

Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 5

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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URL: <http://pirsa.org/20010030>

$$H = \sum_k \Lambda_k d_k^\dagger d_k$$

$$\Lambda_k = \sqrt{\cos\left(\frac{2\pi k}{N} - h\right)^2 + \eta^2 \sin^2 \frac{2\pi k}{N}}$$

$$\Lambda_k \stackrel{?}{=} 0$$

$$e^{i \frac{2\pi k}{N}}$$

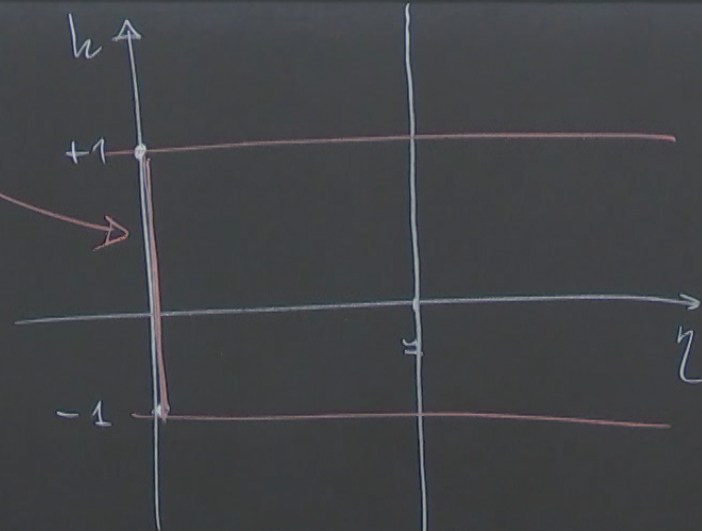
XX criticality

$$\eta = 0$$

$$\lambda_k = \left| \cos\left(\frac{2\pi k}{N}\right) - h \right|$$

$$\cos \frac{2\pi k}{N} = \pm h$$

$$N \rightarrow \infty$$



$$\eta \neq 0$$

Fermi model

$$H = \sum_k \Lambda_k d_k^\dagger d_k$$

$$\Lambda_k = \sqrt{\left(\cos\left(\frac{2\pi k}{N}\right) - h\right)^2 + \eta^2 \sin^2 \frac{2\pi k}{N}}$$

$$\Lambda_k \stackrel{?}{=} 0 \quad \varepsilon_k^2$$

XX criticality
 $\eta = 0$

$$\Lambda_k = \left| \cos\left(\frac{2\pi k}{N}\right) - h \right|$$

$$\cos \frac{2\pi k}{N} = \pm h$$

$$N \rightarrow \infty$$

$$\eta \neq 0$$

η

Feynman model

$$d_k |g\rangle = 0$$

$$d_k = \cos \frac{\theta_k}{2} c_k - i e^{2i\phi} \sin \frac{\theta_k}{2} c_{-k}^\dagger$$

$$\cos \theta_k = \frac{E_k}{\Lambda_k}$$

$$|g\rangle = \bigotimes_k \left(\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_k \right)$$

$$\eta \neq 0$$

η

Feynman model

$$d_k |g\rangle = 0$$

$$d_k = \cos \frac{\theta_k}{2} c_k - i e^{2i\phi} \sin \frac{\theta_k}{2} c_{-k}^\dagger$$

$$\cos \theta_k = \frac{E_k}{\Lambda_k}$$

$$|g\rangle = \bigotimes_k \left(\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_k \right)$$

$$d_k |g\rangle = 0$$

$$c_k = \cos \frac{\theta_k}{2} c_k - i e^{2i\phi} \sin \frac{\theta_k}{2} c_{-k}^+$$

$$\boxed{c_k |0_k\rangle = 0}$$

$$\cos \theta_k = \frac{\epsilon_k}{\Lambda_k}$$

$$|g\rangle = \bigotimes_k \left(\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_k + i e^{2i\phi} \sin \frac{\theta_k}{2} |1\rangle_k |1\rangle_k \right)$$

$$d_k = \cos \frac{\theta_k}{2} c_k - i e^{2i\phi} \sin \frac{\theta_k}{2} c_{-k}^\dagger \quad |c_k | \theta_k \rangle = 0$$

$$\cos \theta_k = \frac{\epsilon_k}{\Lambda_k}$$

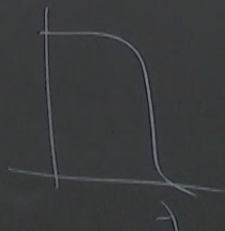
$$d_k |g\rangle = 0$$

$$|g\rangle = \bigotimes_k \left(\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_k + i e^{2i\phi} \sin \frac{\theta_k}{2} |1\rangle_k |1\rangle_k \right)$$

$$\Delta \rightarrow 0$$

Phase as domain
of analyticity of f

CM $\langle 0 \rangle = \frac{\partial}{\partial \lambda^k} f$



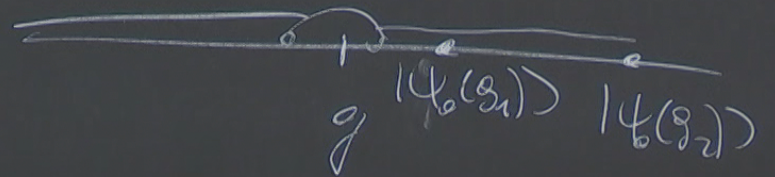
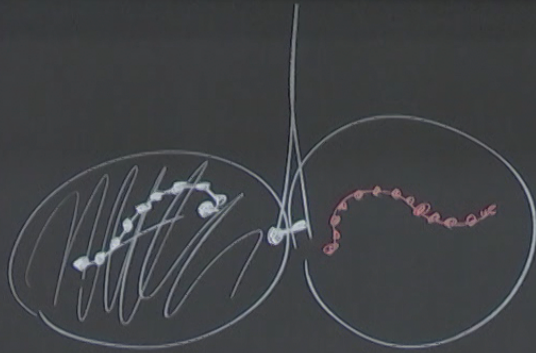
QM

$$\langle 0 \rangle = \text{Tr}[\hat{\phi} \psi]$$

$$H = \sum_n E_n |E_n\rangle \langle E_n| = E_0 |E_0\rangle \langle E_0| + \text{orth.}$$

$$\Delta \rightarrow 0$$

$$\begin{bmatrix} \text{Sym} & \text{Sym} \end{bmatrix}$$



distance between Q. States

$$(x) \rightarrow |\phi(x)|^2$$

$$p_x = \frac{1}{52} \text{ for all } x$$

$$(\text{classical}) \text{ trace distance } D(p_x, q_x) = \frac{1}{2} \sum_x |p_x - q_x|$$

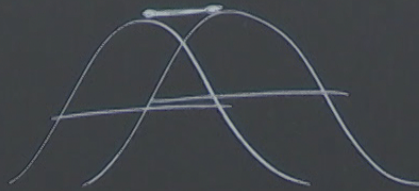
$$= \max_S \left| \sum_{x \in S} p_x - \sum_{x \in S} q_x \right|$$

$$q_x = \begin{cases} \frac{1}{52} - \epsilon & x \neq a_{\text{ces}} \\ \frac{1}{52} + 12\epsilon & x = a_{\text{ces}} \end{cases}$$

$$F(p_2, q_2) = \sum_n \sqrt{p_n q_n} \quad \text{fidelity}$$

$$p, (1-p)$$

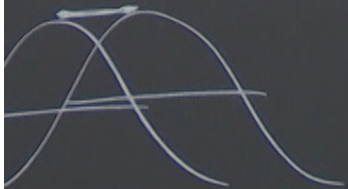
$$q, (1-q)$$



$$|p - q| \leq \Delta p + \Delta q$$

$$\Delta p = \sqrt{\frac{p(1-p)}{n}}$$

fidelity

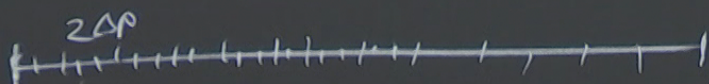


$$|p - q| \leq \Delta p + \Delta q \approx 2\Delta p$$

$$\Delta p = \sqrt{\frac{p(1-p)}{n}}$$

$$d(p, q) = \int_{p_1}^{p_2} \frac{dp}{2\Delta p} = \frac{p_2}{\sqrt{n}} \int_{p_1}^{p_2} \frac{dp}{2\sqrt{p(1-p)}}$$

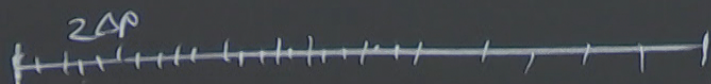
$$\approx 2\Delta p$$



$$d(p, q) = \frac{1}{\sqrt{n}} \int_{p_1}^{p_2} \frac{dp}{2\Delta p} = \frac{\sqrt{n}}{\sqrt{2}} \int_{p_1}^{p_2} \frac{dp}{\sqrt{p(1-p)}}$$

$$d(p, q) = \cos^{-1} \sqrt{\sum_n p_n q_n} = \cos^{-1} F$$

$$\approx 2\Delta p$$



$$d(p, q) = \frac{1}{\sqrt{n}} \int_{p_1}^{p_2} \frac{dp}{2\Delta p} = \frac{\sqrt{n}}{\sqrt{2}} \int_{p_1}^{p_2} \frac{dp}{\sqrt{p(1-p)}}$$

$$d(p, q) = \cos^{-1} \left(\frac{\sum a_i p_i q_i}{\sqrt{\sum a_i p_i \sum a_i q_i}} \right) = \cos^{-1} F$$

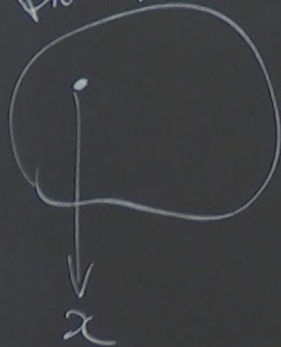
QUANTUM (PURE) STATES

$$\hat{A} = \sum_i a_i |\phi_i^A\rangle \langle \phi_i^A|$$

$$|\psi^{(1)}\rangle$$

$$Pr_i^{(1)}(A) = |\langle \psi^{(1)} | \phi_i^A \rangle|^2 = \text{Tr} [\psi^{(1)} \phi_i^A]$$

Phase space



$\rho(x)$

$$\begin{aligned} \langle f(x) \rangle &= \int f(x) \rho(x) dx \\ \langle g(x) \rangle &= \int g(x) \rho(x) dx \end{aligned}$$

$$\langle \phi_i^A |$$

$$\int \psi^{(1)} \phi_i^A = \int \rho_{\phi_i}^{(1)}(B) = |\langle \psi^{(1)} | \phi_i^A \rangle|^2$$

$$[A, B] = 0$$

$$d(\psi^{(1)}, \psi^{(2)}) = \max_A d_A(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} \sum_i |\langle \psi^{(1)} | \phi_i^A \rangle| |\langle \psi^{(2)} | \phi_i^A \rangle|$$

$$\sum_i |\langle \psi^{(1)} | \phi_i^A \rangle| |\langle \psi^{(2)} | \phi_i^A \rangle|$$

$$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle|$$

F quantum fidelity

$$A = a_1 |\psi^{(1)}\rangle \langle \psi^{(1)}| + \dots$$

$$F_{\psi}^{(\rho, \sigma)} = \text{tr}(\sqrt{\rho \sigma \rho})^{1/2}$$