Title: PSI 2019/2020 - Gravitational Physics - Lecture 1

Speakers: Ruth Gregory

Collection: PSI 2019/2020 - Gravitational Physics

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REVIEW OF GRAVITATIONAL PHYSICS

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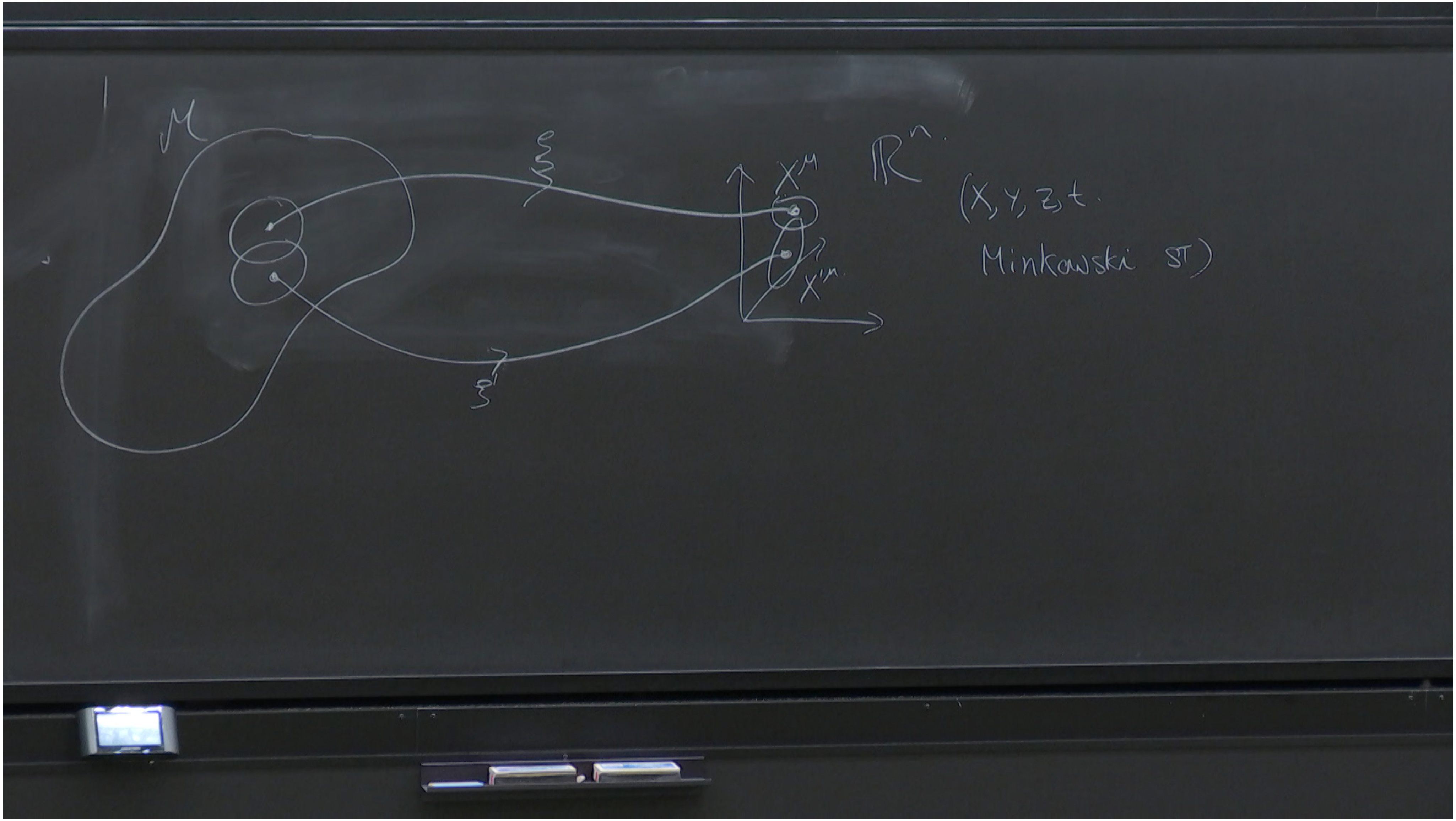
+ David Kubiznak

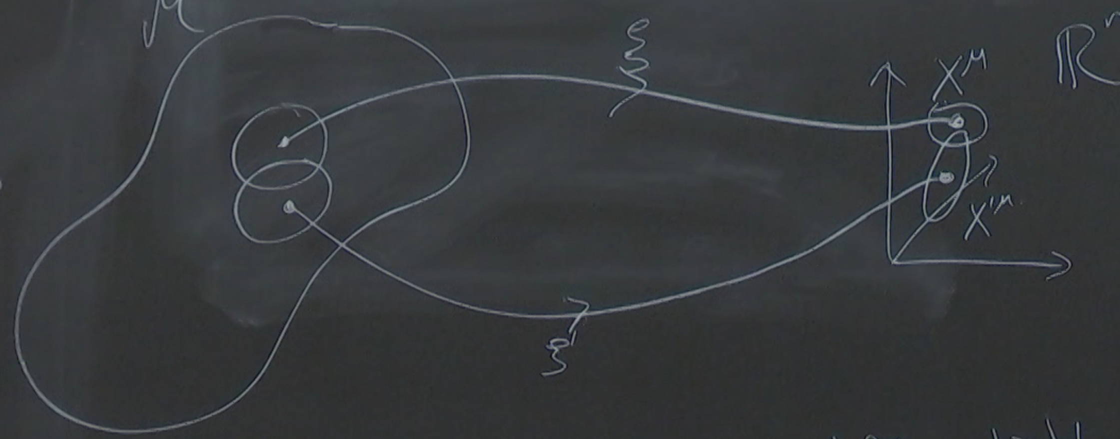
Conventions: $\hbar = c = 1$

Sig: $+ - - -$

$$R^a{}_{bcd} = \Gamma^a_{bd,c} \text{ etc}$$

$$R_{bd} = R^a{}_{bad}$$


M
ξ
ξ'
X^μ
X'^μ
R^n.
(x, y, z, t.
Minkowski ST)



$(x, y, z, t$. Minkowski ST$)$

Consider charts that are ∞'ly differentiable on overlap. i.e. $X'^\mu(X^\nu)$ ∞'ly diff.

LECTURE 1 Differential Geometry Review

A *manifold* (spacetime) is a set of events that looks locally like $\mathbb{R}^n$. The manifold has a covering of open sets (U_i) called charts that map $M \to \mathbb{R}^n$. Give coords for events.

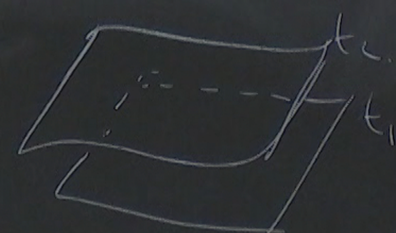
Functions

A C^∞ fn on M is a map

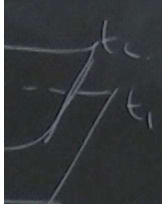
$$f : M \to \mathbb{R}$$

that is C^∞ locally in all charts

e.g. cosmological time



arts

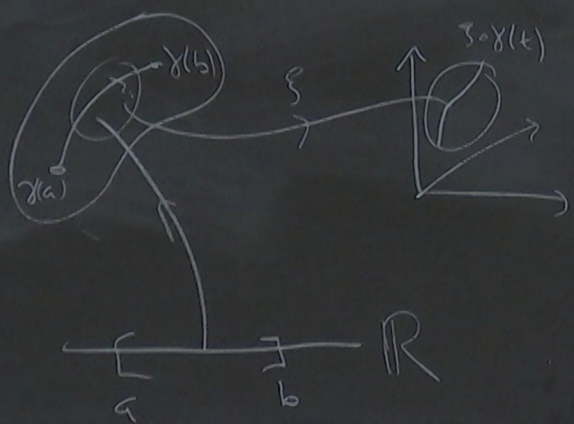


Curves

A C^∞ curve is a map

$$\gamma : \mathbb{R} \to M$$

s.t. image in $\mathbb{R}^n$ is C^∞.



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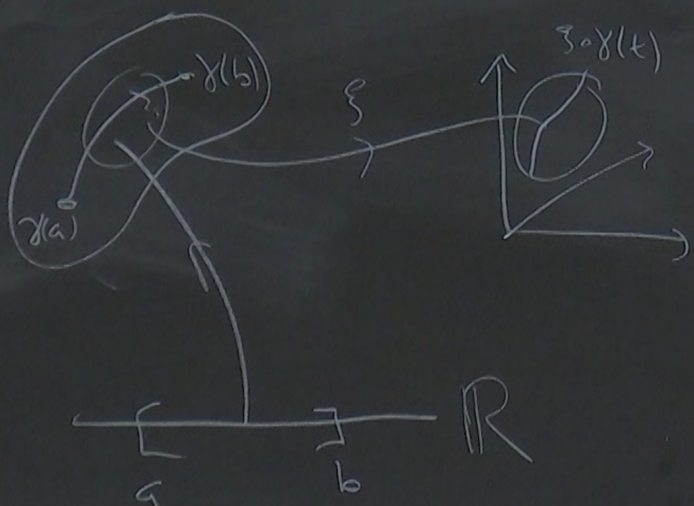
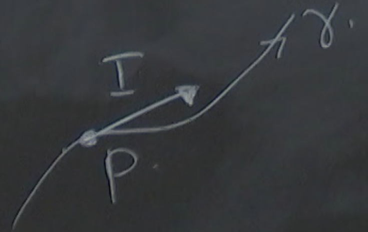


$X^\mu(t)$

for

γ.

__Vectors__ A vector is defined as a linear operator tangent to a curve at P.



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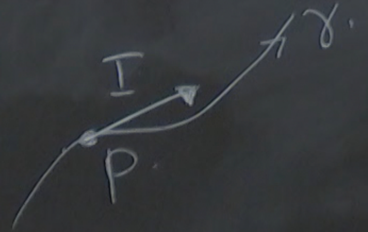
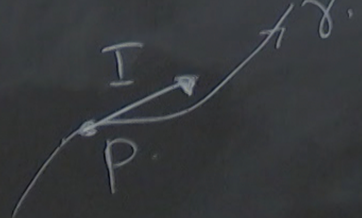


$$T : C^\infty(\mathcal{M}) \longrightarrow C^\infty(\mathcal{M})$$

$$f \longmapsto \left.\frac{df}{dt}\right|_p \quad \forall f \in C^\infty(\mathcal{M})$$

"$f(t)$ along γ"

<u>Vectors</u> A vector is defined as a linear operator tangent to a curve at P.



$$\underline{T} : C^\infty(\mathcal{M}) \longrightarrow C^\infty(\mathcal{M})$$

$$f \longmapsto \left.\frac{df}{dt}\right|_P \qquad \forall f \in C^\infty(\mathcal{M})$$

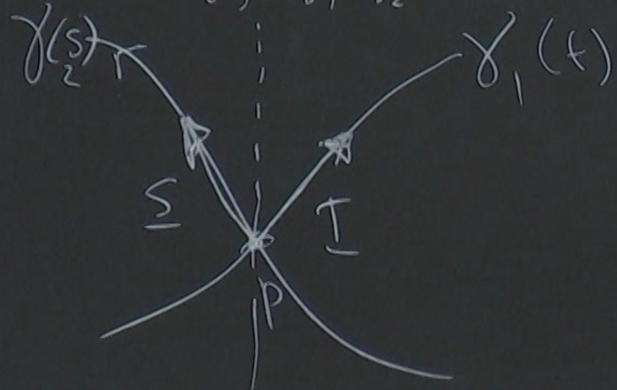
"$f(t)$ along γ"

So $\underline{T} \longleftrightarrow \underset{\text{on } \gamma.}{\dfrac{d}{dt}}$ at P.

Forms a vector space at p:

$$\gamma_\lambda(t) = \gamma(t/\lambda) \quad : \quad \underline{T} \to \lambda \underline{T}$$

"$\gamma_3 = \gamma_1 + \gamma_2$" ← defined in a local chart



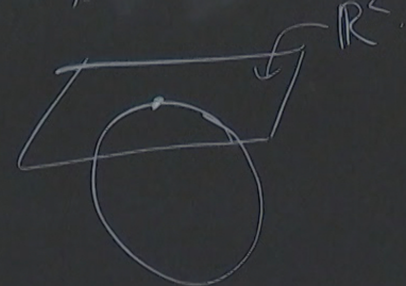
$\underline{S} + \underline{T}$ = tgt to γ_3.

a b

λI

This is the tangent space at P, $T_p(M)$. The collection of tangent spaces is the tangent bundle $T(M)$.

e.g.



a b

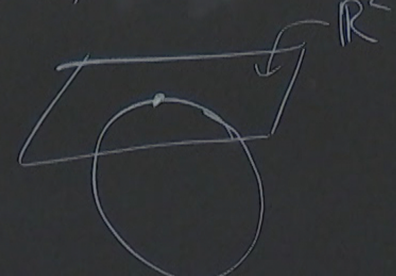
at P:

$I \to \lambda \underline{I}$

t to γ_3.

This is the <u>tangent space</u> at P, $T_p(M)$. The collection of tangent spaces is the <u>tangent bundle</u> $T(M)$.

e.g.



<u>Cov</u>

space at P:

$(\lambda) : \underline{I} \to \lambda \underline{I}$

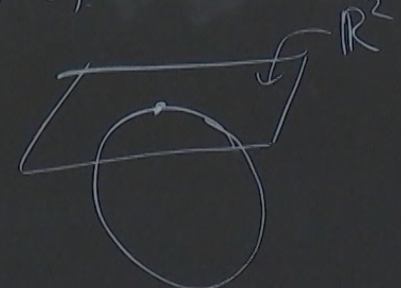
a

art

$\underline{I}$ = tgt to γ_s.

This is the <u>tangent space</u> at P, $T_p(M)$. The collection of tangent spaces is the <u>tangent bundle</u> $T(M)$.

e.g.



ent

Covectors defined as map

from $T_p(M) \longrightarrow \mathbb{R}$

$$\underline{V} \longmapsto \underline{\omega}(\underline{v}) \text{ or } \langle \underline{\omega} | \underline{v} \rangle$$

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<u>Covectors</u> defined as map

from $T_p(M) \rightarrow \mathbb{R}$

$$\underline{V} \longmapsto \underset{\sim}{\omega}(\underline{V}) \text{ or } \langle \underset{\sim}{\omega} | \underline{V} \rangle$$

$\underset{\sim}{\omega} \in T_p^*(M)$ dual tgt space at P.

Bases & Components

$T_p(M)$ & $T_p^*(M)$ are vector spaces, so have a basis for $T_p(M)$ / $T_p^*(M)$ at each P.

– a common basis is coordinate

Co-ords are the labels for p

from charts:

$$\left[T^\mu \frac{\partial}{\partial X^\mu}\right] f$$

$$I f = \frac{df}{dt}\bigg|_p$$

$$= \frac{dX^\mu}{dt}\frac{\partial f}{\partial X^\mu}$$

derivs of f in local chart.

$\frac{d}{dt}$ of coord fns

for p

$$\underbrace{\left[T^{\mu}\frac{\partial}{\partial x^{\mu}}\right]}_{\text{vector}} f.$$

$\underline{T}$

$\underline{T}$ is the geometric object, the vector. T^{μ} are the cpts of the vector in the coord basis $\frac{\partial}{\partial x^{\mu}}$ are the basis vectors.

ws of
n local
chart.

Similarly $\underline{\omega} = \omega_{\mu} dx^{\mu}$

↳ coordinate cotangent basis

Another useful basis is orthormal

$\underset{\sim}{e}_a \longleftrightarrow \quad g(\underset{\sim}{e}_a, \underset{\sim}{e}_b) = \eta_{ab}$ (needs metric)

A general basis will look like

$\underset{\sim}{e}_a = e_a{}^\mu \frac{\partial}{\partial x^\mu}$

Another useful basis is orthormal

$$\underset{\sim}{e}_a \longleftrightarrow \quad g(\underset{\sim}{e}_a, \underset{\sim}{e}_b) = \eta_{ab}. \qquad \left(\begin{array}{c}\text{needs}\\ \text{metric}\end{array}\right)$$

A general basis will look like

$$\underset{\sim}{e}_a = e_a{}^\mu \frac{\partial}{\partial x^\mu}$$

vierbein
(vielbein)

ormal

(needs metric!)

Abstract Index Notation

It is useful (in GR) to write T^μ as the vector. e.g. write

$$\boxed{D_\mu T^\nu = \partial_\mu T^\nu + \Gamma_\mu{}^\nu{}_\lambda T^\lambda}$$

however, strictly speaking, T^μ are the components of $\underline{T}$

ormal

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It is useful (in GR) to write T^μ as the vector. e.g. write

$$\boxed{D_\mu T^\nu = \partial_\mu T^\nu + \Gamma_\mu{}^\nu{}_\lambda T^\lambda}$$

however, strictly speaking, T^μ are the components of T & ∴ scalars.

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The vector is really $\underline{T}$

& so $\underline{\nabla}\,\underline{T} = \underline{\nabla}\left(T^\mu \dfrac{\partial}{\partial x^\mu}\right)$ or $\underline{\nabla}\left(T^a \underline{e}_a\right)$

$$= \underbrace{\left(\underline{\nabla}\, T^a\right)}_{\text{partial derivs}} \underline{e}_a + T^a\, \underbrace{\underline{\nabla}\left(\underline{e}_a\right)}_{\text{connection}}$$

(vielbein)

the componen

Penrose calls the physics habit

Abstract Index Notation

Under a change of coords

$$\frac{\partial}{\partial x^\mu} = \frac{\partial x'^{\nu'}}{\partial x^\mu}\frac{\partial}{\partial x'^{\nu'}}$$

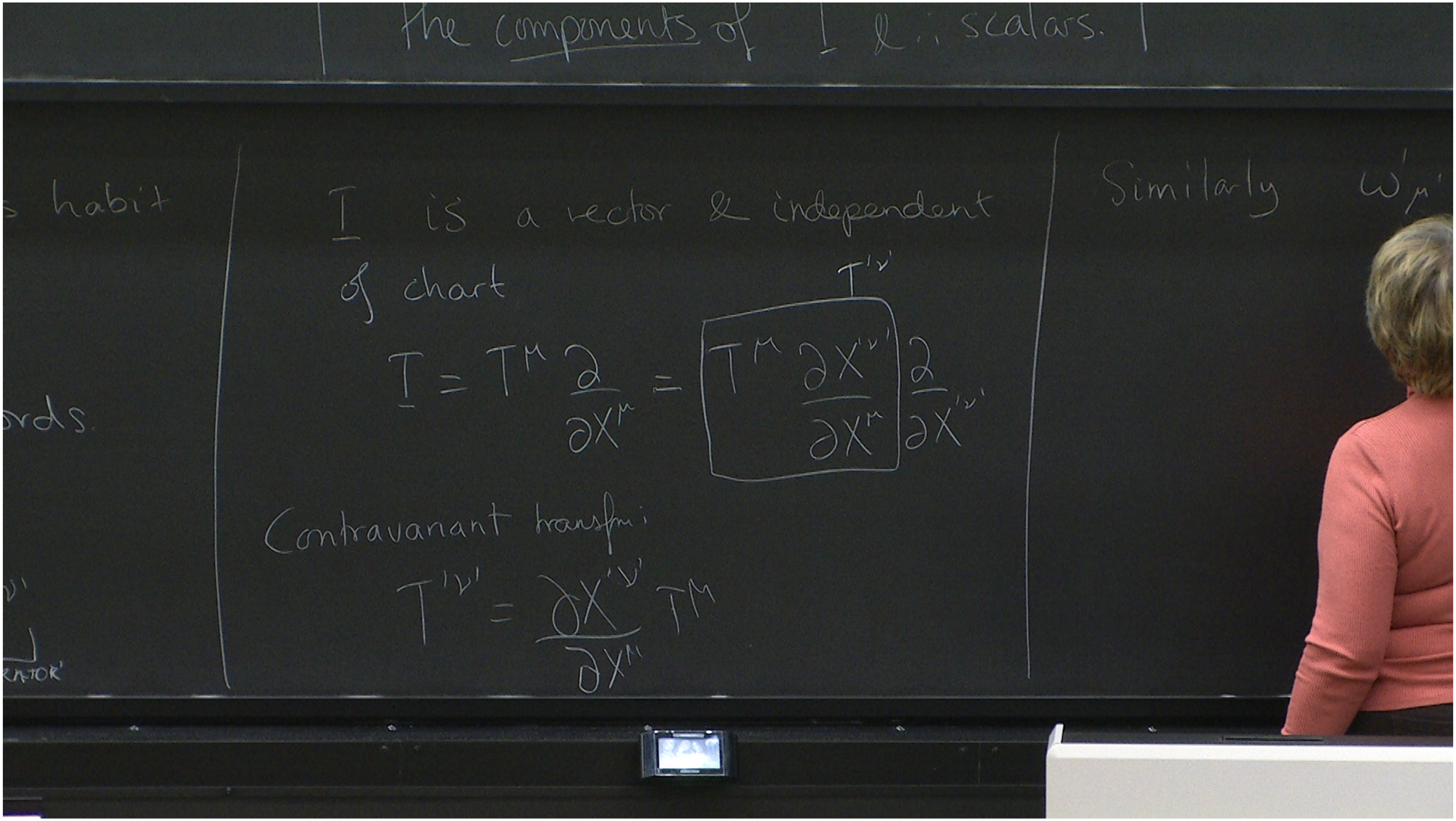
(vielbein) | the components

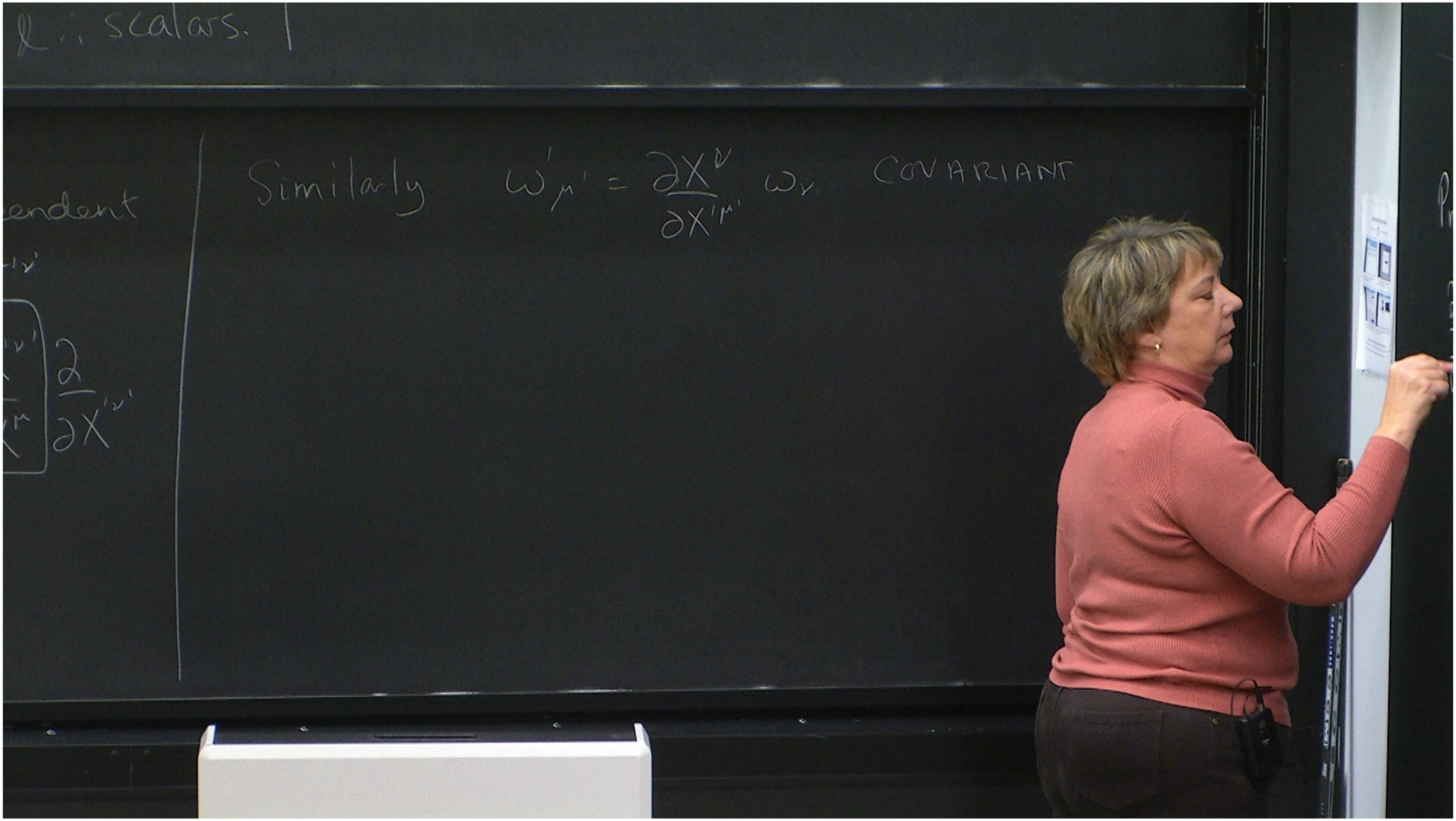
Penrose calls the physics habit

Abstract Index Notation

Under a change of coords

$$\underbrace{\frac{\partial}{\partial X^{\mu}}}_{\text{OPERATOR}} = \underbrace{\frac{\partial X'^{\nu'}}{\partial X^{\mu}}}_{\text{TRANSFM.}} \underbrace{\frac{\partial}{\partial X'^{\nu'}}}_{\text{OPERATOR}'}$$

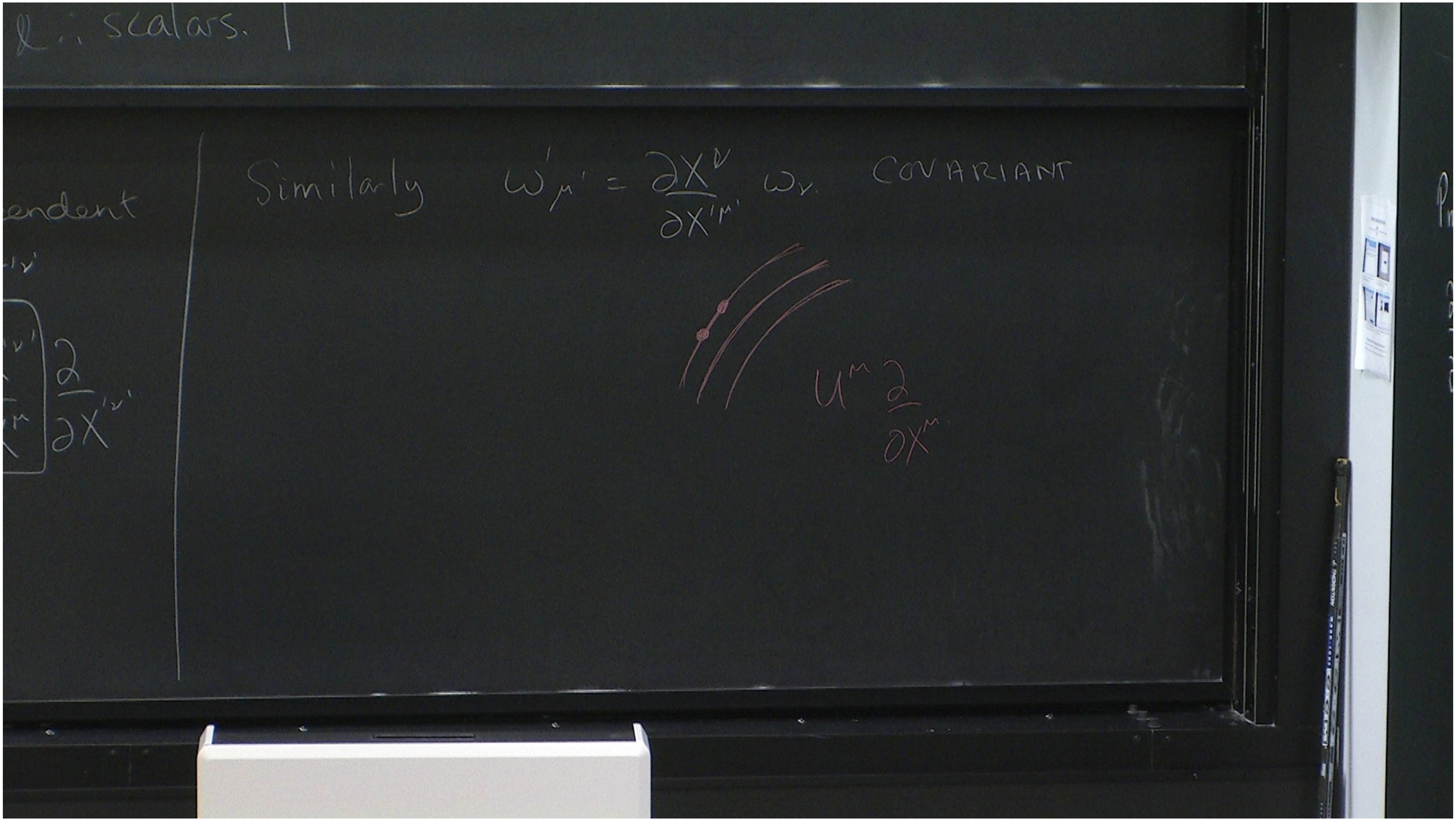

the components of $\underline{T}$ & $\underset{\sim}{\omega}$ scalars.
habit
ords.
$\underline{T}$ is a vector & independent of chart
$\underline{T} = T^{\mu}\frac{\partial}{\partial x^{\mu}} = \boxed{T^{\mu}\frac{\partial x'^{\nu'}}{\partial x^{\mu}}}\frac{\partial}{\partial x'^{\nu'}}$
$T'^{\nu'}$
Contravariant transfm:
$T'^{\nu'} = \frac{\partial x'^{\nu'}}{\partial x^{\mu}} T^{\mu}$
Similarly $\omega'_{\mu'}$


scalars.
Similarly $\omega'_{\mu'} = \frac{\partial X^{\nu}}{\partial X'^{\mu'}}\, \omega_{\nu}$. COVARIANT
endent

Problem!

$$\frac{\partial T^{\mu}}{\partial x^{\nu}} \rightarrow \frac{\partial x'^{\nu'}}{\partial x^{\nu}} \frac{\partial}{\partial x'^{\nu'}} \left(T'^{\mu'} \frac{\partial x^{\mu}}{\partial x'^{\mu'}} \right)$$

$$= \underbrace{\frac{\partial x'^{\nu'}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x'^{\mu'}} \frac{\partial T'^{\mu'}}{\partial x'^{\nu'}}}_{\text{covariant}} + \underbrace{\frac{\partial x'^{\nu'}}{\partial x^{\nu}} \frac{\partial^2 x^{\mu}}{\partial x'^{\nu'} \partial x'^{\mu'}}}_{\text{coord dependent}} T'^{\mu'}$$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endent
Similarly $\omega'_{\mu'} = \frac{\partial X^{\nu}}{\partial X'^{\mu'}} \omega_{\nu}$ COVARIANT
$\frac{\partial}{\partial X'^{\nu'}}$
$u^{\mu} \frac{\partial}{\partial X^{\mu}}$