Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 13

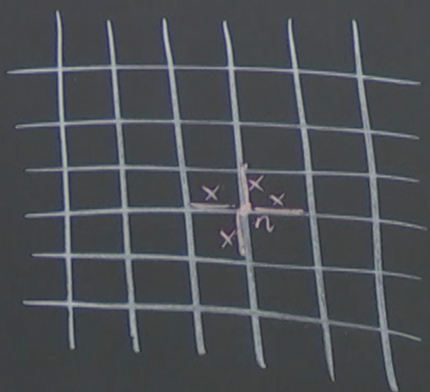
Speakers: Alioscia Hamma

Collection: PSI 2019/2020 - Quantum Matter Part 1

Date: January 21, 2020 - 9:00 AM

URL: http://pirsa.org/20010038

$\mathbb{Z}_2$ Quantum Lattice Gauge theory

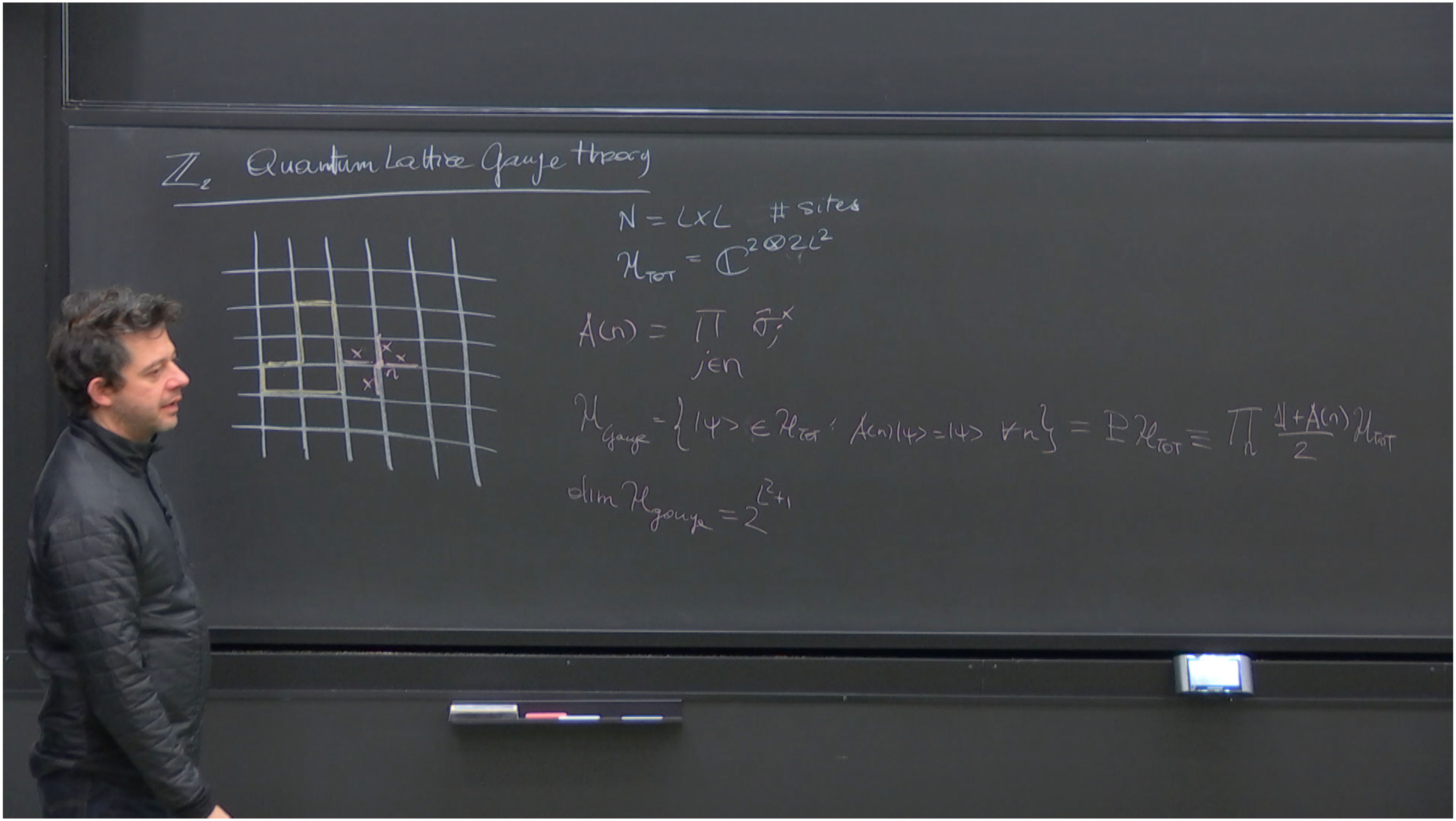


$N = L \times L$ # sites

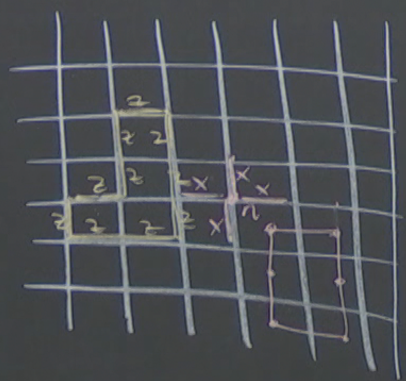
$\mathcal{H}_{TOT} = \mathbb{C}^{2 \otimes 2L^2}$

$$A(n) = \prod_{j \in n} \sigma_j^x$$

$$\mathcal{H}_{Gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \ \forall n \} = \mathbb{P}\,\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2}$$


$\mathbb{Z}_2$ Quantum Lattice Gauge theory
$N = L \times L$ # sites
$\mathcal{H}_{TOT} = \mathbb{C}^{2 \otimes 2L^2}$
$A(n) = \prod_{j \in n} \sigma_j^x$
$\mathcal{H}_{Gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \ \forall n \} = \mathbb{P}\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT}$
$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$
n

$\mathbb{Z}_2$ Quantum Lattice Gauge theory



$N = L \times L$ # sites

$$\mathcal{H}_{TOT} = \mathbb{C}^{2\otimes 2L^2}$$

$$A(n) = \prod_{j\in n} \sigma_j^x$$

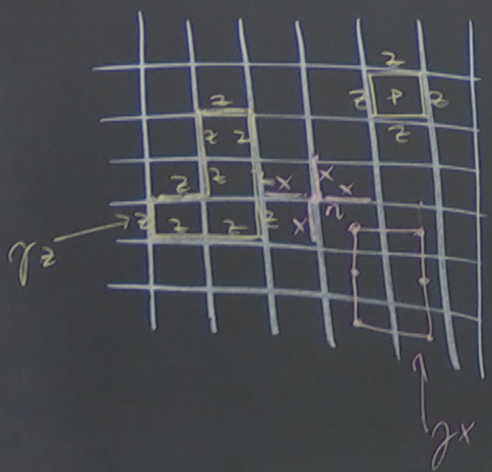
$$\mathcal{H}_{Gauge} = \{|\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \ \forall n\} = P\,\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1}+A(n)}{2}\mathcal{H}_{TOT}$$

$$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$$

$$W_{\gamma^z} = \prod_{j\in\gamma^z} \sigma_j^z$$

$$[W_{\gamma_c^z}, A(n)] = 0$$

$\mathbb{Z}_2$ Quantum Lattice Gauge theory



$N = L \times L$ # sites

$\mathcal{H}_{TOT} = \mathbb{C}^{2 \otimes 2L^2}$

$A(n) = \prod_{j \in n} \hat{\sigma}_j^x$

$W_{\gamma^z} = \prod_{j \in \gamma^z} \hat{\sigma}_j^z$; $W_{\gamma^x} = \prod_{j \in \gamma^x} \sigma_j^x$

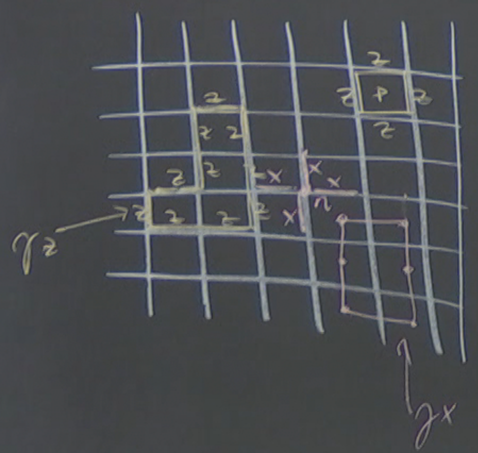
$[W_{\gamma_c^z}, A(n)] = 0$ Gauge invariant quantities

$B_p = \prod_{j \in p} \hat{\sigma}_j^z$

$$\mathcal{H}_{Gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \ \forall n \} = \mathbb{P}\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT}$$

$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$

$\mathbb{Z}_2$ Quantum Lattice Gauge theory



$N = L \times L$ # sites

$\mathcal{H}_{TOT} = \mathbb{C}^{2 \otimes 2L^2}$

$A(n) = \prod_{j \in n} \hat{\sigma}_j^x$

$W_{\gamma^z} = \prod_{j \in \gamma^z} \sigma_j^z$; $W_{\gamma^x} = \prod_{j \in \gamma^x} \sigma_j^x$

$[W_{\gamma_c^z}, A(n)] = 0$ Gauge invariant quantities

$B_p = \prod_{j \in p} \hat{\sigma}_j^z \Rightarrow [B_p, A(n)] = 0$

$\mathcal{H}_{Gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \ \forall n \} = P\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes L^2+1}$

$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$

$W_{\gamma^z} = \prod_{j \in \gamma^z} \sigma^z_j \; ; \; W_{\gamma^x} = \prod_{j \in \gamma^x} \sigma^x_j$

$[W_{\gamma_c}^z, A(n)] = 0$ Gauge invariant quantities

$B_p = \prod_{j \in p} \hat{\sigma}^z_j \Rightarrow [B_p, A(n)] = 0$

$= |\psi\rangle \; \forall n \} = P\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes L^2+1}$

$\prod_{p \in \Lambda} \hat{B}_p = \mathbb{1}$

$B_p = \pm 1$

$\mathrm{span}\{ | B_{p_1} ; \ldots B_{p_{L^2-1}} \rangle \} \subseteq \mathcal{H}_{Gauge}$

$W_{\gamma^z} = \prod_{j\in\gamma^z} \sigma^z_j \quad ; \quad W_{\gamma^x} = \prod_{j\in\gamma^x} \sigma^x_j$

$\prod_{p\in\Lambda} \hat{B}_p = \mathbb{1}$

$B_p = \pm 1$

$[W_{\gamma_c^z}, A(n)] = 0$ Gauge invariant quantities

$\mathrm{span}\{ |\underset{\pm 1}{B_{p_1}}, \dots B_{p_{L^2-1}}\rangle \} \subseteq \mathcal{H}_{gauge} = \bigoplus_{J=1}^{4} \mathcal{H}_J^{gauge}$

$B_p = \prod_{j\in p} \hat{\sigma}^z_j \Rightarrow [B_p, A(n)] = 0$

$= |\psi\rangle \ \forall n\} = P\mathcal{H}_{TOT} = \prod_n \frac{\mathbb{1}+A(n)}{2} \mathcal{H}_{TOT} \simeq \mathbb{C}^{2\otimes L^2+1}$

$W_{\gamma^z} = \prod_{j \in \gamma^z} \sigma^z_j \quad ; \quad W_{\gamma^x} = \prod_{j \in \gamma^x} \sigma^x_j$

$\prod_{p \in \Lambda} \hat{B}_p = \mathbb{1}$

$B_p = \pm 1$

$[W_{\gamma_c}^z, A(n)] = 0$ Gauge invariant quantities

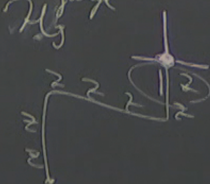
$\text{span}\{ | \underbrace{B_{p_1}, \ldots\ldots B_{p_{L^2-1}}}_{\mathcal{H}_J^{gauge}} \rangle \} \subseteq \mathcal{H}_{gauge} = \bigoplus_{J=1}^{4} \mathcal{H}_J^{gauge}$

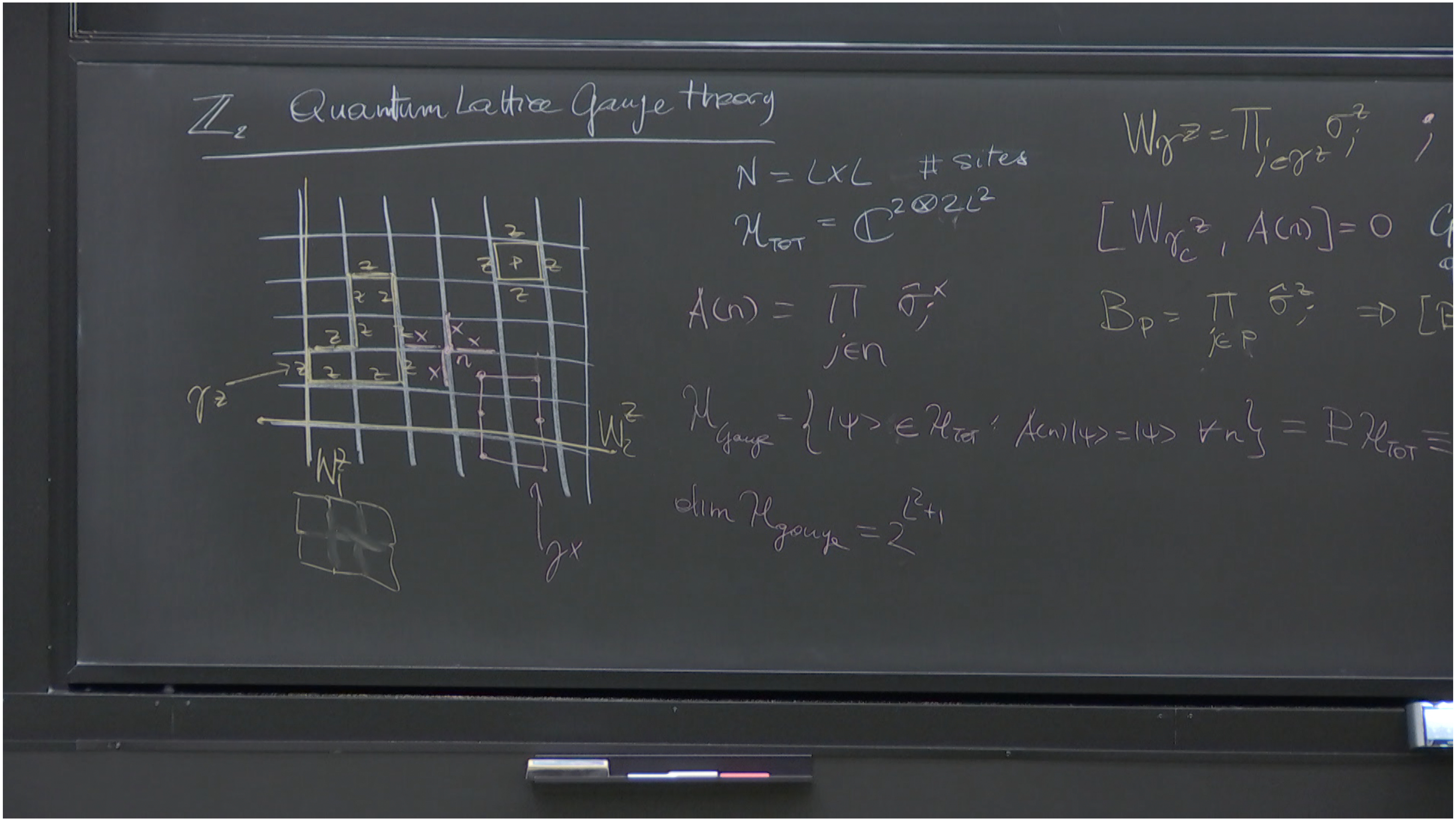
$B_{p_1} = \pm 1$

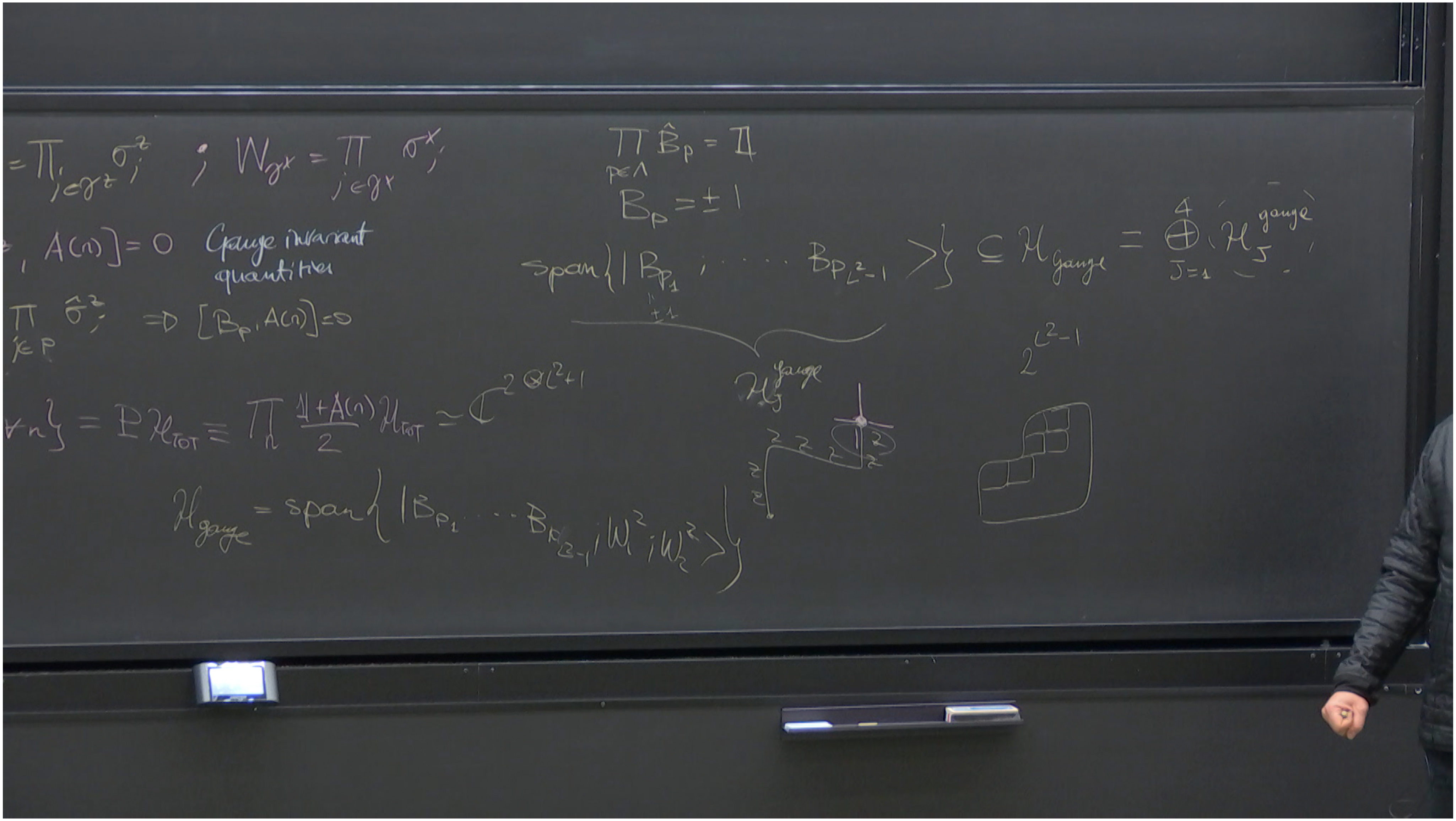
$B_p = \prod_{j \in p} \hat{\sigma}^z_j \Rightarrow [B_p, A(n)] = 0$

2^{L^2-1}

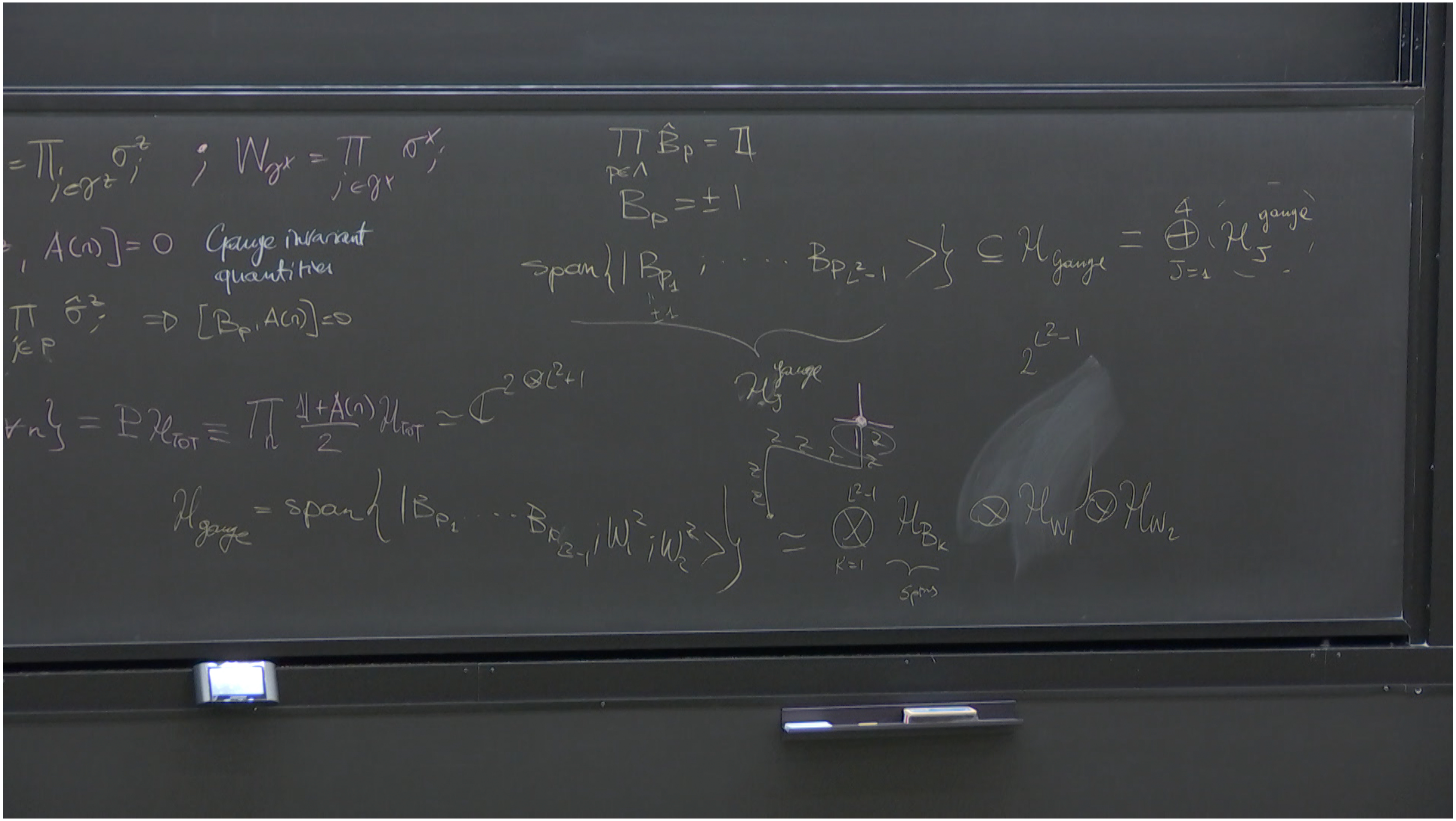
$|\psi\rangle \ \forall n \} = \mathbb{P}\mathcal{H}_{TOT} = \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes L^2+1}$




$\mathbb{Z}_2$ Quantum Lattice Gauge theory
$N = L \times L$ # sites
$\mathcal{H}_{TOT} = \mathbb{C}^{2 \otimes 2L^2}$
$A(n) = \prod_{j \in n} \hat{\sigma}_j^x$
$\mathcal{H}_{Gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle \ \forall n \} = \mathbb{P}\mathcal{H}_{TOT} =$
$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$
$W_{\gamma^z} = \prod_{j \in \gamma^z} \sigma_j^z$;
$[W_{\gamma_c^z}, A(n)] = 0$
$B_p = \prod_{j \in p} \hat{\sigma}_j^z \Rightarrow [$
γ^z
γ^x
W_1^z
W_2^z
p
n



Π_{p∈Λ} B̂_p = 1
B_p = ±1
Gauge invariant quantities
span{|B_{p_1}, . . . B_{p_{L^2-1}}>} ⊆ H_gauge = ⊕_{J=1}^{4} H_J^gauge
H_gauge = span{|B_{p_1} . . . B_{p_{L^2-1}}; W_1^2; W_2^2>}


$= \prod_{j \in \gamma^z} \sigma^z_j$; $W_{\gamma^x} = \prod_{j \in \gamma^x} \sigma^x_j$
$\prod_{p \in \Lambda} \hat{B}_p = \mathbb{1}$
$B_p = \pm 1$
$, A(n)] = 0$
Gauge invariant quantities
$\mathrm{span}\{|B_{p_1}, \ldots, B_{p_{L^2-1}}\rangle\} \subseteq \mathcal{H}_{gauge} = \bigoplus_{J=1}^{4} \mathcal{H}_J^{gauge}$
$\prod_{j \in p} \hat{\sigma}^z_j \Rightarrow [B_p, A(n)] = 0$
$\mathcal{H}_g^{gauge}$
2^{L^2-1}
$\forall n\} = P\mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes L^2+1}$
$\mathcal{H}_{gauge} = \mathrm{span}\{|B_{p_1} \ldots B_{p_{L^2-1}}; W_1^z; W_2^z\rangle\} \simeq \bigotimes_{k=1}^{L^2-1} \mathcal{H}_{B_k} \otimes \mathcal{H}_{W_1} \otimes \mathcal{H}_{W_2}$
spins

$\sigma^z_i \; ; \; W_{\gamma^x} = \prod_{j \in \gamma^x} \sigma^x_j$

$\prod_{p \in \Lambda} \hat{B}_p = \mathbb{1}$

$B_p = \pm 1$

$]=0$ Gauge invariant quantities

$\Rightarrow [B_p, A(n)] = 0$

$\mathrm{span}\{ \underbrace{| B_{p_1} ; \cdots B_{p_{L^2-1}} \rangle}_{\mathcal{H}^{gauge}_g} \} \subseteq \mathcal{H}_{gauge} = \bigoplus_{J=1}^{4} \mathcal{H}^{gauge}_J$

2^{L^2-1}

$= P \mathcal{H}_{TOT} \equiv \prod_n \frac{\mathbb{1} + A(n)}{2} \mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes L^2+1}$

$\mathcal{H}_{gauge} = \mathrm{span}\{ | B_{p_1} \cdots B_{p_{L^2-1}} ; W_1^2 ; W_2^2 \rangle \} \simeq \bigotimes_{K=1}^{L^2-1} \underbrace{\mathcal{H}_{B_K}}_{\text{spins}} \otimes \mathcal{H}_{W_1} \otimes \mathcal{H}_{W_2}$

H for $\mathcal{H}_{\text{gauge}}$

$[H, A(n)] = 0$ Gauge structure

$$H_{\mathbb{Z}_2} = -J \sum_p B_p - g \sum_i \hat{\sigma}_i^x$$

H for $\mathcal{H}_{gauge}$

$[H, A(n)] = 0$ Gauge structure

$$H_{\mathbb{Z}_2} = -J\sum_p B_p - g\sum_i \hat{\sigma}_i^x$$

$\boxed{J=0}$

Paramagnet

$|\Psi_0\rangle = \bigotimes_i |+\rangle_i$

$\boxed{J=0}$ Paramagnet (disordered phase)

$|\psi_0\rangle = \bigotimes_i |+\rangle_i$

Gauge structure

$\boxed{g=0}$

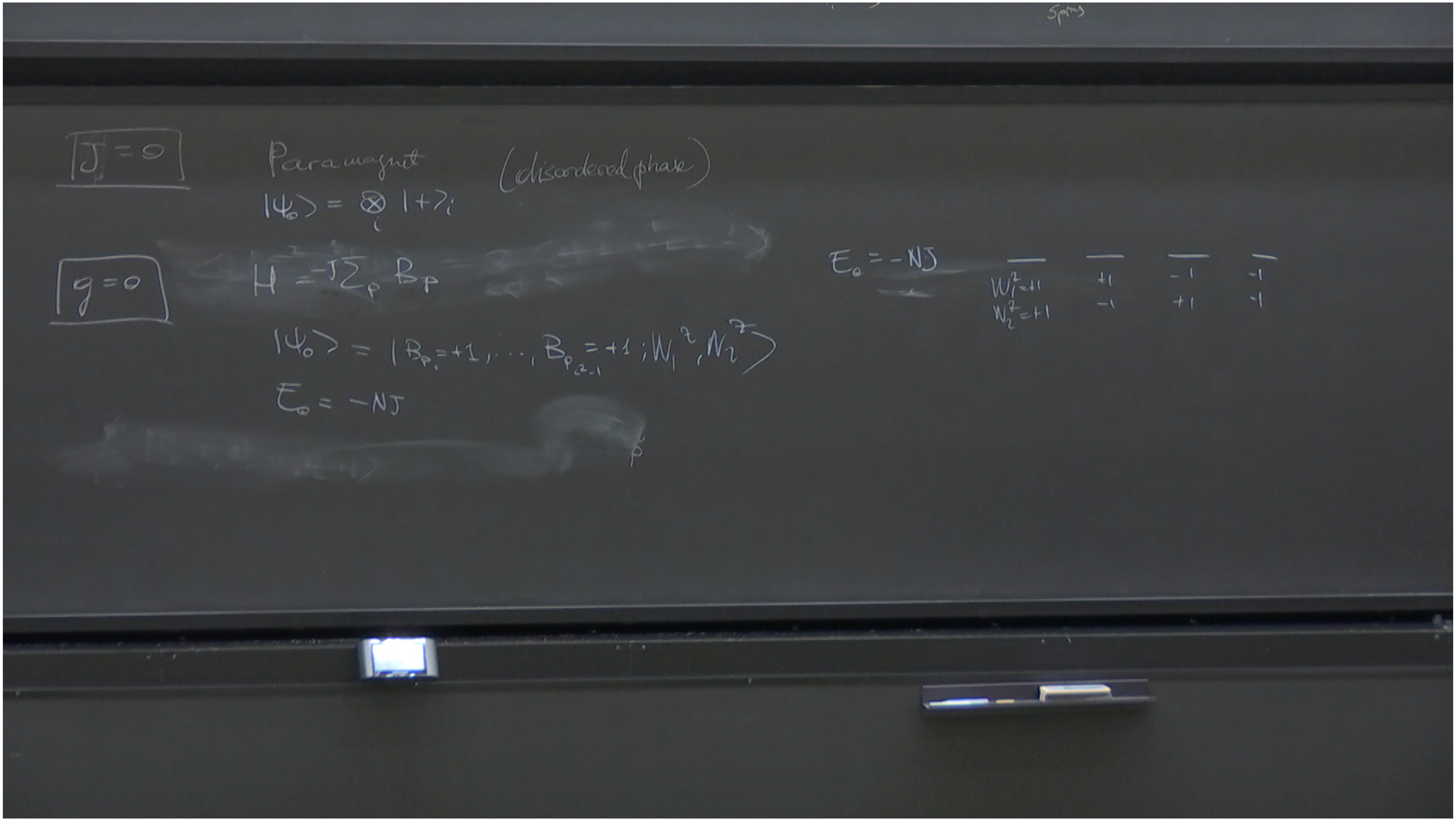
$H = -J\sum_p B_p$

$J\sum_p B_p - g\sum_i \hat{\sigma}_i^x$

$|\psi_0\rangle = |B_{p_1}=+1, \ldots, B_{p_{L^2-1}}=+1; W_1^?, W_2^?\rangle$

$E_0 = -NJ$

$|B_p=1; W_1=-1, W_2=+1\rangle$


J = 0
Paramagnet (disordered phase)
|ψ₀> = ⊗ᵢ |+>ᵢ
g = 0
H = -J Σ_p B_p
|ψ₀> = |B_p1 = +1, ..., B_p(L²-1) = +1; W₁², W₂²>
E₀ = -NJ
E₀ = -NJ
W₁² = +1 +1 -1 -1
W₂² = +1 -1 +1 -1

$= 0$

$= 0$

Paramagnet (disordered phase)

$|\psi_0\rangle = \bigotimes_i |+\rangle_i$

$H = -J\sum_p B_p$

$|\psi_0\rangle = |B_{p_1}=+1, \dots, B_{p_{L^2-1}}=+1 ; W_1^z, W_2^z\rangle$

$E_0 = -NJ$

$B_{\tilde{p}} = -1 \qquad \prod_p B_p = 1$

$\} 4J$

$\} 4J$

$E_0 = -NJ$

$W_1^z=+1$	$+1$	-1	-1	
$W_2^z=+1$	-1	$+1$	-1	

$= 0$

$= 0$

Paramagnet (disordered phase)

$|\psi_0\rangle = \bigotimes_i |+\rangle_i$

$H = -J\sum_p B_p$

$|\psi_0\rangle = |B_{p_1} = +1, \dots, B_{p_{L^2-1}} = +1; W_1^z, W_2^z\rangle$

$E_0 = -NJ$

$B_{\tilde{p}} = -1 \qquad \prod_p B_p = 1$

$\} 4J$

$\} 4J$

$E_0 = -NJ$

$W_1^z = +1$	$+1$	-1	-1
$W_2^z = +1$	-1	$+1$	-1

$A(n)|\psi\rangle = |\psi\rangle$

$\Leftrightarrow [A(n), \psi] = 0$

$\langle\psi_0|\sigma_\ell^z|\psi_0\rangle = \langle\psi_0|\sigma_\ell^z A(n)|\psi_0\rangle = -\langle\psi_0|A(n)\sigma_\ell^z|\psi_0\rangle$

$= -\langle\psi_0|\sigma_\ell^z|\psi_0\rangle$