Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 14

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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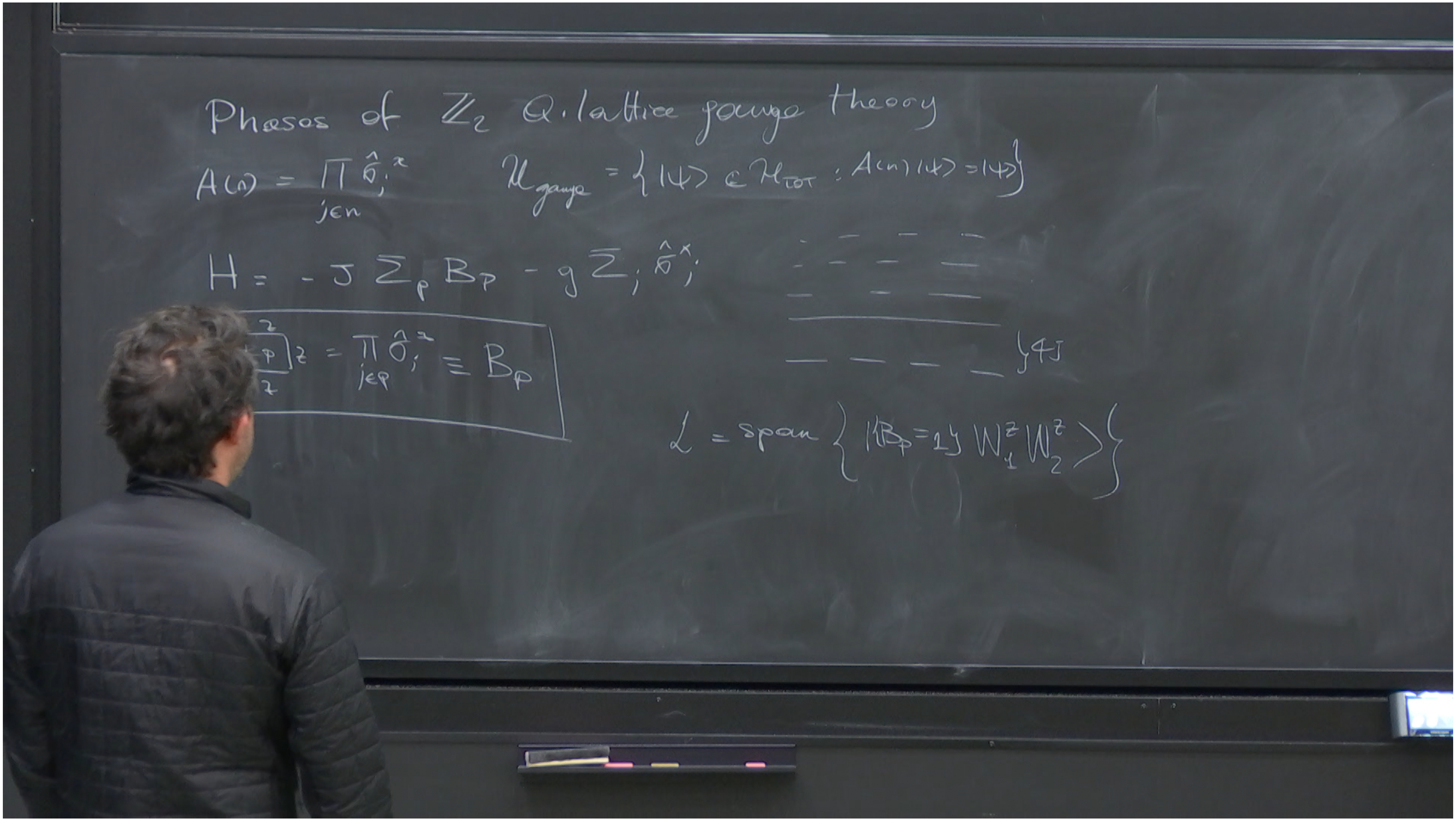
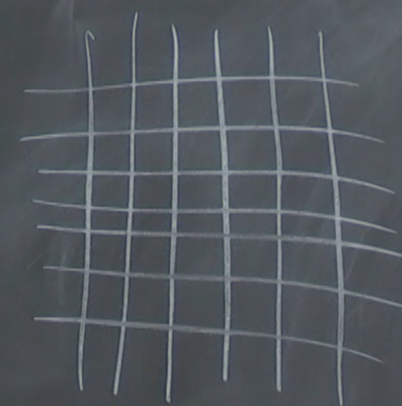

Phases of Z_2 Q. lattice gauge theory
A(n) = Π_{j∈n} σ̂_j^x
H_gauge = {|ψ⟩ ∈ H_TOT : A(n)|ψ⟩ = |ψ⟩}
H = -J Σ_p B_p - g Σ_j σ̂_j^x
= Π_{j∈p} σ̂_j^z ≡ B_p
{4}
L = span{ |{B_p = 1} W_1^z W_2^z⟩ }

Phases of $\mathbb{Z}_2$ Q. lattice gauge theory

$$A(n) = \prod_{j \in n} \hat{\sigma}_j^x \qquad \mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \}$$

$$H = -J \sum_p B_p - g \sum_j \hat{\sigma}_j^x$$

$$\boxed{ z\,\boxed{p}\,z = \prod_{j \in p} \hat{\sigma}_j^z \equiv B_p }$$



$$\mathcal{L} = \mathrm{span} \{ |\{B_p = 1\} W_1^z W_2^z \rangle \}$$

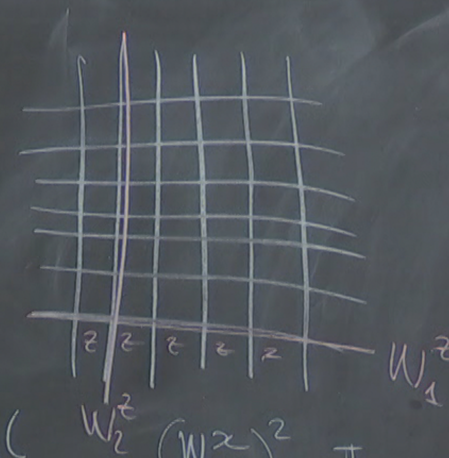
Phases of $\mathbb{Z}_2$ Q. lattice gauge theory

$A(n) = \prod_{j\in n} \hat{\sigma}_j^x$ $\qquad$ $\mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \}$

$H = -J\sum_p B_p - g\sum_j \hat{\sigma}_j^x$

$$\boxed{\; z\,\boxed{p}\,z \;=\; \prod_{j\in p} \hat{\sigma}_j^z \equiv B_p \;}$$

$\mathcal{L} = \text{span}\left\{ |\{B_p = 1\}\, W_1^z W_2^z\rangle \right\}$



$(W_{1,2}^x)^2 = \mathbb{1}$

$\{W_{1,2}^x, W_{1,2}^z\} = 0$

$[W_{1,2}^x, B_p] = 0$

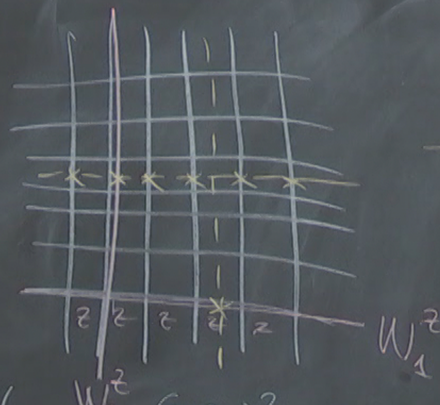
lattice gauge theory

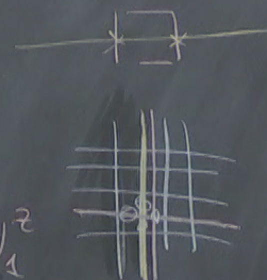
$\mathcal{L} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \}$

$Z;\ \hat{\sigma}^x;$

$|00\rangle \quad |10\rangle \quad |01\rangle \quad |11\rangle \} 4J$

$W_2^x \quad W_1^x \quad W_1^x W_2^x$





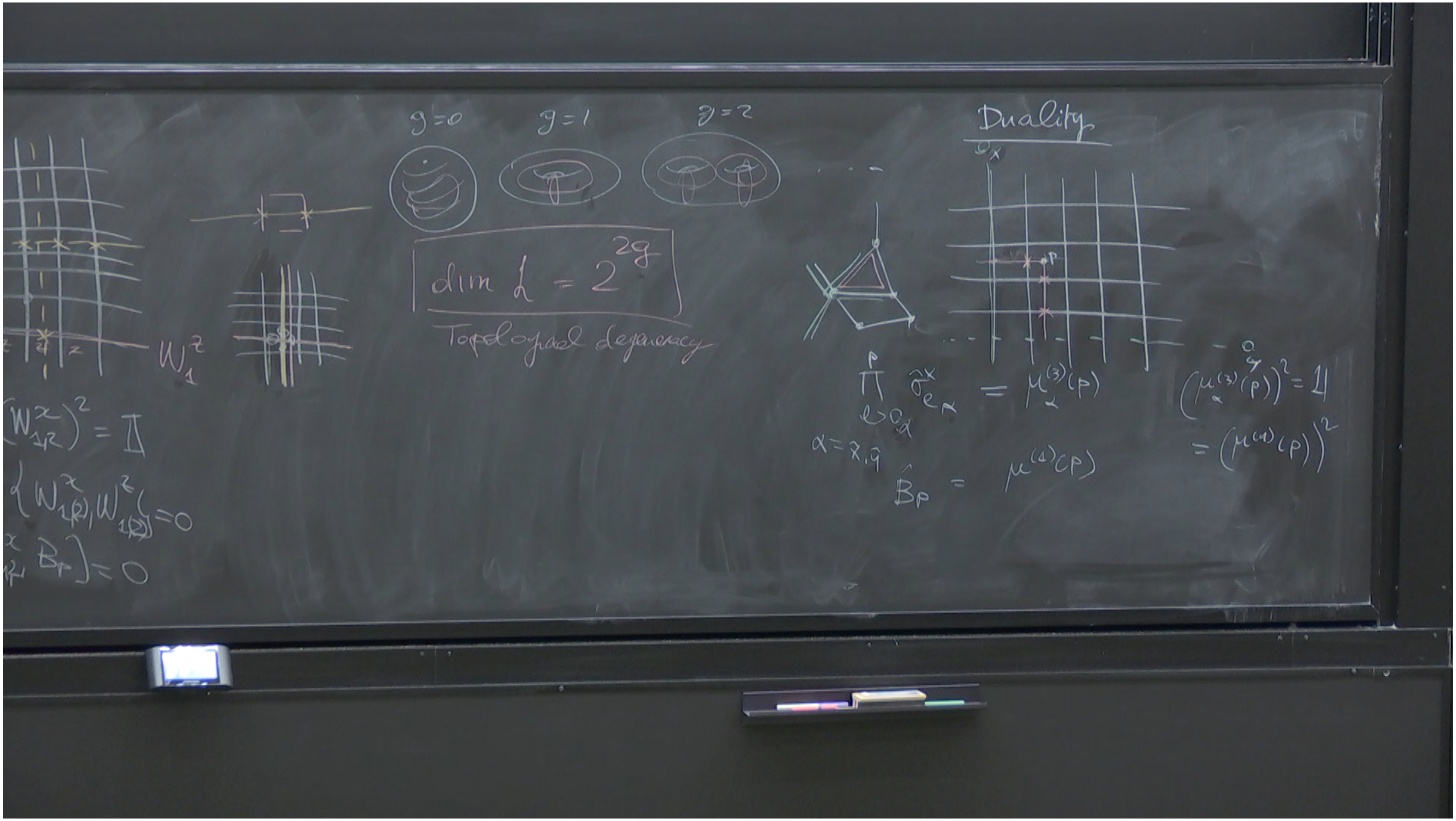
$\mathcal{L} = \text{span}\{ |\{B_p = 1\} W_1^z W_2^z\rangle \}$

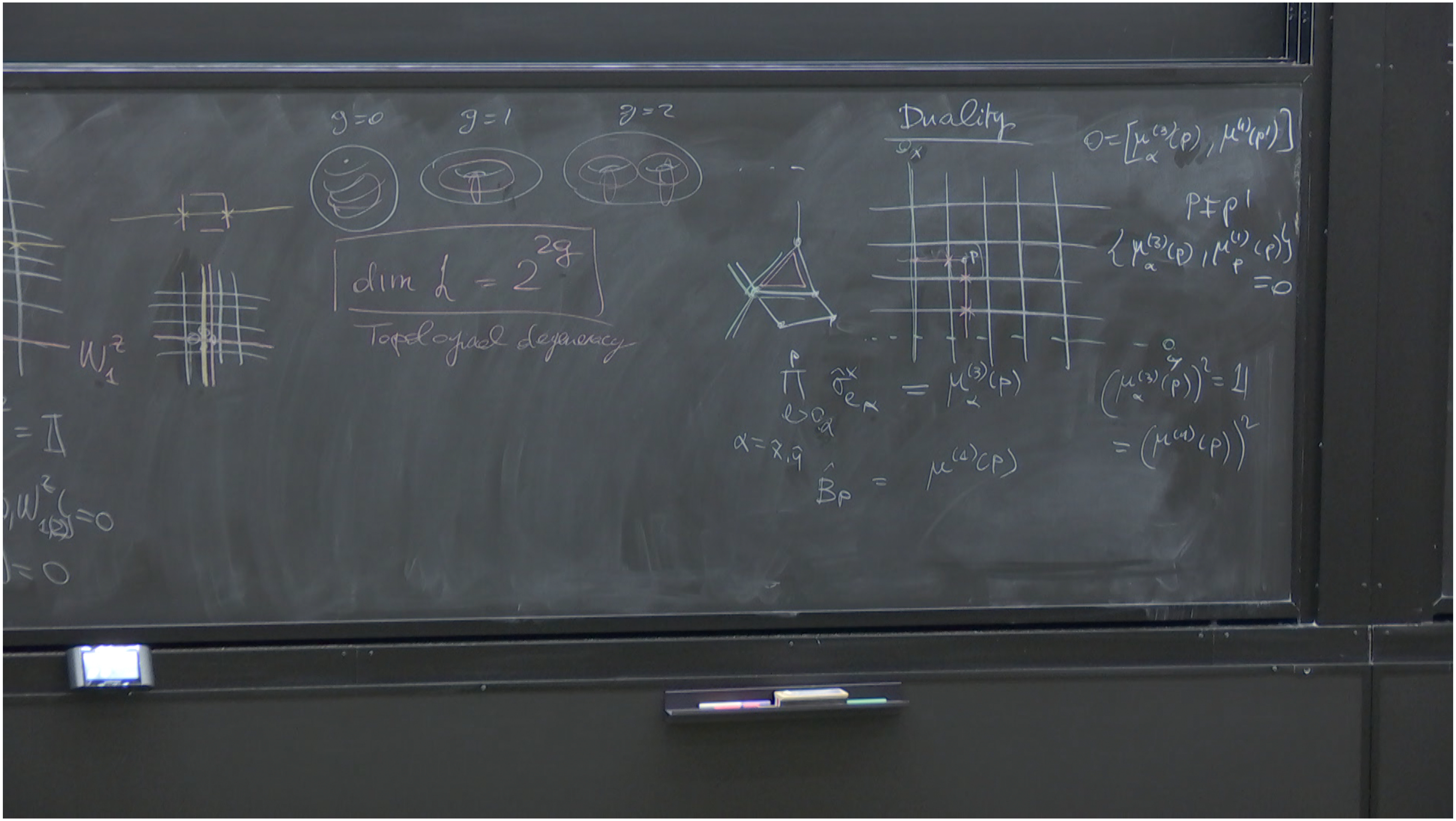
$= \text{span}\{ (W_1^x)^a (W_2^x)^b |00\rangle ;\ a,b = 0,1 \}$

$(W^x_{1,2})^2 = \mathbb{1}$

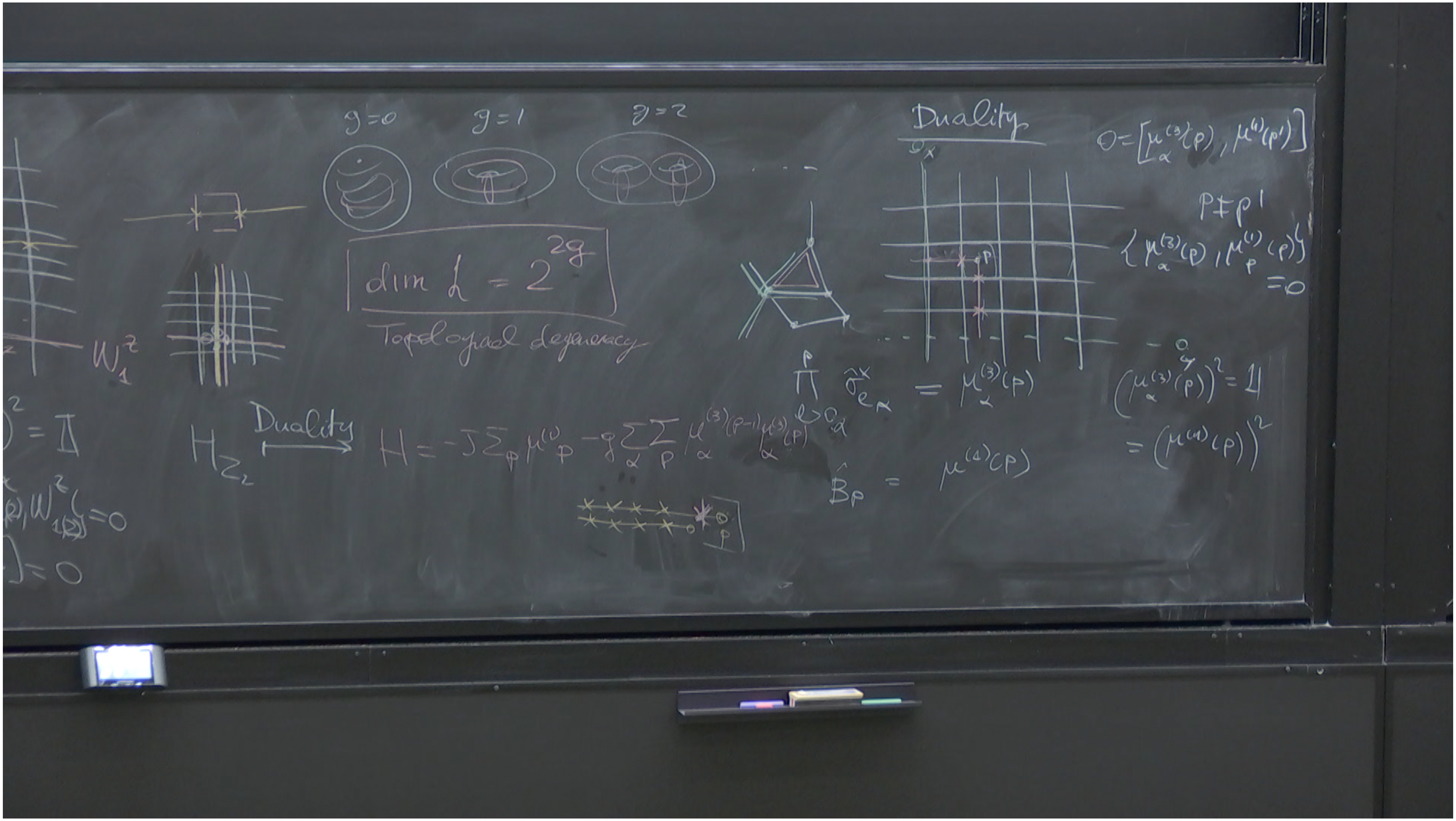
$\{ W^x_{1(2)}, W^z_{1(2)} \} = 0$

$[W^x_{1,2}, B_p] = 0$


g=0
g=1
g=2
Duality
dim L = 2^2g
Topological degeneracy



g=0
g=1
g=2
dim L = 2^2g
Topological degeneracy
Duality
P ≠ P'
=0
= 𝟙


g=0
g=1
g=2
Duality
dim L = 2^{2g}
Topological degeneracy
Duality
P ≠ P'
=0
= 1
=0
=0

$[W^x_{1p}, B_p] = 0$

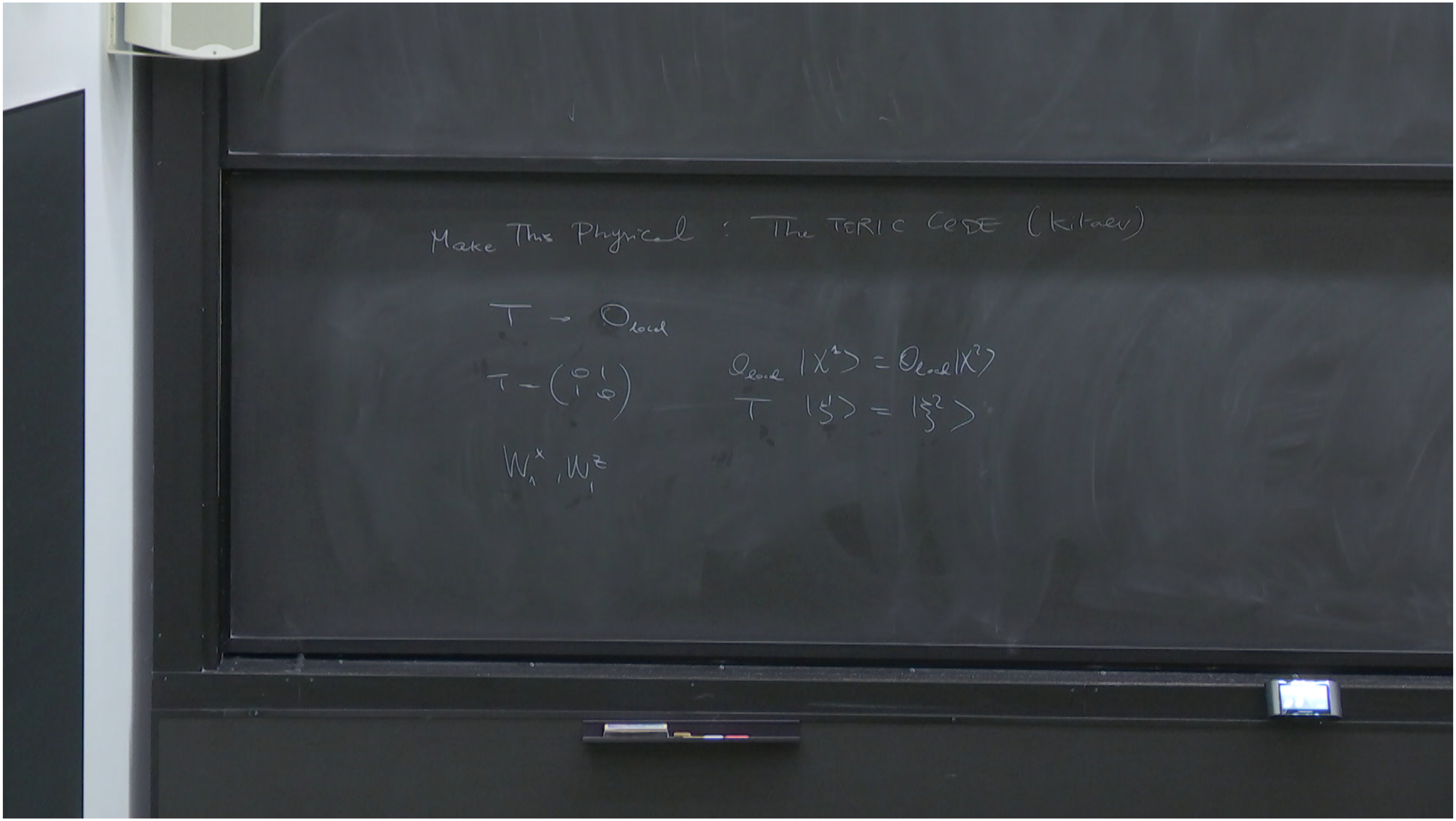
Q. Ising 2D

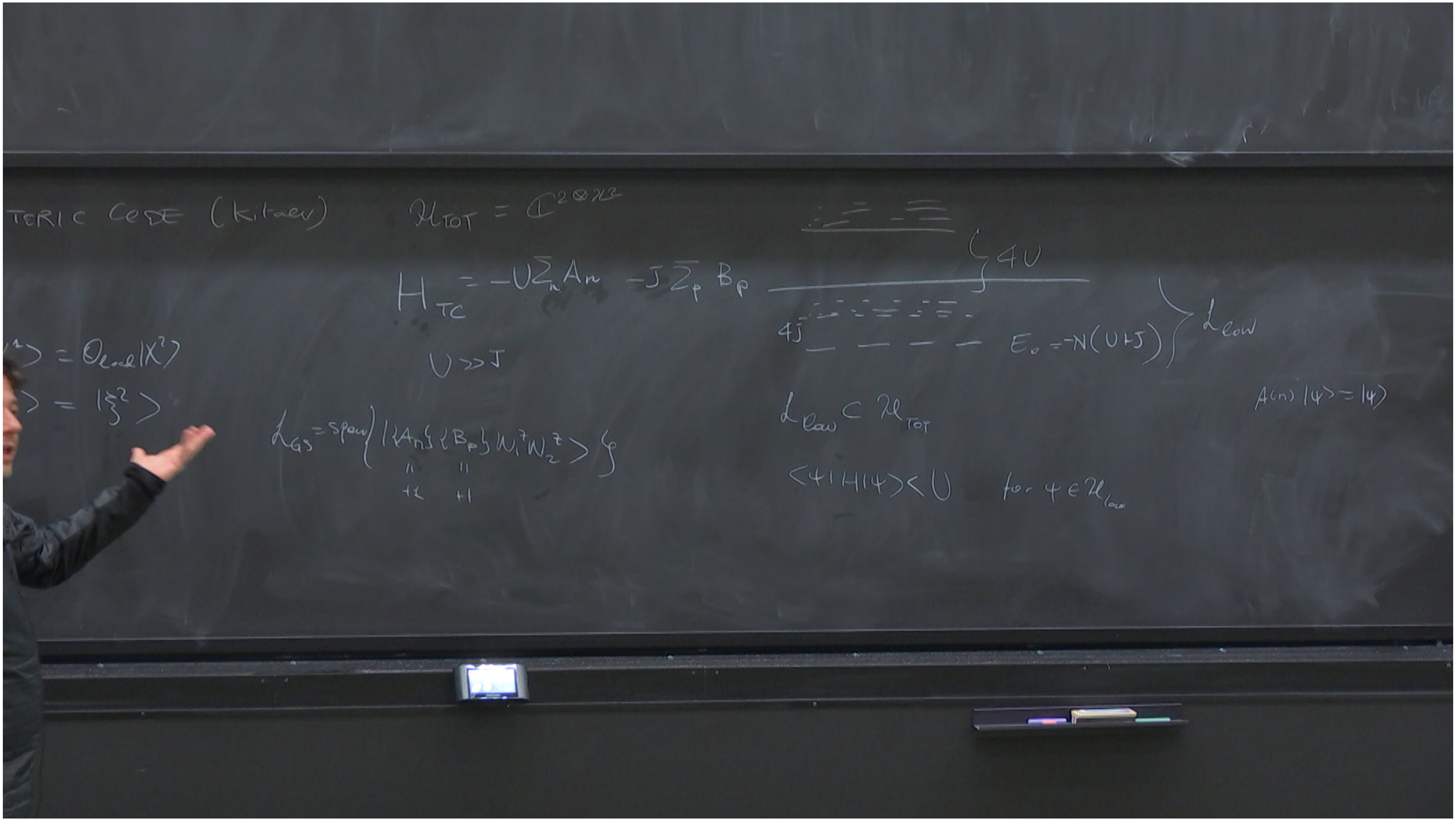
$\frac{J}{g} > (J/g)_c$ paramagnet $\langle \mu^{(3)}(p) \rangle = 0$ $\leadsto$ Top. Order

$\langle \mu^{(3)}(p)\, \mu^{(3)}(p') \rangle \sim e^{-|p-p'|/\xi}$

$J/g <$ crit. $\langle \mu^{(3)}(p) \rangle \neq 0 \xrightarrow{\text{Duality}} \langle \prod_{\ell,\alpha}^{p} \hat{\tau}_{\ell\alpha} \rangle \neq 0$

Condensation of "Kinks"


Make This Physical : The TORIC CODE (Kitaev)
$T \to O_{local}$
$T \sim \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$O_{local} |\chi^1\rangle = O_{local} |\chi^2\rangle$
$T \ |\xi^1\rangle = |\xi^2\rangle$
W_1^x , W_1^z


TORIC CODE (Kitaev)
$\mathcal{H}_{TOT} = \mathbb{C}^{2\otimes N}$
$H_{TC} = -U\sum_n A_n - J\sum_p B_p$
$4U$
$4J$
$E_0 = -N(U+J)$
$\mathcal{L}_{low}$
$U \gg J$
$\mathcal{L}_{low} \subset \mathcal{H}_{TOT}$
$A(n)|\psi\rangle = |\psi\rangle$
$\mathcal{L}_{GS} = \mathrm{span}\{|\{A_n\}\{B_p\} W_1^z W_2^z\rangle\}$
$\langle\psi|H|\psi\rangle < U$ for $\psi \in \mathcal{H}_{low}$

$$H_{TC} = -U\sum_n A(n) - J\sum_p B_p + \lambda \sum_j V_j$$

$$[V_j, A(n)] \neq 0$$

$$[V_j, B_p] \neq 0$$

E_{GS} $\}\,\Delta$

$\Delta E \overset{?}{\sim}$

$$\langle \psi_0 | \textstyle\sum_j V_j | \psi_0 \rangle =$$

$$H_{TC} = -U\sum_n A(n) - J\sum_p B_p + \lambda\sum_j V_j$$

$$[V_j, A(n)] \neq 0$$

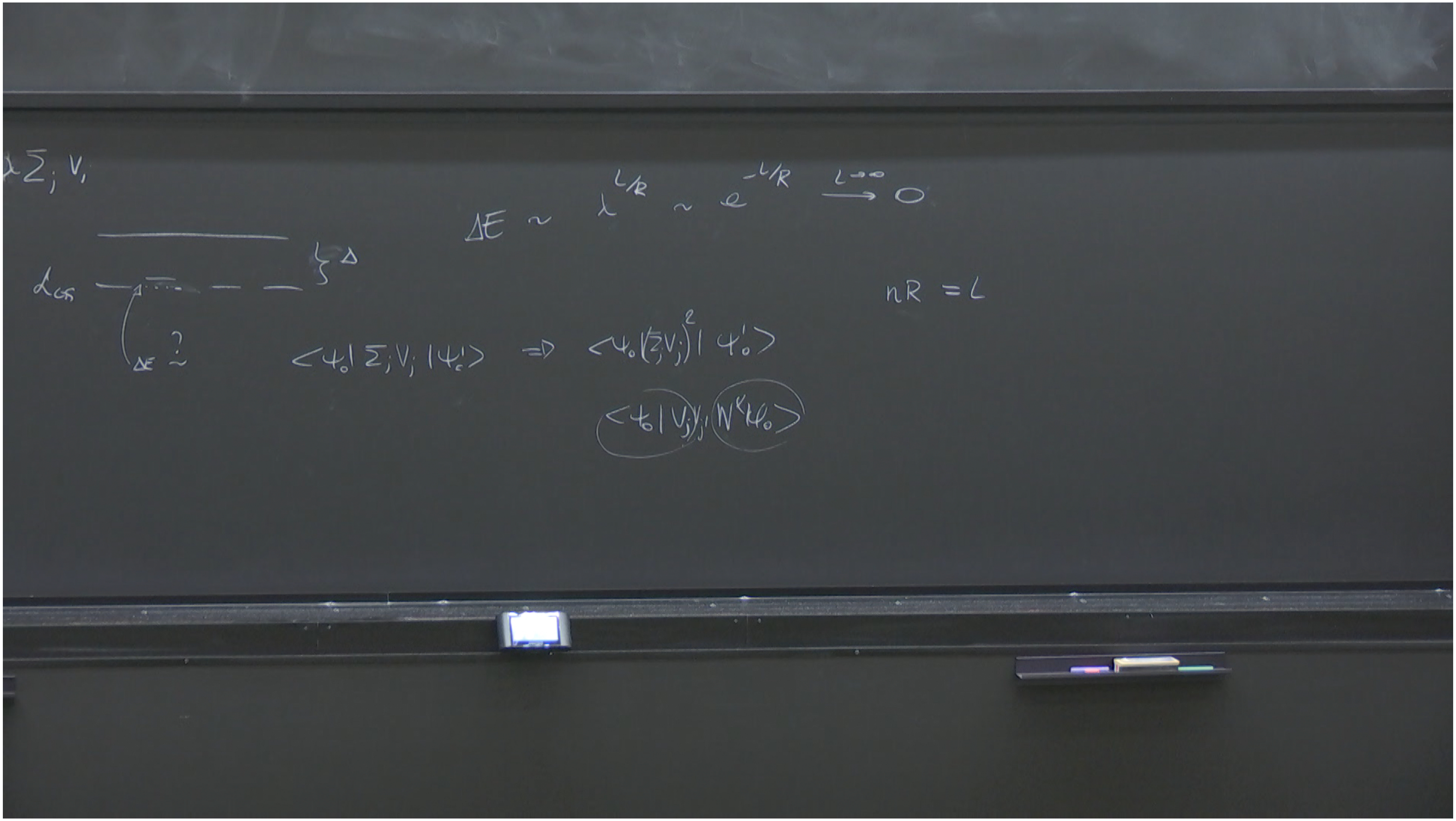
$$[V_j, B_p] \neq 0$$

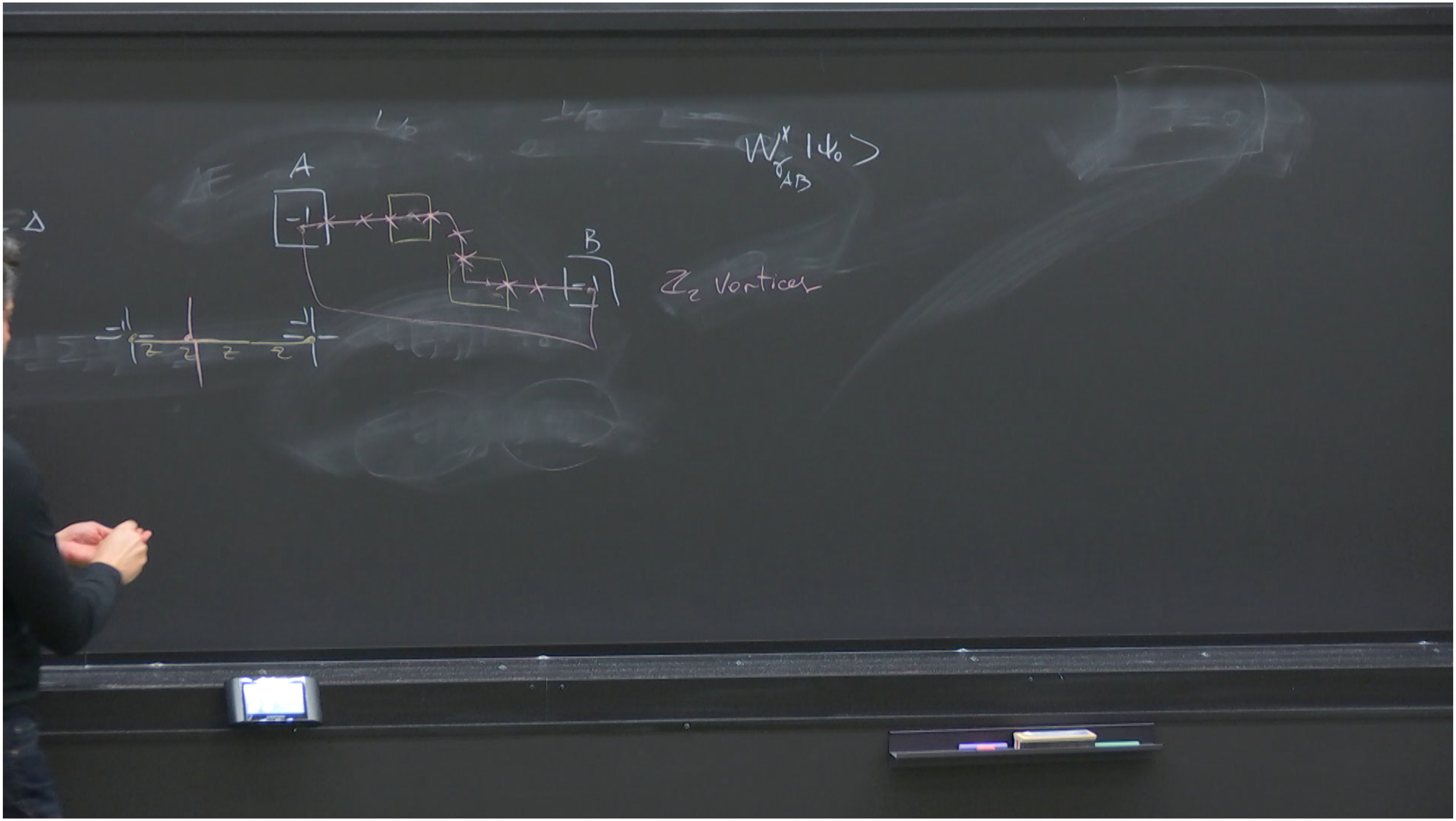
d_{GS} $\} \Delta$

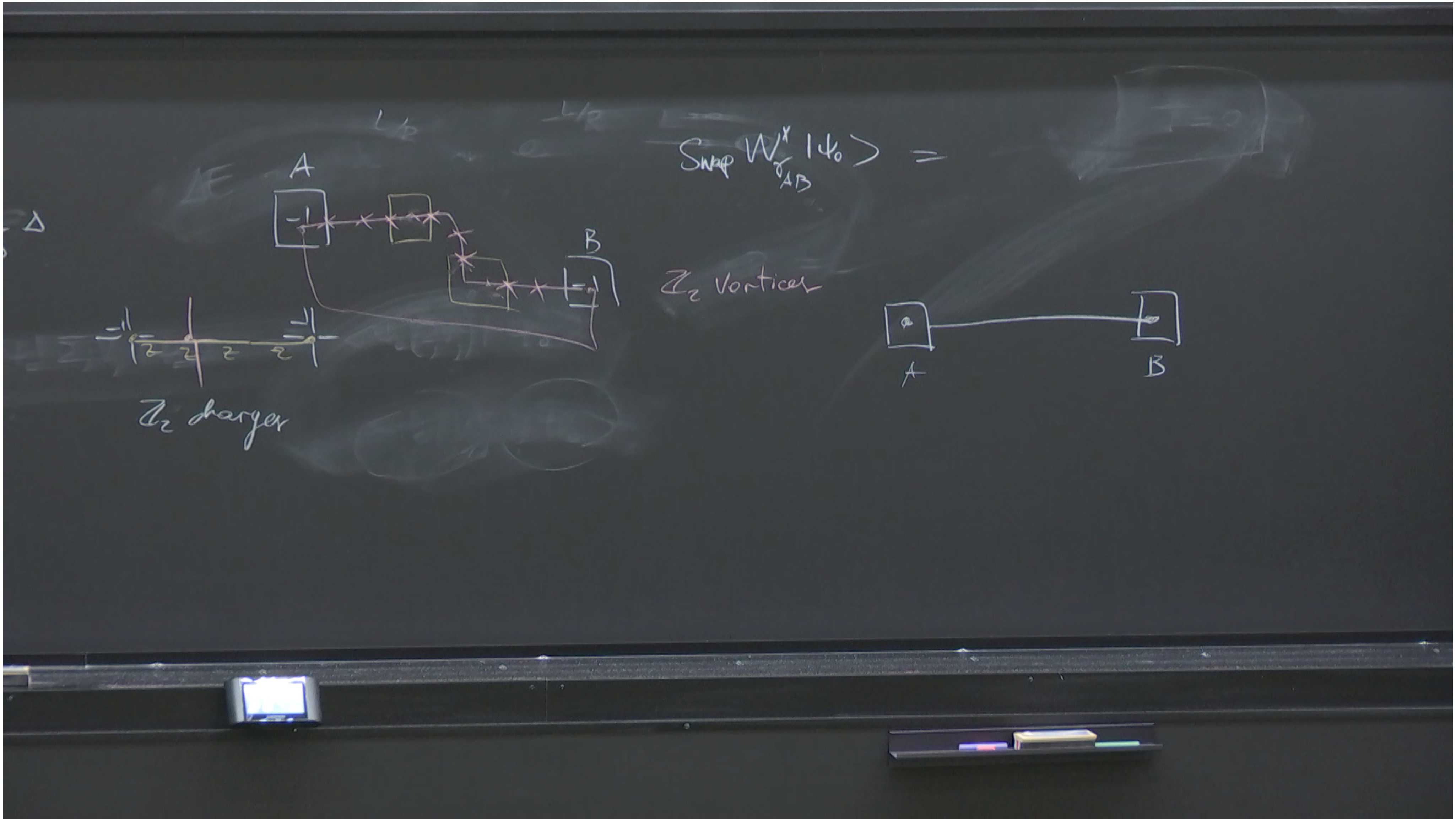
$\Delta E \sim ?$

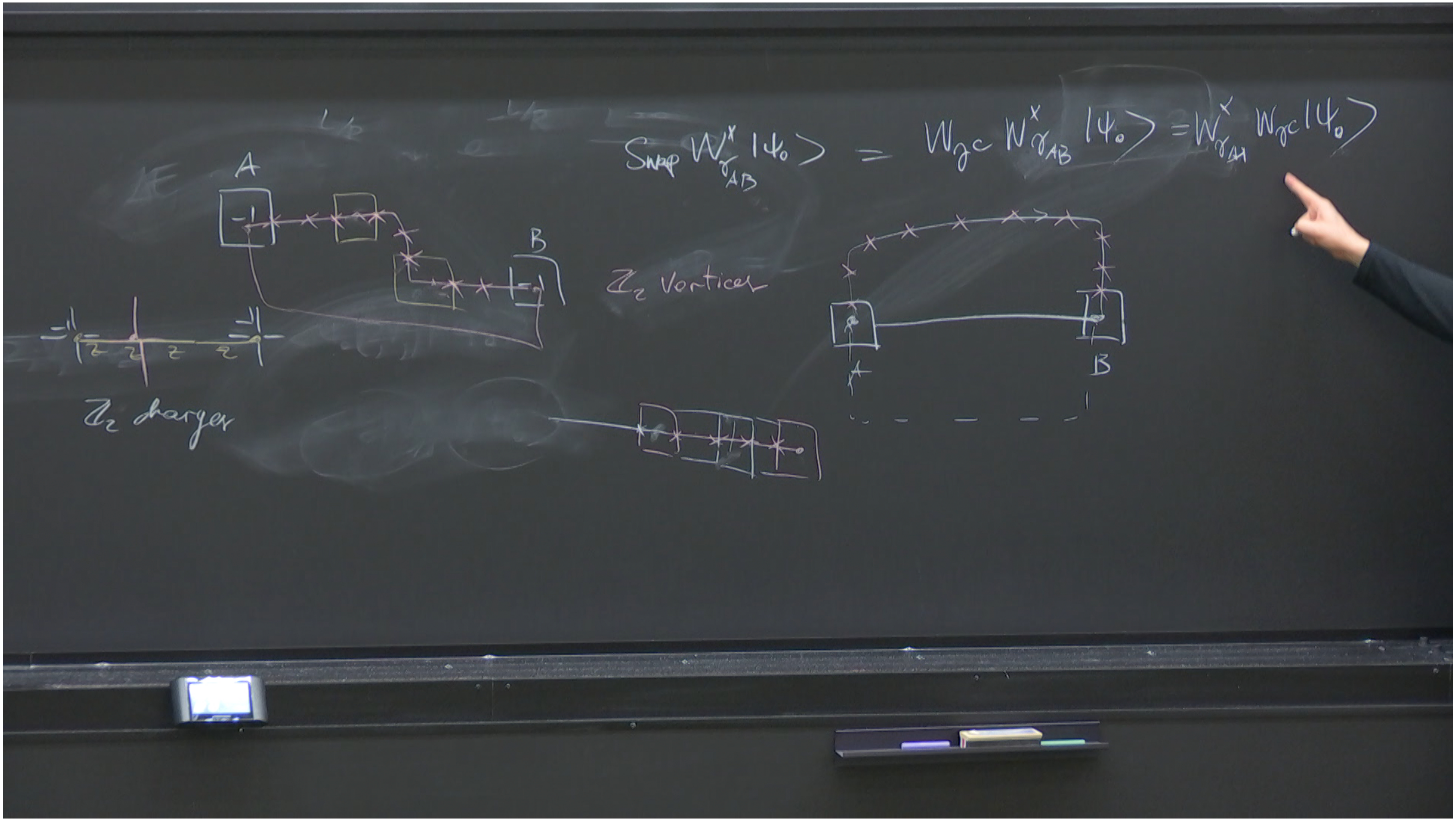
$$\langle \psi_0 | \sum_j V_j | \psi_0' \rangle \Rightarrow \langle \psi_0 | V_j | \psi_0' \rangle$$

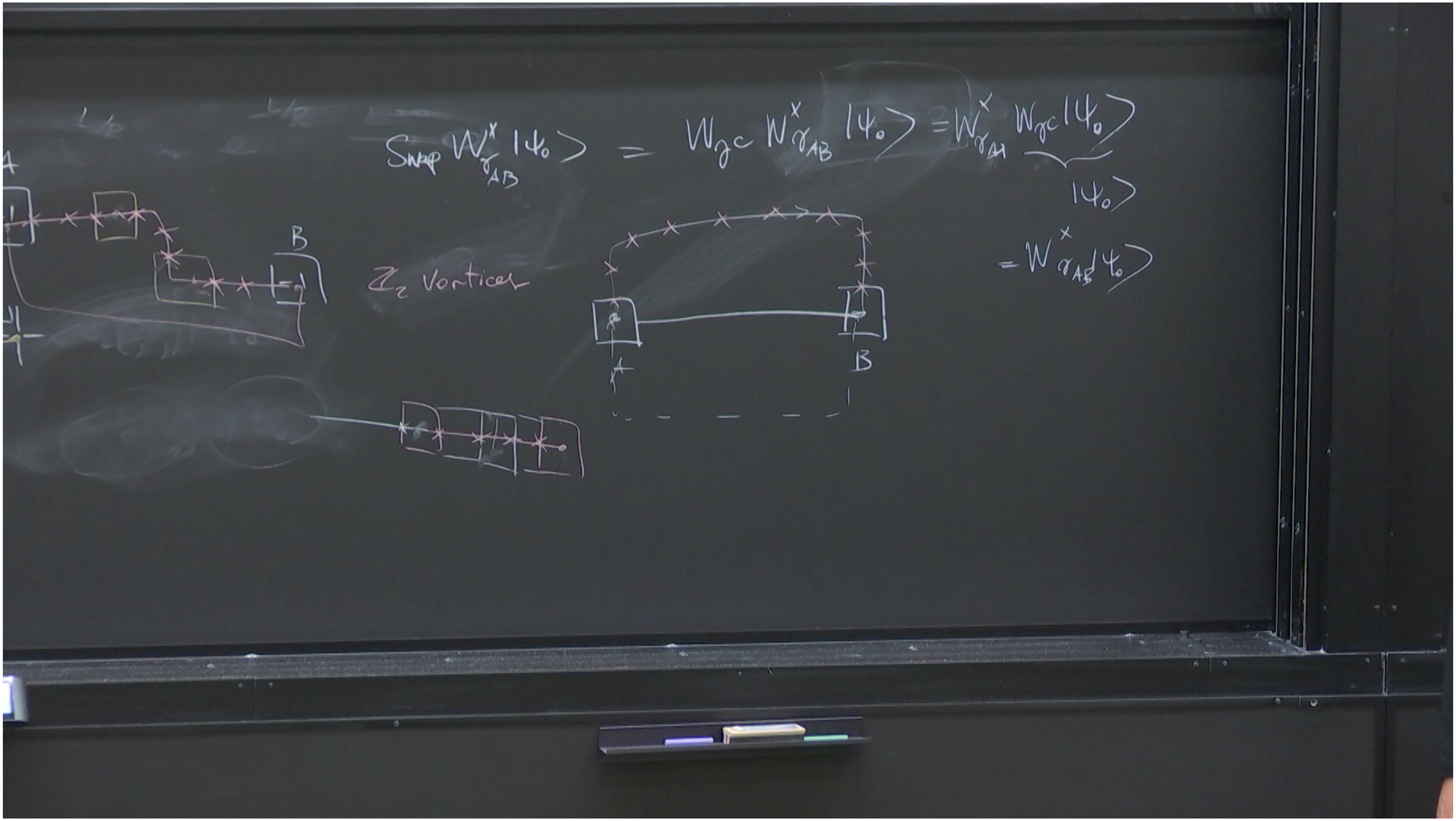
$$\langle \psi_0 | V_j \quad N^k | \psi_0 \rangle$$


$\lambda \sum_j V_j$
$\Delta E \sim \lambda^{L/R} \sim e^{-L/R} \xrightarrow{L\to\infty} 0$
λ_{GS} Δ
$\Delta E \sim ?$
$nR = L$
$\langle \psi_0 | \sum_j V_j | \psi_0' \rangle \Rightarrow \langle \psi_0 | (\sum_j V_j)^2 | \psi_0' \rangle$
$\langle \psi_0 | V_j V_{j'} N^k | \psi_0 \rangle$

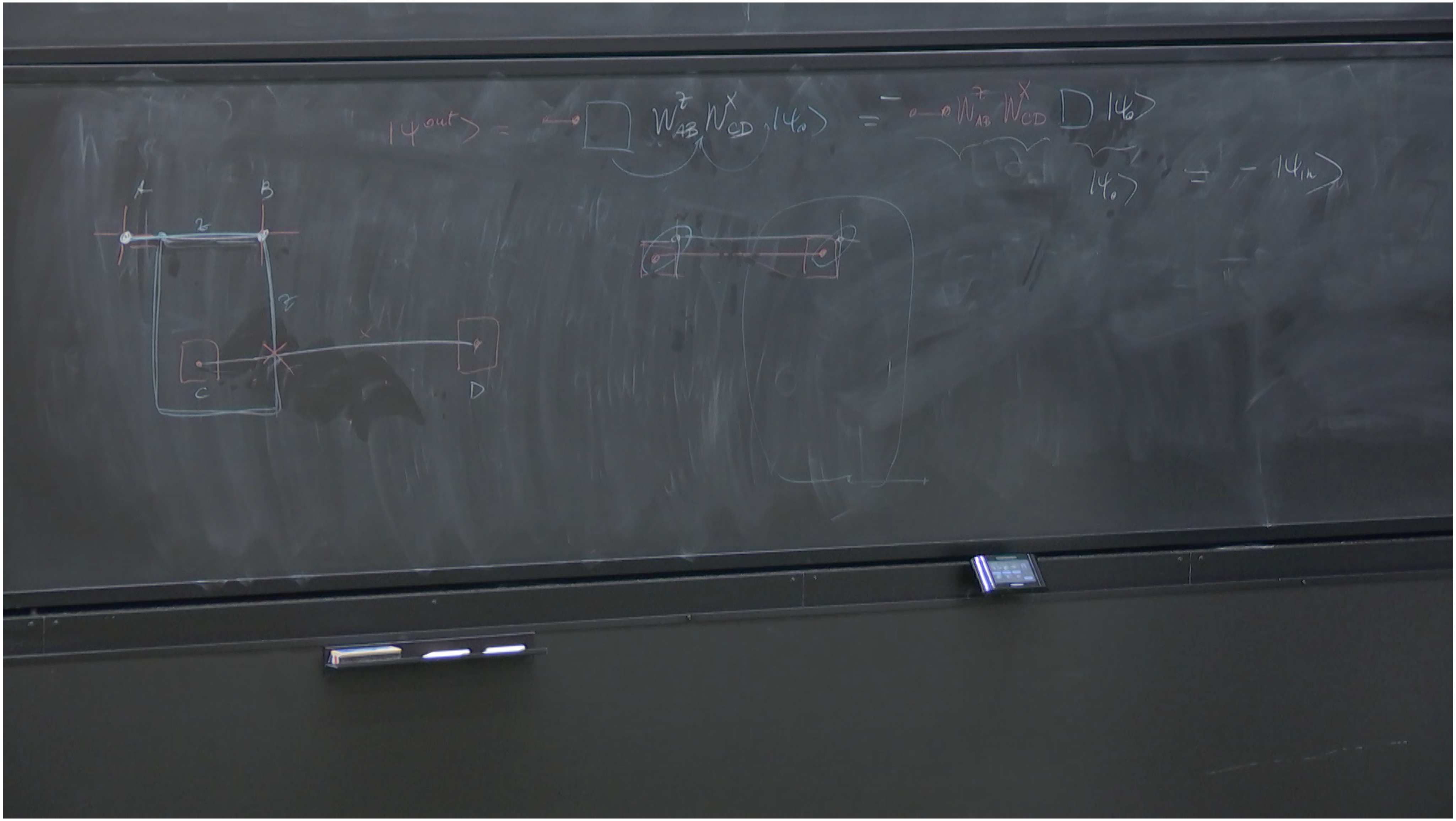

A
B
-1
$W^{x}_{\gamma_{AB}}|\psi_0\rangle$
Z_2 Vortices


Swap $W^x_{\gamma_{AB}} |\psi_0\rangle =$
A
B
Z_2 Vortices
Z_2 charges
A
B


Swap $W^{x}_{\gamma_{AB}}|\psi_0\rangle = W_{\gamma_C} W^{x}_{\gamma_{AB}}|\psi_0\rangle = W^{x}_{\gamma_{AB}} W_{\gamma_C}|\psi_0\rangle$
A
B
$\mathbb{Z}_2$ vortices
$\mathbb{Z}_2$ charges
A
B



Swap $W^x_{\gamma_{AB}}|\Psi_0\rangle = W_{\gamma c} W^x_{\gamma_{AB}}|\Psi_0\rangle = W^x_{\gamma_{AB}} \underbrace{W_{\gamma c}|\Psi_0\rangle}_{|\Psi_0\rangle}$
$= W^x_{\gamma_{AB}}|\Psi_0\rangle$
B
Z_2 Vortices
A
B


A
B
C
D