

Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 7

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Collection: PSI 2019/2020 - Quantum Matter Part 1

Date: January 14, 2020 - 9:00 AM

URL: <http://pirsa.org/20010032>

$n \times n$
matrix

$g_{\mu\nu}$

$$|\phi\rangle = \phi_{SH}^\mu |e_\mu\rangle$$

$$\langle \partial_\mu H \phi^\mu \rangle_{\psi_0}$$

$$|g_{\mu\nu}| \leq \Delta^{-2} \left[\langle SH^2 \rangle_{\psi_0} - \langle SH \rangle_{\psi_0}^2 \right]$$

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$$\langle \partial_\mu H \phi^\mu \rangle_{\psi_0}$$

$$|g_{\mu\nu}| \leq \Delta^{-2} [\langle SH^2 \rangle_{\psi_0} - \langle SH \rangle_{\psi_0}^2]$$

$$= \Delta^{-2} \sum_{I, J} [\langle SV_I SV_J \rangle - \langle SV_I \rangle \langle SV_J \rangle]$$

$$= \Delta^{-2} L^d \sum_i C(r)$$

not. Inv.
& tr. //

Locality $SH = \sum_j SV_j$

If there is a
(finite) ξ

$\Rightarrow$

$$\frac{|g_{\mu\nu}|}{L^d}$$

$$\leq CL^d \Delta^{-2}$$

$n \times n$
matrix

$g_{\mu\nu}$

$$|\phi\rangle = \phi_{SH}^\mu |e_\mu\rangle$$

$$\langle \partial_\mu H \phi^\mu \rangle_{\psi_0}$$

$$|g_{\mu\nu}| \leq \Delta^{-2} [\langle SH^2 \rangle_{\psi_0} - \langle SH \rangle_{\psi_0}^2]$$

$$= \Delta^{-2} \sum_{I, J} [\langle SV_I SV_J \rangle - \langle SV_I \rangle \langle SV_J \rangle]$$

$$= \Delta^{-2} L^d \left(\sum_i C(i) \right)$$

not. Inv.
& tr. //

Locality $SH = \sum_j SV_j$

If there is a
(finite) ξ

$$\Rightarrow \frac{|g_{\mu\nu}|}{L^d} \leq C \Delta^{-2}$$

$$\leq C L^d \Delta^{-2}$$

When is ξ finite?

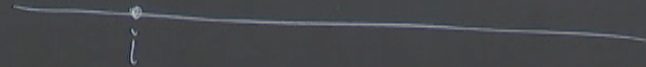
$\longleftrightarrow$? Paramagnetic Phase
 $\Delta > 0$

Ground State is Unique (No Symm. breaking)

Degeneracy ?
+ gap $\rightarrow \xi$ finite

Locality

H



$t < t_0$ $|\psi_0\rangle$

$t = t_0$ LOCAL QUANTUM QUENCH $H \rightarrow H + V_i$

$t > t_0$ $|\psi(t)\rangle$

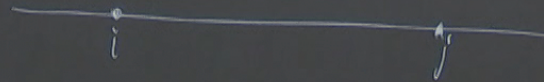
Locality

H

$$t < t_0 \quad |\psi_0\rangle$$

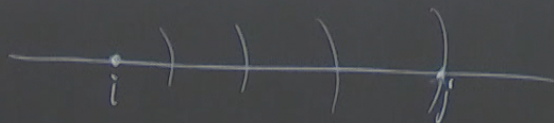
$$t = t_0 \quad \text{LOCAL QUANTUM QUENCH} \quad H \rightarrow H + V_i$$

$$t > t_0 \quad |\psi(t)\rangle = e^{-i(H+V_i)t} |\psi_0\rangle$$



$$\hat{O} = \hat{O}_i \otimes \mathbb{1}_j$$

$$O(t) = \text{Tr}[\psi(t) \hat{O}]$$



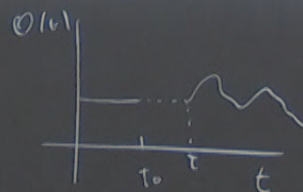
QUENCH $H \rightarrow H + V_i \equiv H'$

$$e^{-i(H+V_i)t} | \psi_0 \rangle$$

$$\hat{O} = \hat{O}_i \otimes 1_j$$

$$O(t) = \text{Tr}[\psi(t) \hat{O}]$$

$$V_i(t) = e^{iH't} V_i e^{-iH't}$$



$$\text{supp}[V_i(t)] = \mathcal{H}$$

① Why do we have instant spreading?

EMERGENCE OF A LIGHT CONE

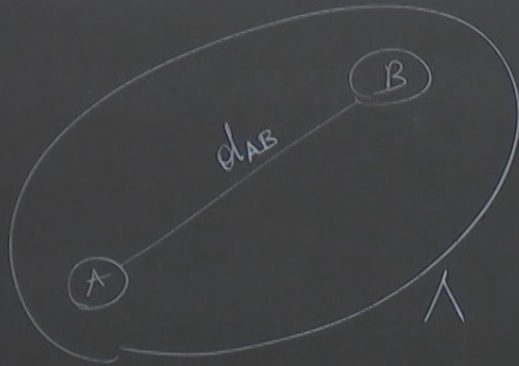
LIEB-ROBINSON BOUNDS

Local Q. System

$$\|\bar{\Phi}_x\| \leq k$$

$$H = \sum_{X \subset \Lambda} \bar{\Phi}_X \in \mathcal{B}(\mathcal{H}_\Lambda)$$

$$\left\{ \begin{array}{l} |X| \leq \sigma \\ \text{diam } X \leq R \end{array} \right.$$



$$\hat{A} \in \mathcal{B}(\mathcal{H}_A)$$

$$\hat{B} \in \mathcal{B}(\mathcal{H}_B)$$

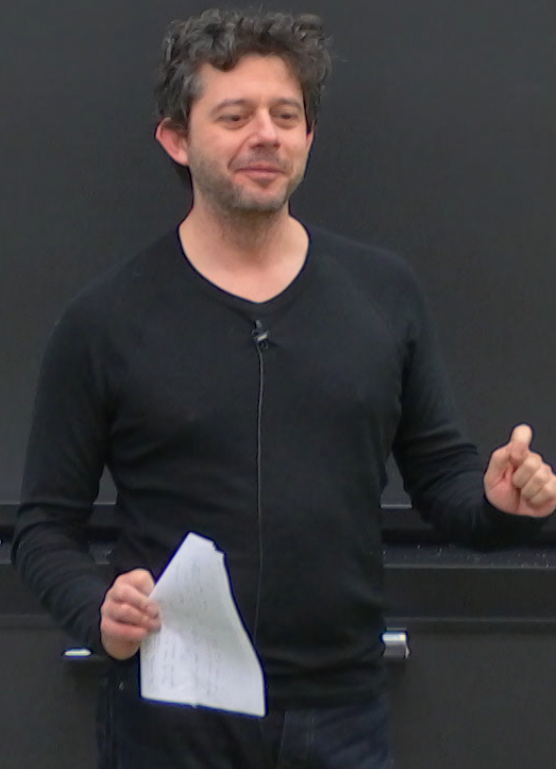
$$[\hat{A}, \hat{B}] = 0$$

$$f(t) := [\hat{A}(t), \hat{B}]$$

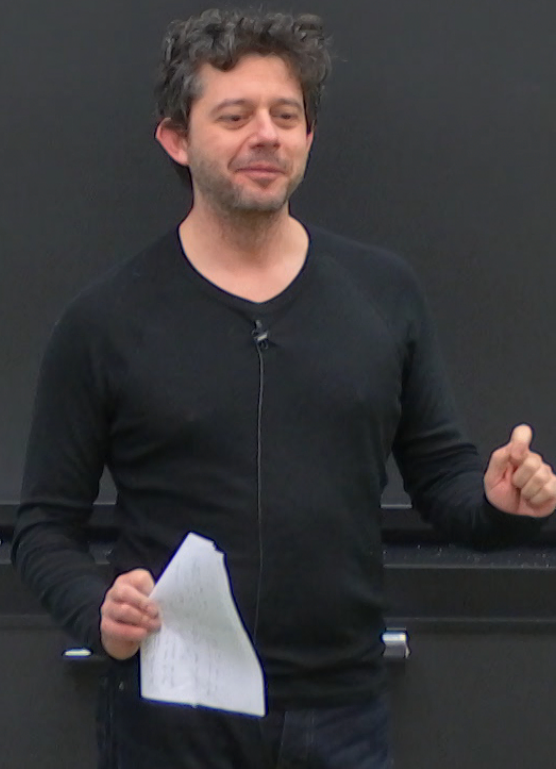
$$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$$

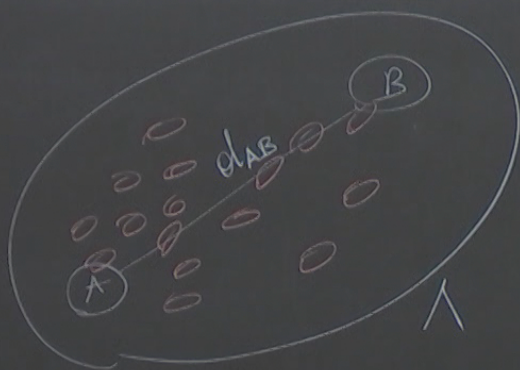
$$\|f(t)\|$$

$$f'(t) = \left[\frac{d\hat{A}(t)}{dt}, B \right] = i \left[[H, \hat{A}(t)], B \right] =$$



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$$\hat{A} \in \mathcal{B}(\mathcal{H}_A)$$

$$\hat{B} \in \mathcal{B}(\mathcal{H}_B)$$

$$[\hat{A}, \hat{B}] = 0$$

$$f(t) := [\hat{A}(t), \hat{B}]$$

$$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$$

$$\|f(t)\|$$

$$e^{-iHt} = U$$

$$\|P_X\| \leq h$$

$$\begin{cases} |X| \leq \infty \\ \dim X \leq R \end{cases}$$

$$f(t) := [A(t), B]$$

$$e^{-iHt} = U$$

$$f'(t) = \left[\frac{dA(t)}{dt}, B \right] = i [[H, A(t)], B] = i [U^\dagger [H, A(t)] U, B] = i \sum_{X_1 \in Z_1} [U^\dagger [\Phi_{X_1}, A] U, B]$$

$$Z_1 = \{ X_1 \subset \Lambda : X_1 \cap A \neq \emptyset \} \quad \left| \right. = i \sum_{X_1 \in Z_1} [[\Phi_{X_1}(t), A(t)], B] = i \sum_{X_1} \left\{ [\Phi_{X_1}(t), \underbrace{[A(t), B]}_F] + \underbrace{[A(t), [B, \Phi_{X_1}(t)]]}_G \right\}^{X \cap A = \emptyset}$$

$$f' = Lf + G \quad L(\cdot) = i[H_1, \cdot] \quad Lf \quad G$$

$$\|E_{\lambda}\| \leq h$$

$$\begin{cases} |X| \leq 0 \\ \dim X \leq R \end{cases}$$

$$f(t) := [A(t), B]$$

$$e^{-iHt} = U$$

$$f'(t) = \left[\frac{dA(t)}{dt}, B \right] = i [[H, A(t)], B] = i [U^\dagger [H, A(t)] U, B] = i \sum_{X_1 \in Z_1} [U^\dagger [E_{X_1}, A] U, B]$$

$$Z_1 = \{ X_1 \subset \Lambda : X_1 \cap A \neq \emptyset \} \quad = i \sum_{X_1 \in Z_1} [[E_{X_1}(t), A(t)], B] = i \sum_{X_1} \left\{ \underbrace{[E_{X_1}(t), [A(t), B]]}_{\mathcal{F}} + \underbrace{[A(t), [B, E_{X_1}(t)]]}_{\mathcal{G}} \right\}^{X \cap A = \emptyset}$$

$$f' = \mathcal{L}f + G \quad \mathcal{L}(\cdot) = i[H_{\Lambda_1}, \cdot] \quad \mathcal{L}f$$

$$\gamma_t : f(0) \rightarrow f_t = \gamma_t(f(0))$$