Title: PSI 2019/2020 - Gravitational Physics - Lecture 11

Speakers: Ruth Gregory

Collection: PSI 2019/2020 - Gravitational Physics

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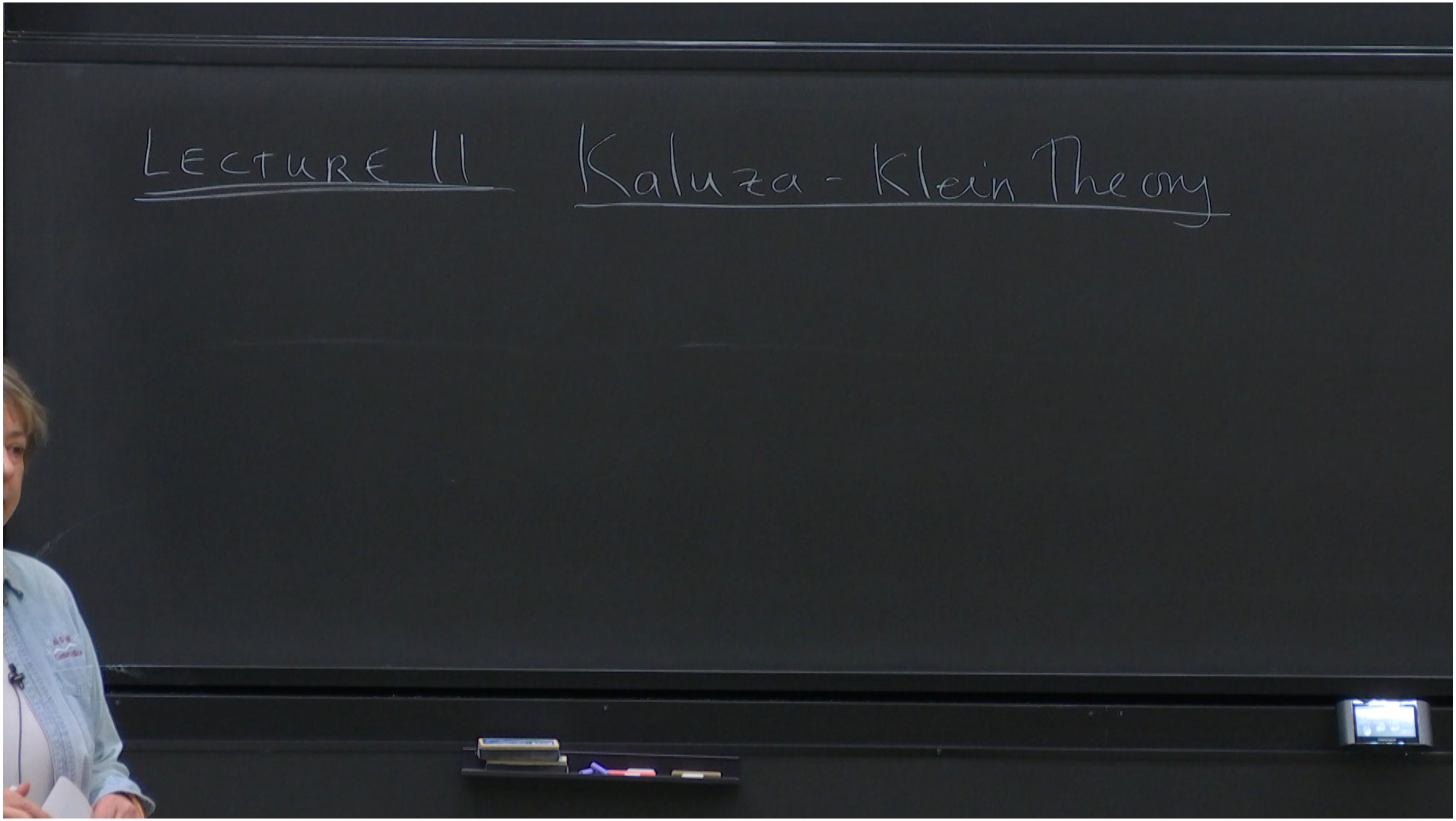
LECTURE II
Kaluza - Klein Theory

LECTURE 11 Kaluza-Klein Theory

We should explore extra dims, but keep in mind that spacetime "looks" 4D.

KK achieves this by having an extra dim curled up on a small scale: "$ds_5^2 = ds_4^2 + L^2 d\theta^2$"

LECTURE 11

Kaluza-Klein Theory

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Consider a field on this spacetime & expand in Fourier modes:

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Then a 5D massless field
becomes:

$$\nabla_5^2 \Psi = e^{2\pi i\theta/L}\left[\Box_4 \psi_0 + \frac{(2\pi)^2}{L^2}\psi_0\right]$$

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a 5D massless field

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$$\partial_5^2 \Psi = e^{2\pi i\theta}\left[\Box_4 \psi_0 + \frac{(2\pi)^2}{L^2}\psi_0\right]$$

$\Delta\theta = 1$ (sorry!)

Length of extra dim is L.

Ψ has an effective mass $\propto 1/L$.

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Low energy physics does not depend on extra dimension

i.e. $\dfrac{\partial}{\partial X}$ is a Killing vector $(\theta \to X)$

g_{ab} is indep of X

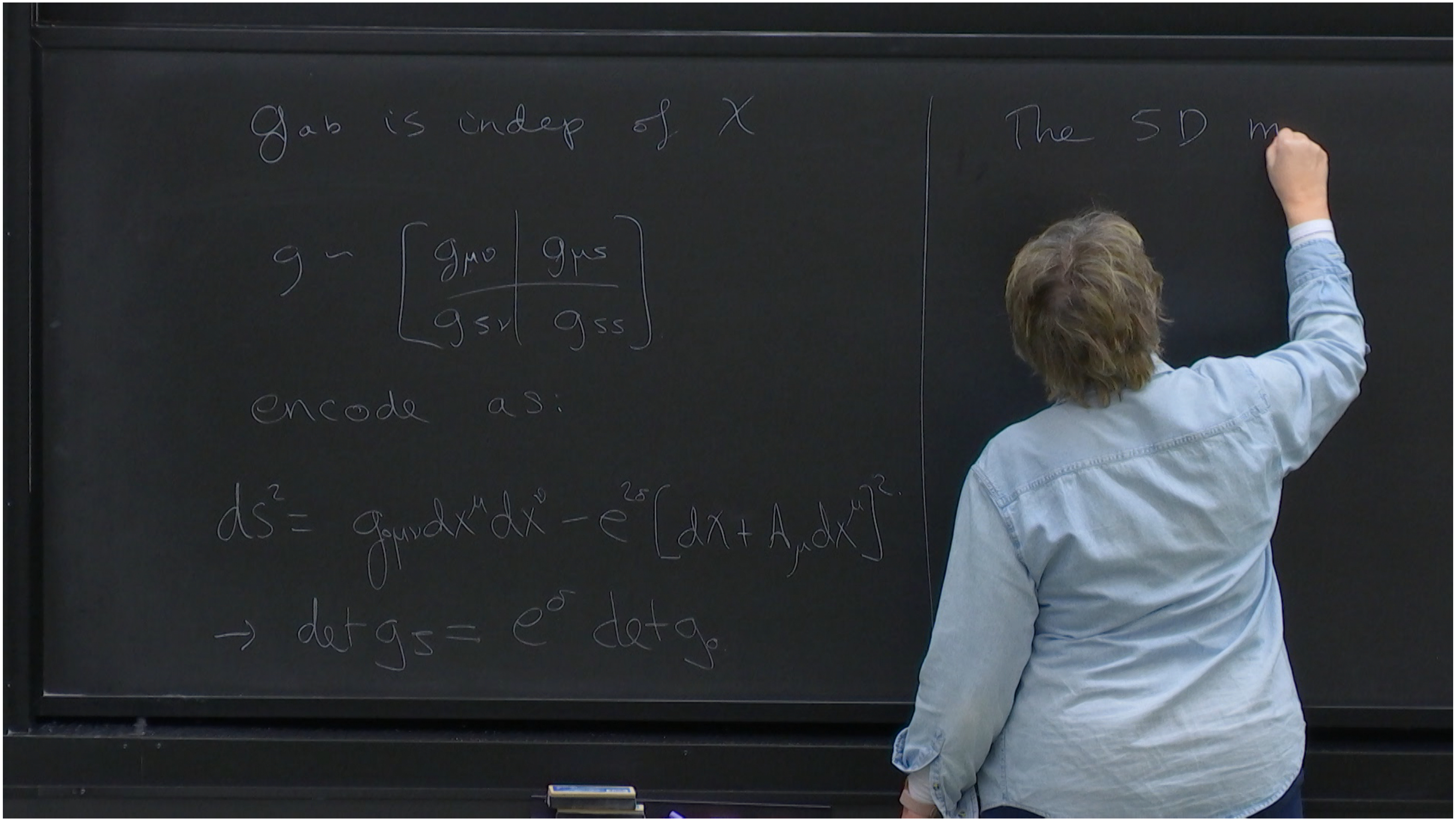
$$g \sim \left[\begin{array}{c|c} g_{\mu\nu} & g_{\mu s} \\ \hline g_{s\nu} & g_{ss} \end{array}\right]$$

g_{ab} is indep of χ

$$g \sim \left[\begin{array}{c|c} g_{\mu\nu} & g_{\mu s} \\ \hline g_{s\nu} & g_{ss} \end{array}\right]$$

encode as:

$$ds^2 = g_{0\mu\nu} dx^\mu dx^\nu - e^{2\sigma}\left[d\chi + A_\mu dx^\mu\right]^2.$$



g_{ab} is indep of χ
$g \sim \left[\begin{array}{c|c} g_{\mu\nu} & g_{\mu 5} \\ \hline g_{5\nu} & g_{55} \end{array}\right]$
encode as:
$ds^2 = g_{0\mu\nu} dx^\mu dx^\nu - e^{2\sigma}\left[d\chi + A_\mu dx^\mu\right]^2$
$\rightarrow \det g_5 = e^{\sigma} \det g_0$
The 5D m

$dx^\mu]^2$.

The 5D metric splits

into 4D $g_{\mu\nu}$, A_μ, σ

$\swarrow$ spin-2. $\downarrow$ vector $\searrow$ scalar.

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To compute 4D effective

theory, must see how 5D

Einstein-Hilbert action decomposes.

Use Cartan - example of off-diagonal terms.

- $\underset{\sim}{\omega}^{\hat{a}} = e_0{}^{\hat{a}}{}_\mu \underline{d} x^\mu$
- $\underline{d}\underset{\sim}{\omega}^{\hat{a}} = -\underset{\sim}{\Omega}_0{}^{\hat{a}}{}_{\hat{b}} \wedge \underset{\sim}{\omega}_0{}^{\hat{b}}$

$\underset{\sim}{\omega}^{\hat{\chi}} = e^{\sigma}[d\chi + A_\mu dx^\mu]$

$\underline{d}\underset{\sim}{\omega}^{\hat{\chi}} = \sigma_{,\hat{a}}\, \underset{\sim}{\omega}^{\hat{a}} \wedge \underset{\sim}{\omega}^{\hat{\chi}} + e^{\sigma}\, \underline{d}\underline{A}$

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$$= - \underset{\sim}{\Theta}^{\hat{\chi}}{}_{\hat{a}} \wedge \underset{\sim}{\omega}^{\hat{a}}$$

$$\Rightarrow \underset{\sim}{\Theta}^{\hat{\chi}}{}_{\hat{a}} = \sigma_{\hat{a}} \underset{\sim}{\omega}^{\hat{\chi}} + \frac{1}{2} e^\sigma F_{\hat{a}\hat{b}} \underset{\sim}{\omega}^{\hat{b}}$$

Use Cartan - example of off-diagonal terms.

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$$= -\underset{\sim}{\Theta}^{\hat a}{}_{\hat b} \wedge \underset{\sim}{\omega}^{\hat b} - \underset{\sim}{\Theta}^{\hat a}{}_{\hat \chi} \wedge \underset{\sim}{\omega}^{\hat \chi}$$

$$\underset{\sim}{\omega}^{\hat\chi} = e^\sigma \left[d\chi + A_\mu dx^\mu \right]$$

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$$= -\underline{\Theta}^{\hat{a}}{}_{\hat{b}} \wedge \underline{\omega}^{\hat{b}} - \underline{\Theta}^{\hat{a}}{}_{\hat{\chi}} \wedge \underline{\omega}^{\hat{\chi}}$$

$$\underline{\omega}^{\hat{\chi}} = e^{\sigma}\left[d\chi + A_{\mu} dx^{\mu}\right]$$

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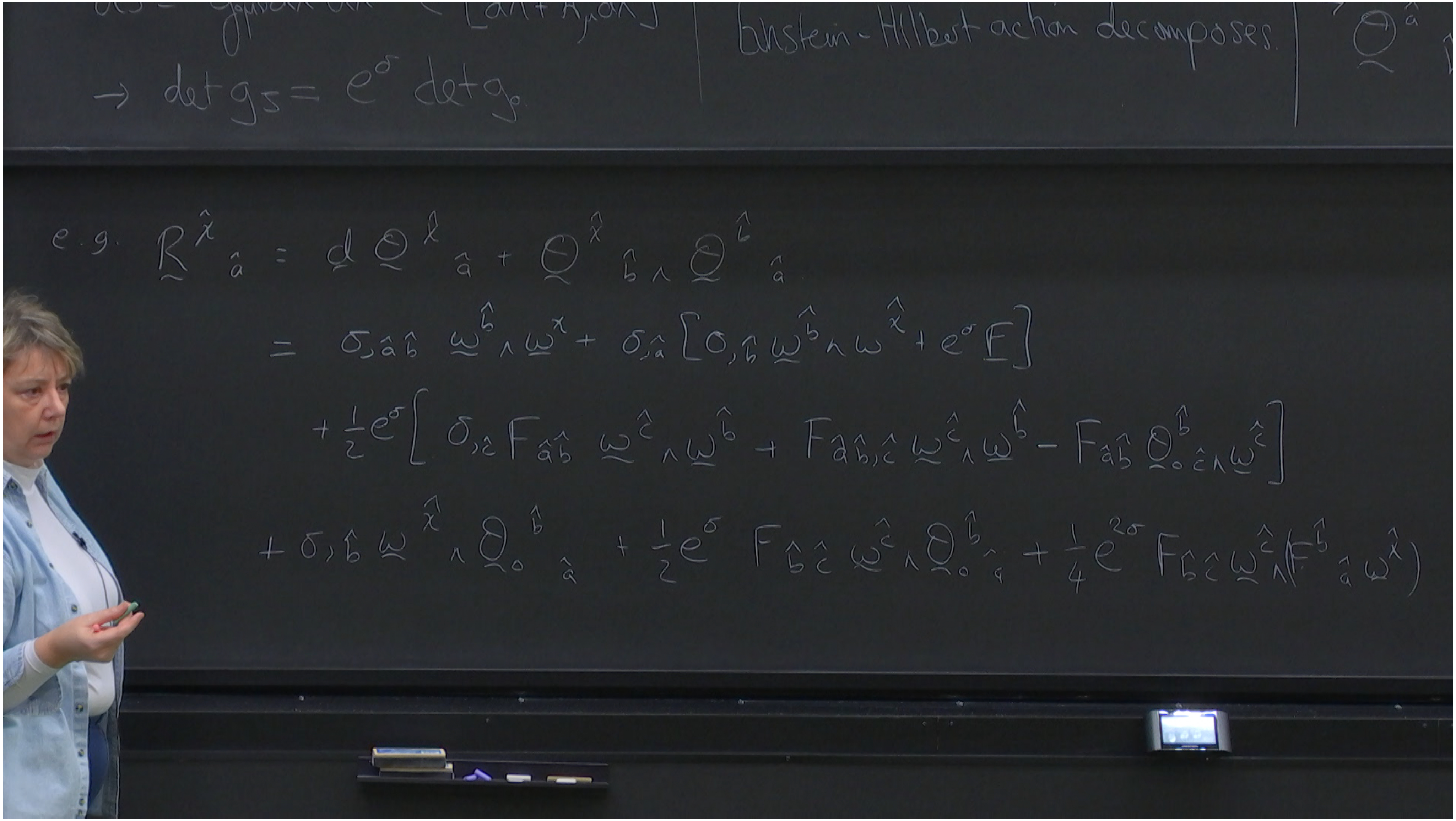
$$\Rightarrow \underline{\Theta}^{\hat{a}}{}_{\hat{b}} = \underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}} + \frac{1}{2} e^{\sigma} F^{\hat{a}}{}_{\hat{b}}\, \underline{\omega}^{\hat{\chi}}$$

$\rightarrow \det g_5 = e^{\sigma} \det g_0$

e.g. $R^{\hat{x}}{}_{\hat{a}} = d\theta^{\hat{x}}{}_{\hat{a}} + \theta^{\hat{x}}{}_{\hat{b}} \wedge \theta^{\hat{b}}{}_{\hat{a}}$

$$= \sigma_{,\hat{a}\hat{b}}\, \omega^{\hat{b}} \wedge \omega^{x} + \sigma_{,\hat{a}} \left[\sigma_{,\hat{b}}\, \omega^{\hat{b}} \wedge \omega^{\hat{x}} + e^{\sigma} F \right]$$

$$+ \frac{1}{2} e^{\sigma} \left[\sigma_{,\hat{c}} F_{\hat{a}\hat{b}}\, \omega^{\hat{c}} \wedge \omega^{\hat{b}} + F_{\hat{a}\hat{b},\hat{c}}\, \omega^{\hat{c}} \wedge \omega^{\hat{b}} - F_{\hat{a}\hat{b}}\, \theta_{0}{}^{\hat{b}}{}_{\hat{c}} \wedge \omega^{\hat{c}} \right]$$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Einstein-Hilbert action decomposes.
e.g. $R^{\hat{x}}{}_{\hat{a}} = d\Theta^{\hat{x}}{}_{\hat{a}} + \Theta^{\hat{x}}{}_{\hat{b}} \wedge \Theta^{\hat{b}}{}_{\hat{a}}$
$= \sigma_{,\hat{a}\hat{b}}\, \omega^{\hat{b}} \wedge \omega^{x} + \sigma_{,\hat{a}} [\sigma_{,\hat{b}}\, \omega^{\hat{b}} \wedge \omega^{\hat{x}} + e^{\sigma} F]$
$+ \frac{1}{2} e^{\sigma} [\sigma_{,\hat{c}} F_{\hat{a}\hat{b}}\, \omega^{\hat{c}} \wedge \omega^{\hat{b}} + F_{\hat{a}\hat{b},\hat{c}}\, \omega^{\hat{c}} \wedge \omega^{\hat{b}} - F_{\hat{a}\hat{b}}\, \Theta_0{}^{\hat{b}}{}_{\hat{c}} \wedge \omega^{\hat{c}}]$
$+ \sigma_{,\hat{b}}\, \omega^{\hat{x}} \wedge \Theta_0{}^{\hat{b}}{}_{\hat{a}} + \frac{1}{2} e^{\sigma} F_{\hat{b}\hat{c}}\, \omega^{\hat{c}} \wedge \Theta_0{}^{\hat{b}}{}_{\hat{a}} + \frac{1}{4} e^{2\sigma} F_{\hat{b}\hat{c}}\, \omega^{\hat{c}} \wedge (F^{\hat{b}}{}_{\hat{a}}\, \omega^{\hat{x}})$

$\to \det g_5 = e^{\sigma} \det g_0$

Einstein-Hilbert action decomposes.

$\underset{\sim}{\Theta}^{\hat{}}{}_{\hat{a}}$

e.g.

$$\underset{\sim}{R}^{\hat{x}}{}_{\hat{a}} = \underset{\sim}{d}\,\underset{\sim}{\Theta}^{\hat{x}}{}_{\hat{a}} + \underset{\sim}{\Theta}^{\hat{x}}{}_{\hat{b}} \wedge \underset{\sim}{\Theta}^{\hat{b}}{}_{\hat{a}}$$

$$= \sigma_{,\hat{a}\hat{b}}\, \underset{\sim}{\omega}^{\hat{b}} \wedge \underset{\sim}{\omega}^{x} + \sigma_{,\hat{a}}\left[\sigma_{,\hat{b}}\, \underset{\sim}{\omega}^{\hat{b}} \wedge \omega^{\hat{x}} + e^{\sigma} \underset{\sim}{F}\right]$$

$$+ \frac{1}{2} e^{\sigma}\left[\sigma_{,\hat{c}} F_{\hat{a}\hat{b}}\, \underset{\sim}{\omega}^{\hat{c}} \wedge \underset{\sim}{\omega}^{\hat{b}} + F_{\hat{a}\hat{b},\hat{c}}\, \underset{\sim}{\omega}^{\hat{c}} \wedge \underset{\sim}{\omega}^{\hat{b}} - F_{\hat{a}\hat{b}}\, \underset{\sim}{\Theta}_0{}^{\hat{b}}{}_{\hat{c}} \wedge \underset{\sim}{\omega}^{\hat{c}}\right]$$

$$+ \sigma_{,\hat{b}}\, \underset{\sim}{\omega}^{\hat{x}} \wedge \underset{\sim}{\Theta}_0{}^{\hat{b}}{}_{\hat{a}} + \frac{1}{2} e^{\sigma} F_{\hat{b}\hat{c}}\, \underset{\sim}{\omega}^{\hat{c}} \wedge \underset{\sim}{\Theta}_0{}^{\hat{b}}{}_{\hat{a}} + \frac{1}{4} e^{2\sigma} F_{\hat{b}\hat{c}}\, \underset{\sim}{\omega}^{\hat{c}} \wedge \left(F^{\hat{b}}{}_{\hat{a}}\, \omega^{\hat{x}}\right)$$

$$= \Big[\sigma_{,\hat{a}\hat{b}} + \sigma_{,\hat{a}}\sigma_{,\hat{b}} - \sigma_{,\hat{c}}\Gamma_{0\hat{b}\hat{a}}^{\hat{c}}$$

$$+ \frac{1}{4} e^{2\sigma} F_{\hat{c}\hat{b}} F^{\hat{c}}{}_{\hat{a}} \Big] \underline{\omega}^{\hat{b}} \wedge \underline{\omega}^{\hat{a}}$$

$$+ \frac{1}{2} e^{\sigma}\, \underline{\omega}^{\hat{b}} \wedge \underline{\omega}^{\hat{c}} \Big[\sigma_{,\hat{a}} F_{\hat{b}\hat{c}} - \sigma_{,\hat{c}} F_{\hat{a}\hat{b}} - F_{\hat{a}\hat{b},\hat{c}}$$

$$- F_{\hat{a}\hat{d}} \Gamma^{\hat{d}}_{\hat{b}\hat{c}} + F_{\hat{d}\hat{b}} \Gamma^{\hat{d}}_{\hat{c}\hat{a}} \Big]$$

$$\hat{a}\, \omega^{\hat{\lambda}})$$

Need $R^{\hat{x}\hat{a}}{}_{\hat{x}\hat{b}} = -\nabla^{\hat{a}}\nabla_{\hat{b}}\sigma - \nabla^{\hat{a}}\sigma\,\nabla_{\hat{b}}\sigma - \frac{e^{2\sigma}}{4}F^{\hat{a}\hat{c}}F_{\hat{b}\hat{c}}$ (A.I.N.)

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A similar board's-worth of computation (see notes) gives

$$R^{\hat{a}\hat{b}}{}_{\hat{c}\hat{d}} = R_0{}^{\hat{a}\hat{b}}{}_{\hat{c}\hat{d}} + \frac{1}{4} e^{2\sigma}\left(F^{\hat{a}}{}_{\hat{c}} F^{\hat{b}}{}_{\hat{d}} - F^{\hat{a}}{}_{\hat{d}} F^{\hat{b}}{}_{\hat{c}} + 2F^{\hat{a}\hat{b}} F_{\hat{c}\hat{d}}\right)$$

Need $R^{\hat{\chi}\hat{a}}{}_{\hat{\chi}\hat{b}} = -\nabla^{\hat{a}}\nabla_{\hat{b}}\sigma - \nabla^{\hat{a}}\sigma\,\nabla_{\hat{b}}\sigma - \frac{e^{2\sigma}}{4}F^{\hat{a}\hat{c}}F_{\hat{b}\hat{c}}$ (A. I. N.)

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$$\left.\begin{array}{l} \Rightarrow R^{\hat{\chi}}{}_{\hat{\chi}} = -\Box\sigma - (\partial\sigma)^2 - \frac{1}{4}e^{2\sigma}F^2 \\ R^{\hat{a}}{}_{\hat{b}} = R_0{}^{\hat{a}}{}_{\hat{b}} + \frac{1}{4}e^{2\sigma}\left(2F^{\hat{a}\hat{c}}F_{\hat{b}\hat{c}}\right) - \nabla^{\hat{a}}\nabla_{\hat{b}}\sigma - \sigma^{,\hat{a}}\sigma_{,\hat{b}} \end{array}\right\} \begin{array}{l} R = R_0 - 2\Box\sigma \\ \quad - 2(\nabla\sigma)^2 + \frac{1}{4}e^{2\sigma}F^2 \end{array}$$

(A.I.N.)

gives

$F_{\hat{c}\hat{d}})$

$_0 - 2\Box\sigma$

$(\nabla\sigma)^2 + \frac{1}{4}e^{2\sigma}F^2$

Thus $\sqrt{g_5}\, R_5 = e^{\sigma}\sqrt{g_0}\left[R_0 - 2e^{-\sigma}\Box e^{\sigma} + \frac{1}{4}e^{2\sigma}F^2\right]$

i.e. $S_5 = -\frac{1}{16\pi G_5}\int d^4x \underbrace{dx}_{0\to L} \sqrt{g_0}\left[e^{\sigma}R_0 - \underbrace{\cancel{2\Box e^{\sigma}}}_{\text{BOUNDARY}} + \frac{1}{4}e^{3\sigma}F^2\right]$

$$= -\frac{L}{16\pi G_5}\int d^4x\,\sqrt{g_0}\left[e^{\sigma}R_0 + \frac{1}{4}e^{3\sigma}F^2\right]$$

(A. I. N.)

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Looks like Einstein-Maxwell theory.

For constant σ, this is Einstein-Maxwell, but we need a mechanism to fix σ. But in general σ can vary.

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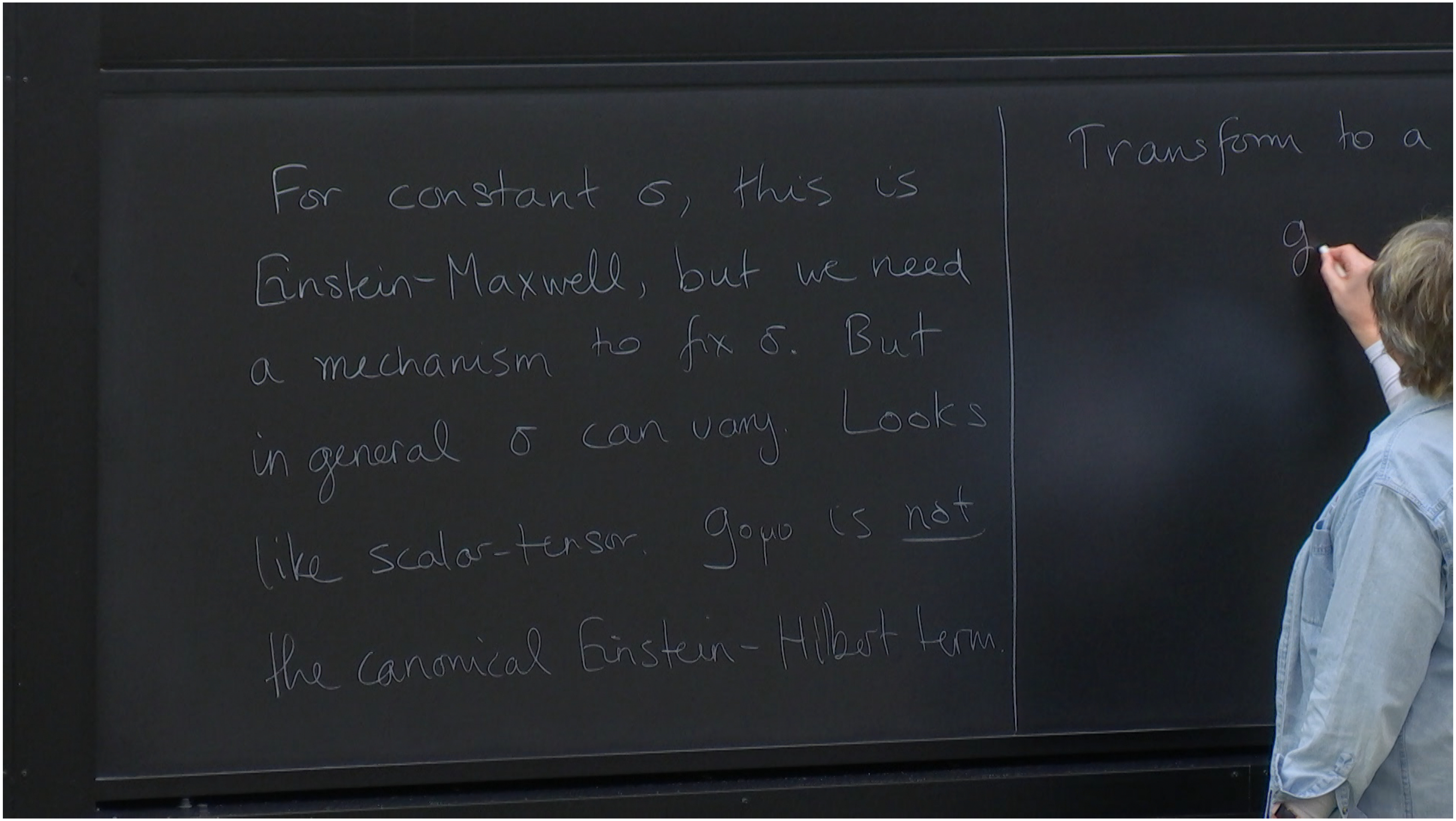
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the canonical Einstein-Hilbert term.
Transform to a
g

Transform to a conformal frame:

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$$R_0 = \Omega^{-2}\left(R - 6\Omega^{-1}\Box\Omega\right)$$

4 DIMS.

ne:

ot

$\Box\Omega)$

$$\sqrt{g_0}\, R_0 = \Omega^4 \sqrt{g}\, \Omega^{-2}\left(R - 6\Omega^{-1}\Box\Omega\right)$$

Set $\Omega = e^{-\sigma/2}$:

$$= e^{-\sigma}\sqrt{g}\left(R + 3\Box\sigma - \frac{3}{2}(\nabla\sigma)^2\right)$$

ne:

nt

$\Box\Omega)$

$$\sqrt{g_0}\, R_0 = \Omega^4 \sqrt{g}\, \Omega^{-2}\left(R - 6\Omega^{-1}\Box\Omega\right)$$

Set $\Omega = e^{-\sigma/2}$ $\therefore$ $= e^{-\sigma}\sqrt{g}\left(R + 3\Box\sigma - \frac{3}{2}(\nabla\sigma)^2\right)$

Finally, redefine $\phi = \sigma/\sqrt{3}$ to canonically normalise scalar:

$$S_5 = \frac{1}{16\pi(G_5/L)} \int d^4x \sqrt{g}\left[-R + \frac{1}{2}(\partial\phi)^2 - \frac{e^{\sqrt{3}\phi}}{4} F^2\right]$$

ume.

not

$\Omega^{-1}\Box\Omega)$

S.

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the canonical Einstein-Hilbert term.

$$ds^2 = e^{-\sqrt{3}\phi} g_{\mu\nu} dx^\mu dx^\nu - e^{2\sqrt{3}\phi} [d\chi + A_\mu dx^\mu]^2$$

Note: G_5 is renormalised: $G_4 = G_N = G_5/L$.

If L is not at Planck scale, we have a hierarchy between G_5 & G_4.

4 DIMS

G_4

Can perform KK reduction for n extra dims.

4 DIMS

$$\underbrace{G_5/L}_{G_4}$$

Can perform KK reduction for n extra dims.

- If internal space has curvature, this introduces a potential for $\sigma \sim e^{\#\sigma} R_{int}$.

4 DIMS $\underbrace{16\pi (G_5/L)}_{G_4}$

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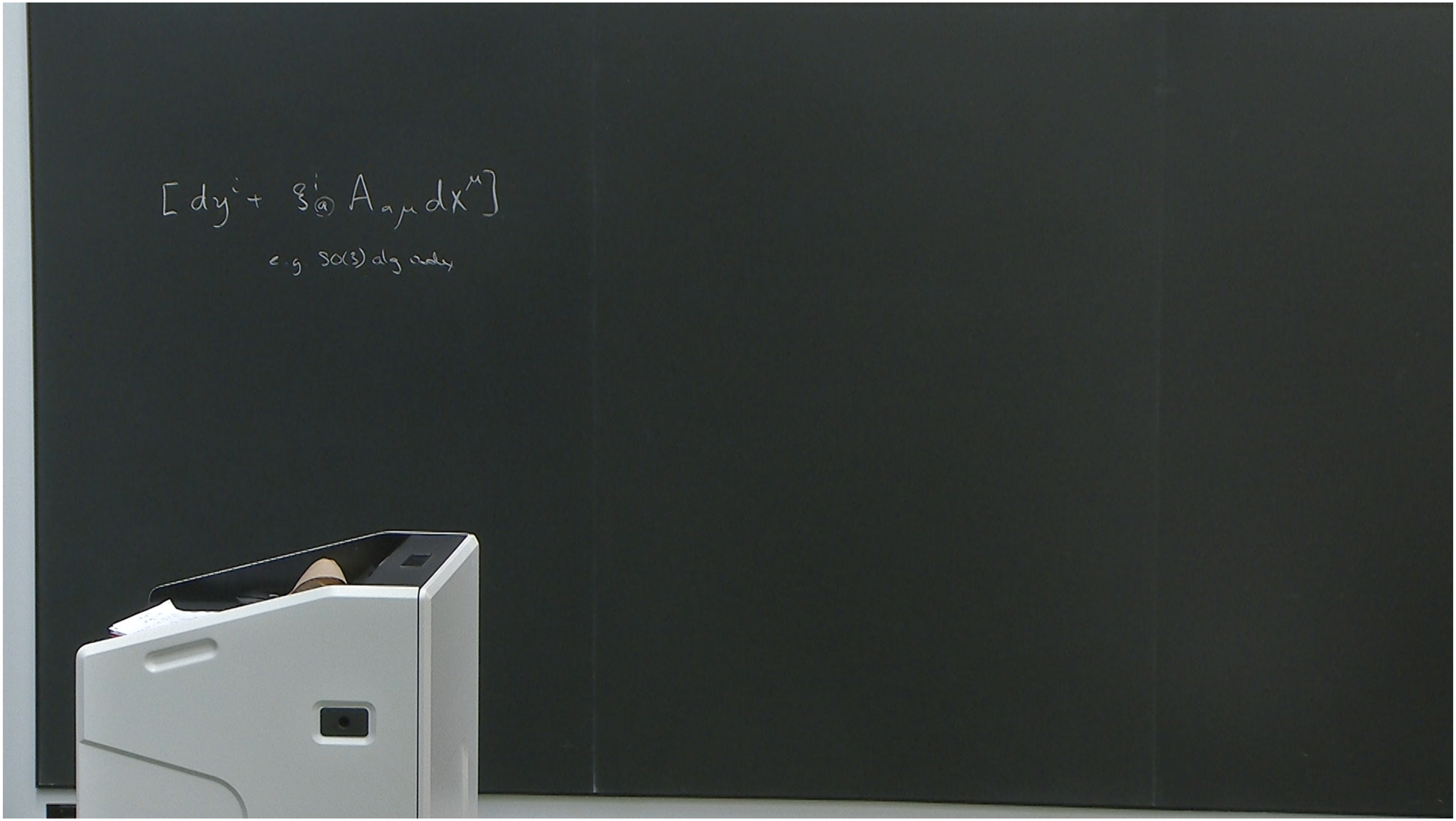
- If internal manifold has algebra of Killing vectors,
-

$16\pi(G_5/L)$ ... $\underbrace{G_5/L}_{G_4}$

4 DIMS.

Can perform KK reduction for n extra dims.

- If internal space has curvature, this introduces a potential for $\sigma \sim e^{\#\sigma} R_{int}$.
- If internal manifold has algebra of Killing vectors, have gauge field for each Killing vector $\rightarrow$ nonabelian gauge grp.


$[dy^i + \xi^i_{a} A_{a\mu} dx^\mu]$
e.g. SO(3) alg