Title: PSI 2019/2020 - Gravitational Physics - Lecture 12

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LECTURE 12 KK black holes

Recall $ds^2 = e^{-\sqrt{3}\phi} g_{\mu\nu} dx^\mu dx^\nu - e^{2\sqrt{3}\phi}[d\chi + \underline{A}]^2$

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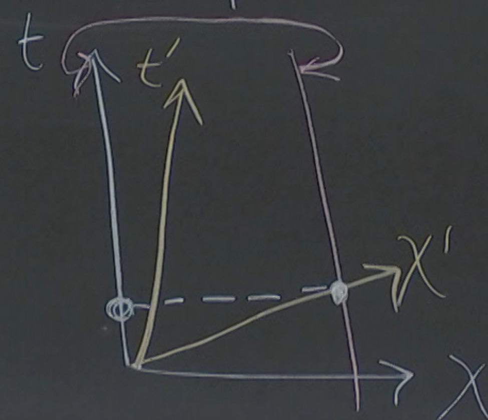
Recall $ds^2 = e^{-\sqrt{3}\phi} g_{\mu\nu} dx^\mu dx^\nu - e^{2\sqrt{3}\phi}[d\chi + \underline{A}]^2$

Note that compactification picks a rest frame.



$\chi \sim \chi + L$

χ at const t

$$t' = \gamma(t - vx)$$
$$x' = \gamma(x - vt)$$

$\therefore \quad x \sim x + L$ at const t

$$t' \sim t' - v\gamma L$$
$$x' \sim x' + \gamma L$$

We cannot "gauge away" A_μ.

Build a black hole from a soln of 5D gravity: SCH $\times \mathbb{R}$

$$ds^2 = \left(1 - \frac{r_4}{r}\right)dt^2 - \frac{dr^2}{1-\frac{r_4}{r}} - r^2 d\Omega_{4}^2 - dz^2$$

$r_4 = 2G_4 M$

Black String

Lorentz boost:

$$ds^2 = \left(1 - \frac{r_4}{r}\right)\frac{(dt + v\,dz)^2}{1-v^2} - \frac{(dz + v\,dt)^2}{1-v^2}$$

$$- \left(1 - \frac{r_4}{r}\right)^{-1} dr^2 - r^2 d\Omega_{II}^2$$

Identify $z \sim z + L$ at const t.

$$g_{zz} = \frac{1}{1-v^2}\left[-1 + v^2 - \frac{r_4 v^2}{r}\right] = -\left[1 + \frac{r_4 v^2}{(1-v^2)\,r}\right].$$

$$ds^2 = \left(1 - \frac{r_4}{(1-v^2)r + r_4 v^2}\right) dt^2 - \frac{dr^2}{1 - r_4/r} - r^2 d\Omega_{II}^2 - \left(1 + \frac{r_4 v^2}{(1-v^2)r}\right)\left[dz + \frac{v r_4 dt}{(1-v^2)r + r_4 v^2}\right]^2 .$$

Now write $\hat{r} = r + \frac{r_4 v^2}{1-v^2}$

$$Q = \frac{r_4 v}{1-v^2} = v \cdot 2G\hat{M}$$

$$2G\hat{M} = \frac{r_4}{1-v^2}$$

$$\underline{A} = \frac{Q}{\hat{r}} dt \qquad e^{2\sqrt{3}\phi} = \frac{\hat{r}}{\hat{r} - vQ}$$

$$g_{\mu\nu} dx^\mu dx^\nu = \left(1 - \frac{2G\hat{M}}{\hat{r}}\right)\left(1 - \frac{vQ}{\hat{r}}\right)^{-1/2} dt^2 - \left(1 - \frac{2G\hat{M}}{\hat{r}}\right)^{-1}\left(1 - \frac{vQ}{\hat{r}}\right)^{1/2} d\hat{r}^2 - \hat{r}^2\left(1 - \frac{vQ}{\hat{r}}\right)^{3/2} d\Omega_{II}^2$$

$$g_{zz} = \frac{1}{1-v^2}\left[-1+v-\frac{r_4 v^2}{r}\right] = -\left[1+\frac{r_4 v^2}{(1-v^2)r}\right].$$

$$g_{\mu\nu}dx^\mu dx^\nu = \left(1-\frac{2G\hat{M}}{\hat{r}}\right)($$

Here $g_{tt} = -1/g_{rr}$, but area of S^2 is not $4\pi\hat{r}^2$.

Extremal limit is $v \to 1$, $Q = 2G\hat{M}$

This is a singular limit.

<u>Magnetic Soln:</u>

In 4D have RN soln.

$$g_{tt} \to 1 - \frac{2GM}{r} + \frac{Q^2}{r^2}.$$

Since $T_{EM\,\mu\nu} = -F_{\mu\lambda}F_\nu{}^\lambda + \frac{1}{4}F^2 g_{\mu\nu}$.

inv't under $F \to *F$. $\frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$

$+\frac{1}{4}F^2 g_{\mu\nu}$

$\frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$

$$\frac{Q}{r^2}\, dt \wedge dr \longleftrightarrow \underline{Q \sin\theta \, d\theta \wedge d\varphi}$$

$F_{\theta\varphi}$ is a <u>magnetic monopole</u>

$(B)_i = (*F)_i \qquad F_{\theta\varphi} \longleftrightarrow B_r$

$dF = 0$! $\qquad F \propto$ area form on S^2

Can we write $\underline{F} = \underline{d}\,\underline{A}$?

$$\underline{A}_{N} = Q\,(1-\cos\theta)\,\underline{d}\varphi$$

regular at $\theta = 0$, but not at $\theta = \pi$.

however,

$$\underline{A}_{S} = -Q\,(1+\cos\theta)\,\underline{d}\varphi$$

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Locally, can integrate $\underline{E}$,

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$$\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi$$

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$)\,\underline{d}\varphi$

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$$\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi$$

From KK perspective, look

for a soln with $[d\varphi_{N,S} + A_{N,S}]^2$.

The so

$ds^2 = \Big($

grate $\underline{F}$,

$\underline{d}\,\varphi$

hie, look

$\ldots_{N,S} + A_{N,S}\Big]^2.$

The soln is.

$$ds^2 = \left(\frac{r-r_+}{r-r_-}\right)dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - r(r-r_-)\,d\Omega_{\mathrm{II}}^2$$
$$-\left(1-\frac{r_-}{r}\right)\Big[d\psi_{N,S} \pm \underline{A}_{N,S}\Big]^2.$$

$$Q^2 = r_+ r_-$$

Patches for $\underline{A}$ translate into coord charts for $\psi_N < \pi$ & $\psi_S > 0$.

From defn of manifold, different coord systems have to be related by C^∞ coord transfm.

$$\psi_S = \psi_N + 2Q\varphi = \psi_N + 2Q(\varphi + 2\pi)$$

i.e. $\psi \sim \psi + \frac{4\pi Q}{n}$.

i.e. Q quantized for fixed $\Delta\psi$.

t

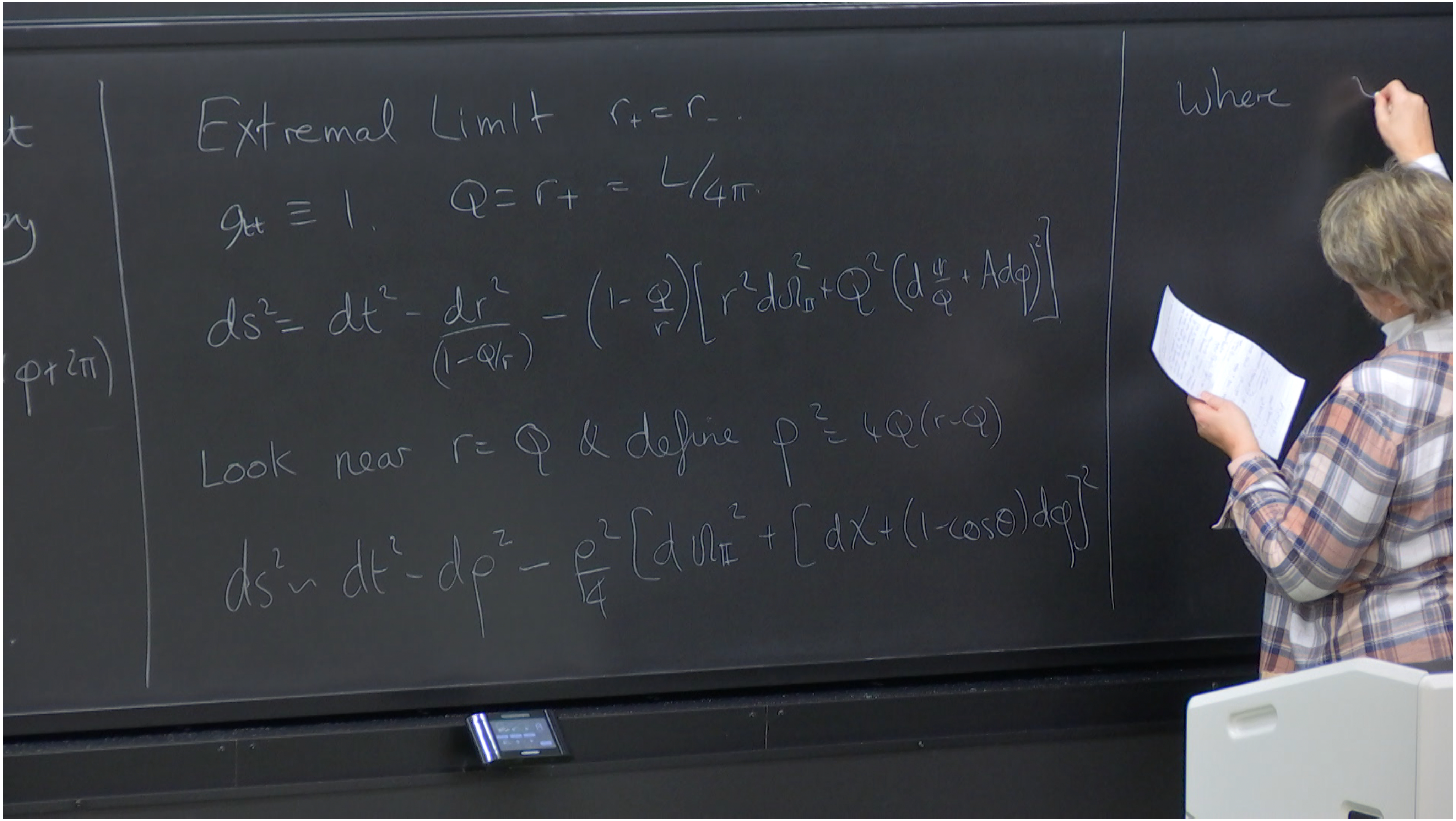
ny

$\varphi + 2\pi)$

Extremal Limit $r_+ = r_-$.

$g_{tt} \equiv 1$. $\quad Q = r_+ = L/4\pi$

$$ds^2 = dt^2 - \frac{dr^2}{(1-Q/r)} - \left(1-\frac{Q}{r}\right)\left[r^2 d\Omega_{II}^2 + Q^2\left(d\frac{\psi}{Q} + A\,d\varphi\right)^2\right]$$


Extremal Limit $r_+ = r_-$.
$g_{tt} \equiv 1$. $Q = r_+ = L/4\pi$
$ds^2 = dt^2 - \frac{dr^2}{(1-Q/r)} - \left(1-\frac{Q}{r}\right)\left[r^2 d\Omega_{II}^2 + Q^2\left(d\frac{\psi}{Q} + A\,d\phi\right)^2\right]$
Look near $r = Q$ & define $\rho^2 = 4Q(r-Q)$
$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\left[d\Omega_{II}^2 + [d\chi + (1-\cos\theta)d\phi]^2\right]$
Where

for a ... $[d\varphi_{N,S} \ldots A_{N,S}]$

Charts for $\varphi_N < \pi$ & $\varphi_S > 0$.

$2\pi)$

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Look near $r = Q$ & define $\rho^2 = 4Q(r-Q)$

$$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\underbrace{\left[d\Omega_{II}^2 + \left[d\chi + (1-\cos\theta)\,d\varphi\right]^2\right]}_{\text{METRIC ON } S^3}$$

Where $\chi = \psi/Q$, $\Delta\chi = 4\pi$.

$$x + iy = \rho\cos\theta/2\, e^{i\chi/2}$$

$$z + iw = \rho\sin\theta/2\, e^{i(\varphi + \chi/2)}$$

$$x^2 + y^2 + z^2 + w^2 = \rho^2$$

S^3

i.e. Q quantized for fixed $\Delta\psi$.

$$ds^2 \sim dt^2 - d\rho^2 - \frac{\rho^2}{4}\Big[d\Omega_{II}^2 + \big[d\chi + (1-\cos$$

METRIC ON S^3

Ext limit is nonsingular

S^3 is a Hopf fibration of S^2 by S^1.

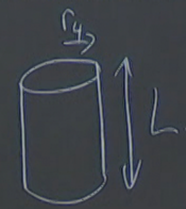
S^3 is a group manifold.

2 copies of $SO(3) \sim SU(2)$ acting on it

Gross-Perry-Sorkin.

? Are these solns stable?

SCH $\times \mathbb{R}$.

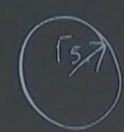


$$M_{BS} = \frac{r_4 L}{2G_5}$$

$$S_{BS} = \frac{4\pi r_4^2 L}{4G_5} = \frac{4\pi M^2 G_5}{L}$$

$$M = \frac{3\times 2\pi^2 r_5^2}{16\pi G_5} = \frac{3\pi r_5^2}{8G_5}$$

SCH_5



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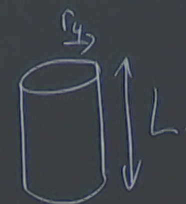

$$M_{BS} = \frac{r_4 L}{2G_5}$$

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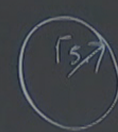
SCH$_5$



$$M = \frac{3\times 2\pi^2 r_5^2}{16\pi G_5} = \frac{3\pi r_5^2}{8G_5}$$

$$S_{BH} = \frac{2\pi^2 r_5^3}{4G_5} = \frac{\pi^2}{2G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

METRIC ON S^3

ble?

$$= \frac{r_4 L}{2G_5}$$

$$\frac{4\pi r_4^2 L}{4G_5} = \frac{4\pi M^2 G_5}{L}$$

$$\times 2\pi^2 r_5^2 \over 16\pi G_5$$

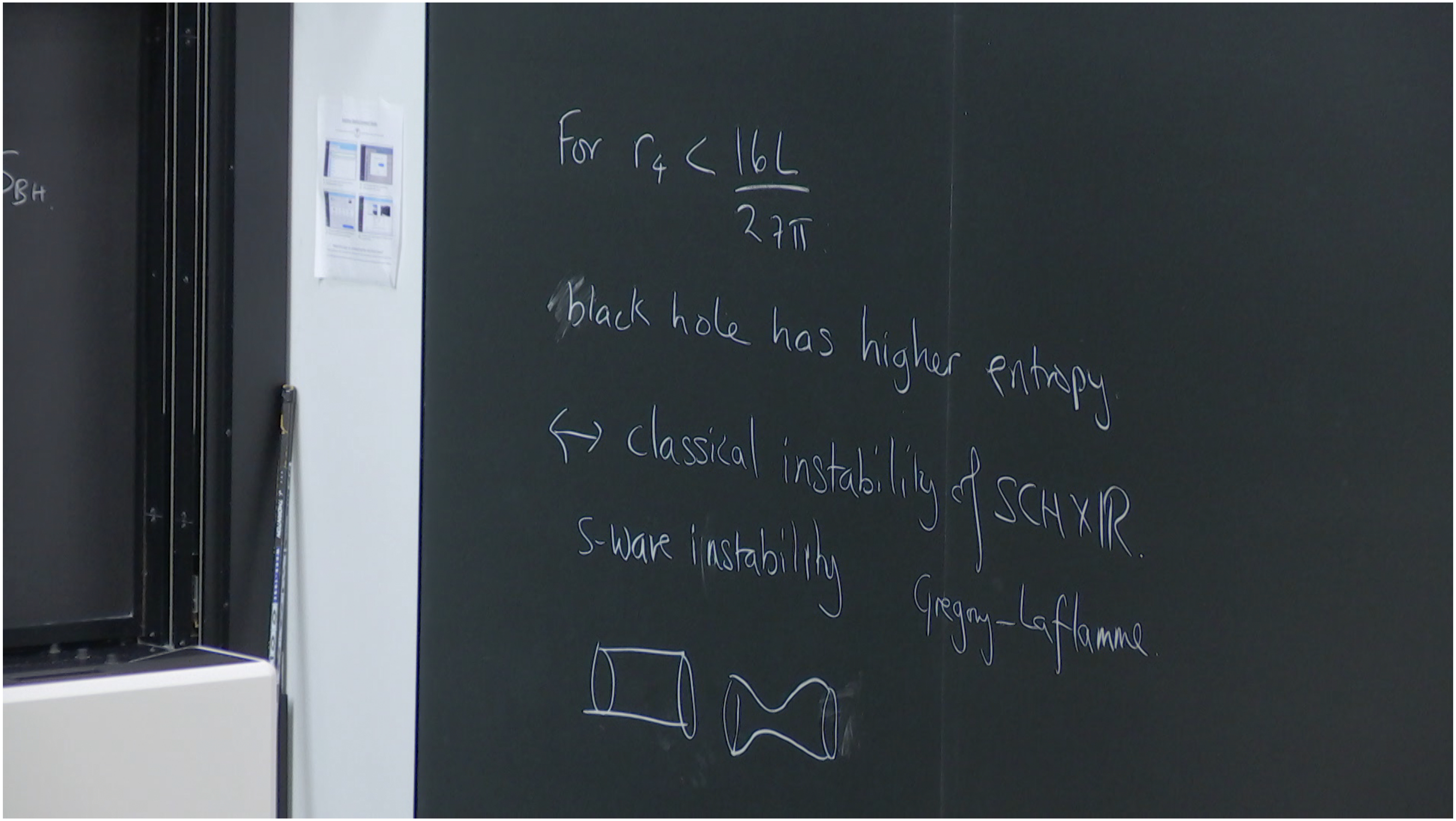
$$= \frac{3\pi r_5^2}{8G_5}$$

$$\frac{2\pi^2 r_5^3}{4G_5} = \frac{\pi^2}{2G_5}\left(\frac{8G_5 M}{3\pi}\right)^{3/2}$$

Compare entropies.

$$S_{BS} = \frac{4\pi M^2 G_5}{L} = \frac{3\sqrt{3\pi}}{2\sqrt{2}}\sqrt{\frac{G_5 M}{L}}\, S_{BH}$$

$$= \sqrt{\frac{27\pi}{8}}\sqrt{\frac{r_4}{2L}}\, S_{BH}$$


For r4 < 16L/27π
black hole has higher entropy
↔ classical instability of SCH × ℝ
S-wave instability
Gregory–Laflamme