Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 12

Speakers: Alioscia Hamma

Collection: PSI 2019/2020 - Quantum Matter Part 1

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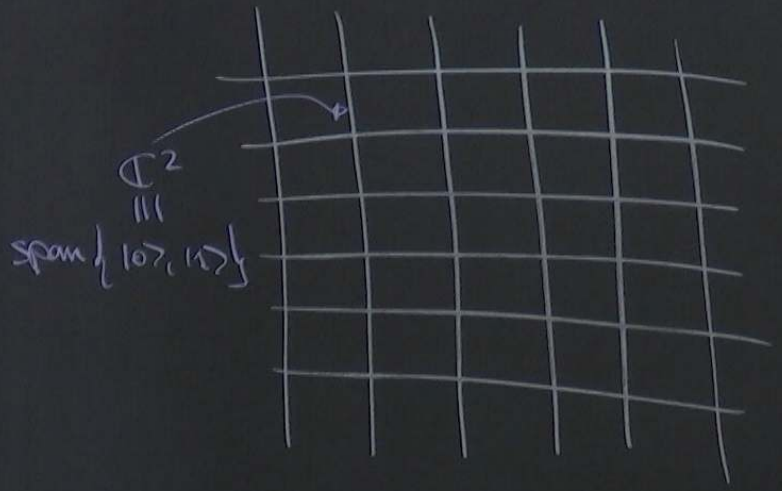
URL: http://pirsa.org/20010037

Quantum lattice $\mathbb{Z}_2$ gauge theory



$N = L \times L$ sites

$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$

$|\psi\rangle = |\psi'\rangle$ in $\mathcal{H}_{tot}$

$= |\psi\rangle, \forall n\}$

$\tilde{\mathcal{H}} = \{ |\psi\rangle \in \mathcal{H}_{tot} : A(\vec{n})|\psi\rangle = |\psi\rangle \}$

$\tilde{\mathcal{H}} = \mathbb{P}\,\mathcal{H}_{tot}$

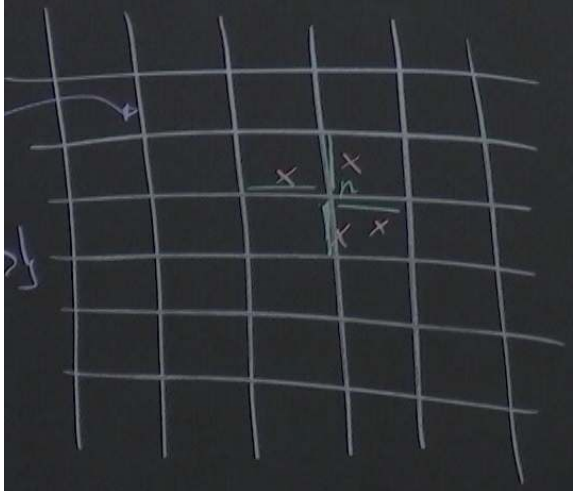
$\mathbb{P} = \frac{\mathbb{I} + A(\vec{n})}{2}$

$\mathbb{P}^2 = \mathbb{P}$

$[\mathbb{P}, A(n)] = 0$

$A(\vec{n})\mathbb{P}|\psi\rangle = \frac{1}{2}\left(A(\vec{n}) + \underbrace{A^2(\vec{n})}_{=\mathbb{I}}\right)|\psi\rangle = \mathbb{P}|\psi\rangle$

Quantum lattice $\mathbb{Z}_2$ gauge theory



$N = L \times L$ sites

$\mathcal{H}_{TOT} \simeq \mathbb{C}^2 \otimes 2L^2$

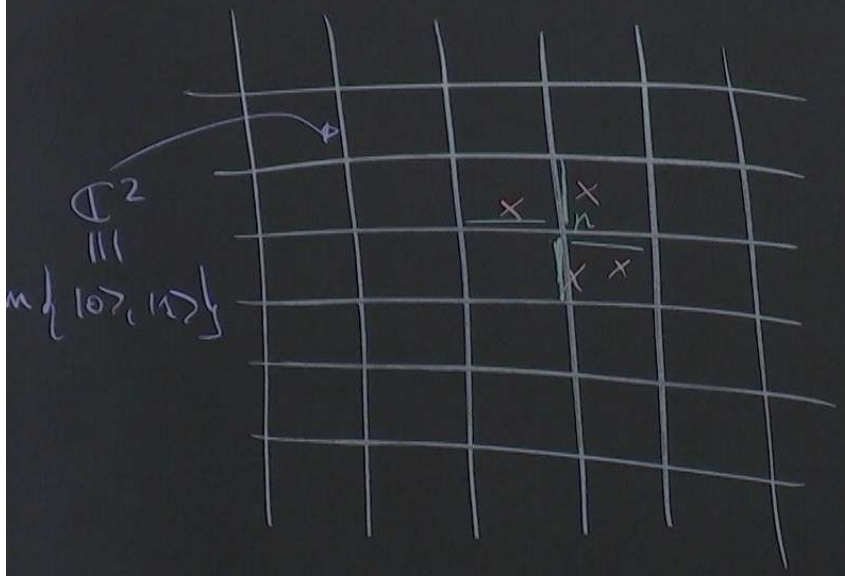
$A(n)\,|\psi\rangle = |\psi\rangle \quad \forall n$ in $\mathcal{H}_{TOT}$

$\mathcal{H}_{gauge} = \{|\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle, \forall n\}$

$A(n) = \prod_{l \in n} \hat{\sigma}_l^x$ local

$[A(n), A(n')] = 0$

Quantum lattice $\mathbb{Z}_2$ gauge theory



$N = L \times L$ sites

$\mathcal{H}_{TOT} \cong \mathbb{C}^{2 \otimes 2L^2}$

$\mathcal{H}_{gauge} = \{|\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle, \ldots$

$A(n)\,|\psi\rangle$

$$\boxed{A(n) = \prod_{l \in n} \hat{\sigma}_l^x}$$

local

$P(n) = \ldots$

$[A(n), A(n')] = 0$

$\#\, A(n) = L^2$

$\prod_{n \in \Lambda} A(n)$

$A(n)\,|\psi\rangle = |\psi'\rangle$ in $\mathcal{H}_{tot}$

$\mathbb{C}^2 \otimes \mathbb{C}^2$

$|\psi\rangle \in \mathcal{H}_{tot}$: $A(n)|\psi\rangle = |\psi\rangle, \forall n\} = \mathcal{P}\mathcal{H}_{tot} = \prod_{n\in\Lambda} P(n)\mathcal{H}_{tot}$

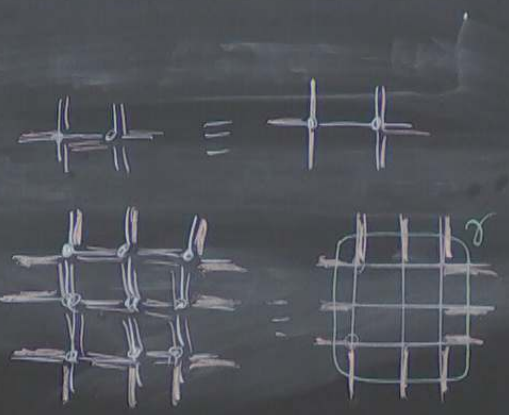
$\hat{\sigma}_l^x$ $\in n$

$n)] = 0$

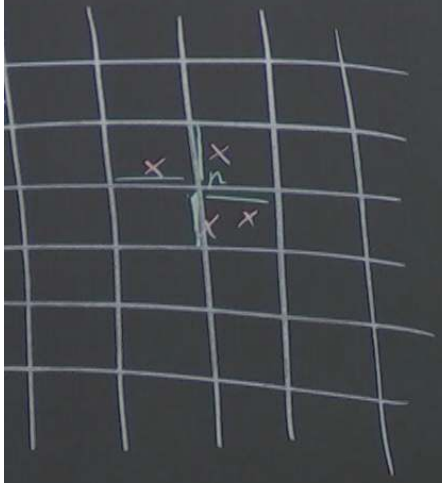
local $P(n) = \frac{\mathbb{1} + A(n)}{2}$ $A(n)$

$\#\, A(n) = L^2$ $\forall\, n_1$ $\prod_{n \neq n_1} A(n) = A(n_1)$

$\prod_{n\in\Lambda} A(n) = \mathbb{1}$



Lattice $\mathbb{Z}_2$ gauge theory



$N = L \times L$ sites

$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$

$\hat{A}(n)|\psi\rangle = |\psi'\rangle$ in $\mathcal{H}_{TOT}$

$\mathcal{H}_{gauge} = \{|\psi\rangle \in \mathcal{H}_{TOT} : \hat{A}(n)|\psi\rangle = |\psi\rangle, \forall n\} = \mathcal{P}\mathcal{H}_{TOT} = \prod_{n \in \Lambda} P(n)$

$$\boxed{\hat{A}(n) = \prod_{l \in n} \hat{\sigma}_l^x}$$

local

$$P(n) = \frac{\mathbb{1} + \hat{A}(n)}{2}$$

$[\hat{A}(n), \hat{A}(n')] = 0$

$\hat{A}^2(n) = \mathbb{1}$

$\#\,\hat{A}(n) = L^2$

$\prod_{n \in \Lambda} \hat{A}(n) = \mathbb{1}$

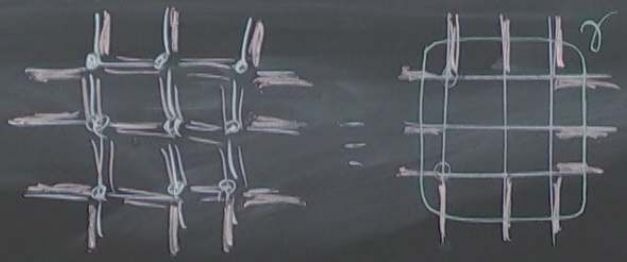
$\forall\, n_1 \Rightarrow \prod_{n \neq n_1} \hat{A}(n) = \hat{A}(n_1)$

$2^{2L^2} \to 2^{2L^2-(L^2-1)} = 2^{L^2+1} = \dim \mathcal{H}_{\text{gauge}}$

$\mathcal{H}_{\text{tot}}$

L^2-1 ind. projections

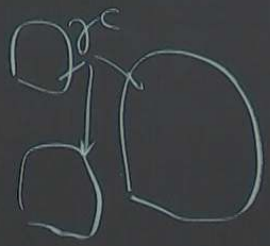
$= \prod_{n \in \Lambda} P(n) \mathcal{H}_{\text{tot}}$



$W^x_\gamma = \prod_{l \in \gamma} \sigma^x_l$ string operator

(loop if γ is closed)

$\prod_{n \in I} A(n) = W^x_\gamma$



$A(n) = \pm 1$

$\{A(n), \mathbb{1}\} = \mathbb{Z}_2$

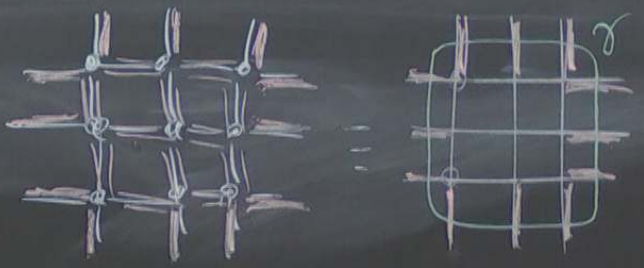
$(n) = \hat{A}(n_4)$

$\dim \mathcal{H}_{TOT} = 2^{2L^2} \to 2^{2L^2-(L^2+1)} = 2^{L^2+1} = \dim \mathcal{H}_{gauge}$

$|\psi\rangle = |\psi'\rangle \in \mathcal{H}_{TOT}$

$L^2 - 1$ ind. projections

$A(n) = +1$



$\forall \{n\} \quad P\mathcal{H}_{TOT} = \prod_{n \in \Lambda} P(n)\mathcal{H}_{TOT}$

$\prod_{n \in I} A(n)$

$= \frac{\mathbb{1} + \hat{A}(n)}{2}$

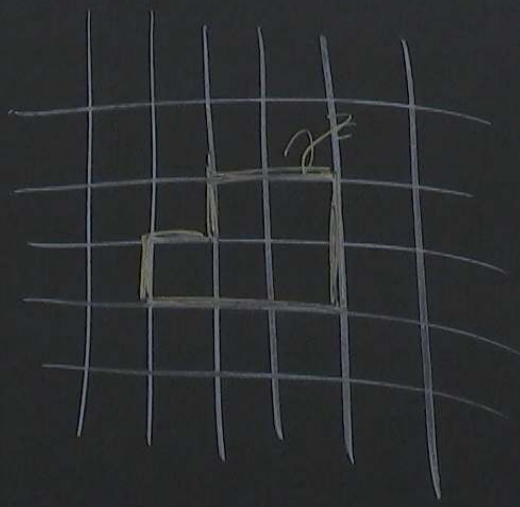
$A(n) = \pm 1$

$\{A(n), \mathbb{1}\} = \mathbb{Z}_2$

$\forall n_1 \quad \prod_{n \neq n_2} \hat{A}(n) = \hat{A}(n_2)$

basis in $\mathcal{H}_{gauge}$?

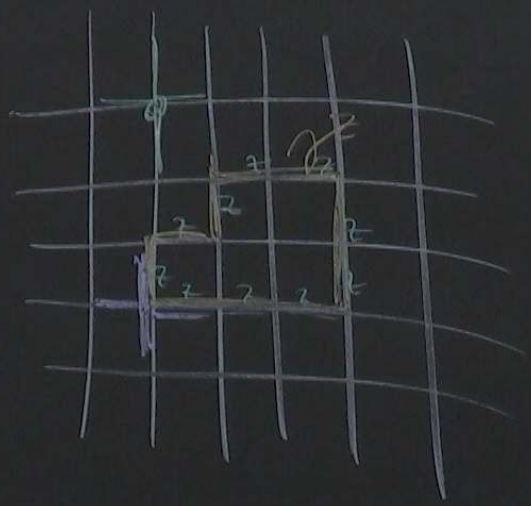
$[B, A(n)] = 0$



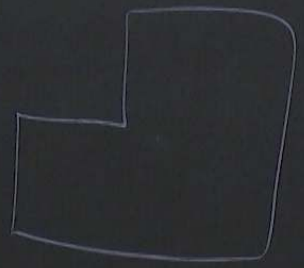
$W_{\gamma^2} = \prod_{e \in \gamma^2} \hat{\sigma}_e^z$

$[W_{\gamma_c^2}, A(n)] = 0$?

?



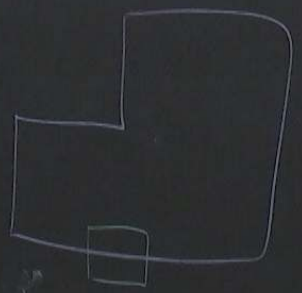
$$W_{\gamma^2} = \prod_{e \in \gamma^2} \hat{\sigma}_e^{\tau}$$

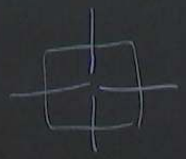


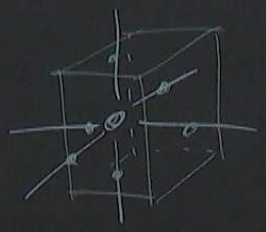
$$[W_{\gamma_c^2}, A(n)] = 0 \; ?$$

Wilson loops

$$W_{\gamma^2} = \prod_{e \in \gamma^2} \hat{\sigma}_e^z$$

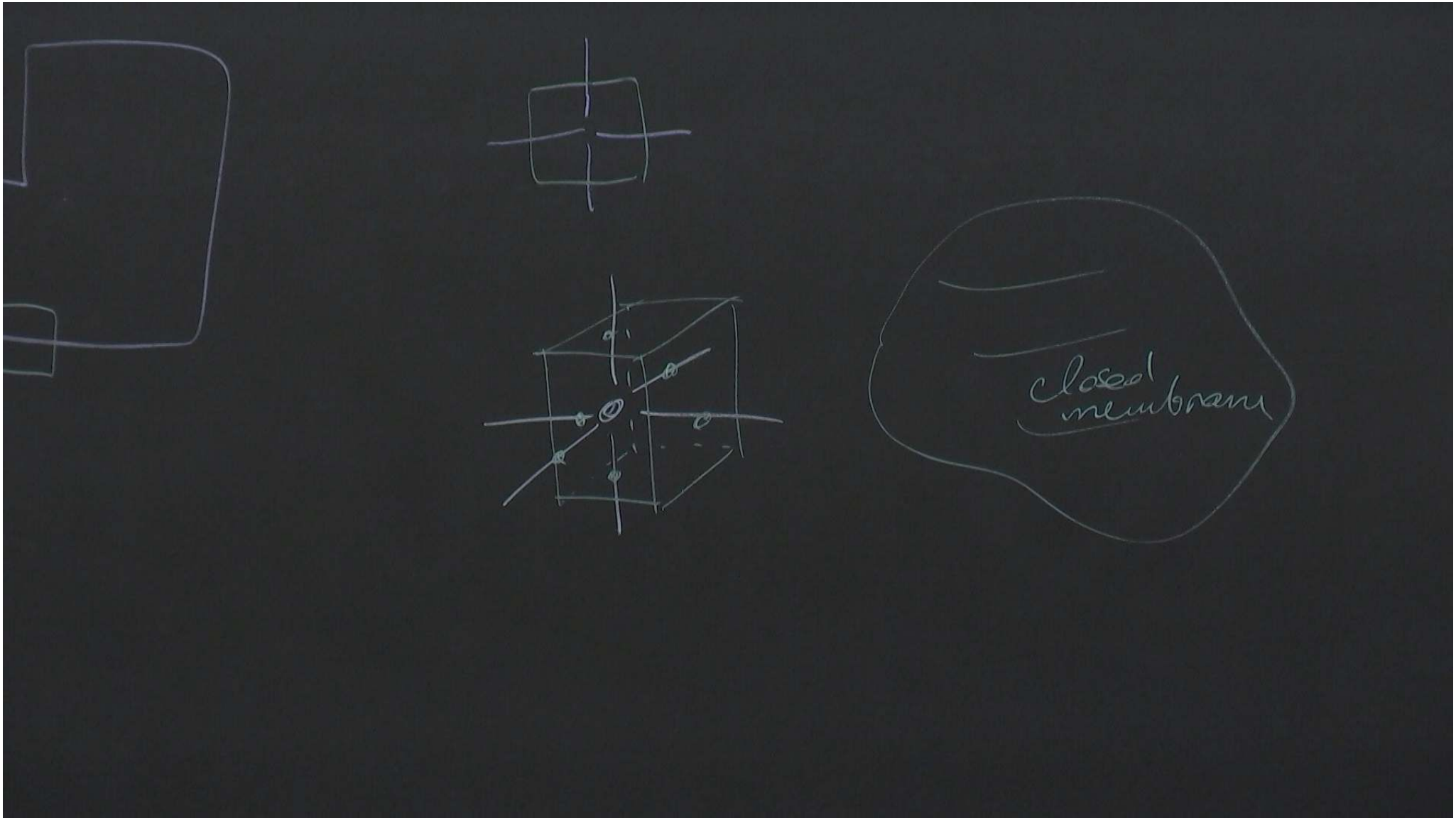


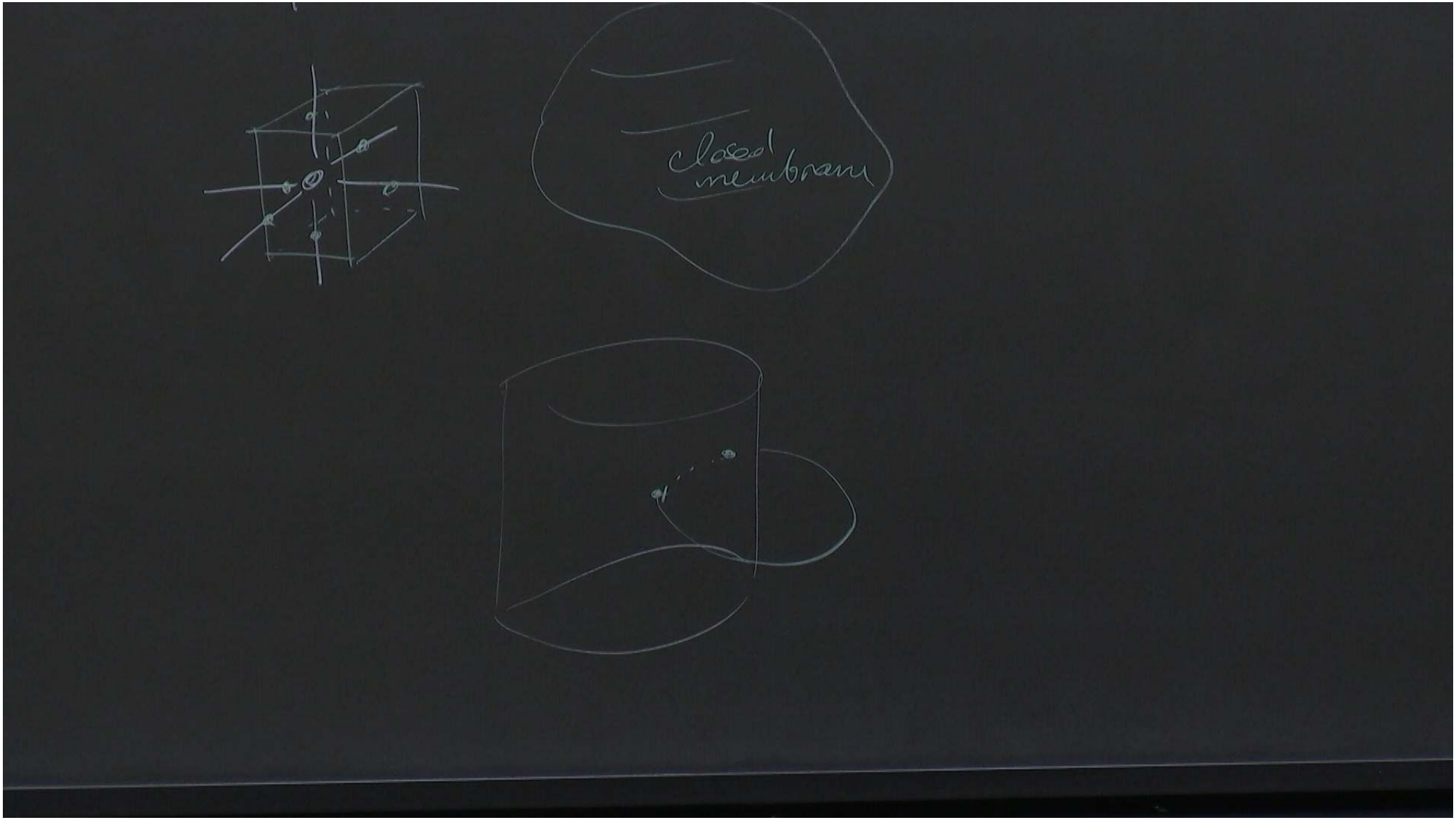




$$[W_{\gamma_c^2}, A(n)] = 0$$

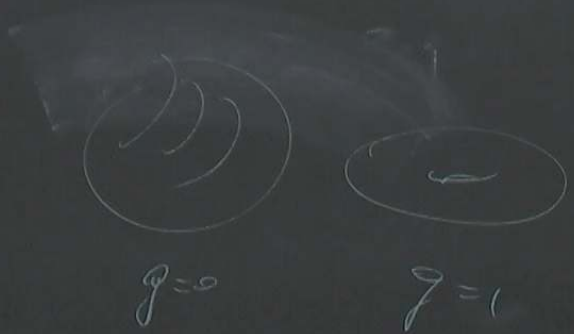
→ are gauge-invariant quantities

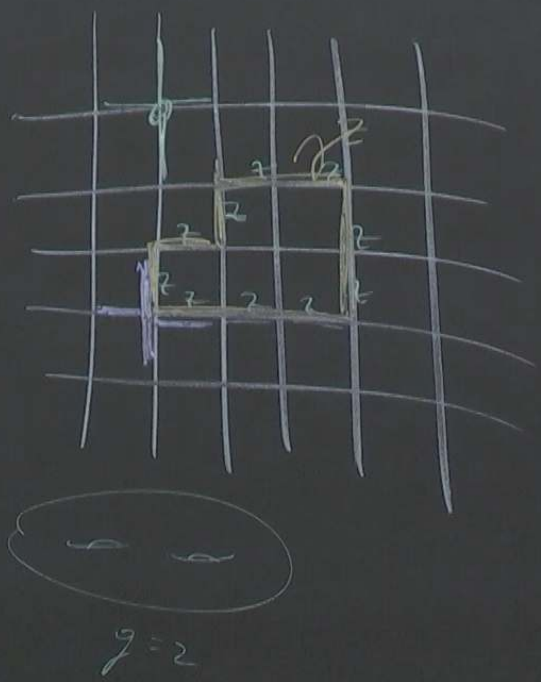

closed
membrane


closed
membrane

What is a basis in $\mathcal{H}_{gauge}$?

$[B, A(n)] = 0$



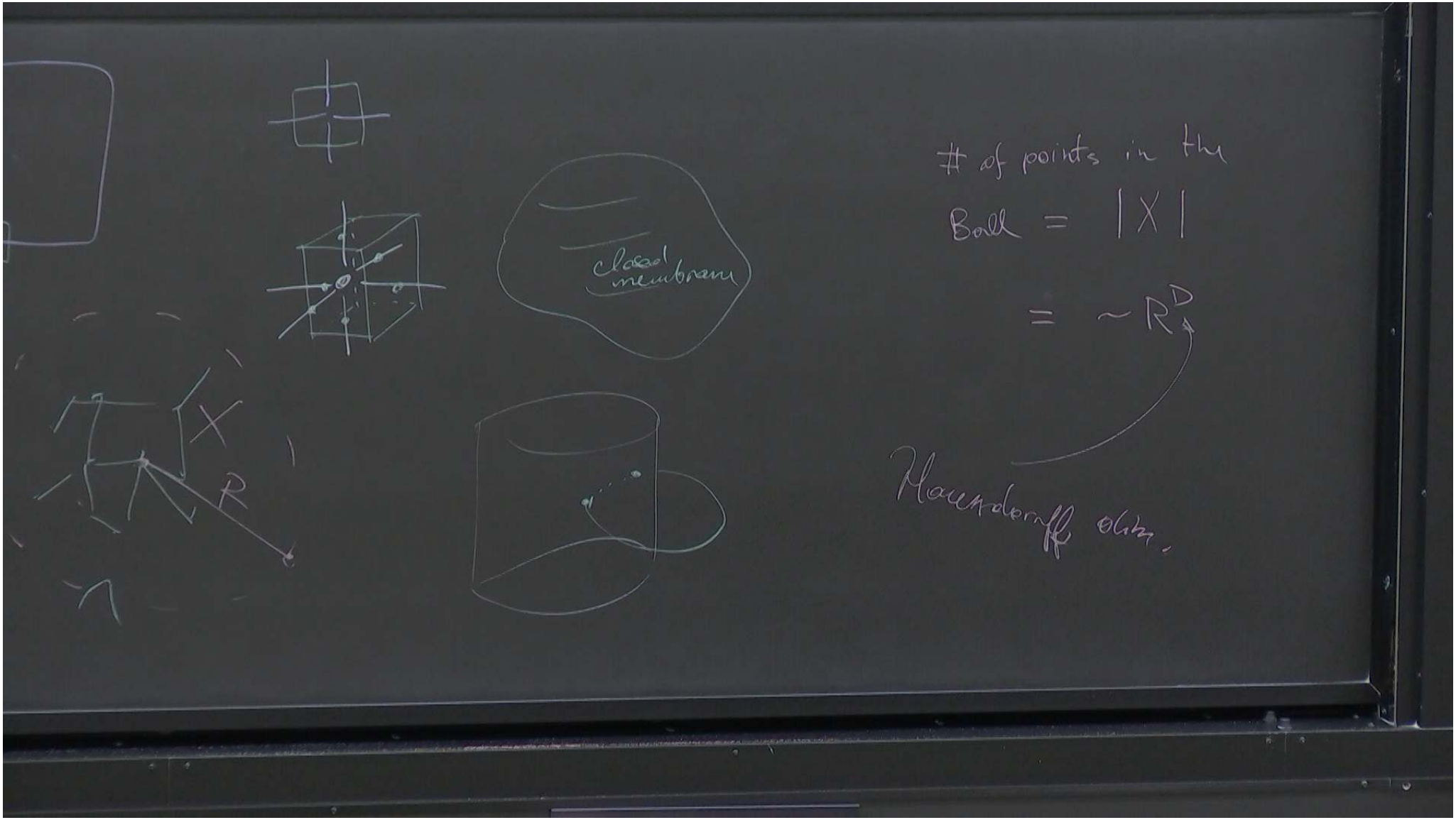


Wilson loops

$W_{\gamma^2} = \prod_{e \in \gamma^2} \hat{U}_e^{\pm}$

$[W_{\gamma_C^2}, A(n)]$

→ are gauge-quantities


# of points in the
Ball = |X|
= ~ R^D
closed membrane
X
R
Hausdorff dim.