

Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 6

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Collection: PSI 2019/2020 - Quantum Matter Part 1

Date: January 13, 2020 - 9:00 AM

URL: <http://pirsa.org/20010031>

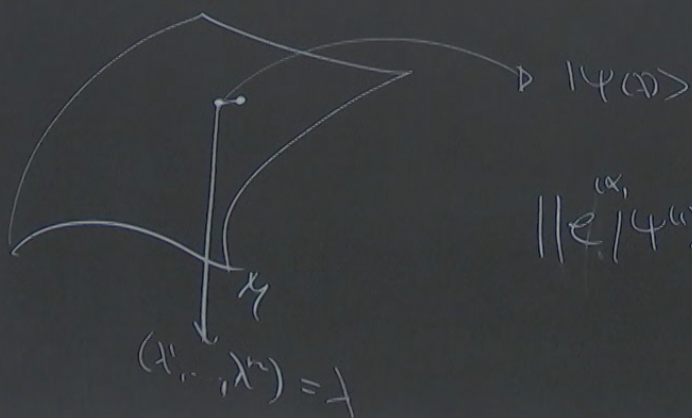
$$F = |\langle \psi^{(1)} | \psi^{(2)} \rangle|$$

$$d(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} F$$

$$F = |\langle \psi(\lambda) | \psi(\lambda + \delta\lambda) \rangle| \simeq 1$$

$$d(\psi^{(1)}, \psi^{(2)}) \simeq \sqrt{1 - F^2} \simeq \sqrt{2(1 - F)} \equiv d_{FS}$$

Fubini-
Study
Metric



$$d\ell^2 = 2(1 - F)$$

$$e^{i\alpha} |\psi\rangle$$

$$\|e^{i\alpha_1} |\psi^{(1)}\rangle e^{i\alpha_2} |\psi^{(2)}\rangle\|$$

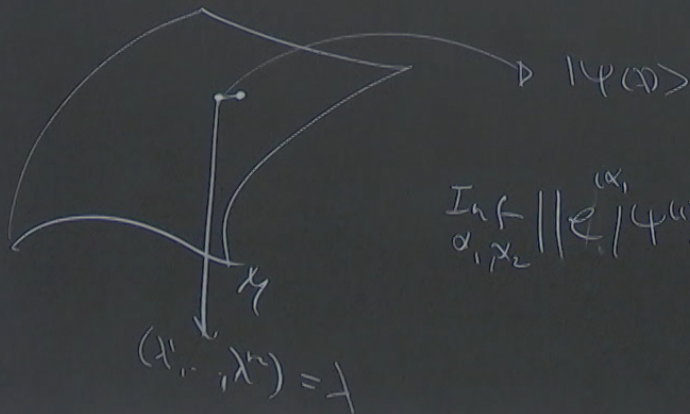
$$F = |\langle \psi^{(1)} | \psi^{(2)} \rangle|$$

$$d(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} F$$

$$F = |\langle \psi(x) | \psi(x + \delta x) \rangle| \approx 1$$

$$d(\psi^{(1)}, \psi^{(2)}) \approx \sqrt{1 - F^2} \approx \sqrt{2(1 - F)} \equiv d_{FS}$$

Fubini-Study
Metric



$$d\ell^2 = 2(1 - F)$$

$$e^{i\alpha} |\psi\rangle$$

$$\inf_{\alpha, x^i} \| e^{i\alpha} |\psi^{(1)}\rangle - e^{i\alpha} |\psi^{(2)}\rangle \| = 2(1 - F)$$

$\equiv d_{FS}$ Fubini-Study Metric

$$|\psi(\lambda + \delta\lambda)\rangle = |\psi(\lambda)\rangle + \partial_i |\psi(\lambda)\rangle d\lambda^i + \frac{1}{2} \partial_i \partial_j |\psi(\lambda)\rangle d\lambda^i d\lambda^j + \dots$$

$$\langle \psi(\lambda) | \psi(\lambda + \delta\lambda) \rangle = \frac{1}{2} \left[\text{Re} \langle \psi | \partial_i \partial_j \psi \rangle + \beta_i \beta_j \right] d\lambda^i d\lambda^j + O_3$$

$$\beta_i = \langle \psi | \partial_i \psi \rangle$$

d_{FS} Fubini-Study Metric

$$\begin{aligned}
 |\psi(\lambda + \delta\lambda)\rangle &= |\psi(\lambda)\rangle + \partial_i |\psi(\lambda)\rangle d\lambda^i + \frac{1}{2} \partial_i \partial_j |\psi(\lambda)\rangle d\lambda^i d\lambda^j + \dots \\
 \langle \psi(\lambda) | \psi(\lambda + \delta\lambda) \rangle &= \frac{1}{2} \left[\text{Re} \left(\langle \psi | \partial_i \partial_j \psi \rangle \right) + \beta_i \beta_j \right] d\lambda^i d\lambda^j + O_3 \\
 \beta_i &= -i \langle \psi | \partial_i \psi \rangle \\
 &\quad \underbrace{-\text{Re} \langle \partial_i \psi | \partial_j \psi \rangle}_{-\gamma_{ij}}
 \end{aligned}$$

d_{FS}

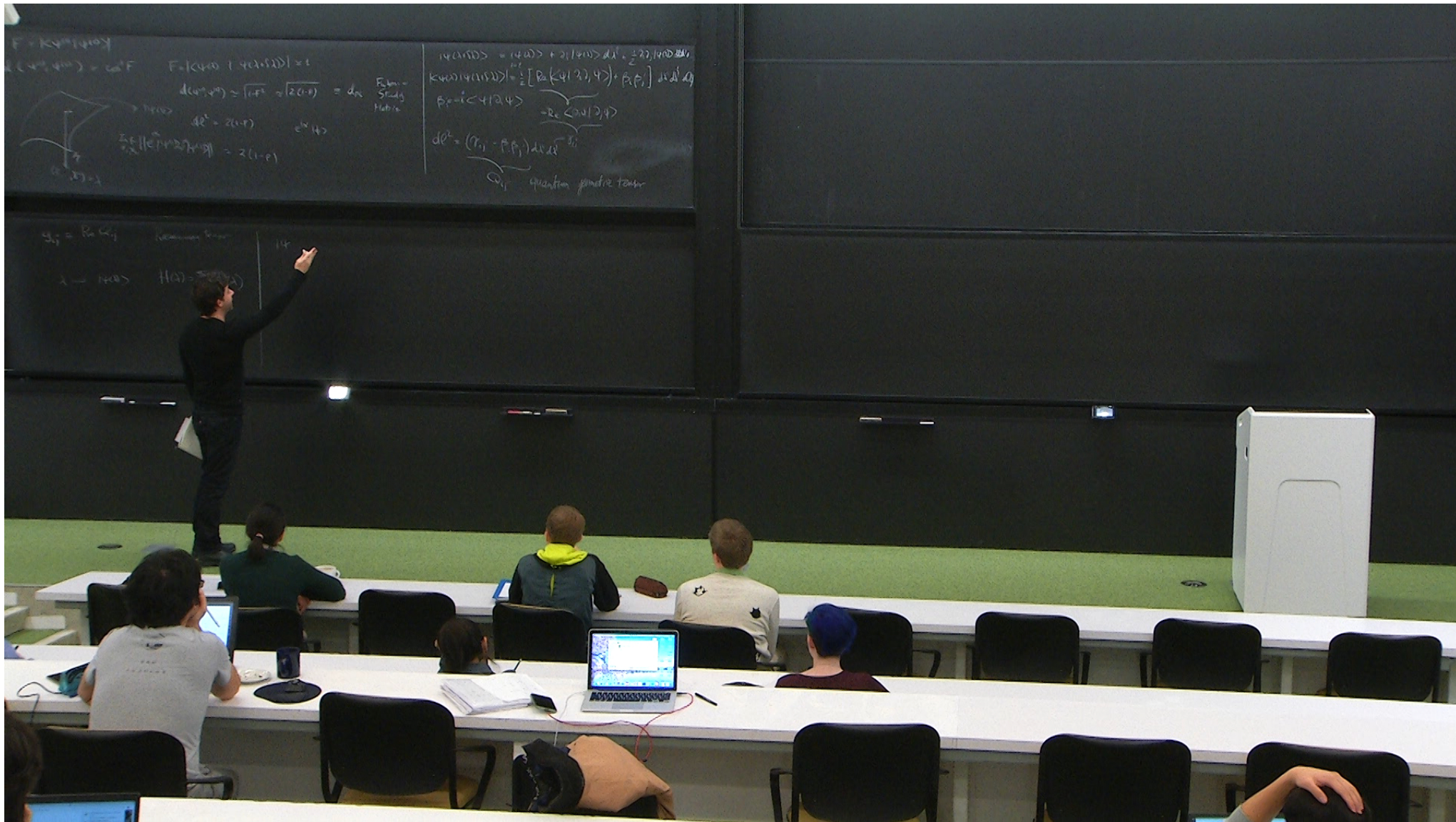
Fubini-
Study
Metric

$$|\psi(\lambda + \delta\lambda)\rangle = |\psi(\lambda)\rangle + \partial_i |\psi(\lambda)\rangle d\lambda^i + \frac{1}{2} \partial_i \partial_j |\psi(\lambda)\rangle d\lambda^i d\lambda^j + \dots$$
$$\langle \psi(\lambda) | \psi(\lambda + \delta\lambda) \rangle = \frac{1}{2} \left[\text{Re} \left(\langle \psi | \partial_i \partial_j \psi \rangle \right) + \beta_i \beta_j \right] d\lambda^i d\lambda^j + O_3$$

$$\beta_i = -i \langle \psi | \partial_i \psi \rangle$$
$$- \text{Re} \langle \partial_i \psi | \partial_j \psi \rangle$$

$$d\ell^2 = (\gamma_{ij} - \beta_i \beta_j) d\lambda^i d\lambda^j \equiv \gamma_{ij}$$

Q_{ij} quantum geometric tensor



$$g_{ij} = \operatorname{Re} Q_{ij}$$

Riemann tensor

$$|\psi_0(\lambda + i\delta\lambda)\rangle$$

$$\lambda \rightarrow |\psi(\lambda)\rangle$$

$$H(\lambda) = \sum_x \Phi_x(\lambda)$$



$$|\psi_0(\lambda)\rangle$$

quantum geometric tensor

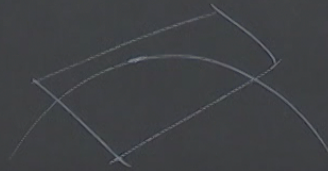
$$|\psi_0(\lambda + \delta\lambda)\rangle \approx |\psi_0(\lambda)\rangle + \sum_{n \neq 0} (E_n(\lambda) - E_0(\lambda))^{-1} \langle \psi_n(\lambda) | \delta H | \psi_0(\lambda) \rangle |\psi_n(\lambda)\rangle$$

$$H(\lambda + \delta\lambda) - H(\lambda) \equiv \delta H$$

$$g_{\mu\nu} = \text{Re} \sum_{n \neq 0} [E_n(\lambda) - E_0(\lambda)]^{-2} \cdot$$

$$\cdot \langle \psi_0 | \partial_\mu H | \psi_n \rangle \langle \psi_n | \partial_\nu H | \psi_0 \rangle$$

$|\partial_\mu \psi\rangle$



$$H = - \sum_{i=1}^M \left(\frac{1+r}{2} x x + \frac{1-r}{2} y y + \lambda z \right)$$

$$N = 2M+1$$

$$|g(r, \lambda)\rangle = \bigotimes_k \left[\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_{-k} + i \sin \frac{\theta_k}{2} |1\rangle_k |1\rangle_{-k} \right]$$

$$|g(\bar{r}, \bar{\lambda})\rangle$$

$$H = - \sum_{i=1}^M \left(\frac{1+r}{2} x x + \frac{1-r}{2} y y + h_i z \right)$$

$$N = 2M+1$$

$$|g(r, h)\rangle = \bigotimes_k \left[\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_{-k} + i \sin \frac{\theta_k}{2} |1\rangle_k |1\rangle_{-k} \right]$$

$$|g(\bar{r}, \bar{h})\rangle$$

$$\lambda = (r, h)$$

$$\langle g(\bar{\lambda}) | g(\lambda) \rangle = \pi_{\kappa} \cos \frac{\bar{\theta}_{\kappa} - \theta_{\kappa}}{2}$$

$$\left[\langle \lambda_{\kappa} | \lambda_{-\kappa} \rangle \right]$$

$$g_{\mu\nu} = \text{Re} Q_{\mu\nu} = \frac{1}{4} \sum_{\kappa} \frac{\partial \theta_{\kappa}}{\partial \lambda^{\mu}} \frac{\partial \theta_{\kappa}}{\partial \lambda^{\nu}}$$

$$\langle g(\bar{\lambda}) | g(\lambda) \rangle = \pi_{\kappa} \cos \frac{\bar{\theta}_{\kappa} - \theta_{\kappa}}{2}$$

$$g_{\mu\nu} = \text{Re} Q_{\mu\nu} = \frac{1}{4} \sum_{\kappa} \left(\frac{\partial \theta_{\kappa}}{\partial \lambda^{\mu}} \frac{\partial \theta_{\kappa}}{\partial \lambda^{\nu}} \right)$$

$$= \frac{1}{4} \frac{M}{2\pi} \int_0^{\pi} dx$$

$$|h| < 1$$

$$\langle g(\bar{\lambda}) | g(\lambda) \rangle = \pi_{\kappa} \cos \frac{\bar{\theta}_{\kappa} - \theta_{\kappa}}{2}$$

$$\left[\begin{matrix} \lambda_{\kappa} \\ \bar{\lambda}_{\kappa} \end{matrix} \right]$$

$$g_{\mu\nu} = \text{Re} Q_{\mu\nu} = \frac{1}{4} \sum_{\kappa}$$

$$\left(\begin{matrix} \frac{\partial \theta_{\kappa}}{\partial \lambda^{\mu}} & \frac{\partial \theta_{\kappa}}{\partial \lambda^{\nu}} \end{matrix} \right)$$

↓ dx

$$= \frac{1}{4} \frac{M}{2\pi} \int_0^{\pi} e^{i\theta} \begin{pmatrix} (1-h^2)^{-1} & 0 \\ 0 & (1+|g|^2)^{-2} \end{pmatrix}$$

$$|h| < 1$$

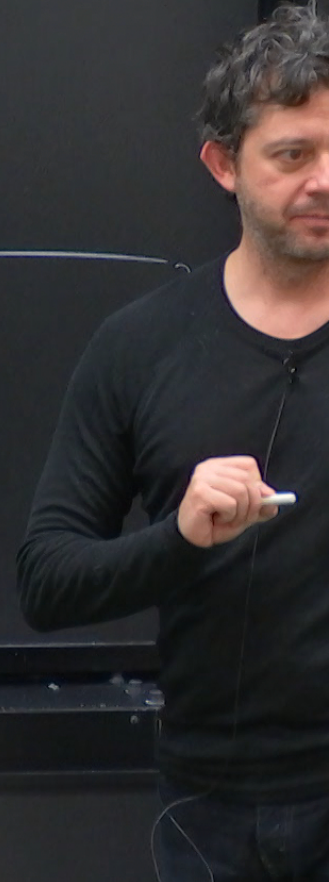
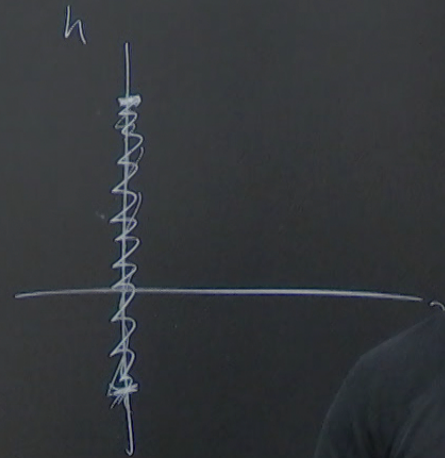
$$\langle g(\bar{\lambda}) | g(\lambda) \rangle = \pi \sum_k \cos \frac{\bar{\theta}_k - \theta_k}{2}$$

$$\left[\begin{array}{c} \lambda_k \\ \lambda_{-k} \end{array} \right]$$

$$g_{\mu\nu} = \text{Re} Q_{\mu\nu} = \frac{1}{4} \sum_k \left(\frac{\partial \theta_k}{\partial \lambda^\mu} \frac{\partial \theta_k}{\partial \lambda^\nu} \right)$$

$$= \frac{1}{4} \frac{M}{2\pi} \int_0^\pi e^{i\theta} \left(\begin{array}{cc} (1-h^2)^{-1} & 0 \\ 0 & (1+h^2)^{-2} \end{array} \right)$$

$\downarrow$
 $\frac{1}{6|\gamma|}$



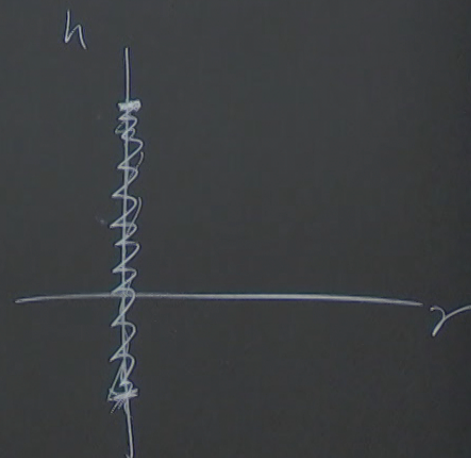
$$\langle g(\bar{\lambda}) | g(\lambda) \rangle = \pi_{\kappa} \cos \frac{\bar{\theta}_{\kappa} - \theta_{\kappa}}{2}$$

$$\left[\begin{matrix} \lambda_{\kappa} \\ \bar{\lambda}_{\kappa} \end{matrix} \right]$$

$$g_{\mu\nu} = \text{Re} Q_{\mu\nu} = \frac{1}{4} \sum_{\kappa} \left(\frac{\partial \theta_{\kappa}}{\partial \lambda^{\mu}} \frac{\partial \theta_{\kappa}}{\partial \lambda^{\nu}} \right) dx$$

$$= \frac{1}{4} \frac{M}{2\pi} \int_{-\pi}^{\pi} e^{i\theta} \begin{pmatrix} (1-h^2)^{-1} & 0 \\ 0 & (1+h^2)^{-2} \end{pmatrix} d\theta$$

$\downarrow N$
 $\frac{1}{|6\lambda|}$



$$\frac{1-\gamma}{2} \gamma \gamma + h, z)$$

$$\left[\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_{-k} + i \sin \frac{\theta_k}{2} |1\rangle_k |1\rangle_{-k} \right]$$

$$\lambda = (\gamma, h)$$

$$R = \begin{cases} -\frac{16}{N} \frac{1+|\gamma|}{|\gamma|} & |\lambda| < 1 \\ \frac{16}{N} \frac{|1+\sqrt{h^2+\gamma^2}|}{\sqrt{h^2+\gamma^2-1}} & |\lambda| > 1 \end{cases}$$

$$\langle g(\bar{\lambda}) | g(\lambda) \rangle = \pi_k \cos \bar{\theta}_k$$

$$g_{\mu\nu} = \text{Re} Q_{\mu\nu} = \frac{1}{4} \sum_k$$

$$= \frac{1}{4} \frac{M}{2\pi} \int \frac{1}{(1-h^2)^{-1}} \frac{1}{16|\gamma|} \begin{pmatrix} (1-h^2)^{-1} & 0 \\ 0 & (1+h^2)^{-1} \end{pmatrix}$$



$$g_{\mu\nu} = \text{Re} \sum_{n \neq 0} \Delta_{n0}^{-2} \langle \psi_0 | \partial_\mu H | \psi_n \rangle \langle \psi_n | \partial_\nu H | \psi_0 \rangle$$

$$|g_{\mu\nu}| \leq \langle \phi | g_{\mu\nu} | \phi \rangle$$

$$\begin{array}{l} |\phi_i\rangle \quad \phi_i \\ |\phi\rangle \end{array}$$

$$|\phi\rangle = d^\mu |e_\mu\rangle$$

$$\delta H = \sum_\mu (\partial_\mu H) d^\mu$$

$$g_{\mu\nu} = \text{Re} \sum_{n \neq 0} \Delta_{n0}^{-2} \underbrace{\langle \psi_0 | \partial_\mu H | \psi_n \rangle \langle \psi_n | \partial_\nu H | \psi_0 \rangle}$$

$$|g_{\mu\nu}| \leq \sum_n$$

$$|g_{\mu\nu}| \leq \langle \phi | g_{\mu\nu} | \phi \rangle$$

$$|\phi\rangle = d^\mu |e_\mu\rangle$$

$$\begin{array}{l} |\phi_i\rangle \quad \phi_i \\ |\phi\rangle \end{array}$$

$$\delta H = \sum_\mu (\partial_\mu H) d^\mu$$

$$|g_{\mu\nu}| \leq \sum_{n \neq 0} \Delta_{n0}^{-2} |\langle \psi_0 | S_H | \psi_n \rangle|^2$$

$$\leq \Delta^{-2} \sum_{n \neq 0} \langle \psi_0 | S_H | \psi_n \rangle \langle \psi_n | S_H | \psi_0 \rangle - \langle \psi_0 | S_H | \psi_0 \rangle^2$$

$$\leq \Delta^{-2} \left[\langle S_H^2 \rangle - \langle S_H \rangle^2 \right]$$