Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 10

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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$$\langle 0|AB|0\rangle = \sum_{i\geq 1} A_{0i} B_{i0} = F^{-1}\left[\theta(\omega) F\left[\langle 0|[A(t),B]|0\rangle\right]\right]\Big|_{t=0}$$

is finite

$$\boxed{\langle 0|[A(t),B]|0\rangle = \sum_{i\geq 1} \left(e^{-iE_i t} A_{0i} B_{i0} - e^{iE_i t} B_{0i} A_{i0}\right)}$$

$$(*) = \frac{1}{2\pi} \hat{\theta}(t) * \langle 0|[A(t),B]|0\rangle\Big|_{t=0}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \langle 0|[A(t'),B]|0\rangle \hat{\theta}(t-t')\,dt'\Big|_{t=0}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t)\,dt$$

Clustering of Correlations

If the GS is unique (No degeneracy) $\Rightarrow \xi$ is finite

there is a gap Δ above the GS $\overset{?}{\Leftarrow}$

$$\langle 0|A|0\rangle = A_{00} = 0$$

$$\langle 0|B|0\rangle = B_{00} = 0$$

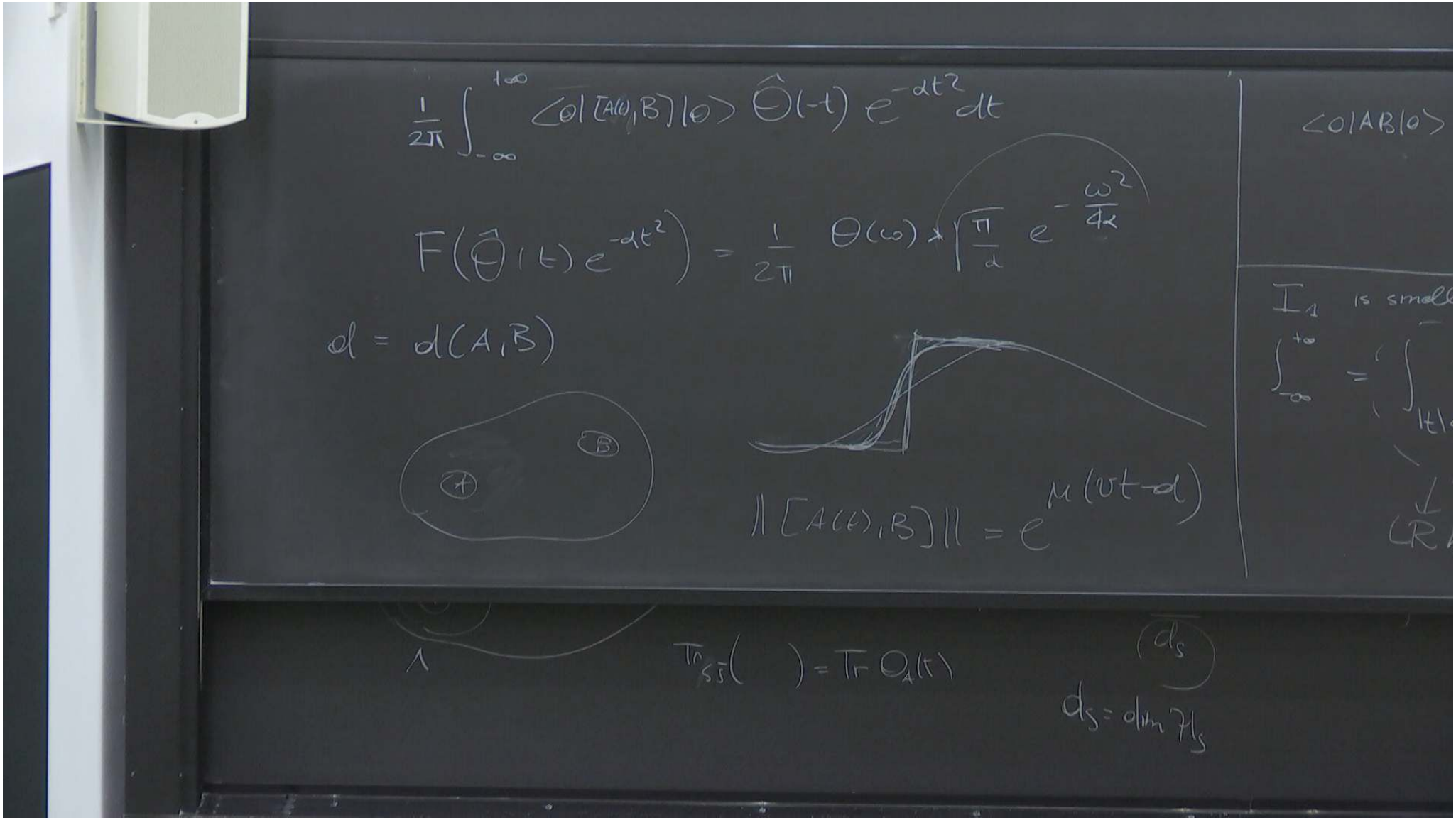
$$\langle 0|AB|0\rangle = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t) e^{-\alpha t^2} + \underbrace{\langle 0|AB|0\rangle - \frac{1}{2\pi}\int_{-\infty}^{+\infty} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t) e^{-\alpha t^2}}_{I_2}$$

I_1

Error due to imperfect filtering

I_1 is small because of LR + Gaussian Envelope

$$\int_{-\infty}^{+\infty} = \int_{|t|<c\ell}$$


$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \langle 0|[A(t),B]|0\rangle \hat{\Theta}(-t)\, e^{-\alpha t^2} dt$
$\langle 0|AB|0\rangle$
$F(\vec{\Theta}(t)\, e^{-\alpha t^2}) = \frac{1}{2\pi}\, \Theta(\omega) * \sqrt{\frac{\pi}{\alpha}}\, e^{-\frac{\omega^2}{4\alpha}}$
I_1 is small
$\int_{-\infty}^{+\infty} = \int_{|t|<}$
$d = d(A,B)$
A
B
$\|[A(t),B]\| = e^{\mu(vt-d)}$
$Tr_{SS}(\quad) = Tr\, \rho_A(t)$
d_S
$d_S = \dim \mathcal{H}_S$

$$-\frac{1}{2\pi}\int_{-\infty}^{+\infty} \langle 0|[A(t'),B]|0\rangle \hat{\Theta}(t-t')\,dt' \Big|_{t=0}$$

$$-\frac{1}{2\pi}\int_{-\infty}^{+\infty} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t)\,dt$$

I_1 $\qquad$ I_2

Error due to imperfect filtering

I_1 is small because of LR + Gaussian Envelope

$$\int_{-\infty}^{+\infty} = \int_{|t|<cd} + \int_{|t|>cd}$$

$|t|<cd$ ↓ LR holds $\qquad$ $|t|>cd$ ↓ e^{-dt^2}

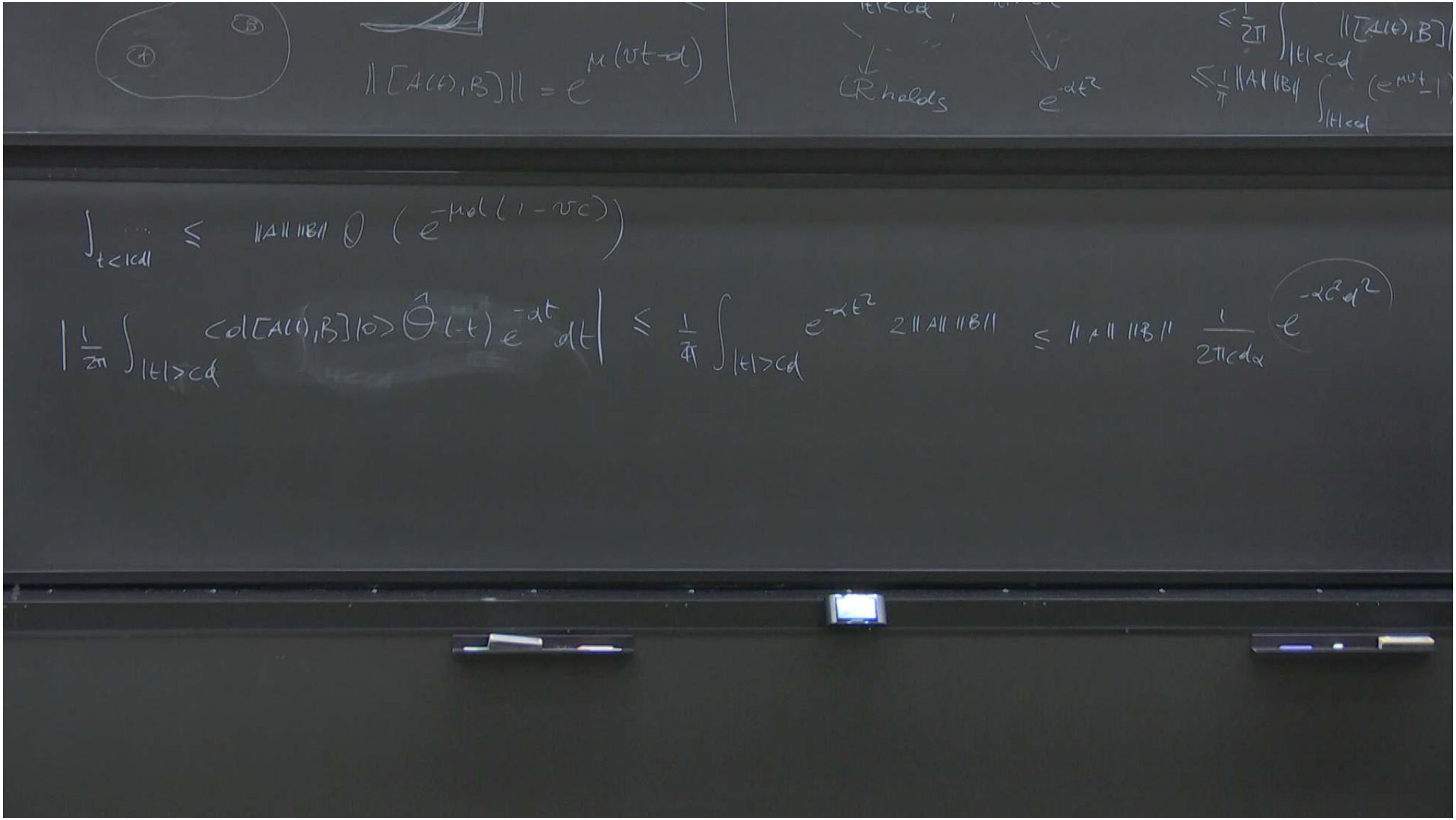
$$\boxed{|t|<cd} \quad \left|\frac{1}{2\pi}\int_{|t|<cd} \langle 0|[A(t),B]|0\rangle \hat{\Theta}(-t) e^{-dt^2}\,dt\right| \leq$$

$$\leq \frac{1}{2\pi}\int_{|t|<cd} \|[A(t),B]\|\left(\delta(t)+\frac{1}{\pi t}\right) dt$$

$$\leq \frac{1}{\pi}\|A\|\|B\|\int_{|t|<cd} (e^{\mu v t}-1)\, e^{\mu d}\left(\delta(t)+\frac{1}{\pi t}\right) dt$$

$$\int_{t<|cd|} \cdots \leq \|A\| \|B\| \, O\left(e^{-\mu_0 d(1-vc)}\right)$$

$$\left| \frac{1}{2\pi} \int_{|t|>cd} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t) e^{-\alpha t} dt \right| \leq \frac{1}{\pi} \int_{|t|>cd} e^{-\alpha t^2} \, 2\|A\| \|B\|$$


A
B
$\|[A(t),B]\| = e^{\mu(vt-d)}$
LR holds
$e^{-\alpha t^2}$
$\int_{t<|cd|} \cdots \leq \|A\|\|B\| O\left(e^{-\mu d(1-vc)}\right)$
$\left|\frac{1}{2\pi}\int_{|t|>cd} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t) e^{-\alpha t} dt\right| \leq \frac{1}{4\pi}\int_{|t|>cd} e^{-\alpha t^2} 2\|A\|\|B\| \leq \|A\|\|B\| \frac{1}{2\pi c d \alpha} e^{-\alpha c^2 d^2}$

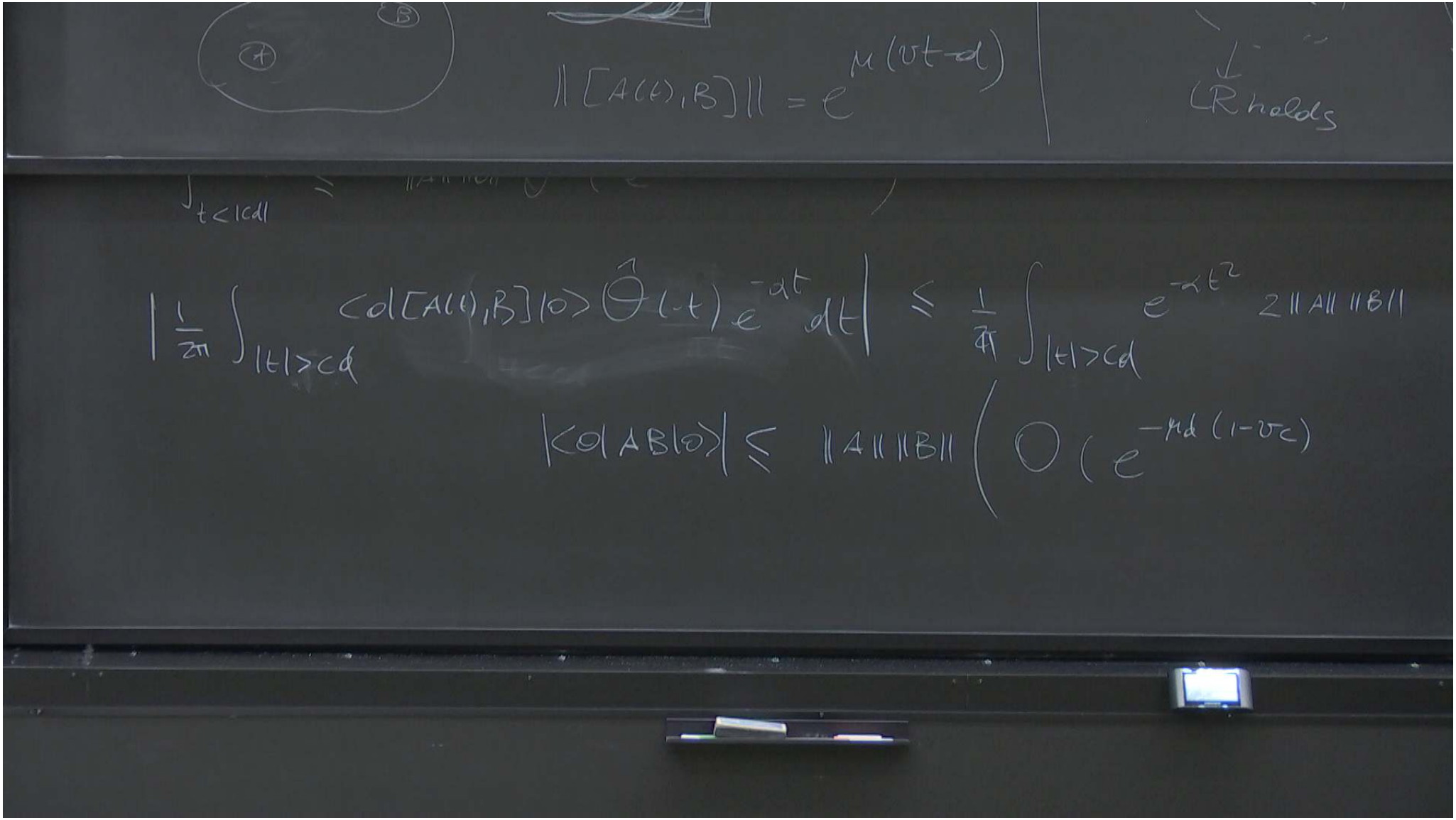
I_2 is small

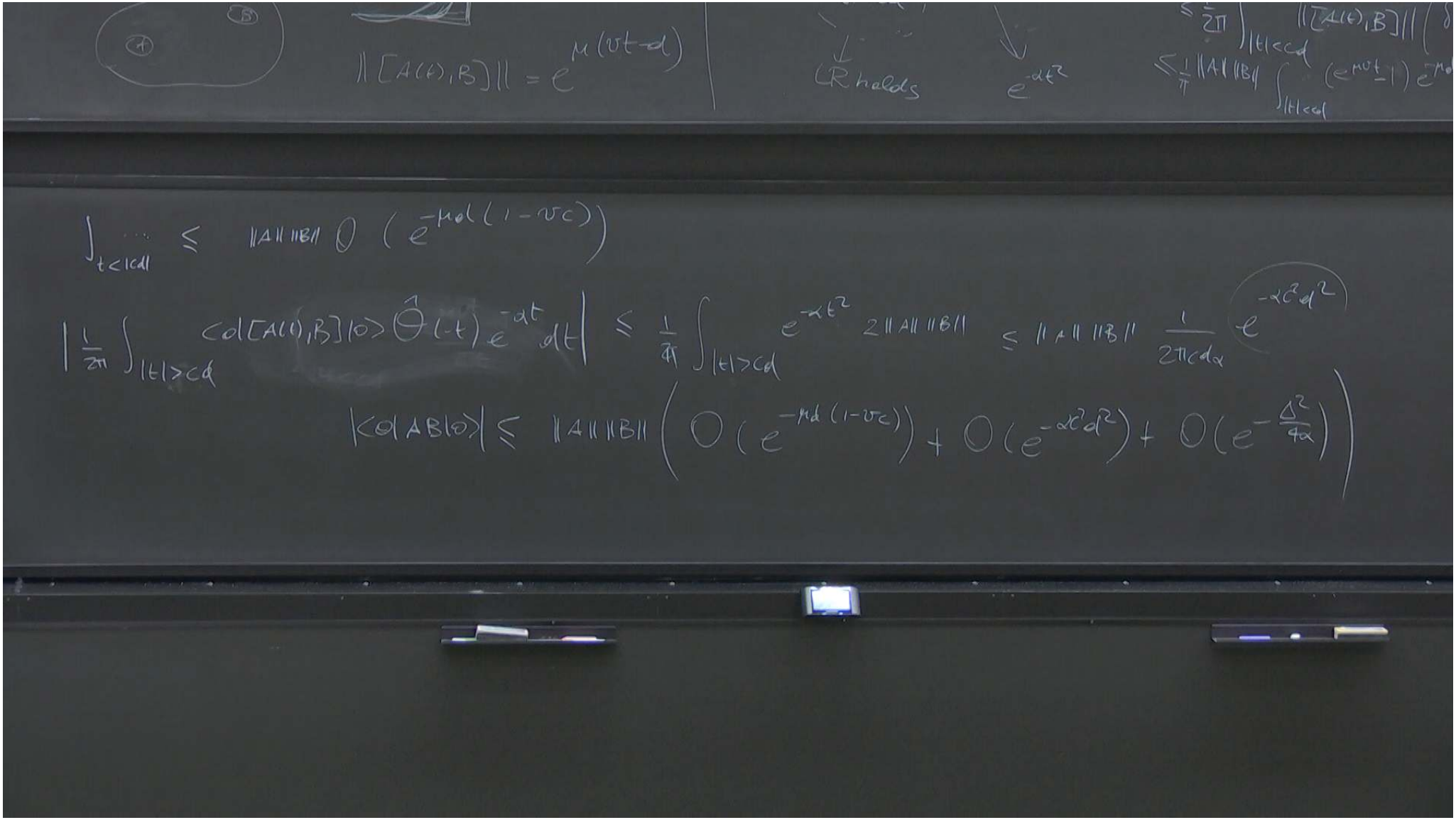
$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t) e^{-\alpha t^2} dt = \sum_{i\geq 1} A_{0i}B_{i0}\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iE_i t}\hat{\theta}(-t)e^{-\alpha t^2}dt \quad \left[1+O\left(e^{-\frac{\Delta^2}{4\alpha}}\right)\right]$$

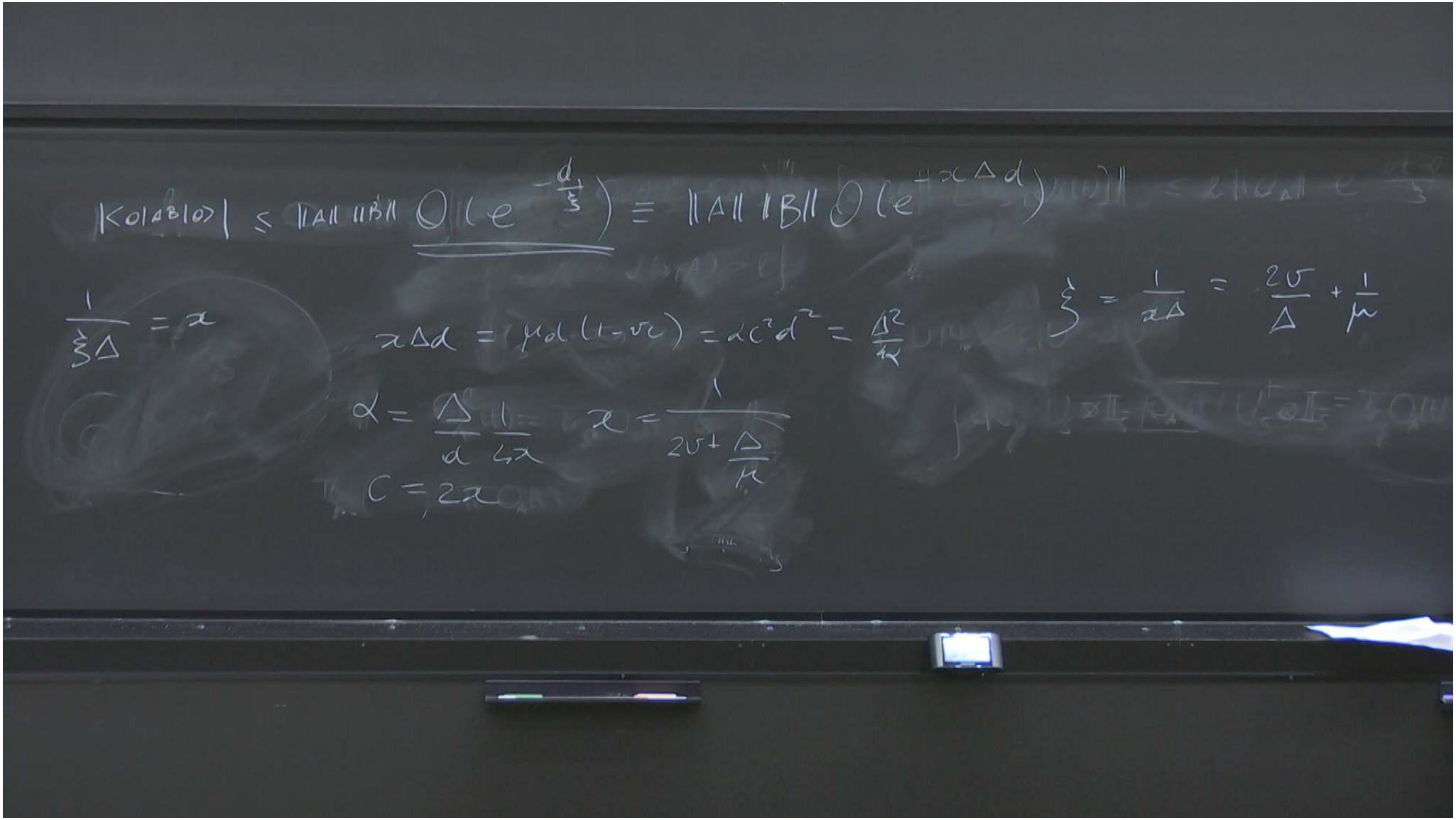
$$- B_{0i}A_{i0}\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i(-E_i)t}\hat{\theta}(-t)e^{-\alpha t^2}dt$$

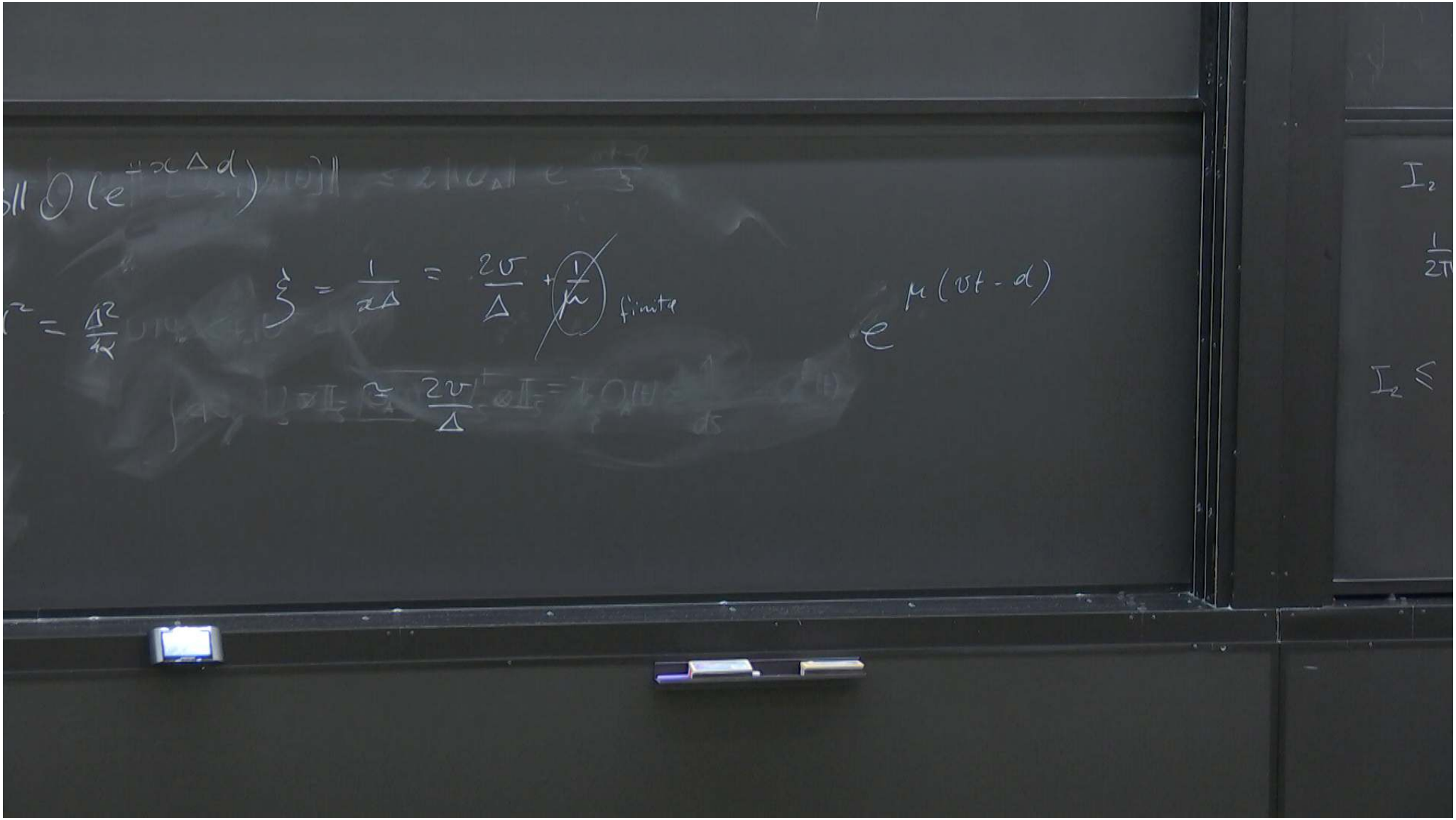
$$I_2 \leq O\left(e^{-\frac{\Delta^2}{4\alpha}}\right)$$

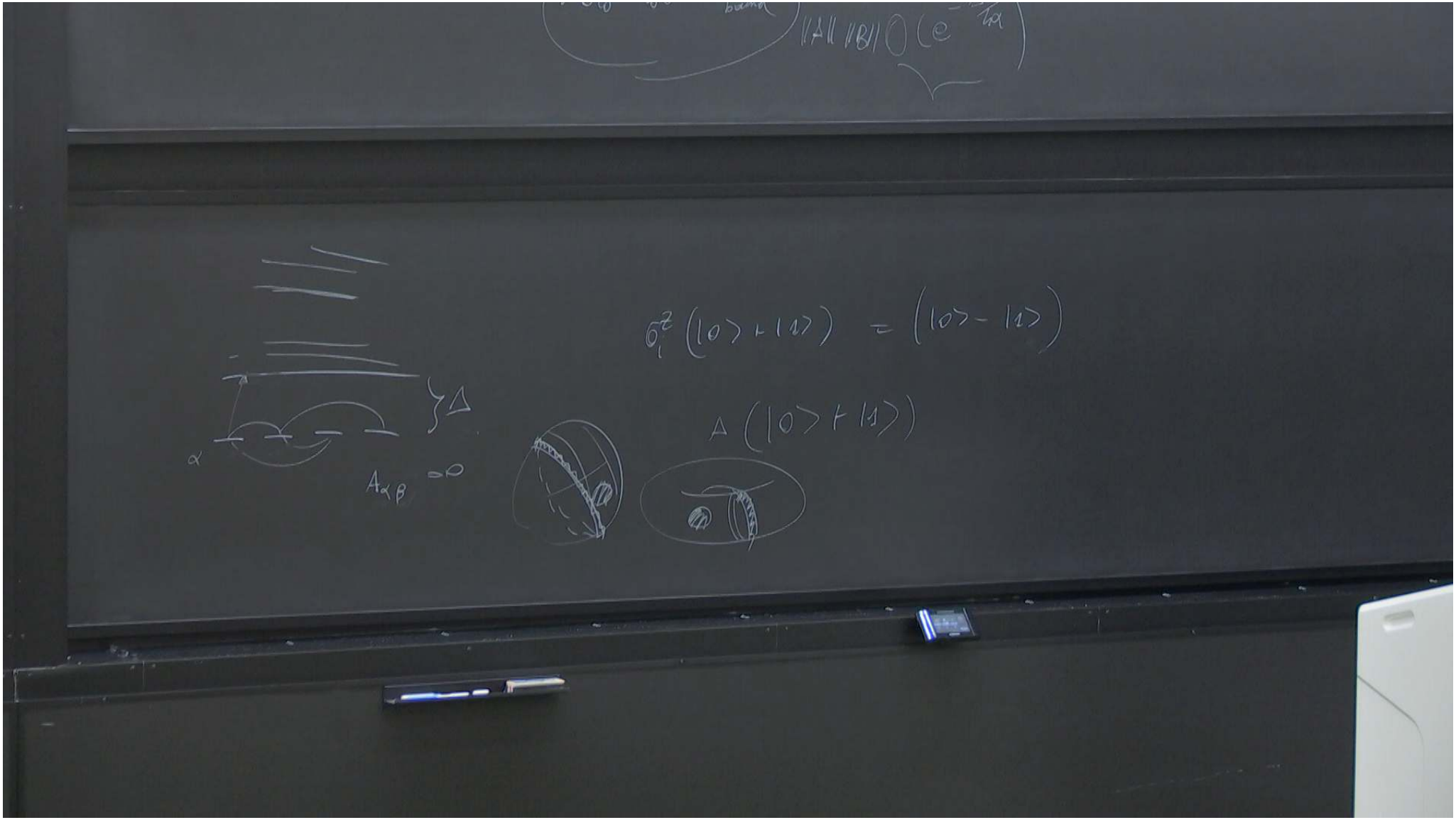
$$\|A\|\|B\| O\left(e^{-\frac{\Delta^2}{4\alpha}}\right)$$


A
B
$\|[A(t),B]\| = e^{\mu(vt-d)}$
LR holds
$\int_{t<|cd|}$
$\left|\frac{1}{2\pi}\int_{|t|>cd} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t)\, e^{-\alpha t}\, dt\right| \leq \frac{1}{4\pi}\int_{|t|>cd} e^{-\alpha t^2}\, 2\|A\|\|B\|$
$|\langle 0|AB|0\rangle| \leq \|A\|\|B\|\left(O(e^{-\mu d(1-vc)}\right.$


$\|[A(t),B]\| = e^{\mu(vt-d)}$
LR holds
$e^{-\alpha t^2}$
$\int_{|t|<cd} \ldots \leq \|A\|\|B\| O\left(e^{-\mu d(1-vc)}\right)$
$\left|\frac{1}{2\pi}\int_{|t|>cd} \langle 0|[A(t),B]|0\rangle \hat{\theta}(-t) e^{-\alpha t} dt\right| \leq \frac{1}{2\pi}\int_{|t|>cd} e^{-\alpha t^2} 2\|A\|\|B\| \leq \|A\|\|B\| \frac{1}{2\pi cd\alpha} e^{-\alpha c^2 d^2}$
$|\langle 0|AB|0\rangle| \leq \|A\|\|B\|\left(O\left(e^{-\mu d(1-vc)}\right) + O\left(e^{-\alpha c^2 d^2}\right) + O\left(e^{-\frac{\Delta^2}{4\alpha}}\right)\right)$


$|\langle 0|AB|0\rangle| \le \|A\|\,\|B\|\, O(e^{-\frac{d}{\xi}}) \equiv \|A\|\,\|B\|\, O(e^{\pm \alpha \Delta d})$
$\frac{1}{\xi\Delta} = \alpha$
$\alpha \Delta d = \mu_0 d (1 - v_c) = \alpha c^2 d^2 = \frac{\Delta^2}{4\alpha}$
$\xi = \frac{1}{\alpha\Delta} = \frac{2v}{\Delta} + \frac{1}{\mu}$
$\alpha = \frac{\Delta}{d}\frac{1}{4\alpha}$
$\alpha = \frac{1}{2v + \frac{\Delta}{\mu}}$
$C = 2\alpha$


$\xi = \frac{1}{2\Delta} = \frac{2v}{\Delta} + \frac{1}{\mu}$
finite
$e^{\mu(vt-d)}$
$\frac{\Delta^2}{4\alpha}$
$\simeq \frac{2v}{\Delta}$
I_2
$I_2 \leq$


$\sigma_i^z(|0\rangle + |1\rangle) = (|0\rangle - |1\rangle)$
$A(|0\rangle + |1\rangle)$
Δ
α
$A_{\alpha\beta} \sim 0$