^2(d\chi^2 + \sin^2\chi\, d\Omega_{III}^2) \quad (S^4)$$

$$ds_-^2 = dp^2 + p^2 d\Omega_{III}^2 \quad (\mathbb{R}^4)$$

Boundary (wall): $p_w = l\sin\chi_w$

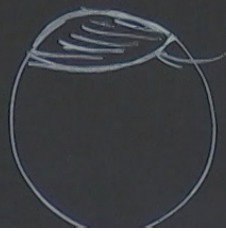
⊕ Granity.

$V(\phi_{FV}) > 0$

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Euclidean dS is S^4.

radius l : $\Lambda = 3/l^2$



Wall

but n

ds^2_+

ds^2_-

Bound

✓ $n_+ = \frac{1}{\ell}\frac{\partial}{\partial \chi}$ $\quad K_{+\alpha\beta} = -\frac{1}{\ell}\Gamma^{\chi}_{\alpha\beta} = \frac{1}{\ell}\cot\chi_W\, g_{\alpha\beta}$.

⇉ $n_- = \frac{\partial}{\partial \rho}$. $\quad K_{-\alpha\beta} = -\Gamma^{\rho}_{\alpha\beta} = -\frac{1}{\rho_W} g_{\alpha\beta}$

Remember, $T_{ab} \sim \sigma h_{ab}$ for wall.

$\Delta K_{ab} - \Delta K h_{ab} = 4\pi T_{ab}$

ISRAEL:

$$-\frac{2}{\ell}\cot\chi_W + \frac{2}{\rho_W} = 8\pi G\sigma.$$

$$n_+ = \frac{1}{\ell}\frac{\partial}{\partial \chi} \qquad K_{+\alpha\beta} = -\frac{1}{\ell}\Gamma^{\chi}_{\alpha\beta} = \frac{1}{\ell}\cot\chi_W\, g_{\alpha\beta}$$

$$\Rightarrow\ n_- = \frac{\partial}{\partial \rho} \qquad K_{-\alpha\beta} = -\Gamma^{\rho}_{\alpha\beta} = \frac{-1}{\rho_W}\, g_{\alpha\beta}$$

ISRAEL:

$$-\frac{2}{\ell}\cot\chi_W + \frac{2}{\rho_W} = 8\pi G\sigma.$$

$$\rightarrow \frac{2}{\rho_W}\left[1 - \sqrt{1-\rho_W^2/\ell^2}\right] = 8\pi G\sigma.$$

ρ_W

$\sin\chi_W$

$\cos\chi_W$

Remember,

$T_{ab} \sim \sigma\, h_{ab}$ for wall.

$\Delta K_{ab} - \Delta K h_{ab} = 4\pi T_{ab}$ (8πG)

$$I_E = \int_{M_+ \cup M_-} \frac{-R+2\Lambda}{16\pi G} + \int_{WALL} \sigma + \int_{\partial M_+ - \partial M_-} \frac{K}{8\pi G}$$

Bounce action.

$$B = \int_{\chi \leq \chi_0} \frac{-R+2\Lambda}{16\pi G} + \int_{WALL} \sigma_\cdot + \frac{\Delta K}{8\pi G}$$

$$\frac{K}{8\pi G}$$

$$N_W = \Delta K = -12\pi G_4 \sigma$$

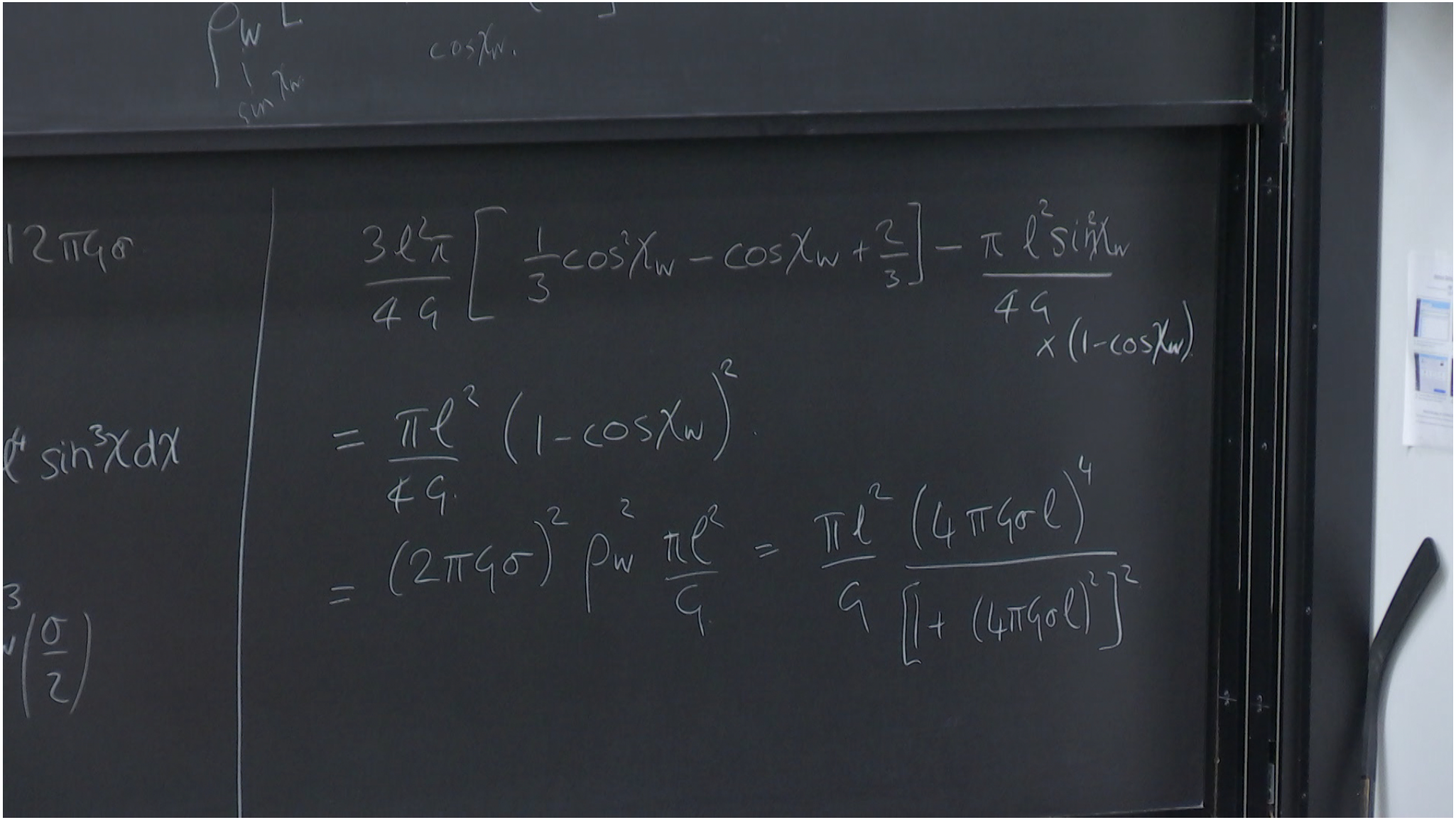
$$R = 4\Lambda$$

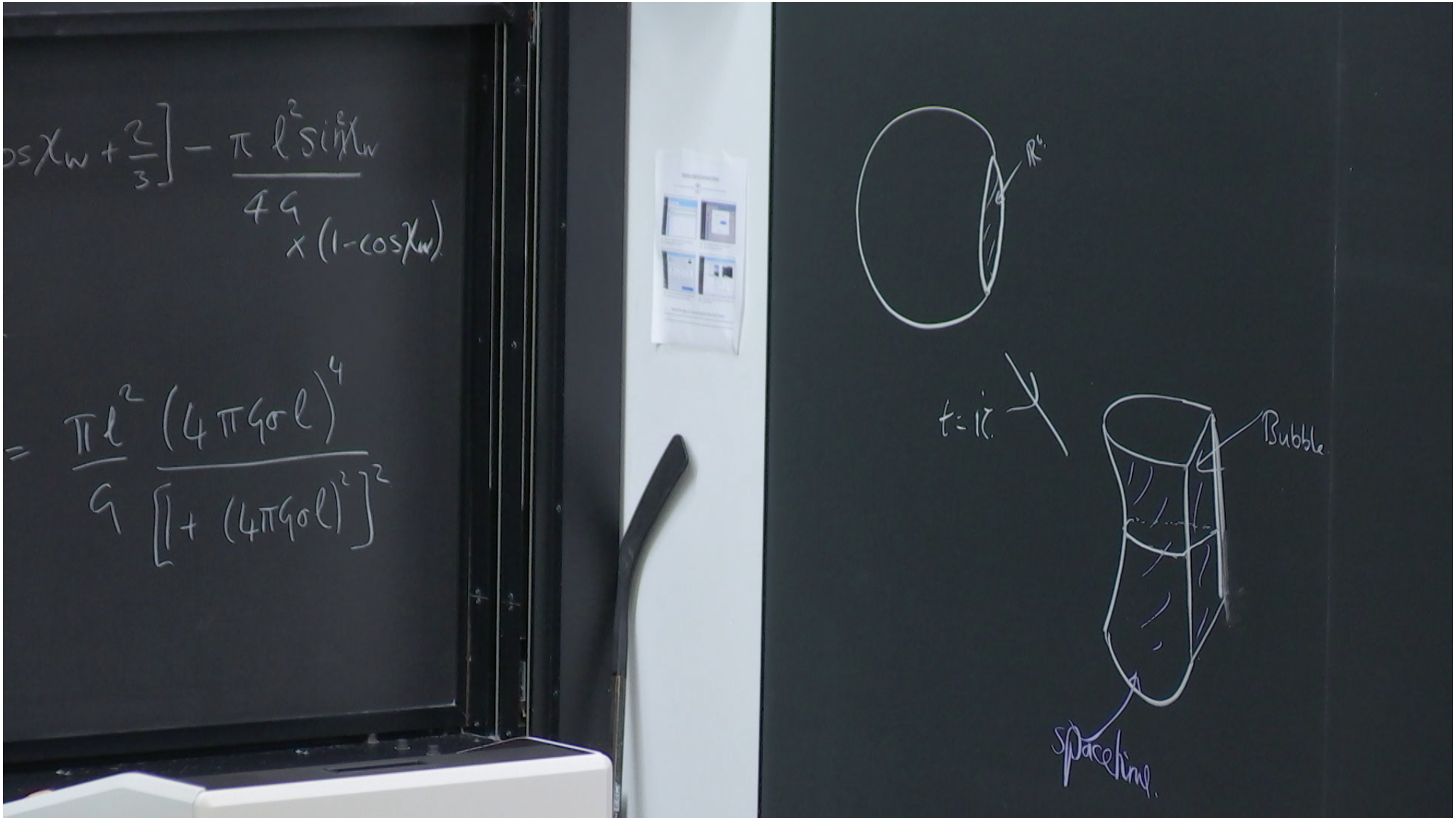
$$\Rightarrow B = \int_0^{\chi_0} \frac{\Lambda}{8\pi G} 2\pi^2 \ell^4 \sin^3\chi \, d\chi$$

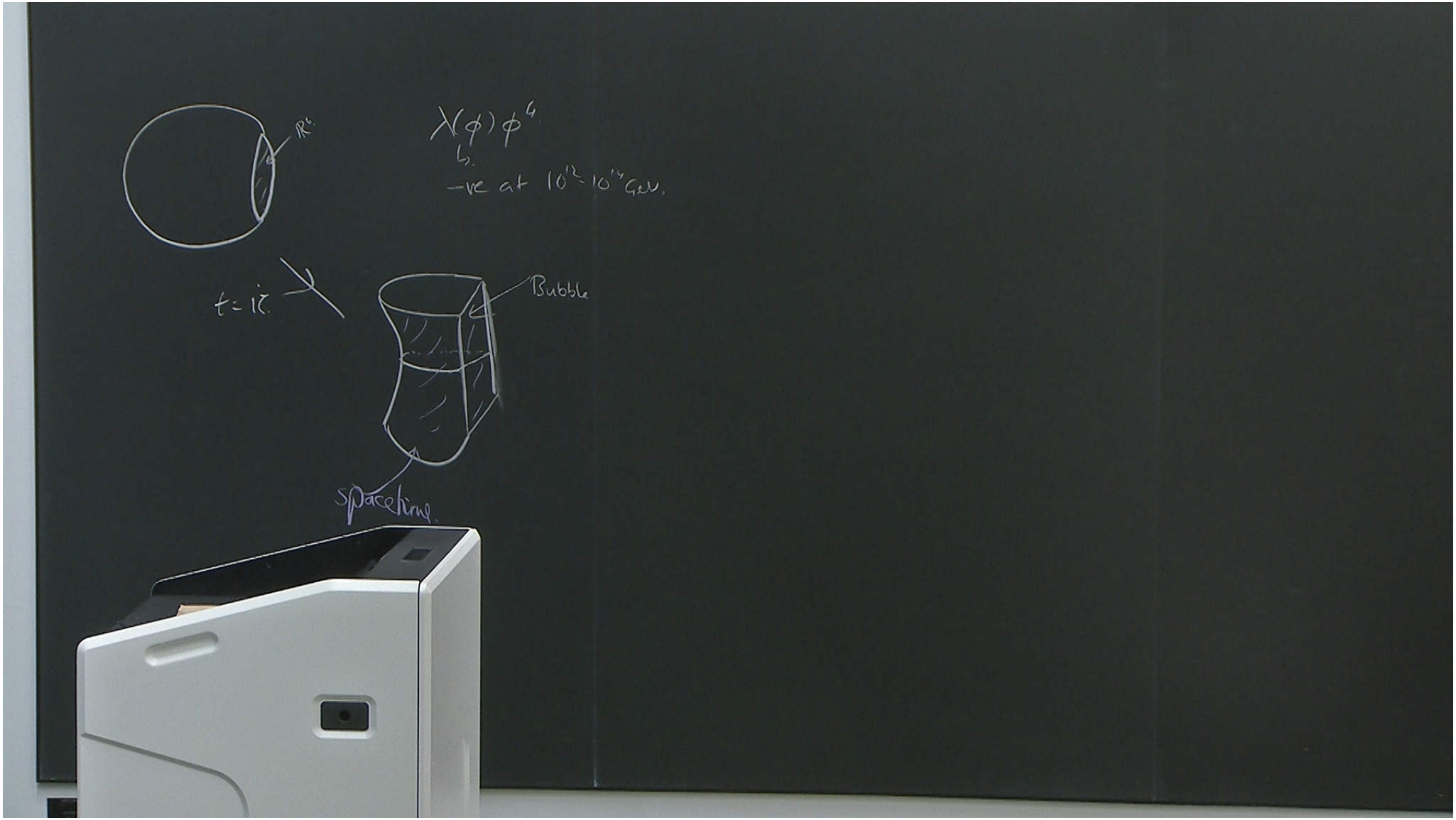
$$- 2\pi^2 \rho_W^3 \left(\frac{\sigma}{2}\right)$$

$$\frac{\Delta K}{8\pi G}$$

$$\frac{3\ell^2\pi}{4G}\left[\frac{1}{3}\cos^3\chi_W - \cos\right.$$


ρ_w L
1
$\sin\chi_w$
$\cos\chi_w$.
$12\pi q\sigma$
$\ell^4 \sin^3\chi d\chi$
$\frac{3}{w}\left(\frac{\sigma}{2}\right)$
$\frac{3\ell^2\pi}{4q}\left[\frac{1}{3}\cos^3\chi_w - \cos\chi_w + \frac{2}{3}\right] - \frac{\pi\ell^2\sin^2\chi_w}{4q}$
$\times(1-\cos\chi_w)$
$= \frac{\pi\ell^2}{4q}(1-\cos\chi_w)^2$
$= (2\pi q\sigma)^2 \rho_w^2 \frac{\pi\ell^2}{q} = \frac{\pi\ell^2}{q}\frac{(4\pi q\sigma\ell)^4}{\left[1+(4\pi q\sigma\ell)^2\right]^2}$


$\cos\chi_w + \frac{2}{3}\Big] - \frac{\pi \ell^2 \sin^2\chi_w}{4G} \times (1-\cos\chi_w)$
$= \frac{\pi \ell^2}{G} \frac{(4\pi G_0 \ell)^4}{\left[1 + (4\pi G_0 \ell)^2\right]^2}$
R^4
$t = it$
Bubble
spacetime


R^6
$\lambda(\phi)\phi^6$
-ve at 10^{12}-10^{14} GeV.
t = it
Bubble
spacetime