Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 12

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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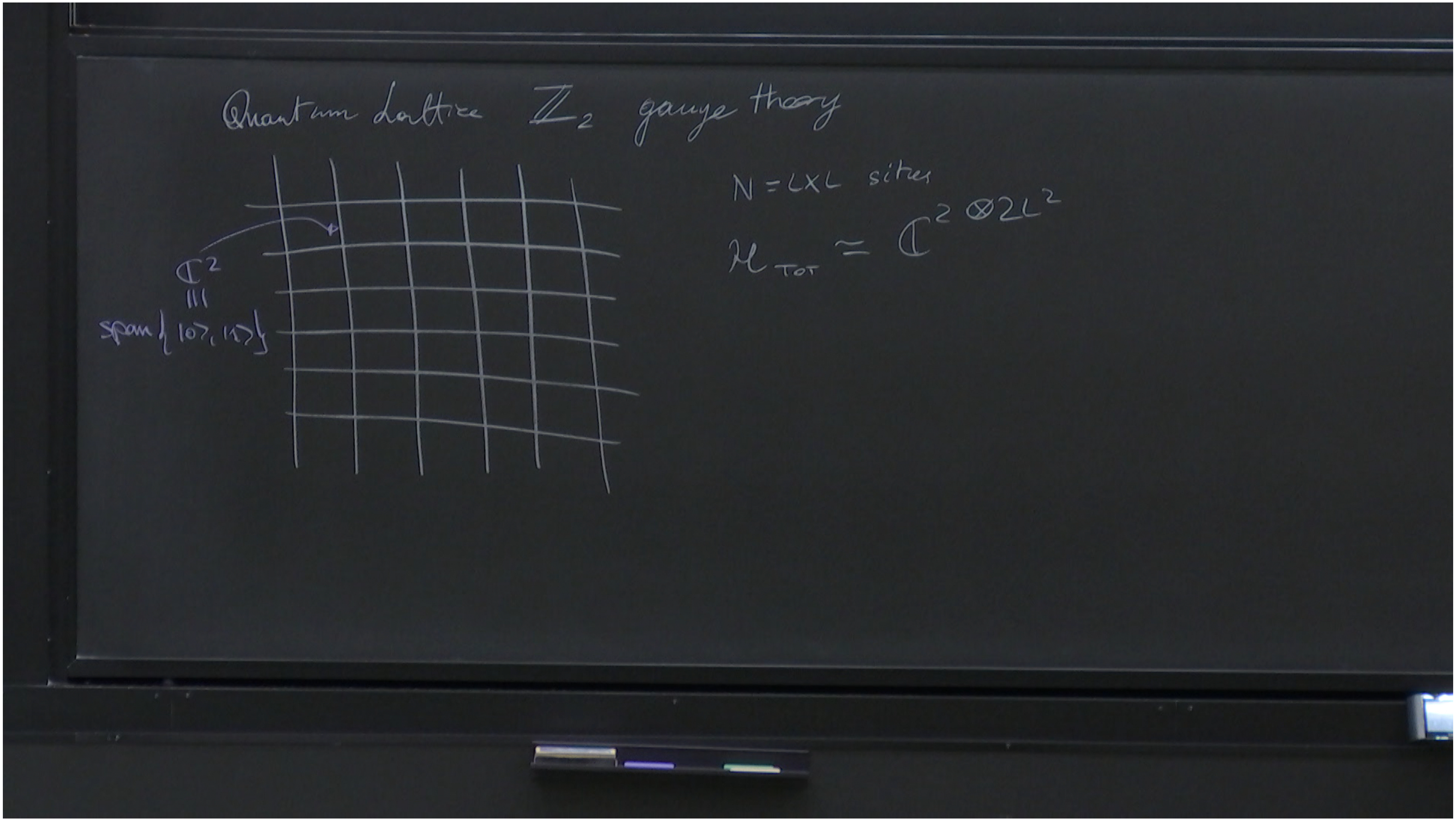

Quantum lattice $\mathbb{Z}_2$ gauge theory
$N = L \times L$ sites
$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathbb{C}^2$
$\cong$
span$\{|0\rangle, |1\rangle\}$

$$|\psi\rangle = |\psi'\rangle \quad \text{in } \mathcal{H}_{TOT}$$

$$= |\psi\rangle, \ \forall n \}$$

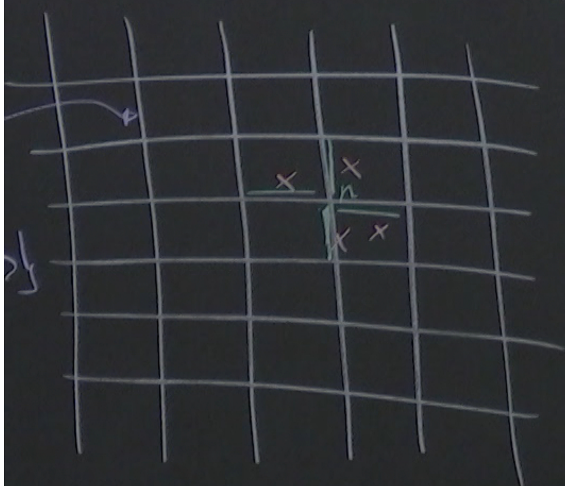
$$\tilde{\mathcal{H}} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : \underline{A(\tilde{n})|\psi\rangle = |\psi\rangle} \}$$

$$P^2 = P$$

$$\tilde{\mathcal{H}} = P\, \mathcal{H}_{TOT} \qquad P = \frac{\mathbb{I} + A(\tilde{n})}{2} \qquad [P, A(n)] = 0$$

$$A(\tilde{n}) P |\psi\rangle = \frac{1}{2}\big(A(\tilde{n}) + \underset{\mathbb{I}}{\underset{\parallel}{A^2(\tilde{n})}} \big)|\psi\rangle = P|\psi\rangle$$

Quantum lattice $\mathbb{Z}_2$ gauge theory



$N = L \times L$ sites

$A(n)|\psi\rangle = |\psi'\rangle$ in $\mathcal{H}_{TOT}$

$$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$$

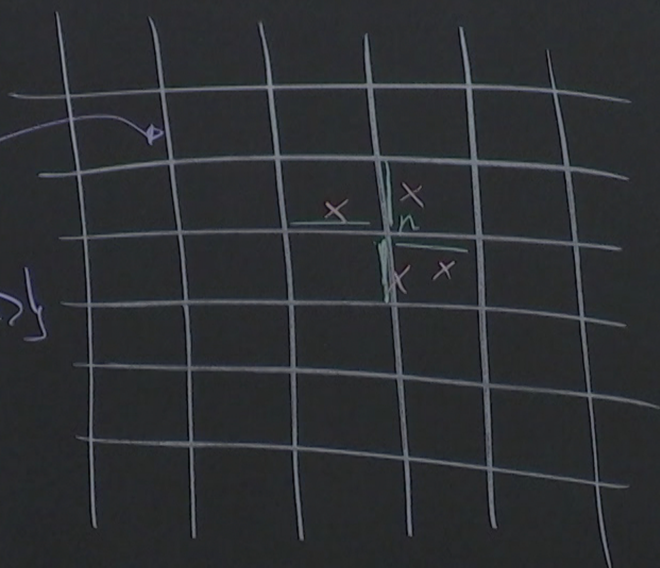
$$\mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle, \forall n \}$$

$$A(n) = \prod_{l \in n} \hat{\sigma}^x_l$$ local

$$[A(n), A(n')] = 0$$

Quantum Lattice $\mathbb{Z}_2$ gauge theory

$\mathbb{C}^2$
$\cong$
$\text{n}\{|0\rangle, |1\rangle\}$



$N = L \times L$ sites

$A(n)|\psi\rangle$

$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$

$\mathcal{H}_{gauge} = \{|\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle,$

$$\boxed{A(n) = \prod_{\ell \in n} \hat{\sigma}^x_\ell}$$

local

$P(n) =$

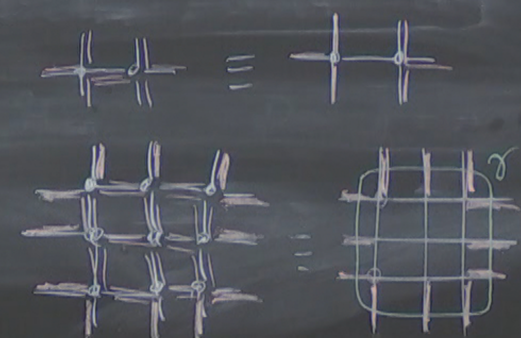
$[A(n), A(n')] = 0$

$\# A(n) = L^2$

$\prod_{n \in \Lambda} A(n)$

$$A(n)|\psi\rangle = |\psi'\rangle \quad \text{in } \mathcal{H}_{TOT}$$

$$\mathbb{C}^2 \otimes 2L^2$$



$$|\psi\rangle \in \mathcal{H}_{TOT} : A(n)|\psi\rangle = |\psi\rangle, \forall n \} = P\,\mathcal{H}_{TOT} = \prod_{n\in\Lambda} P(n)\,\mathcal{H}_{TOT}$$

$$\boxed{\prod_{\ell\in n} \hat{\sigma}^x_\ell}$$

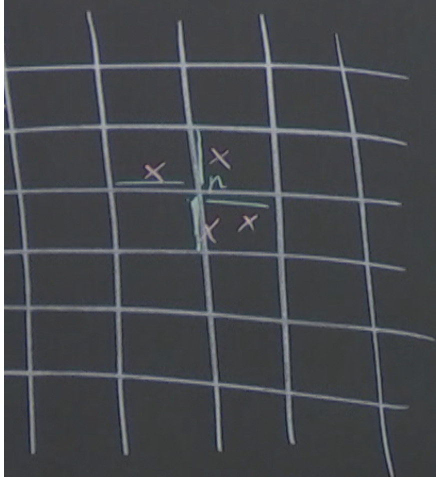
local $\quad P(n) = \dfrac{\mathbb{1} + A(n)}{2} \qquad A(n)$

$$\# A(n) = L^2 \qquad \forall n_1 \quad \prod_{n\neq n_1} A(n) = A(n_1)$$

$$n')] = 0$$

$$\prod_{n\in\Lambda} A(n) = \mathbb{1} \implies$$

lattice $\mathbb{Z}_2$ gauge theory



$N = L \times L$ sites

$\hat{A}(n)|\psi\rangle = |\psi'\rangle$ in $\mathcal{H}_{TOT}$

$$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$$

$$\mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H}_{TOT} : \hat{A}(n)|\psi\rangle = |\psi\rangle , \forall n \} = \mathbb{P}\mathcal{H}_{TOT} = \prod_{n \in \Lambda} \mathbb{P}(n$$

$$\boxed{\hat{A}(n) = \prod_{l \in n} \hat{\sigma}^x_l}$$

local

$$\mathbb{P}(n) = \frac{\mathbb{1} + \hat{A}(n)}{2}$$

$$[\hat{A}(n), \hat{A}(n')] = 0$$

$$\hat{A}^2(n) = \mathbb{1}$$

$$\# \hat{A}(n) = L^2$$

$$\prod_{n \in \Lambda} \hat{A}(n) = \mathbb{1}$$

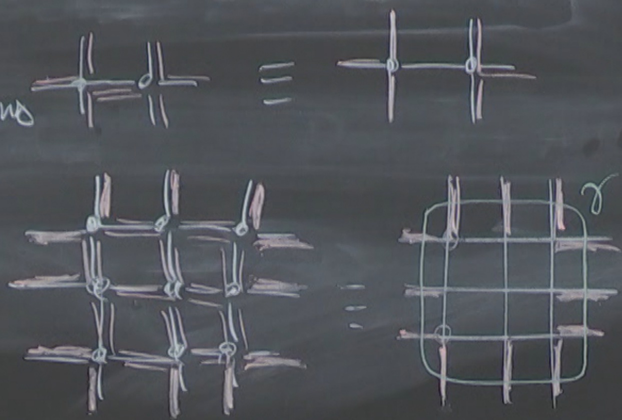
$$\forall n_1 \implies \prod_{n \neq n_1} \hat{A}(n) = \hat{A}(n_1)$$

$$2^{2L^2} \to 2^{2L^2-(L^2+1)} = 2^{L^2+1} = \dim \mathcal{H}_{gauge}$$

$\mathcal{H}_{TOT}$

L^2-1 ind. projections

$$\mathcal{H}_{TOT} = \prod_{n\in\Lambda} P(n)\,\mathcal{H}_{TOT}$$

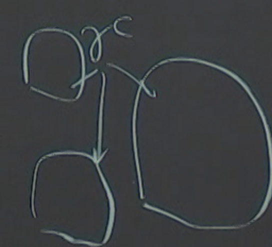


$$W^x_\gamma = \prod_{l\in\gamma} \sigma^x_l$$

String operator

(loop if γ is closed)

$$\prod_{n\in I} A(n) = W^x_\gamma$$



$$A(n) = \pm 1$$

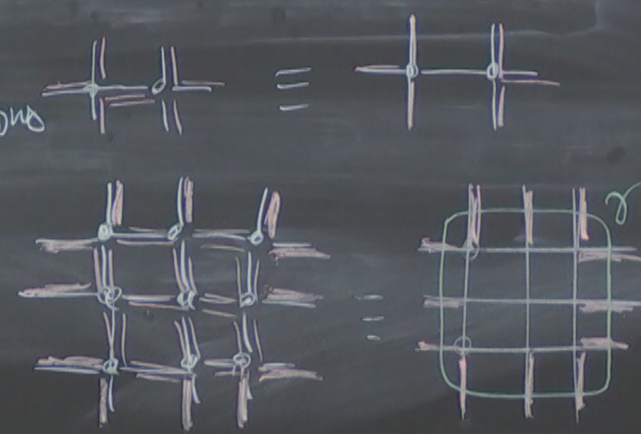
$$\{A(n), \mathbb{1}\} = \mathbb{Z}_2$$

$$(n) = \hat{A}(n_1)$$

$$\dim \mathcal{H}_{TOT} = 2^{2L^2} \to 2^{2L^2-(L^2-1)} = 2^{L^2+1} = \dim \mathcal{H}_{gauge}$$

$> = |\psi'>$ in $\mathcal{H}_{TOT}$

$L^2 - 1$ ind. projections



$A(n) = +1$

$+\rangle), \forall n \} = P\mathcal{H}_{TOT} = \prod_{n \in \Lambda} P(n)\mathcal{H}_{TOT}$

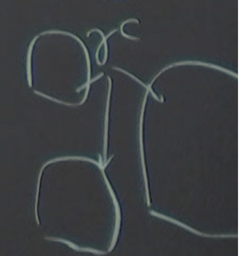
$) = \dfrac{\mathbb{1} + \hat{A}(n)}{2}$

$\forall n_1 \Rightarrow \prod_{n \neq n_1} \hat{A}(n) = \hat{A}(n_1)$

$A(n) = \pm 1$

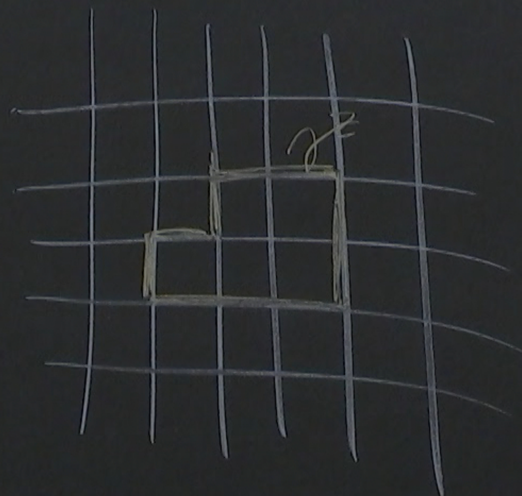
$\{A(n), \mathbb{1}\} = \mathbb{Z}_2$

$\prod_{n \in I} A(n)$



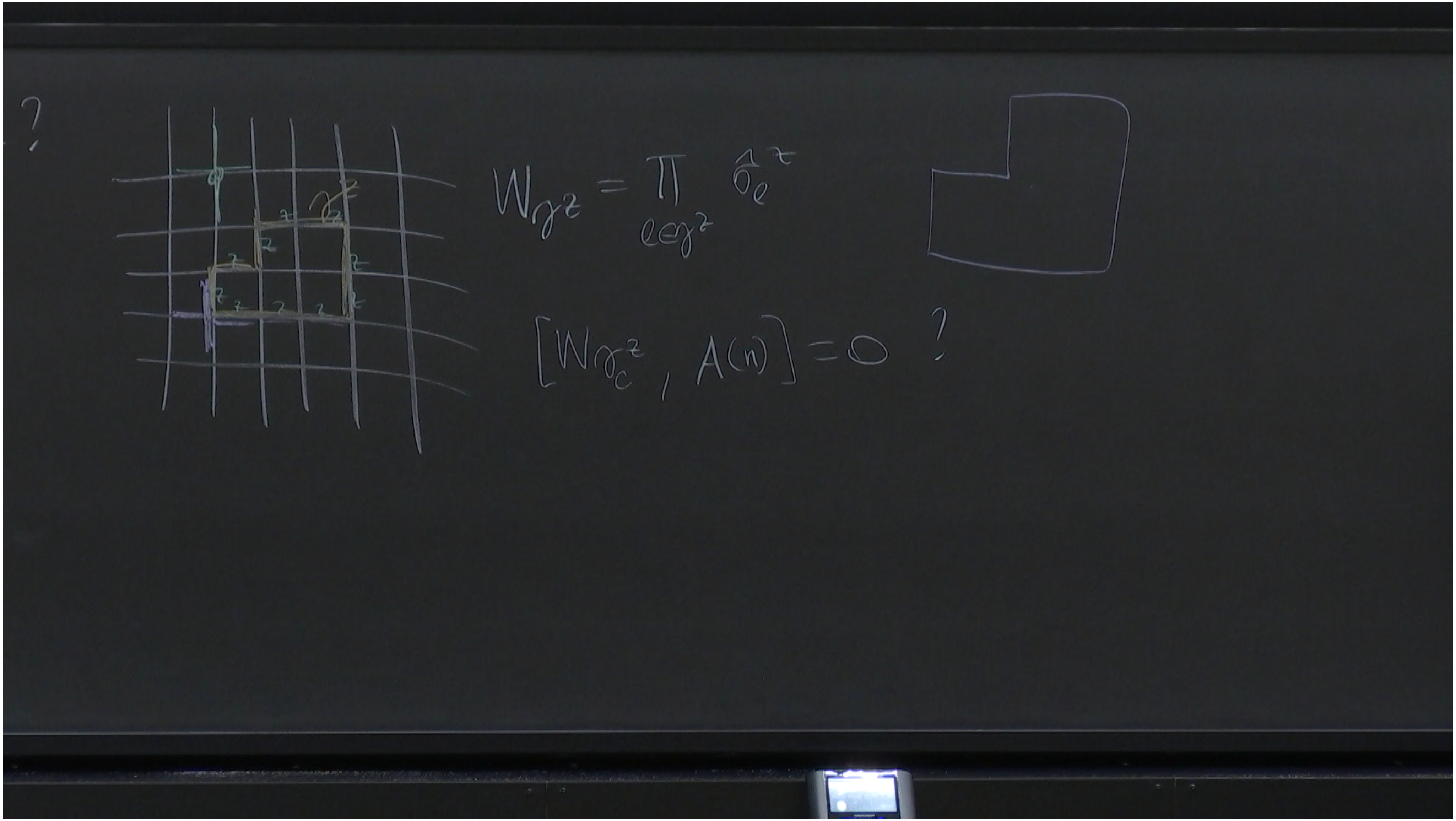
basis in $\mathcal{H}_{gauge}$?

$\ldots, A(n)] = 0$



$$W_{\gamma^z} = \prod_{l \in \gamma^z} \hat{\sigma}_l^z$$

$$[W_{\gamma_c^z}, A(n)] = 0 \;?$$

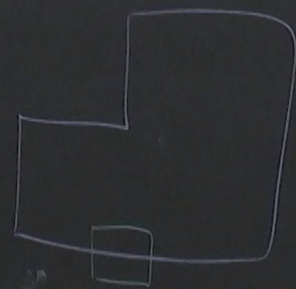

$W_{\gamma^z} = \prod_{l \in \gamma^z} \hat{\sigma}_l^z$
$[W_{\gamma_c^z}, A(n)] = 0$?

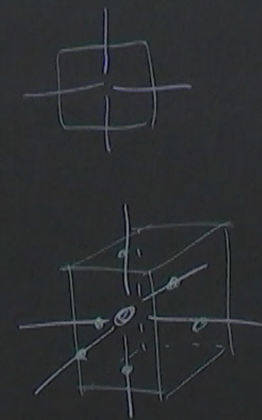
Wilson loops

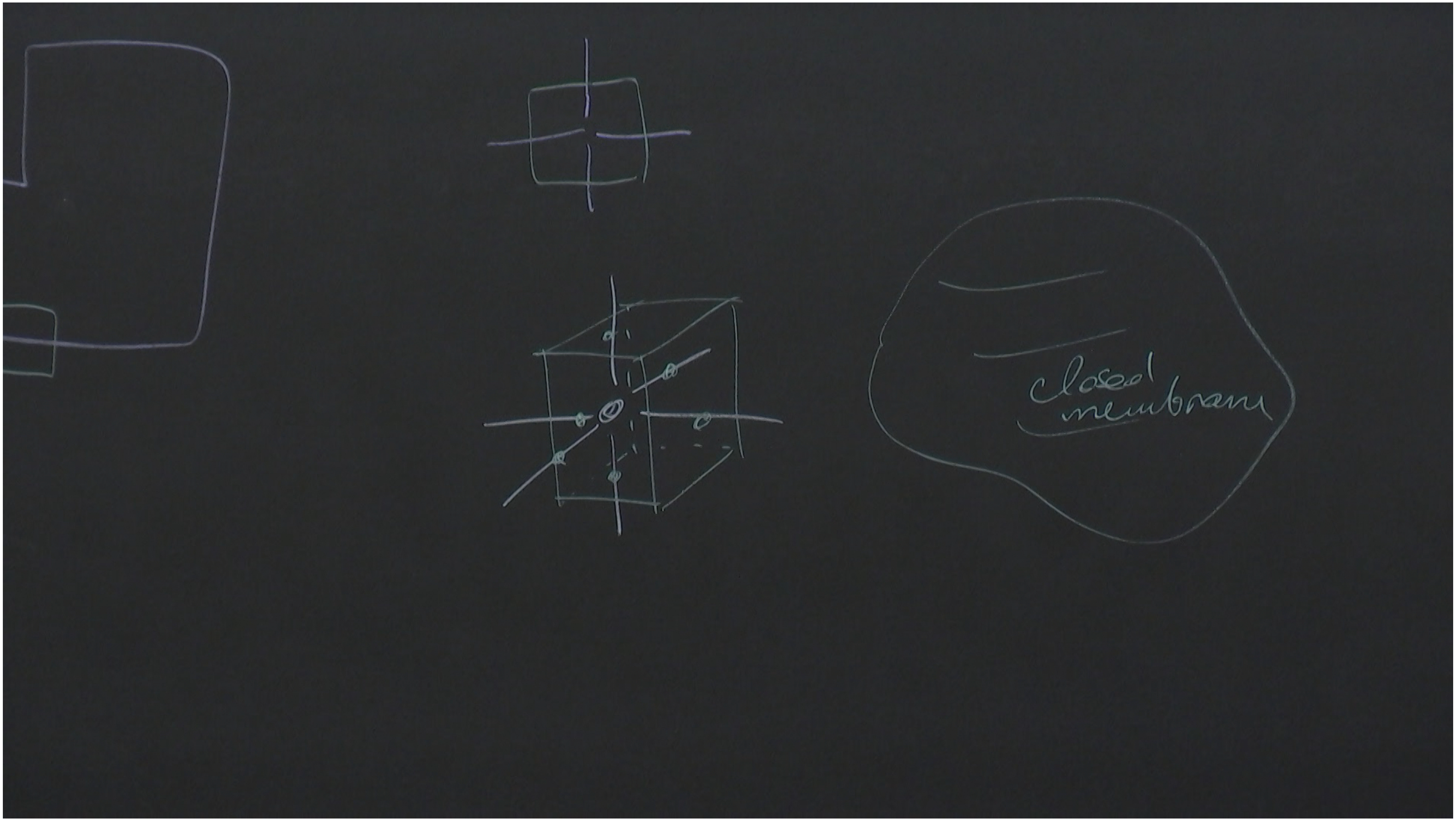
$$W_{\gamma^z} = \prod_{l \in \gamma^z} \hat{\sigma}_l^z$$

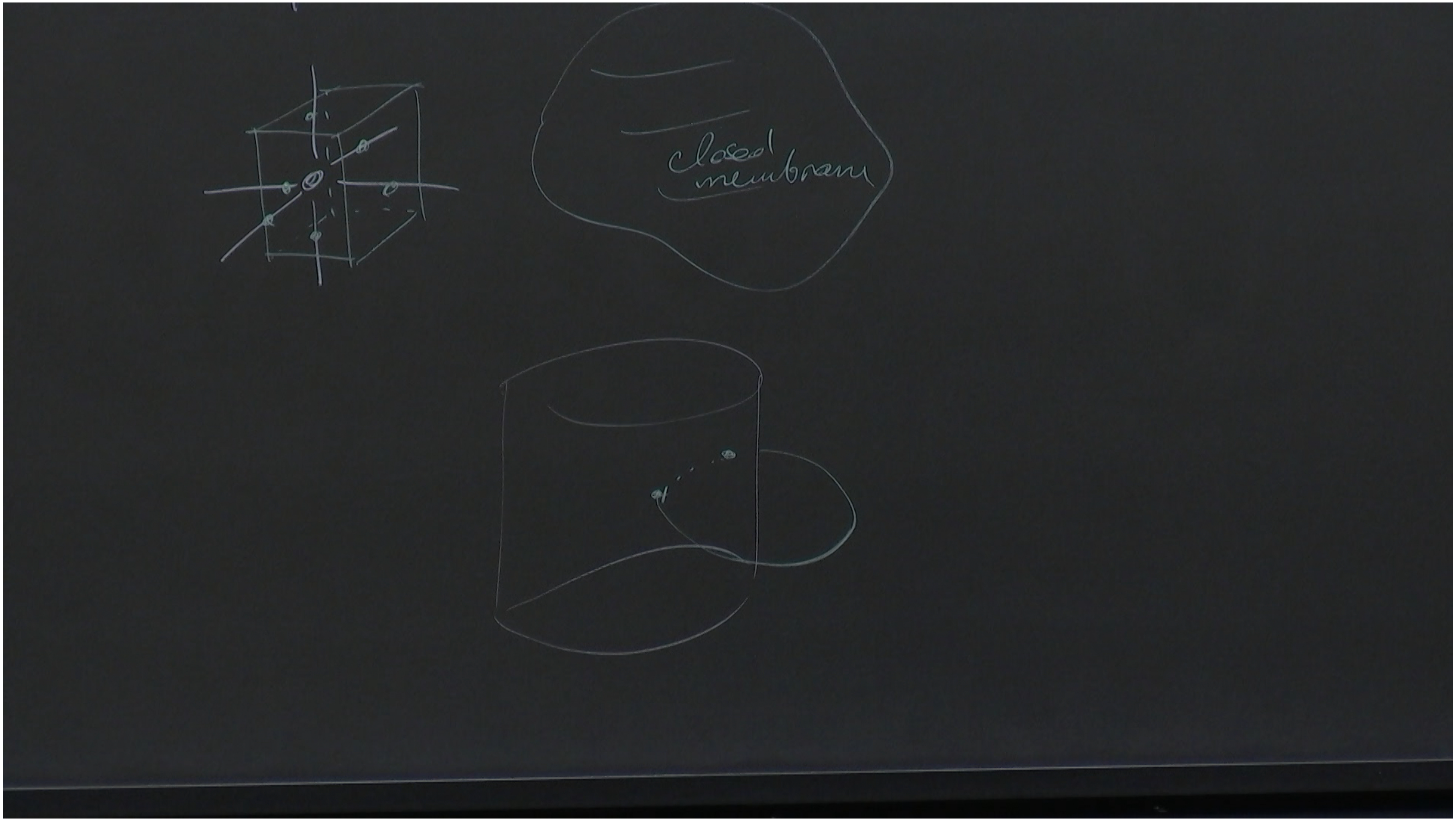
$$[W_{\gamma_c^z}, A(n)] = 0$$

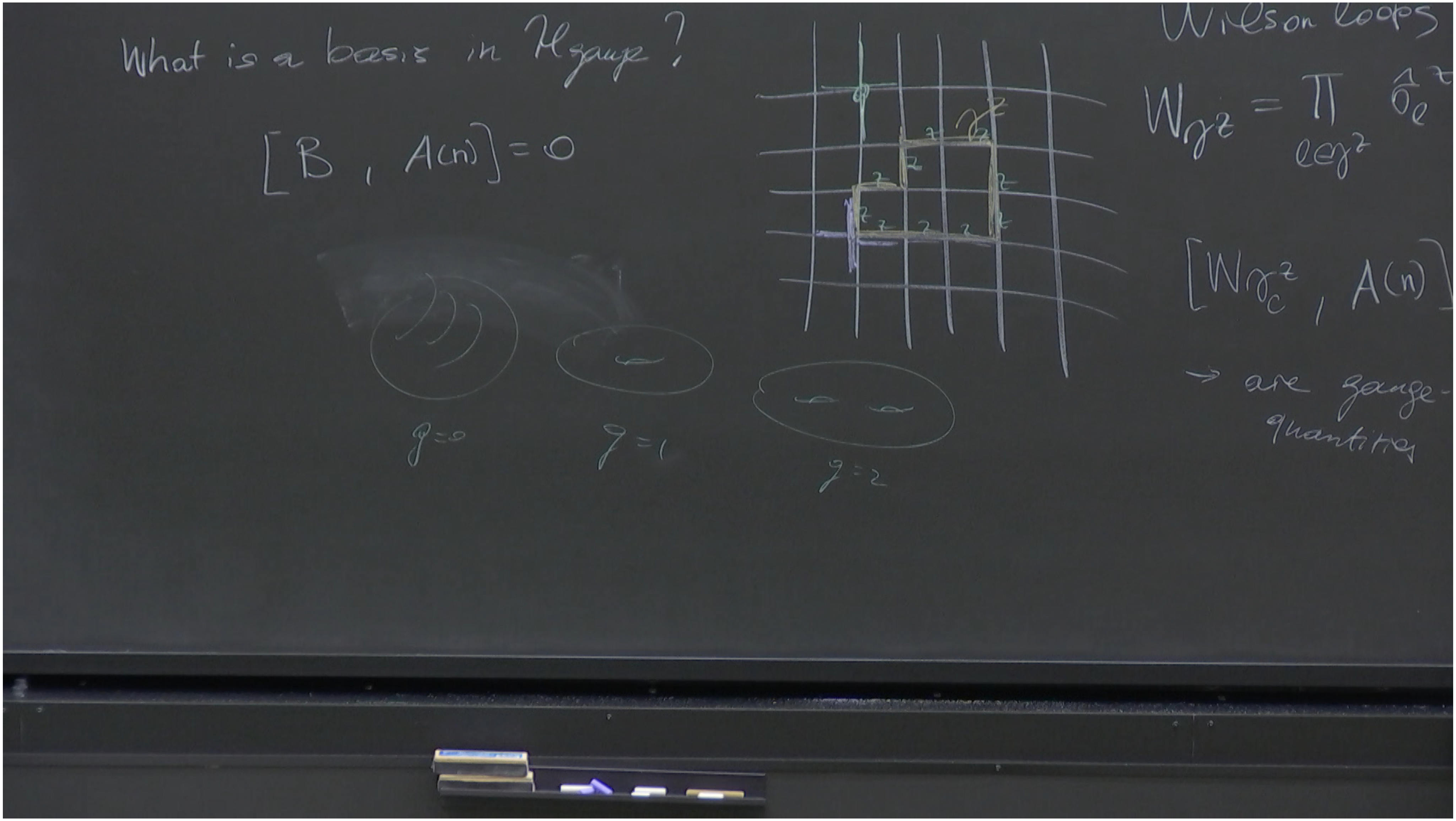
$\rightarrow$ are gauge-invariant quantities

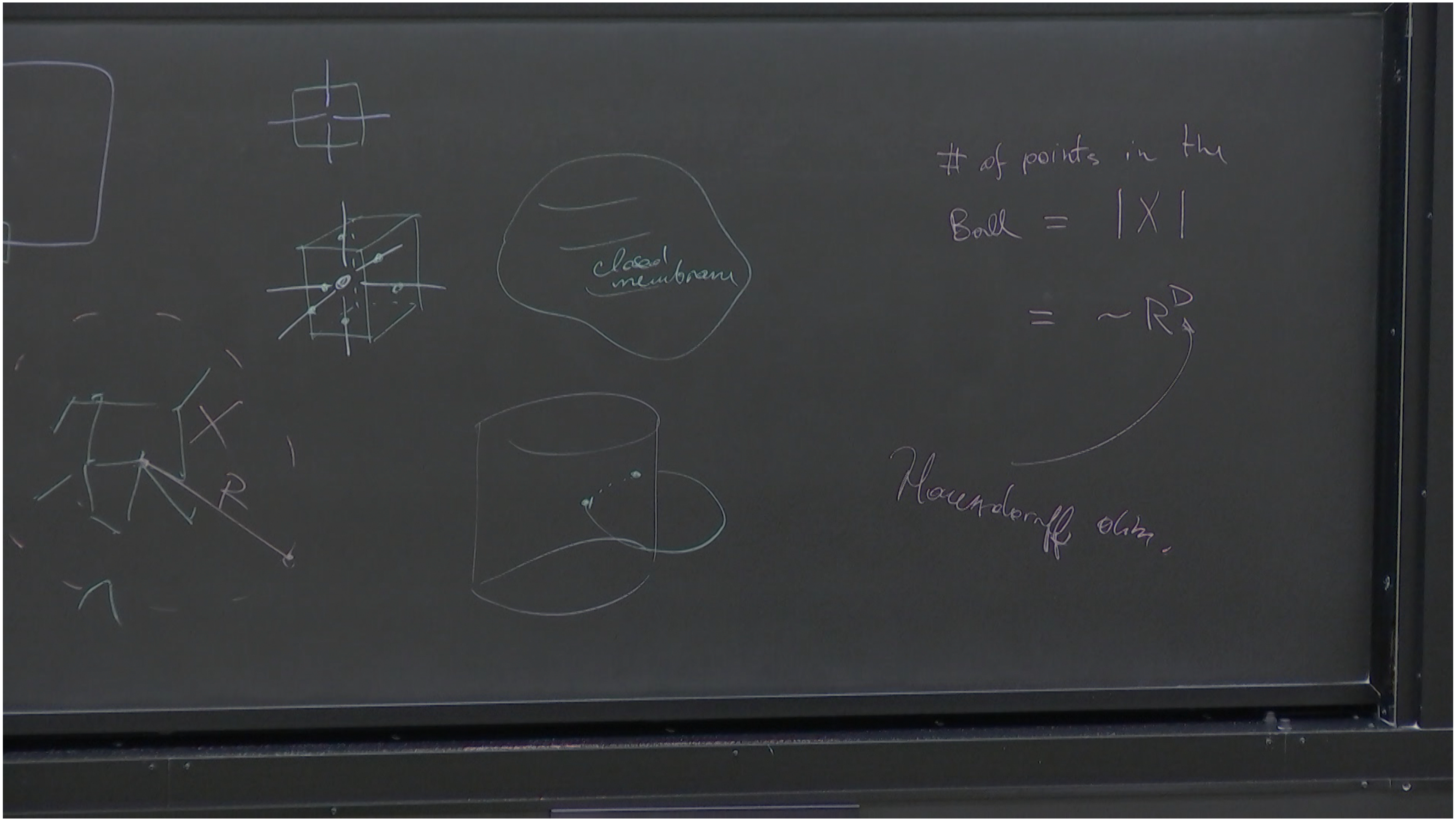





closed
membrane


closed
membrane


What is a basis in $\mathcal{H}_{gauge}$?
$[B, A(n)] = 0$
$g=0$
$g=1$
$g=2$
Wilson loops
$W_{\gamma^z} = \prod_{l \in \gamma^z} \hat{\sigma}_l^z$
$[W_{\gamma_c^z}, A(n)]$
→ are gauge-
quantities



closed membrane
# of points in the
Ball = |X|
= ~R^D
Hausdorff dim.