Title: PSI 2019/2020 - Gravitational Physics - Lecture 13

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LECTURE 13 WALLS & WARP FACTORS

An alternate way of hiding extra dims is to be localized on a submanifold.

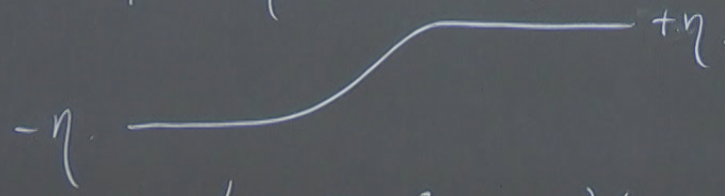
Rubakov
-Shaposhnikov.

Consider

$$\mathcal{L}_{\varphi} = \frac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{2}(\varphi^2 - \eta^2)^2$$

Usually quantize around a vacuum $\phi = \pm\eta$

$$\phi = \eta \tanh(\sqrt{\lambda}\eta z)$$



$$\phi' = \sqrt{\lambda}\eta^2 \operatorname{sech}^2(\sqrt{\lambda}\eta z) = \sqrt{\lambda}(\eta^2 - \phi^2)$$

$$\phi'' = -2\lambda\eta^3 \operatorname{sech}^2(\sqrt{\lambda}\eta z)\tanh(\sqrt{\lambda}\eta z) = -2\lambda\phi(\eta^2 - \phi^2) = \frac{\partial V}{\partial \phi}$$

i.e. $-\phi'' + \frac{\partial V}{\partial \phi} = 0$

ϕ is a soln to eqns of motion. A topologically stable solution.

– DOMAIN WALL



$$T_{ab} = \partial_a \phi \, \partial_b \phi - \left(\tfrac{1}{2}(\partial\phi)^2 - V\right) g_{ab}.$$

$$= \phi'^2 \, \delta^z_a \, \delta^z_b + \left(V + \tfrac{1}{2}\phi'^2\right) g_{ab}$$

$$= 2V\Big(\underbrace{g_{ab} + \delta^z_a \, \delta^z_b}_{h_{ab}}\Big)$$

- a cosmological constant parallel to worldvolume "brane" energy momentum.

$\partial_\phi - \left(\frac{1}{2}(\partial\phi)^2 - V\right) g_{ab}$.

z

$_b + \left(V + \frac{1}{2}\phi'^2\right) g_{ab}$

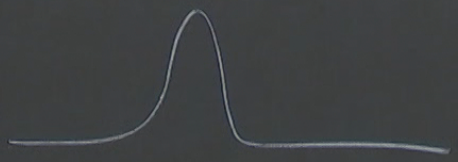
$_{ab} + \delta^z_a \delta^z_b)$

h_{ab}

nstant parallel to

"brane" energy momentum.

$V \propto \mathrm{sech}^4(\sqrt{\lambda}\eta z)$



Strongly localized.

$\int V dz = \sigma = \frac{4}{3}\sqrt{\lambda}\eta^3$.

energy per unit area.

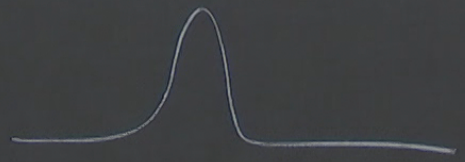
$$T_{ab} = \partial_a\phi\,\partial_b\phi - \left(\tfrac{1}{2}(\partial\phi)^2 - V\right)g_{ab}.$$

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- a cosmological constant parallel to worldvolume "brane" energy momentum.

$V \propto \mathrm{sech}^4(\sqrt{\lambda}\eta z)$.



Strongly localized.

$$\int V\,dz = \sigma = \frac{4}{3}\sqrt{\lambda}\eta^3.$$

Energy per unit area.

Approx a codim 1 submanifold. In 5D the wall has a 4D world volume. Maybe we live on this world volume (Rubakov Shaposhnikov).

Now couple a fermion:

$$\mathcal{L}_{INT} = i\bar{\Psi}\Gamma^a\nabla_a\Psi - g\phi\bar{\Psi}\Psi$$

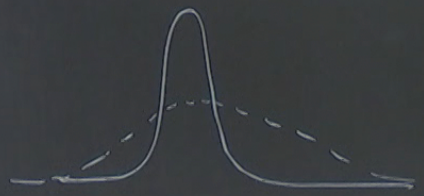
$m_\Psi = 0$ in wall.
$\neq 0$ outside.

Lowest energy state for Ψ is not $\Psi = 0$ but a condensate

$i\gamma^5\Psi' = g\eta \tanh(\sqrt{\lambda}\eta z)\,\Psi$

$\Psi = \psi_0 [\mathrm{sech}(\sqrt{\lambda}\eta z)]^{g/\sqrt{\lambda}}$

for $i\gamma^5\psi_0 = -\psi_0$ - chiral.



Gen soln:

$\not{\partial}\,\psi_0(x) = 0$

$\tilde{\eta}z)\ \Psi$

$\eta z)\Big]^{9/5\pi}$

– chiral.

$\ln$:

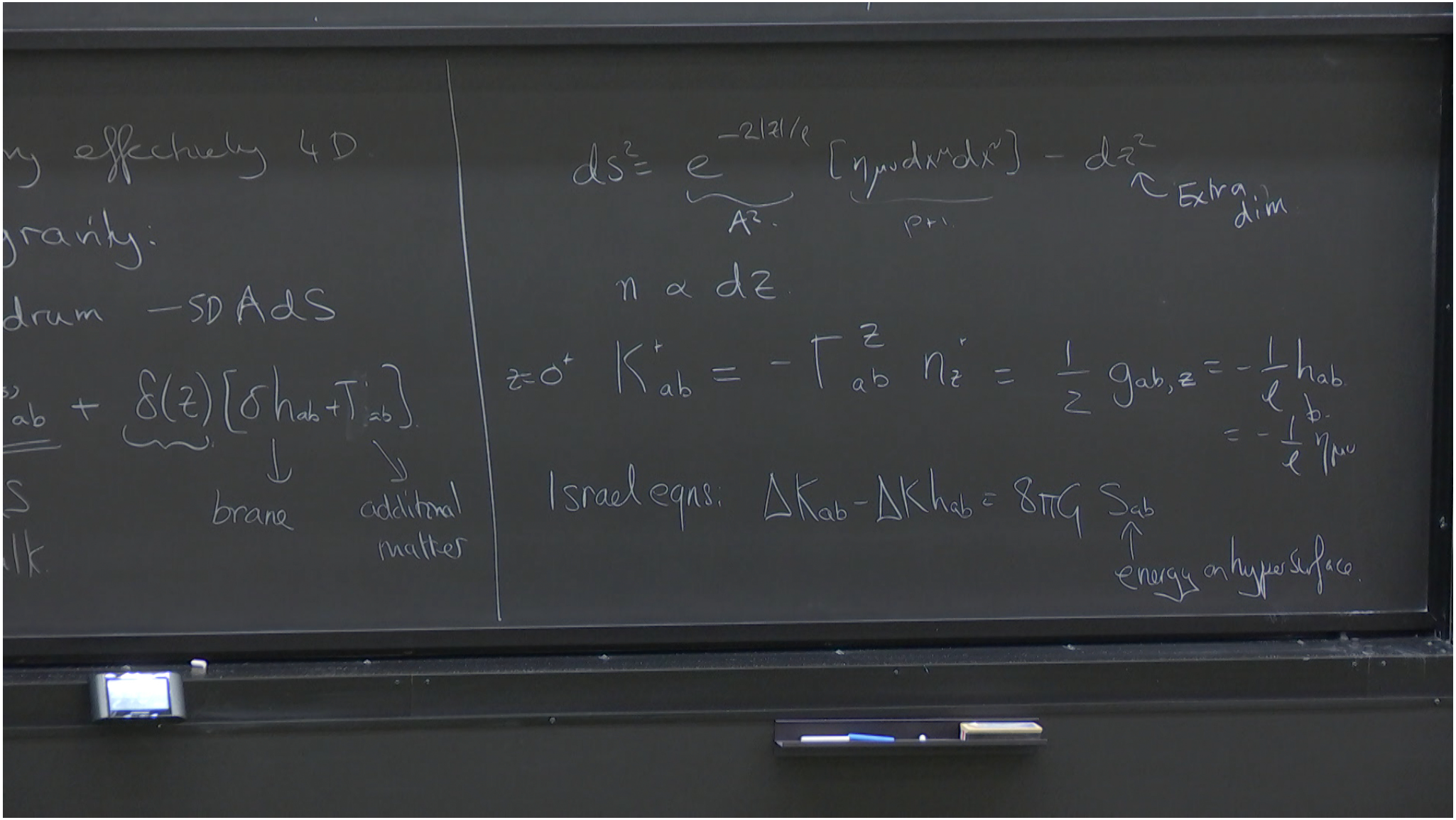
$\psi_0(x) = 0$

Low energy theory effectively 4D.

Now look at gravity:

Randall-Sundrum – 5D AdS

$$\rightarrow T^{(5)}_{ab} = \underbrace{\Lambda g^{(5)}_{ab}}_{\text{AdS bulk}} + \underbrace{\delta(z)}_{\text{brane}}\Big[\underbrace{\delta h_{ab}}_{\text{brane}} + \underbrace{T_{ab}}_{\text{additional matter}}\Big]$$


y effectively 4D
gravity:
drum — 5D AdS
ab + δ(z)[δ h_ab + T_ab]
brane
additional matter
lk
ds² = e^{−2|z|/ℓ} [η_μν dx^μ dx^ν] − dz²
A²
p+1
Extra dim
n ∝ dz
z=0⁺ K⁺_ab = − Γ^z_ab n^z = 1/2 g_ab,z = − 1/ℓ h_ab
= − 1/ℓ η_μν
Israel eqns: ΔK_ab − ΔK h_ab = 8πG S_ab
energy on hypersurface

Here $\Delta K_{ab} = 2K^{+}_{ab}$.

& $S_{ab} = \sigma h_{ab}$ for wall.

$$-\frac{2}{\ell} h_{ab} + \frac{8}{\ell} h_{ab} = 8\pi G \sigma h_{ab}.$$

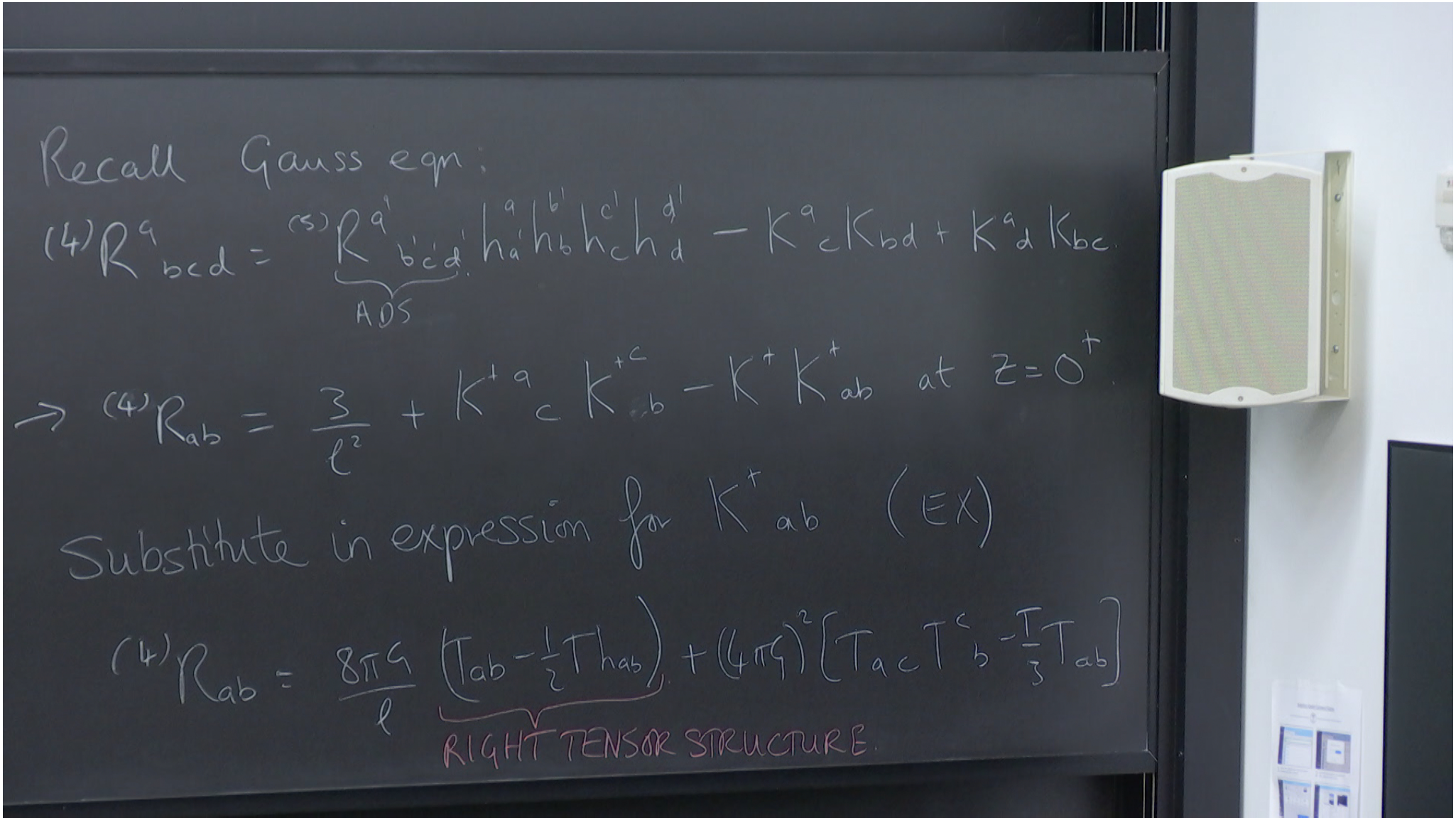
$$\sigma = \frac{3}{4\pi G \ell}.$$

(*) Is a domain wall soln with tension tuned to bulk cos. const.

in 4D $-R = 8\pi G T$.

$$\rightarrow \quad R_{ab} = 8\pi G \left(T_{ab} - \frac{1}{2} T g_{ab} \right)$$

4D


Recall Gauss eqn:
$^{(4)}R^{a}{}_{bcd} = \underbrace{{}^{(5)}R^{a'}{}_{b'c'd'}}_{AdS} h^{a}_{a'} h^{b'}_{b} h^{c'}_{c} h^{d'}_{d} - K^{a}{}_{c} K_{bd} + K^{a}{}_{d} K_{bc}$
$\rightarrow {}^{(4)}R_{ab} = \frac{3}{\ell^2} + K^{+a}{}_{c} K^{+c}{}_{b} - K^{+} K^{+}_{ab}$ at $z = 0^{+}$
Substitute in expression for K^{+}_{ab} (EX)
$^{(4)}R_{ab} = \frac{8\pi G}{\ell} \underbrace{\left(T_{ab} - \frac{1}{2} T h_{ab}\right)}_{\text{RIGHT TENSOR STRUCTURE}} + (4\pi G)^2 \left[T_{ac} T^{c}{}_{b} - \frac{T}{3} T_{ab}\right]$

$2K^+_{ab}$.

for wall

$_{ab} = 8\pi G_5 h_{ab}$.

soln with

cos const.

Check behaviour of gravity

$$K^+_{ab} = 4\pi G\left(S_{ab} - \frac{1}{3} S h_{ab}\right)$$

– Keep Z_2 symm. & reuse Israel.

$$K^+_{ab} = 4\pi G\left(-\frac{\sigma}{3} h_{ab} + T_{ab} - \frac{I}{3} h_{ab}\right)$$

$$= -\frac{1}{\ell} h_{ab} + 4\pi G\left(T_{ab} - \frac{I}{3} h_{ab}\right)$$

5D Einstein

Recall

$^{(4)}R^q_{bc}$

$\rightarrow {}^{(4)}R_{a}$

Subst

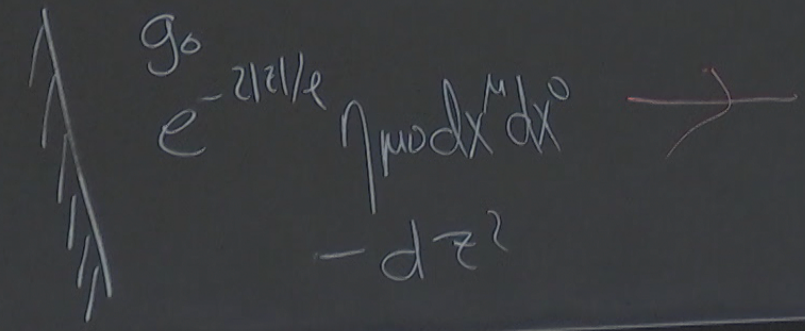
$^{(4)}R$

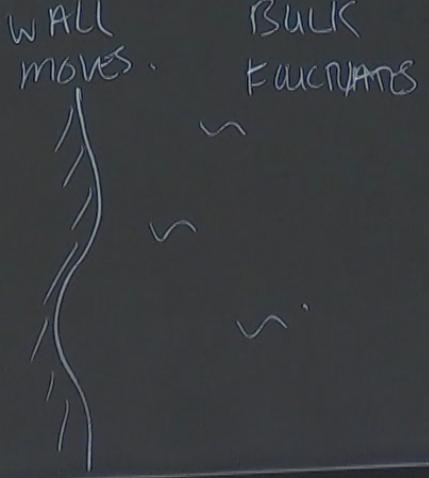
Set $G_N = G/\ell$ 4D Newton const.

& effective eqns for brane curvature are

4D Einstein eqns + T^2 corrections.

Perturbation Theory.



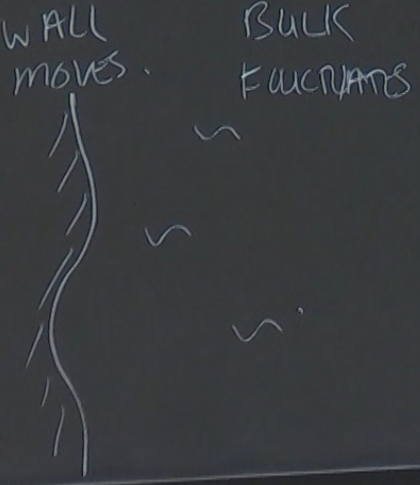


G/ℓ 4D Newton const.

for brane curvature are

$\ldots + T^2$ corrections.

heavy.



$dx^\mu dx^\nu \rightarrow$

z^2

Recall.

$$\delta R_{ab} = -\frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h + \nabla_c \nabla_{(a} h^c{}_{b)}$$

where $\delta g_{ab} = h_{ab}$ (!)

Reverse covariant derivs using Riemann

δR

5D Einstein

RIGHT TENSOR STRUCTURE

$$\delta R_{ab} = \nabla_{(a}\nabla^{c} h_{b)c} - \frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

$$+ \frac{1}{2} R^{c}{}_{dca} h^{d}{}_{b} + \frac{1}{2} R_{bdca} h^{cd}$$

$$- \frac{1}{2} R^{c}{}_{dcb} h^{d}{}_{a} + \frac{1}{2} R_{adcb} h^{cd}$$

$$= -\frac{1}{2}\Delta_L h_{ab} \quad \text{(Lichnerowicz)}$$

$$\Delta_L h_{ab} = \Box h_{ab} + 2R_{acdb} h^{cd} - 2R_{d(a} R^{d}_{b)} - 2\nabla_{(a}\nabla^{c}\bar{h}_{b)c}$$

where $\bar{h}_{ab} = h_{ab} - \frac{1}{2} h g_{ab}$ — gauge

$$-\frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

$$+ \nabla_c \nabla_{(a} h^{c}{}_{b)}$$

$$\bar{h}_{ab} = h_{ab} \quad (!)$$

variant derivs using Riemann

- curved space wave eq for h_{ab}.

Can choose de Donder gauge

$$\nabla_a \bar{h}^a{}_b = 0.$$

More wall back to $z = 0$.

$$\xi^z = f(x^\mu)$$

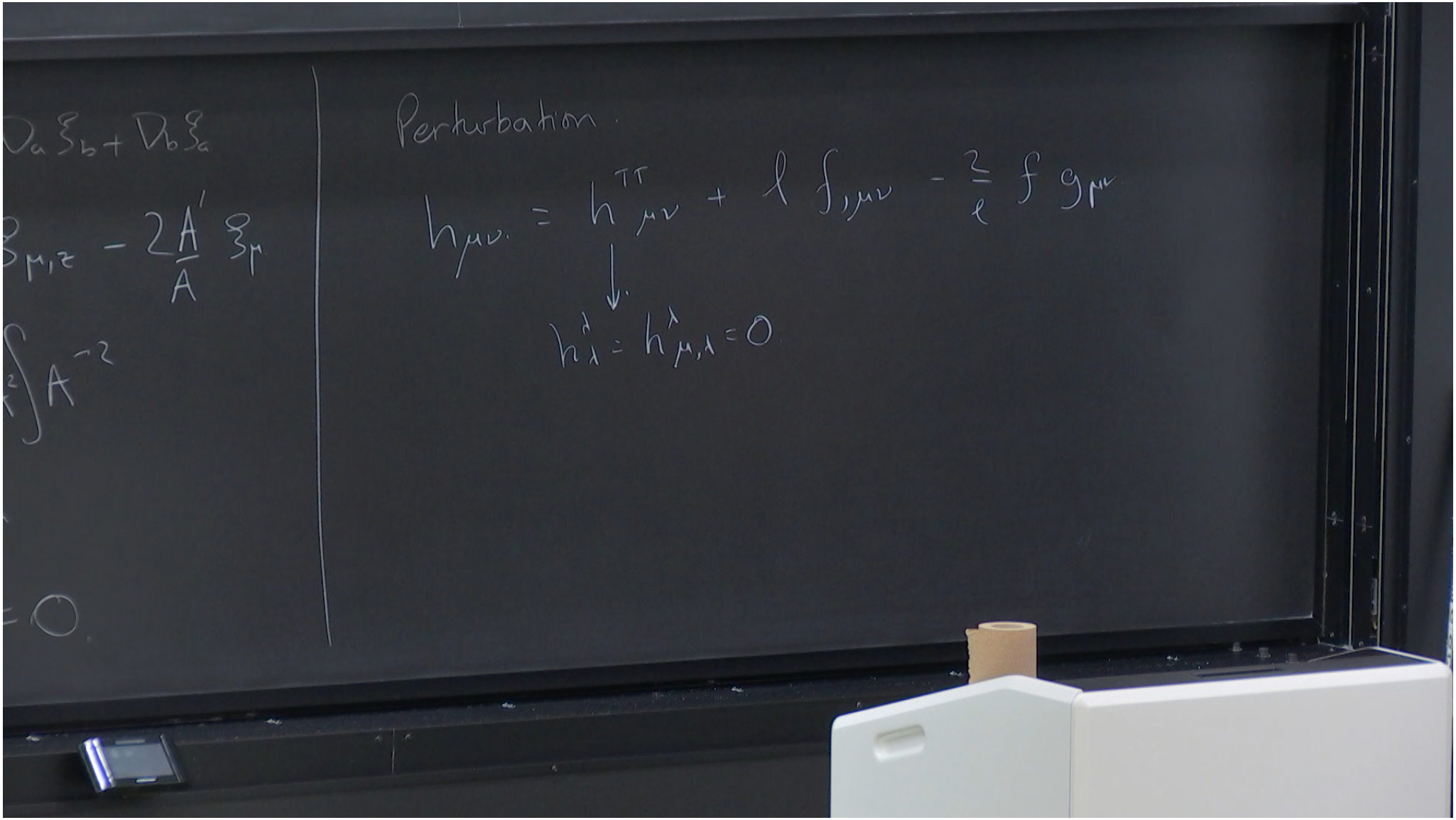
$$\delta g_{ab} = \mathcal{L}_\xi g_{ab} = D_a \xi_b + D_b \xi_a$$

$$\delta g_{z\mu} = -f_{,\mu} + \xi_{\mu,z} - 2\frac{A'}{A}\xi_\mu$$

$$\Rightarrow \xi_\mu = f_{,\mu} A^2 \int A^{-2}$$

$$= \frac{\ell}{2} f_{,\mu}$$

keeps $g_{z\mu} = 0$.


Perturbation
$h_{\mu\nu} = h^{TT}_{\mu\nu} + \ldots$
$h^{\lambda}_{\lambda} = h^{\lambda}_{\mu,\lambda} = 0$

Δ_L : $z \neq z$

$$A^{-2}\left[A^2 (A^{-2} h_{\lambda}^{\lambda})'\right]' = 4\partial^2 f\, \delta(z)$$

$$= -\frac{2}{3}\delta(z)(8\pi G\, T_{\lambda}^{\lambda})$$

$$\rightarrow \quad \partial^2 f = -\frac{8\pi G}{6} T_{\lambda}^{\lambda}$$

Brane bending couples to T_{λ}^{λ}.

μ-ν: $\frac{1}{A^2}\partial^2 h^{TT}_{\mu\nu} - A^{-2}\left[A^4 (A^{-2} h^{TT}_{\mu\nu})'\right]' = -16\pi G \left[T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu}\right] \delta(z)$

Find propagator:

$$h^{TT}_{\mu\nu} = \underbrace{u_m(z)}\, \epsilon_{\mu\nu}\, e^{ip\cdot x}$$

$$m^2 = p^2$$

$$m^2 u_m + \left[e^{-4|z|/\ell} \left(e^{2|z|/\ell} u_m\right)'\right]' = 0$$

Solved by

$$u_m = \frac{\sqrt{m\ell}}{2}\left[J_1(m\ell) N_2(mb) - J_2(mb) N_1(m\ell)\right]$$

$$\sqrt{J_1(m\ell)^2 + N_1(m\ell)^2}$$

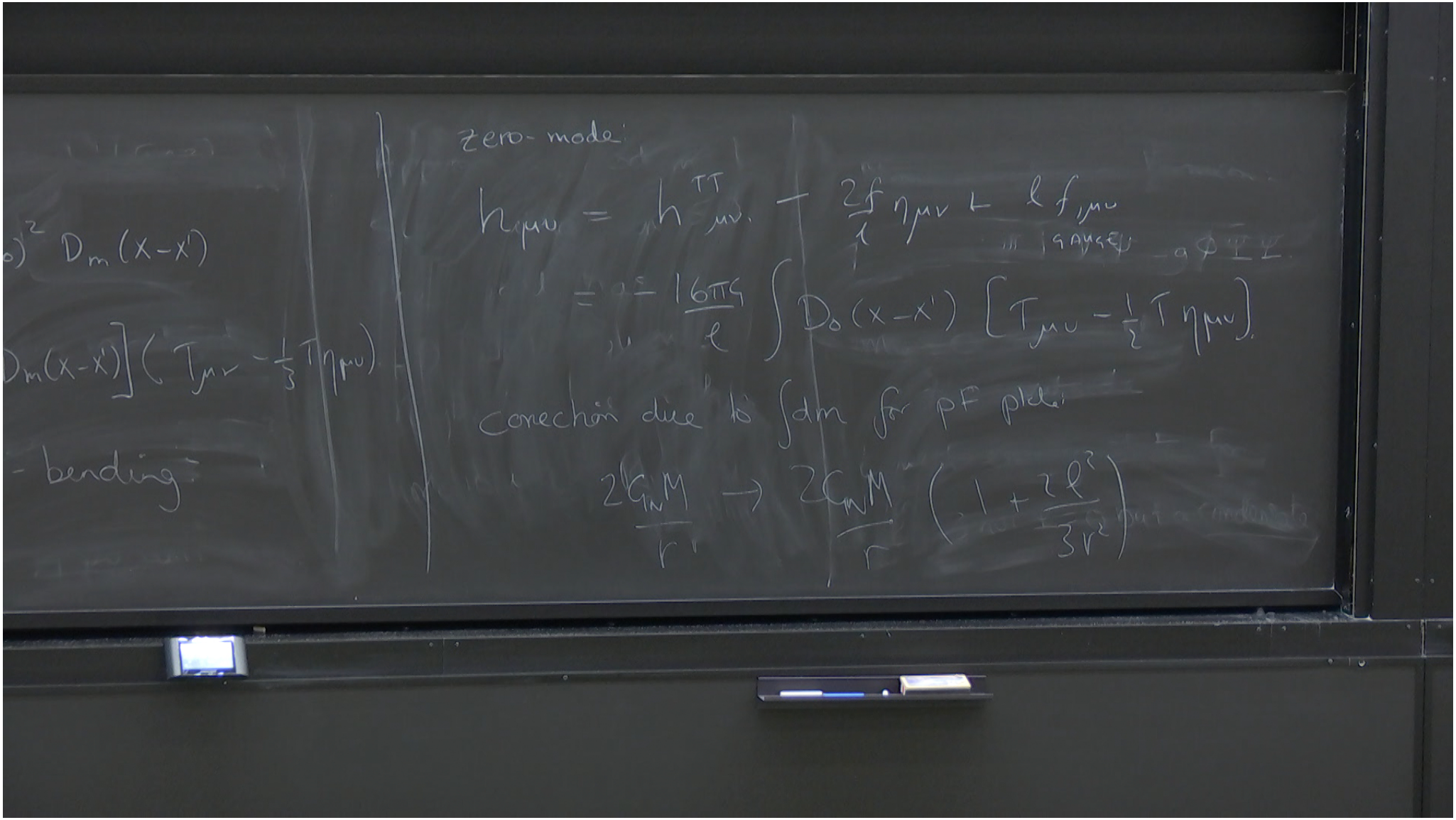
$$\left(b = \ell e^{|z|/\ell}\right)$$

Propagator on brane for matter in brane:

$$G_R(x,0;x',0) = \frac{1}{\ell} D_0(x-x') + \int_0^\infty dm\, u_m(0)^2 D_m(x-x')$$

$$h^{TT}_{\mu\nu} = -\frac{16\pi G}{\ell} \int \left[D_0(x-x') + \int dm\, u_m^2(0) D_m(x-x') \right] \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu} \right)$$

? tensor structure but remember brane-bending


zero-mode:
$h_{\mu\nu} = h^{TT}_{\mu\nu} + \frac{2f}{\ell}\eta_{\mu\nu} + \ell f_{\mu\nu}$
$= -\frac{16\pi G}{\ell}\int D_0(x-x')\left[T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}\right]$
$\frac{2G_N M}{r} \to \frac{2G_N M}{r}\left(1 + \frac{2\ell^2}{3r^2}\right)$