

Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 18

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$$\left| \overline{D(\phi_S(t), \omega_T)} \leq \frac{1}{2} \left\| \text{tr}_S \omega^2 \right\| \right| \begin{array}{l} \omega = \overline{\Phi} = \overline{D_H \Phi} \quad \mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B \\ \omega_S = \text{tr}_B \omega \\ \phi_t = U_t \phi, \quad \phi_{tS} = \text{tr}_B \phi_t \end{array}$$

$$\mathbb{E}(\text{tr} \omega^2) = \mathbb{E}(\text{tr}[T^{(2)} \omega^{\otimes 2}]) = \mathbb{E}(\text{tr}[T^{(2)} D_H^{\otimes 2}(\phi)])$$

$$T^{(2)} \in \mathcal{B}(\mathcal{H}^{\otimes 2})$$

ii

$$\sum_{ij} |i\rangle \langle j|$$



$$\left| \begin{array}{l} \frac{1}{d_S^2} \text{tr} \omega^2 \\ \omega_S = \text{tr}_B \omega \\ \phi_t = U_t \phi, \quad \phi_{t_S} = \text{tr}_B \phi_t \end{array} \right| \quad \mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$$

$$\mathbb{E}(\text{tr}[T^{(2)} \omega^{\otimes 2}]) = \mathbb{E}(\text{tr}[T^{(2)} D_H^{\otimes 2}(\phi^{\otimes 2})]) = \text{tr}[T^{(2)} D_H^{\otimes 2}(\mathbb{E}(\phi^{\otimes 2}))]$$

$$\mathbb{E}(\phi^{\otimes 2}) = \frac{\mathbb{1}_{\mathcal{H} \otimes \mathcal{H}} + T^{(2)}}{b(d+1)}$$



$$(\cdot) = \sum_n \pi_n(\cdot) \pi_n$$

$$\begin{aligned}
 T^{(2)} D_H^{\otimes 2} (E(\phi^{\otimes 2})) &= \text{tr} \left[ T^{(2)} \sum_{kk'} \pi_k \otimes \pi_{k'} (4_{\mathcal{H}^{\otimes 2}} + T^{(2)}) \pi_k \otimes \pi_{k'} \right] \frac{1}{d(d+1)} \\
 &\quad + \text{tr} \left( T^{(2)} \sum_{kk'} |kk'\rangle \underbrace{\langle kk'|kk'\rangle}_1 \langle kk'| \right) \\
 &= \text{tr} \left( \sum_{kk'} |k'k\rangle \langle kk'| \right) = \sum_{kk'} \underbrace{\langle kk'|k'k\rangle}_{\delta_{kk'}} = d \\
 &= \frac{2}{d+1}
 \end{aligned}$$



$$\boxed{D(\phi_S(t), \omega_S) \leq \frac{1}{2} \|d_S^2 \text{tr} \omega^2\|} \quad \begin{array}{l} \omega = \Phi = D_H \phi \quad \mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B \\ \omega_S = \text{tr}_B \omega \\ \phi_t = U_t \phi, \quad \phi_{tS} = \text{tr}_B \phi_t \end{array}$$

$$\boxed{\frac{2}{d+1} = \mathbb{E}(\text{tr} \omega^2)} = \mathbb{E}(\text{tr}[T^{(2)} \omega^{\otimes 2}]) = \mathbb{E}(\text{tr}[T^{(2)}])$$

$$T^{(2)} \in \mathcal{B}(\mathcal{H}^{\otimes 2})$$

ii

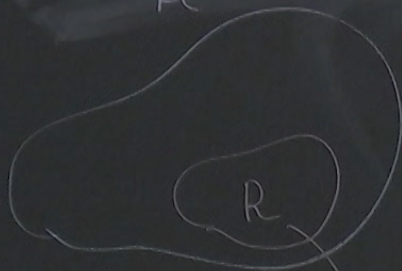
$$\sum_{ij} |i\rangle\langle j|$$

$$\mathbb{E}(\phi^{\otimes 2}) = \frac{\mathbb{1}_{\mathcal{H}^{\otimes 2}} + T^{(2)}}{d(d+1)}$$



$$\Omega_s \equiv \mathbb{E}(\omega_s) = \frac{1}{ds}$$

Remark  $\mathbb{E}_{\mathcal{H}}$



$$\mathbb{E}_R = \mathbb{E}_{\Pi_R \mathcal{H}}$$

$$R = \Pi_R \mathcal{H}$$



$$\overline{D(\phi_S(t), \Omega_S)} \leq \overline{D(\phi_S(t), \omega_S)} + D(\omega_S, \Omega_S)$$

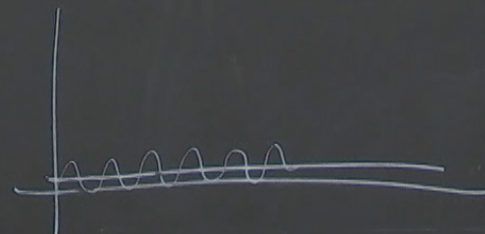
$$\begin{aligned} \mathbb{E} \left( \overline{D(\phi_S(t), \Omega_S)} \right) &\leq \mathbb{E} \left( \overline{D(\phi_S(t), \omega_S)} \right) + \mathbb{E} \left( D(\omega_S, \Omega_S) \right) \\ &\leq \frac{d_S}{2} \mathbb{E} \left( \sqrt{\text{tr} \omega^2} \right) + \end{aligned}$$

$$+ D(\omega_s, \Omega_s) \leq \frac{1}{2} \left| \frac{ds}{dB} \right| \quad |d_B \gg d_S \gg 1|$$

$$\overline{\phi_s(t, \omega_s)} + \left( \mathbb{E} \left( D(\omega_s, \Omega_s) \right) \right)$$

$$\overline{tr \omega_s} + \frac{1}{2} \left| \frac{ds}{dB} \right|$$

$$\frac{ds^2}{(s+1)} + \frac{1}{2} \left| \frac{ds}{dB} \right| \approx \frac{1+\sqrt{2}}{2} \left| \frac{ds}{dB} \right|$$



Markov Inequality

$$X \geq 0$$

$$P_n(|X - \bar{X}| \geq a) \leq \frac{\mathbb{E}(X)}{a}$$



What is the average entropy in  $\mathcal{H}$ ?

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$$

$$|\phi\rangle \in \mathcal{H}, \quad S(|\phi\rangle) = 0$$

for a generic  $\psi$   

$$S(\psi_S)$$

pure

$$\text{Tr} [O_S \otimes \mathbb{I}_S \psi] = \text{Tr}_S [O_S \psi_S]$$

$$|\psi\rangle = |X_S\rangle$$

$$\psi_S = \text{tr}_B (|X_S\rangle\langle X_S|)$$

$$= |X_S\rangle\langle X_S|$$

$$S_{\alpha}(\psi) = \frac{1}{1-\alpha} \log \text{tr} \psi^{\alpha}$$

$$S_1 \geq S_2$$

$$p_1^{\alpha}, \dots, p_n^{\alpha}$$



$$S_\alpha(\psi) = \frac{1}{1-\alpha} \log \operatorname{tr} \psi_S^\alpha$$

$$S_1 \geq S_2$$

$$\mathbb{E}(S_1) \geq \mathbb{E}(S_2) = \mathbb{E}(-\log \operatorname{tr} \psi_S^2) \geq -\log \mathbb{E}(\operatorname{tr} \psi_S^2)$$

$$S_\alpha(\psi_S) = \frac{1}{1-\alpha} \log \text{tr} \psi_S^\alpha$$

$$S_1 \geq S_2$$

$$\begin{aligned} \mathbb{E}(S_1) &\geq \mathbb{E}(S_2) = \mathbb{E}(-\log \text{tr} \psi_S^2) \geq -\log \mathbb{E}(\text{tr} \psi_S^2) = -\log \mathbb{E} \\ &\geq -\log \left( \text{tr} \left[ T_S^{(2)} \otimes \mathbb{I}_B \frac{\mathbb{I}_{2d} + T^{(2)}}{d(d+1)} \right] \right) \end{aligned}$$



$$\begin{aligned}
 -\log \mathbb{E}(\text{tr} \psi_s^2) &= -\log \mathbb{E}(\text{tr} T_s^{(2)} \psi_s^{\otimes 2}) = -\log \mathbb{E}(\text{tr} [T_s^{(2)} \\
 &= -\log \text{tr} \left[ \frac{1}{d(d+1)} \left( T_s^{(2)} \otimes \mathbb{I}_B^{\otimes 2} \right) \right]
 \end{aligned}$$



$$S_\alpha(\psi_S) = \frac{1}{1-\alpha} \log \text{tr} \psi_S^\alpha$$

$$S_1 \geq S_2$$

$$\begin{aligned} \mathbb{E}(S_1) &\geq \mathbb{E}(S_2) = \mathbb{E}(-\log \text{tr} \psi_S^2) \geq -\log \mathbb{E}(\text{tr} \psi_S^2) = -\log \mathbb{E} \\ &\geq -\log \left( \text{tr} \left[ T_S^{(2)} \otimes \mathbb{I}_B \frac{\mathbb{I}_{2d+2} + T^{(2)}}{d(d+1)} \right] \right) = -\log \text{tr} \left[ \frac{1}{d(d+1)} \left( \right. \right. \end{aligned}$$

$$T^{(2)} = T_S^{(2)} \otimes T_B^{(2)}$$



$$\begin{aligned}
 \text{tr } T_S^{(2)} \psi^{\otimes 2} &= -\log \mathbb{E} \left( \text{tr} \left[ T_S^{(2)} \otimes \mathbb{I}_B \psi^{\otimes 2} \right] \right) = -\log \left( \text{tr} \left[ T_S^{(2)} \otimes \mathbb{I}_B \mathbb{E} \psi^{\otimes 2} \right] \right) \\
 &= -\log \left[ \frac{1}{d(d+1)} \left[ d_S d_B^2 + d_S^2 d_B \right] \right] = -\log \left( \frac{d_S + d_B}{d_S d_B + 1} \right) \\
 &\quad \downarrow \quad \quad \quad \downarrow \\
 &\quad d_S d_B \quad \quad \quad d_B \gg d_S \gg 1 \\
 &\quad \quad \quad \mathbb{E}(S_1) \approx \frac{-\log 1}{d_S} \equiv \frac{\log d_S}{d_S}
 \end{aligned}$$