Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 7

Speakers: Alioscia Hamma

Collection: PSI 2019/2020 - Quantum Matter Part 1

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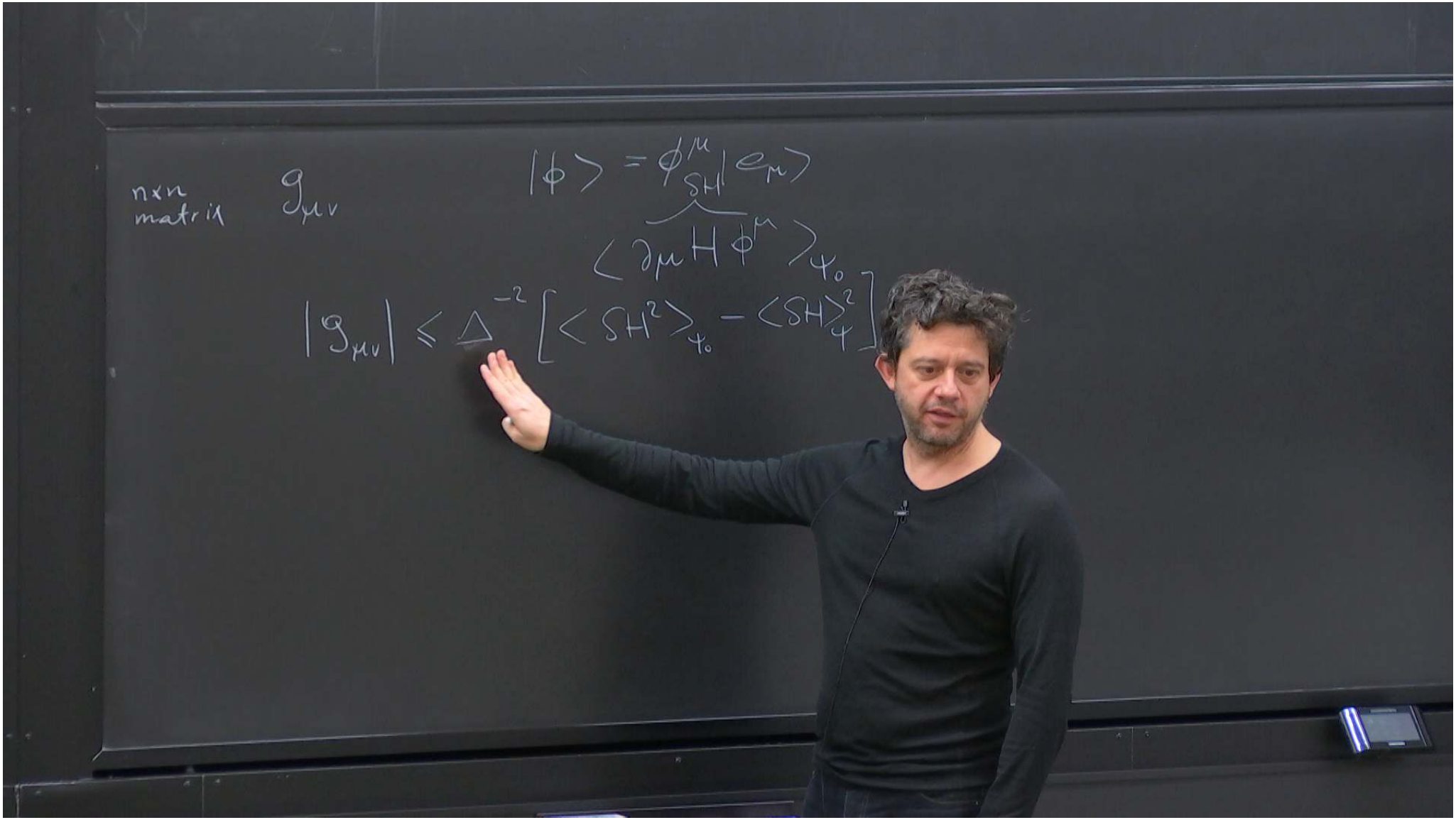
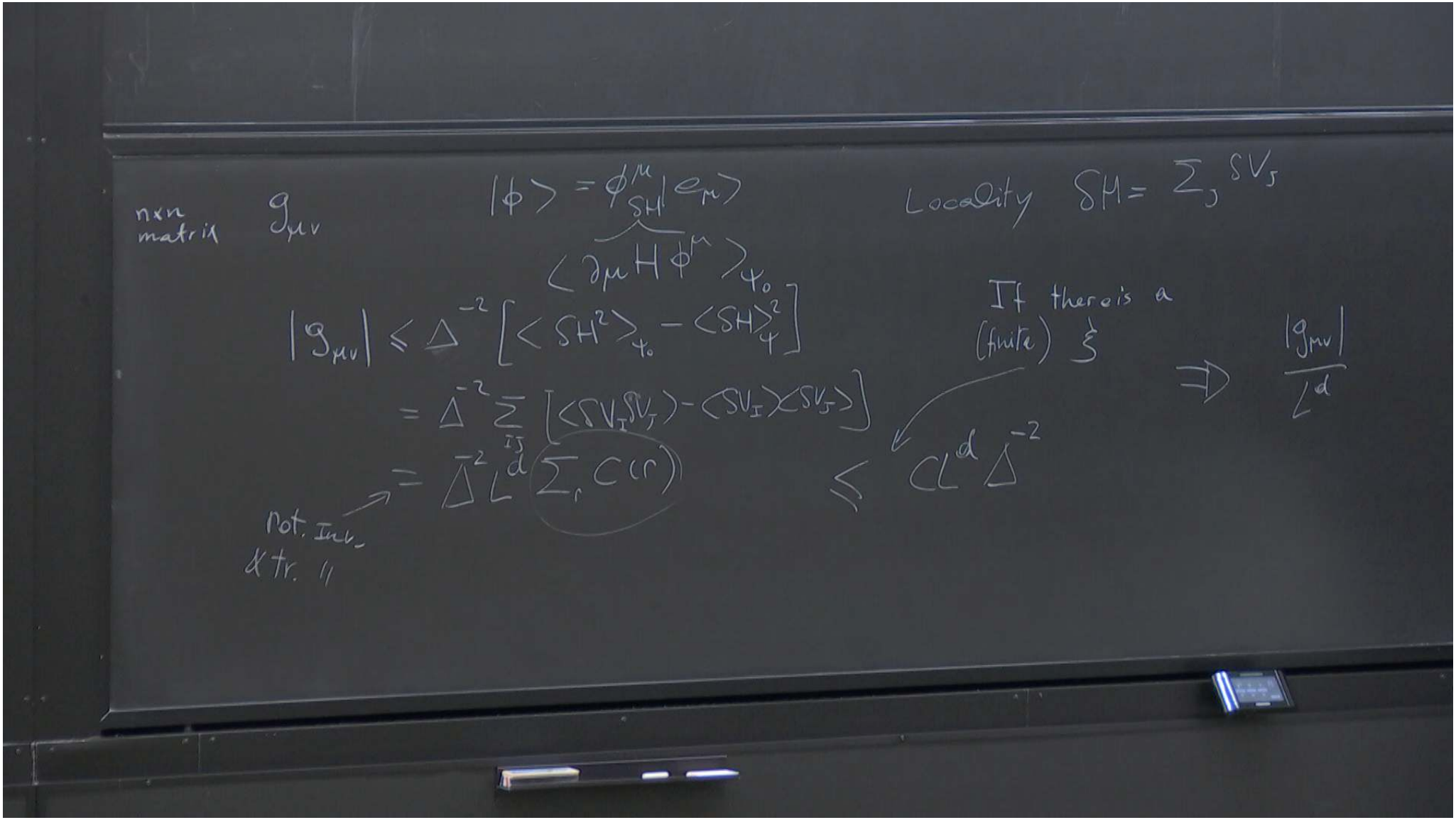
URL: http://pirsa.org/20010032

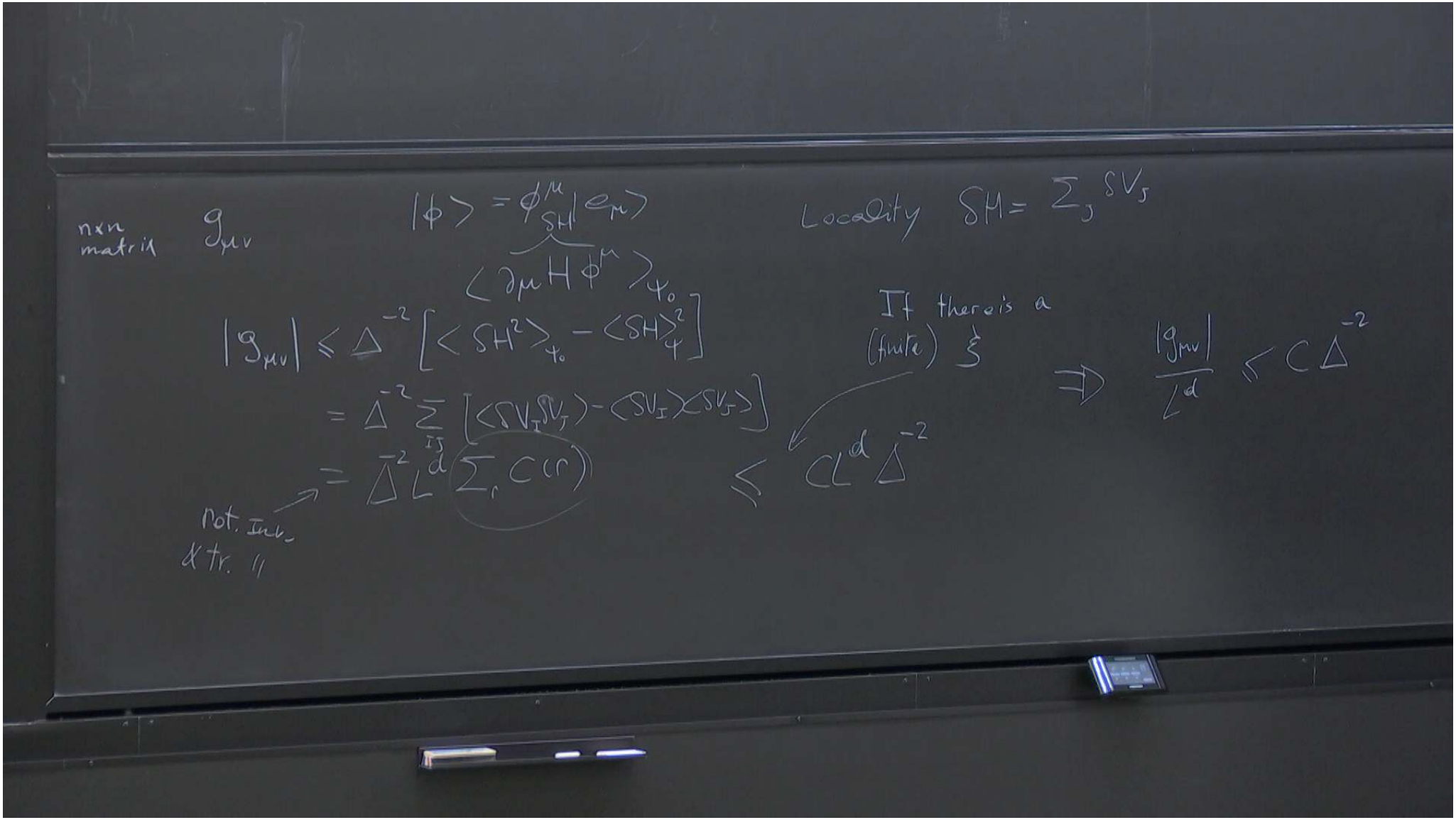

nxn
matrix
$g_{\mu\nu}$
$|\phi\rangle = \phi^{\mu}_{SH} |e_\mu\rangle$
$\langle \partial_\mu H \phi^\mu \rangle_{\psi_0}$
$|g_{\mu\nu}| \leq \Delta^{-2} \left[\langle SH^2 \rangle_{\psi_0} - \langle SH \rangle^2_{\psi_0} \right]$

$n \times n$ matrix $g_{\mu\nu}$

$$|\phi\rangle = \underbrace{\phi^{\mu}_{SH}}_{\langle \partial_\mu H \phi^\mu \rangle_{\psi_0}} |e_\mu\rangle$$

$$|g_{\mu\nu}| \leq \Delta^{-2} \left[\langle SH^2 \rangle_{\psi_0} - \langle SH \rangle^2_{\psi} \right]$$


nxn matrix $g_{\mu\nu}$
$|\phi\rangle = \phi^{\mu}_{SH} |e_\mu\rangle$
$\langle \partial_\mu H \phi^\mu \rangle_{\gamma_0}$
Locality $\delta H = \sum_J \delta V_J$
$|g_{\mu\nu}| \leq \Delta^{-2} \left[\langle \delta H^2 \rangle_{\gamma_0} - \langle \delta H \rangle^2_{\gamma} \right]$
$= \Delta^{-2} \sum_{IJ} \left[\langle \delta V_I \delta V_J \rangle - \langle \delta V_I \rangle \langle \delta V_J \rangle \right]$
$= \Delta^{-2} L^d \sum_r C(r)$
Not. Inv. & tr. II
If there is a (finite) ξ
$\leq C L^d \Delta^{-2}$
$\Rightarrow \frac{|g_{\mu\nu}|}{L^d}$


nxn matrix $g_{\mu\nu}$
$|\phi\rangle = \phi^{\mu}_{SH} |e_\mu\rangle$
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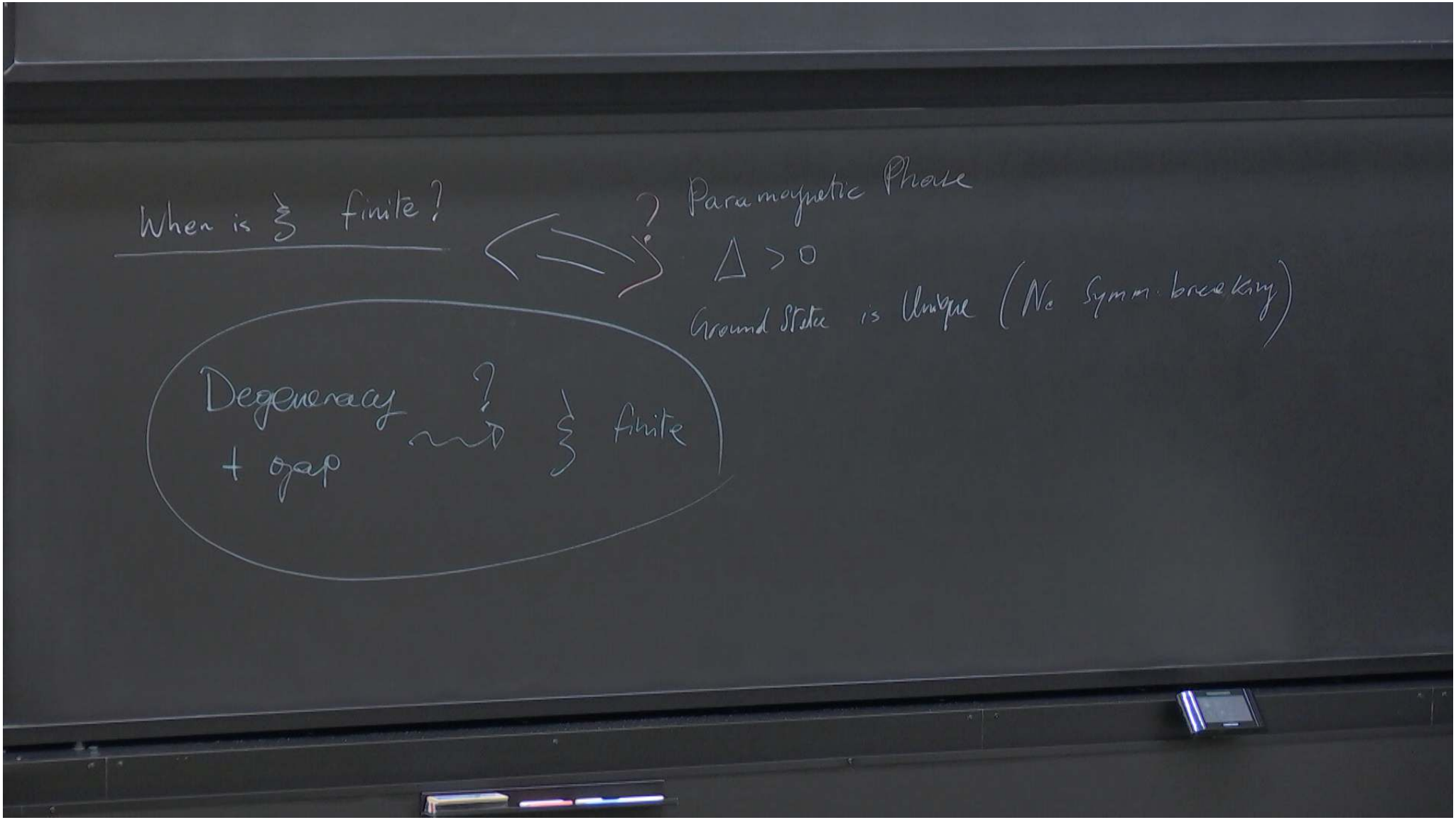
Locality $\delta H = \sum_J \delta V_J$
$|g_{\mu\nu}| \leq \Delta^{-2} \left[\langle \delta H^2 \rangle_{\psi_0} - \langle \delta H \rangle^2_{\psi} \right]$
$= \Delta^{-2} \sum_{IJ} \left[\langle \delta V_I \delta V_J \rangle - \langle \delta V_I \rangle \langle \delta V_J \rangle \right]$
$= \Delta^{-2} L^d \left(\sum_r C(r) \right)$
Pot. Incl. & Tr. Inv.
If there is a (finite) ξ
$\leq C L^d \Delta^{-2}$
$\Rightarrow \frac{|g_{\mu\nu}|}{L^d} \leq C \Delta^{-2}$


When is ξ finite?
?
Paramagnetic Phase
Δ > 0
Ground State is Unique (No Symm. breaking)
Degeneracy + gap
?
ξ finite

Locality

H

$t < t_0$ $\quad |\Psi_0\rangle$

$t = t_0$ $\quad$ LOCAL QUANTUM QUENCH $\quad H \longrightarrow H + V_i$

$t \geq t_0$ $\quad |\Psi(t)\rangle$

Locality

H

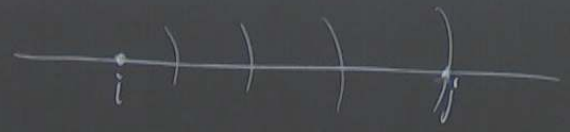
$\hat{O} = \hat{O}_j \otimes \mathbb{1}_{\bar{j}}$

$t < t_0 \quad |\psi_0\rangle$

$O(t) = \mathrm{Tr}[\psi(t)\hat{O}]$

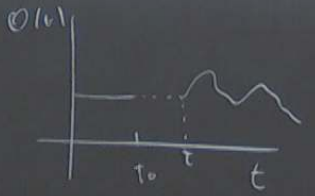
$t = t_0$ LOCAL QUANTUM QUENCH $H \longrightarrow H + V_i$

$t \geq t_0 \quad |\psi(t)\rangle = e^{-i(H+V_i)t} |\psi_0\rangle$



$\hat{O} = \hat{O}_j \otimes \mathbb{1}_{\bar{j}}$

$O(t) = \mathrm{Tr}[\rho(t)\hat{O}]$



QUENCH $H \longrightarrow H + V_i \equiv H'$

$e^{-i(H+V_i)t}|\psi_0\rangle$

$V_i(t) = e^{iH't} V_i e^{-iH't}$

$\mathrm{supp}[V_i(t)] = \mathcal{H}$

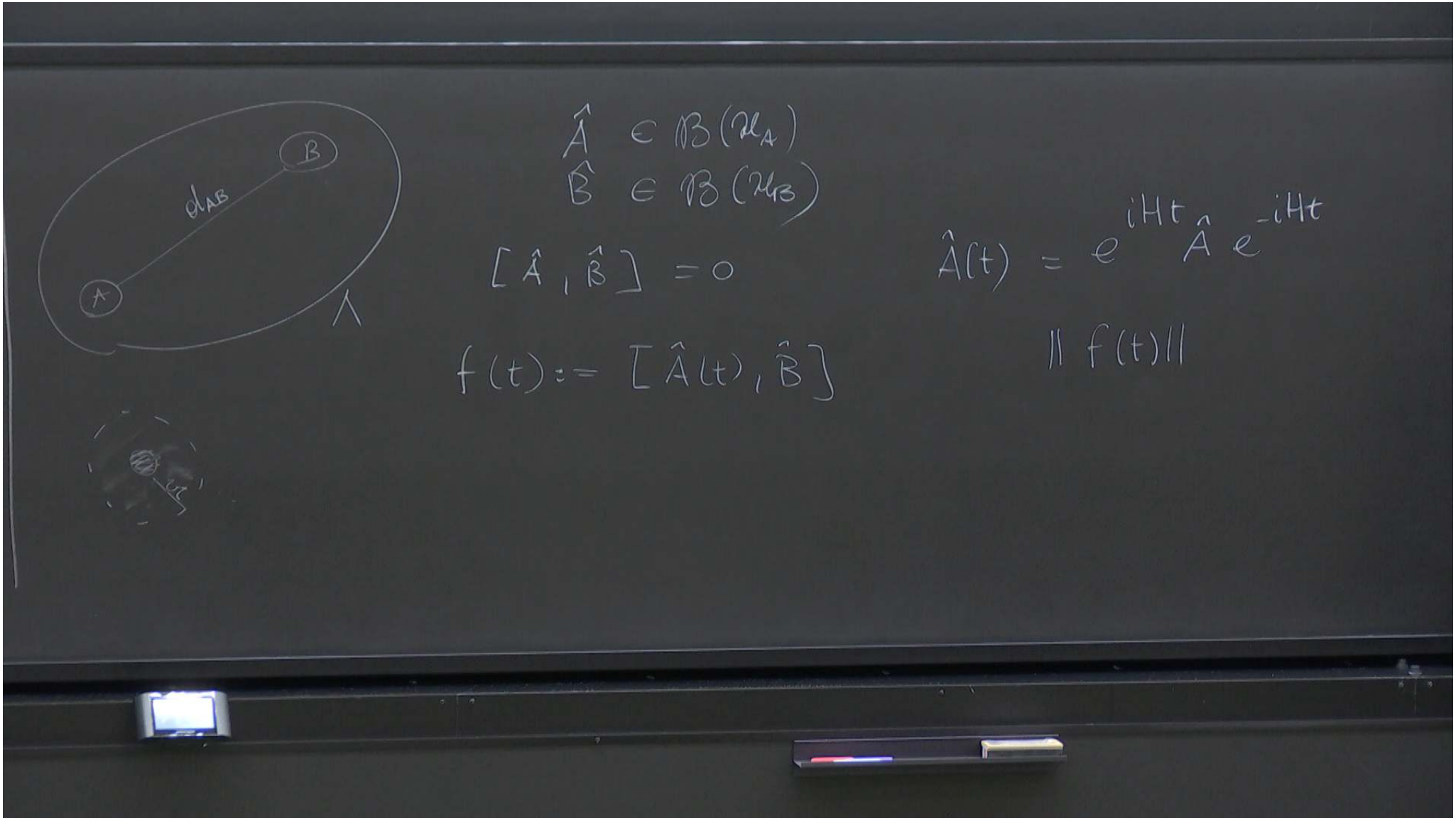
(1) Why do we have instant signalling?

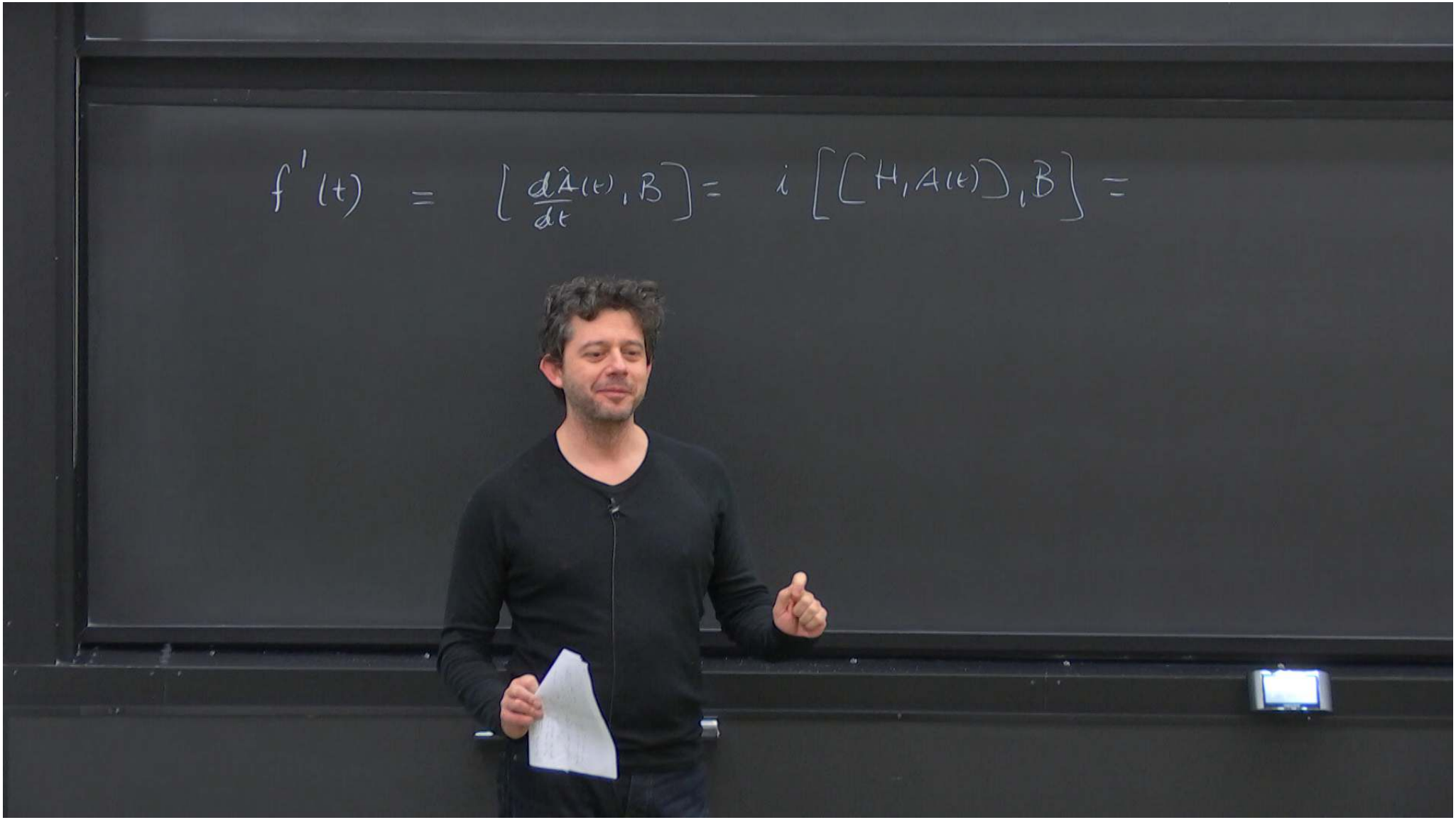
Emergence of a Light Cone

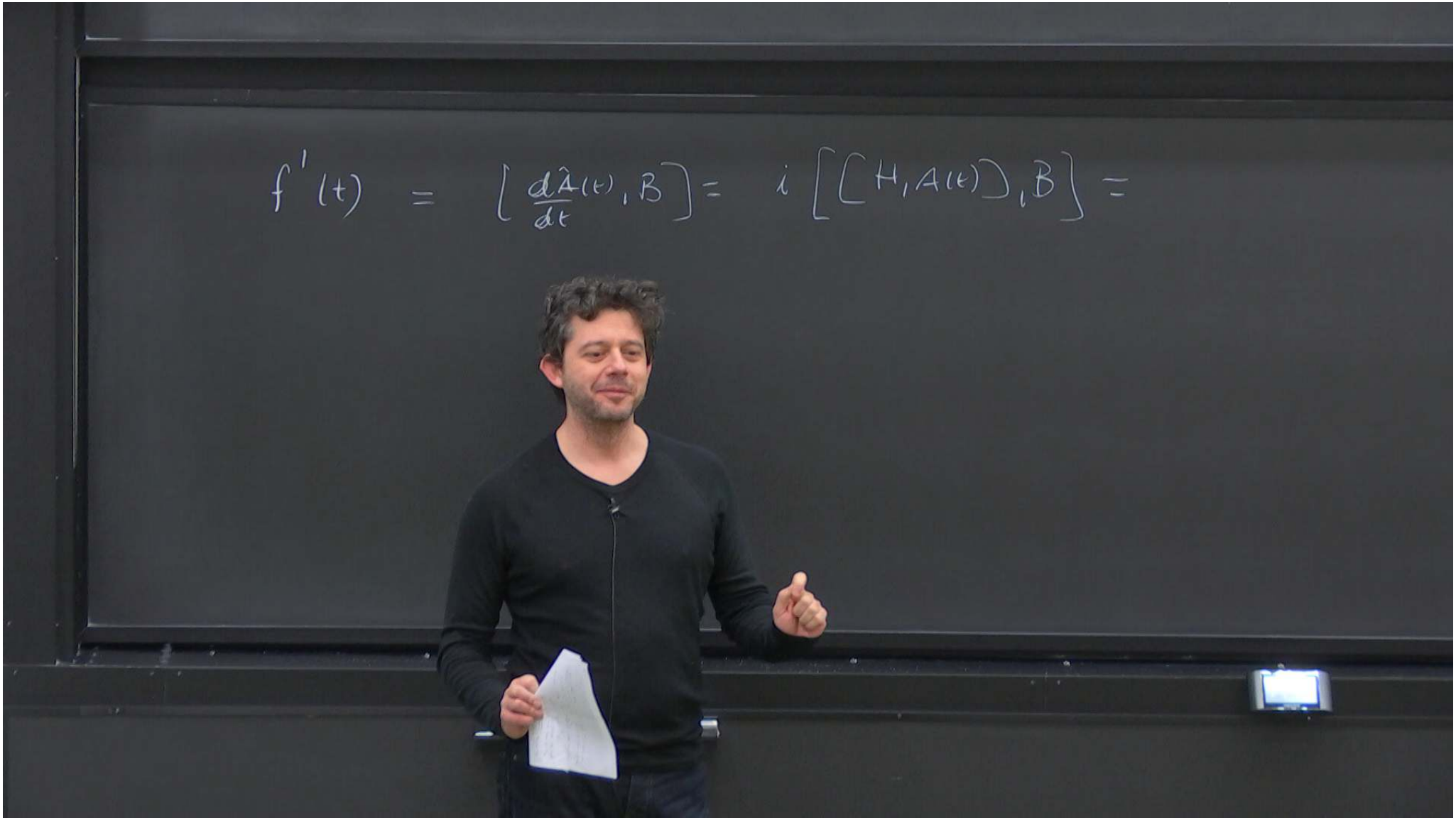
Lieb-Robinson Bounds

Local Q. System $\quad H = \sum_{X \subset \Lambda} \Phi_X \in \mathcal{B}(\mathcal{H}_\Lambda)$

$\|\Phi_X\| \leq h$ $\quad \left\{ \begin{array}{l} |X| \leq a \\ \operatorname{diam} X \leq R \end{array} \right.$


B
d_{AB}
A
$\hat{A} \in \mathcal{B}(\mathcal{H}_A)$
$\hat{B} \in \mathcal{B}(\mathcal{H}_B)$
$[\hat{A}, \hat{B}] = 0$
$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$
$f(t) := [\hat{A}(t), \hat{B}]$
$\| f(t) \|$


$f'(t) = \left[\frac{d\hat{A}(t)}{dt}, B\right] = i\left[\left[H, A(t)\right], B\right] =$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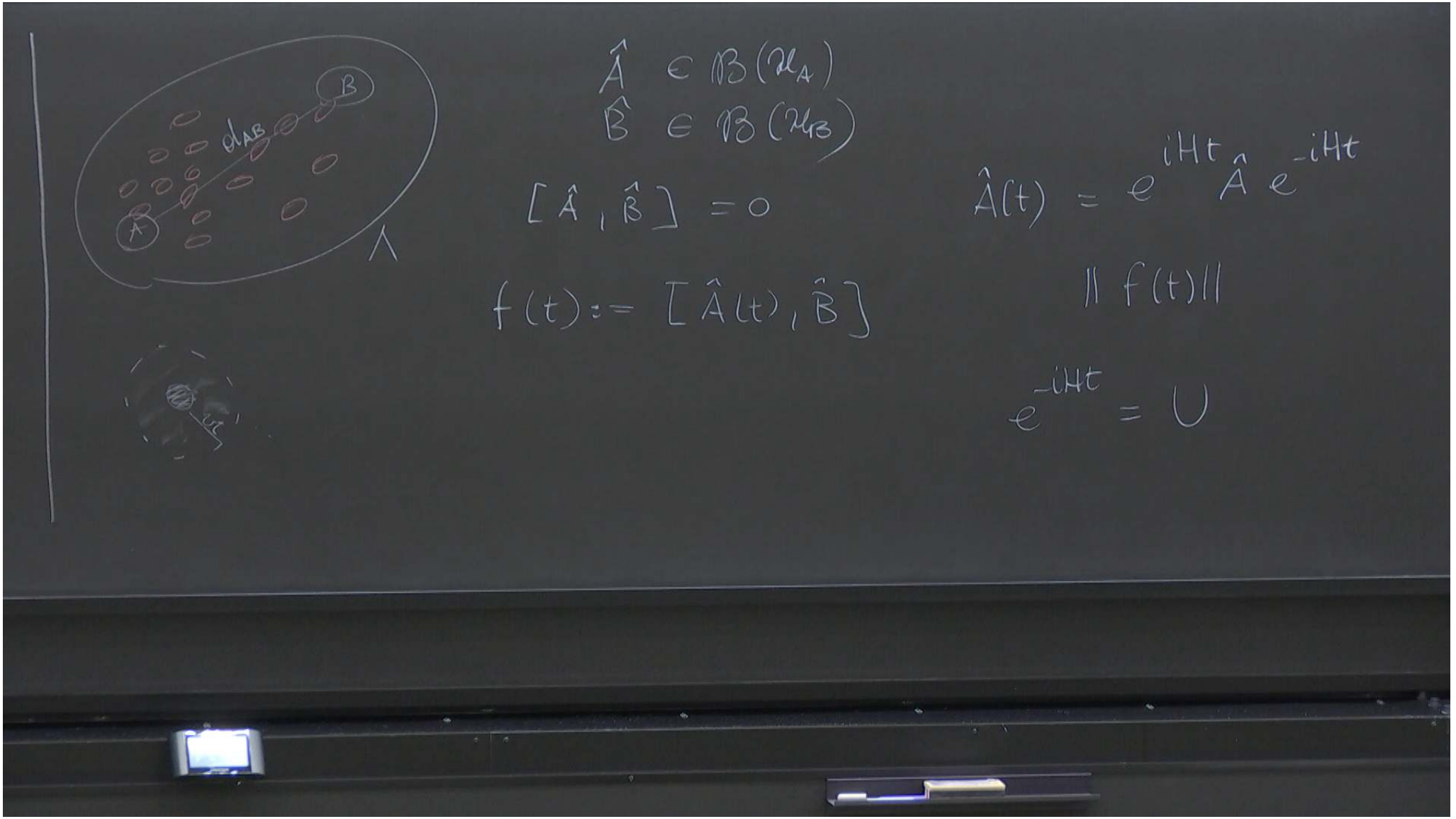
A
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$\hat{A} \in \mathcal{B}(\mathcal{H}_A)$
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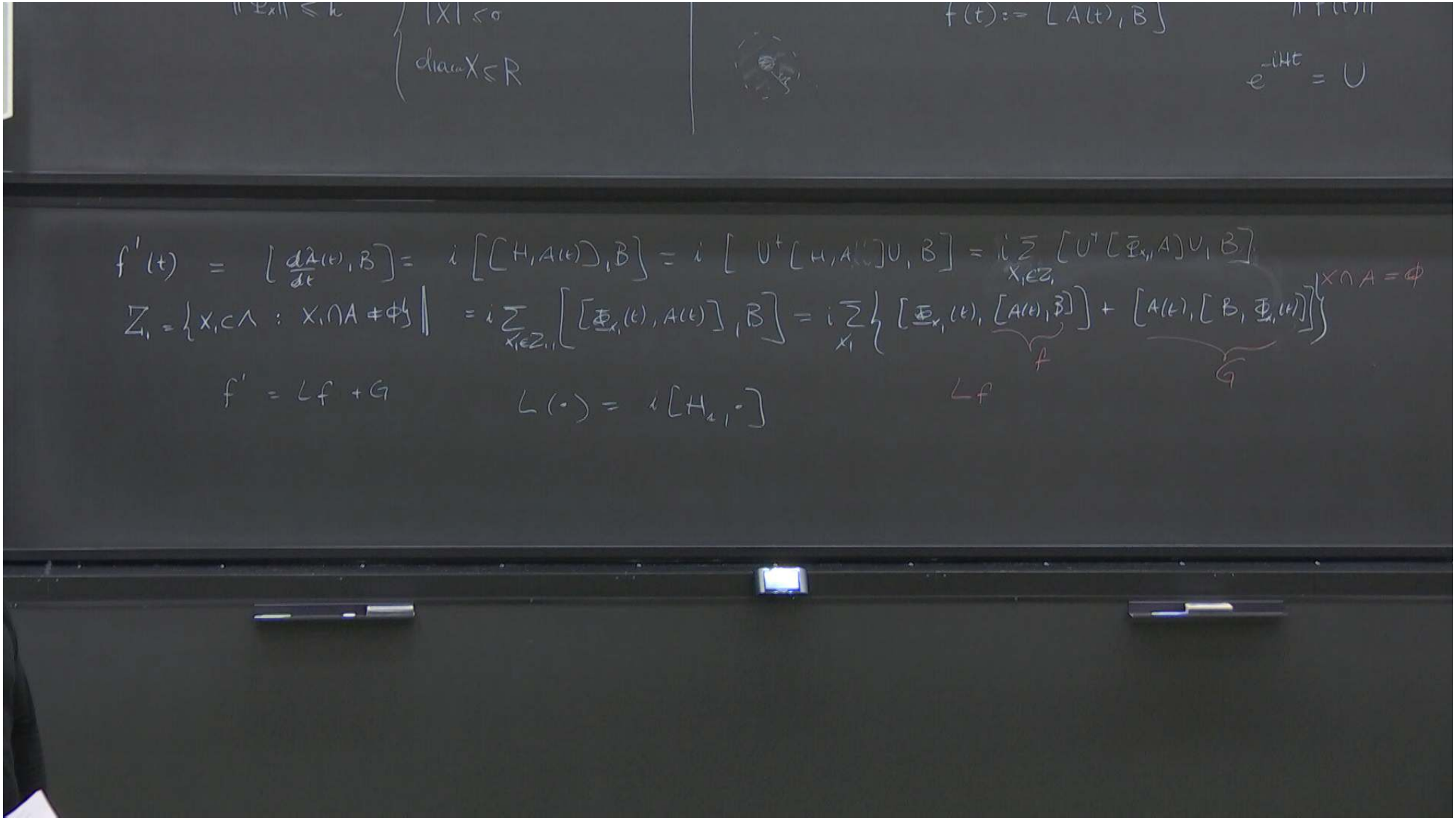
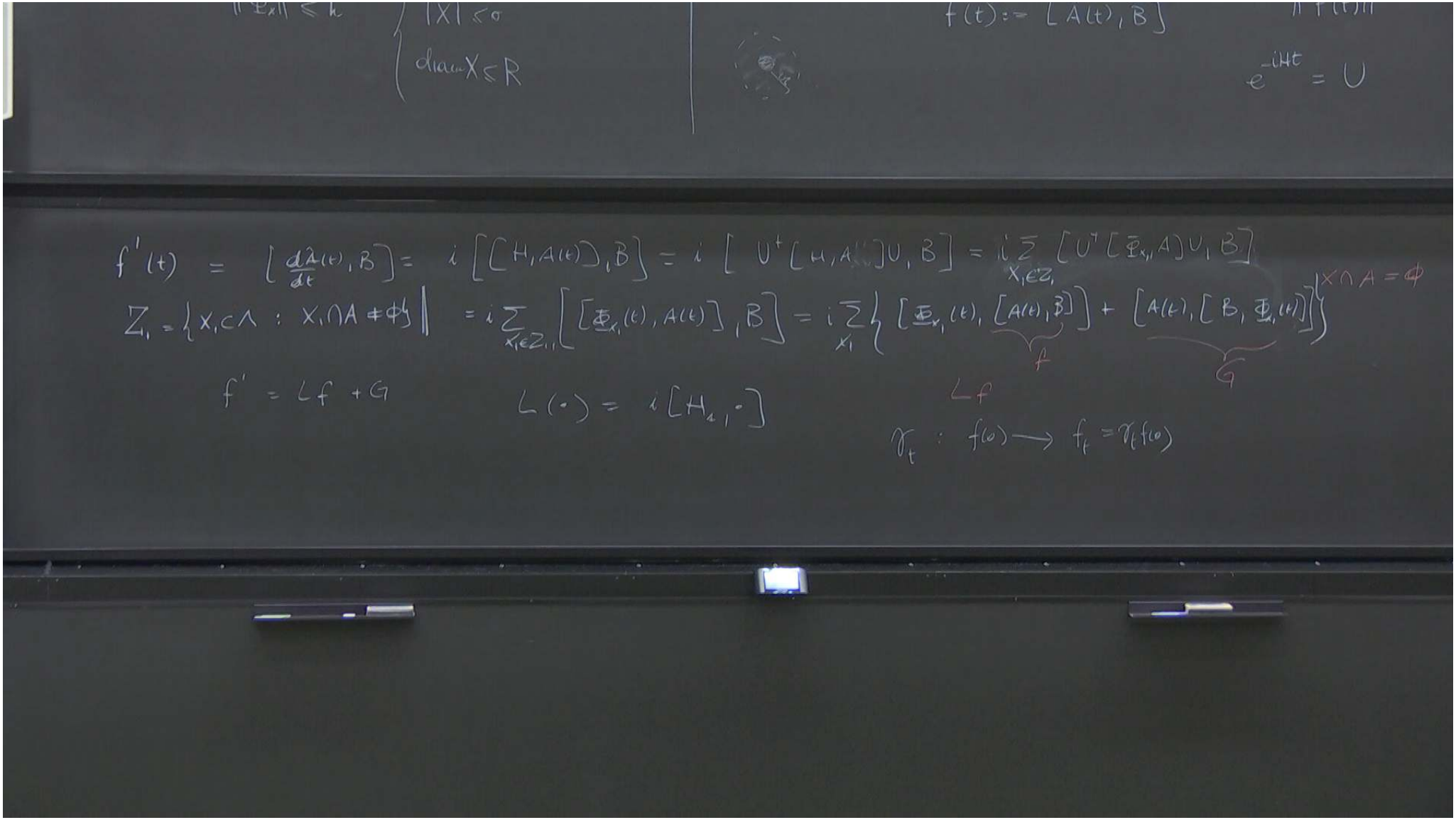
$[\hat{A}, \hat{B}] = 0$
$f(t) := [\hat{A}(t), \hat{B}]$
$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$
$\| f(t) \|$
$e^{-iHt} = U$


$\|\Phi_X\| \le h$
$|X| \le o$
$\mathrm{diam} X \le R$
$f(t) := [A(t), B]$
$e^{-iHt} = U$
$f'(t) = \left[\frac{dA(t)}{dt}, B\right] = i\left[[H, A(t)], B\right] = i\left[U^\dagger [H, A(t)] U, B\right] = i \sum_{X_1 \in \mathcal{Z}_1} \left[U^\dagger [\Phi_{X_1}, A] U, B\right]$
$\mathcal{Z}_1 = \{X_1 \subset \Lambda : X_1 \cap A \neq \phi\}$
$= i \sum_{X_1 \in \mathcal{Z}_1} \left[[\Phi_{X_1}(t), A(t)], B\right] = i \sum_{X_1} \left\{ [\Phi_{X_1}(t), \underbrace{[A(t), B]}_{f}] + [A(t), \underbrace{[B, \Phi_{X_1}(t)]}_{G}] \right\}$
$X \cap A = \phi$
$f' = Lf + G$
$L(\cdot) = i[H_t, \cdot]$
Lf


$\|\Phi_X\| \le h$
$|X| < \infty$
$\mathrm{diam} X \le R$
$f(t) := [A(t), B]$
$\|f(t)\|$
$e^{-iHt} = U$
$f'(t) = \left[\frac{dA(t)}{dt}, B\right] = i\left[[H, A(t)], B\right] = i\left[U^\dagger [H, A] U, B\right] = i \sum_{X_1 \in Z_1} [U^\dagger [\Phi_{X_1}, A] U, B]$
$Z_1 = \{X_1 \subset \Lambda : X_1 \cap A \neq \phi\}$
$= i \sum_{X_1 \in Z_1} \left[[\Phi_{X_1}(t), A(t)], B\right] = i \sum_{X_1} \{ [\Phi_{X_1}(t), [A(t), B]] + [A(t), [B, \Phi_{X_1}(t)]] \}$
$X \cap A = \phi$
f
G
$f' = Lf + G$
$L(\cdot) = i[H_t, \cdot]$
$L_t f$
$\gamma_t : f(0) \longrightarrow f_t = \gamma_t f(0)$