Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 4

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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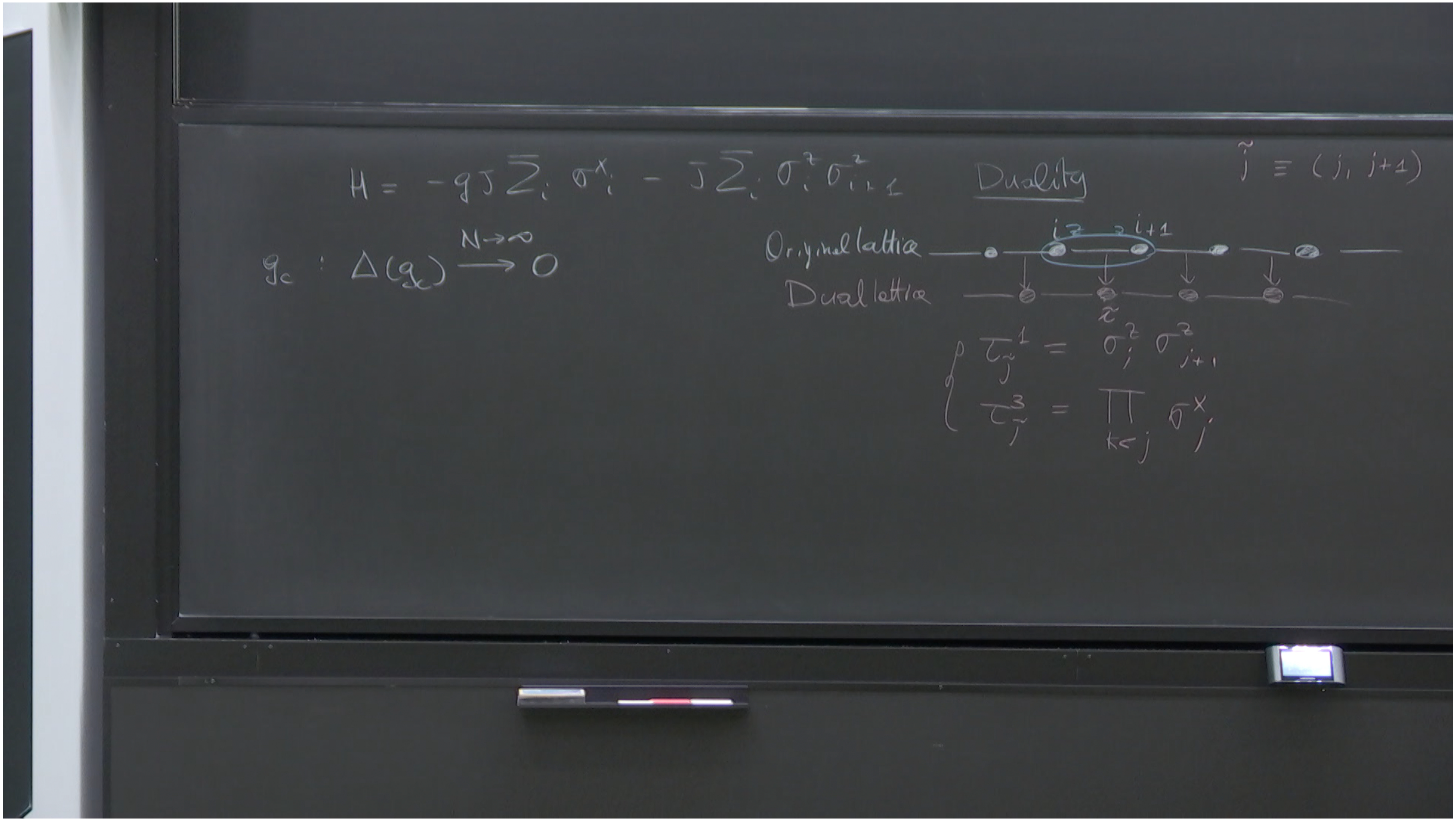
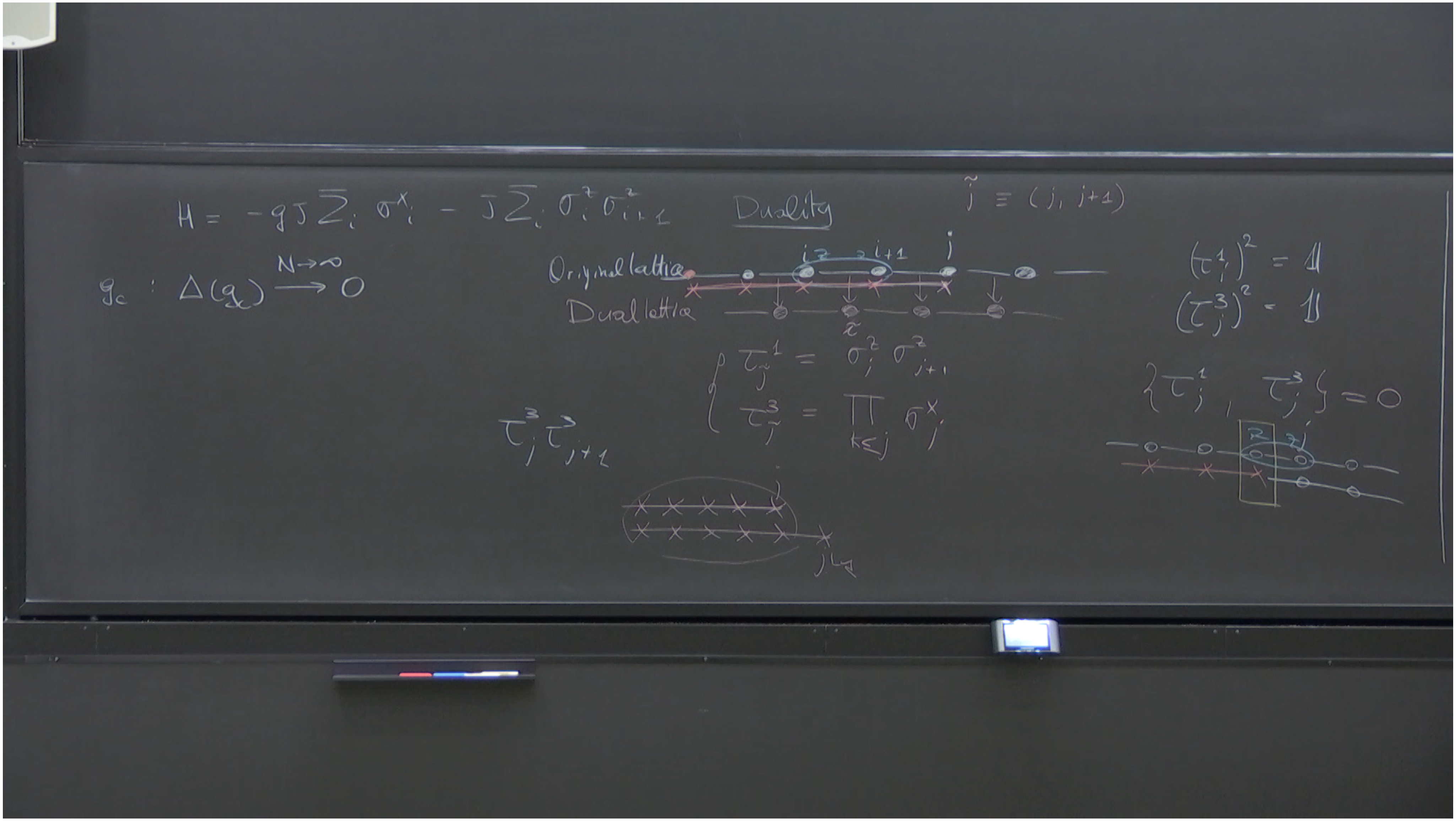

$H = -gJ\sum_i \sigma^x_i - J\sum_i \sigma^z_i \sigma^z_{i+1}$
$g_c : \Delta(g_c) \xrightarrow{N\to\infty} 0$
Duality
$\tilde{j} \equiv (j, j+1)$
Original lattice
i
$i+1$
Dual lattice
$\tilde{x}$
$\tau^1_{\tilde{j}} = \sigma^z_j \sigma^z_{j+1}$
$\tau^3_{\tilde{j}} = \prod_{k<j} \sigma^x_j$


$H = -gJ\sum_i \sigma^x_i - J\sum_i \sigma^z_i \sigma^z_{i+1}$
Duality
$\tilde{j} \equiv (j, j+1)$
$g_c : \Delta(g_c) \xrightarrow{N\to\infty} 0$
Original lattice
Dual lattice
$(\tau^1_j)^2 = 1\!\!1$
$(\tau^3_j)^2 = 1\!\!1$
$\tau^1_{\tilde{j}} = \sigma^z_j \sigma^z_{j+1}$
$\tau^3_{\tilde{j}} = \prod_{k\leq j} \sigma^x_j$
$\{\tau^1_j, \tau^3_j\} = 0$
$\tau^3_j \tau^3_{j+1}$

$$H_\sigma(g) \mapsto H_\tau(g) = -J\sum_j \tau^1_j - gJ\sum_j \tau^3_j \tau^3_{j+1}$$

$$= g\left[-\frac{J}{g}\sum_j \tau^1_j - J\sum_j \tau^3_j \tau^3_{j+1}\right]$$

$$H_\tau(g) = g\,H_\sigma\left(\tfrac{1}{g}\right)$$ Self-duality

$$E(g) = g\,E\left(\tfrac{1}{g}\right)$$

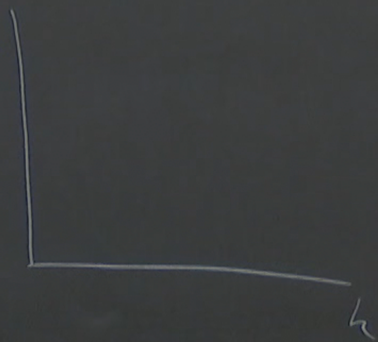
$$\Delta E(g) = g\,\Delta E(g^{-1})$$

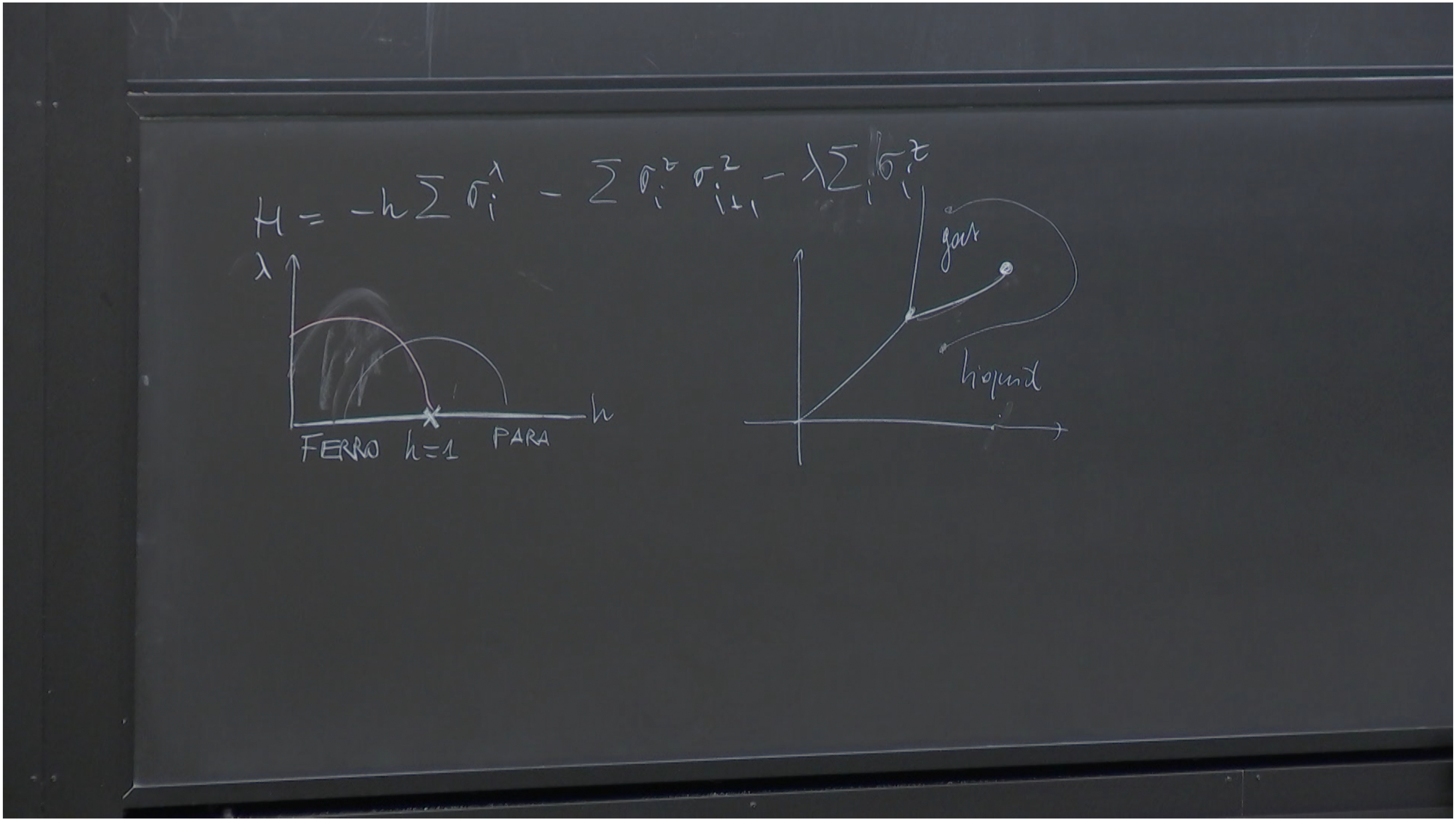
$$\Delta E = 0$$
$$\rightarrow g = g^{-1}$$
$$\rightarrow g = 1$$

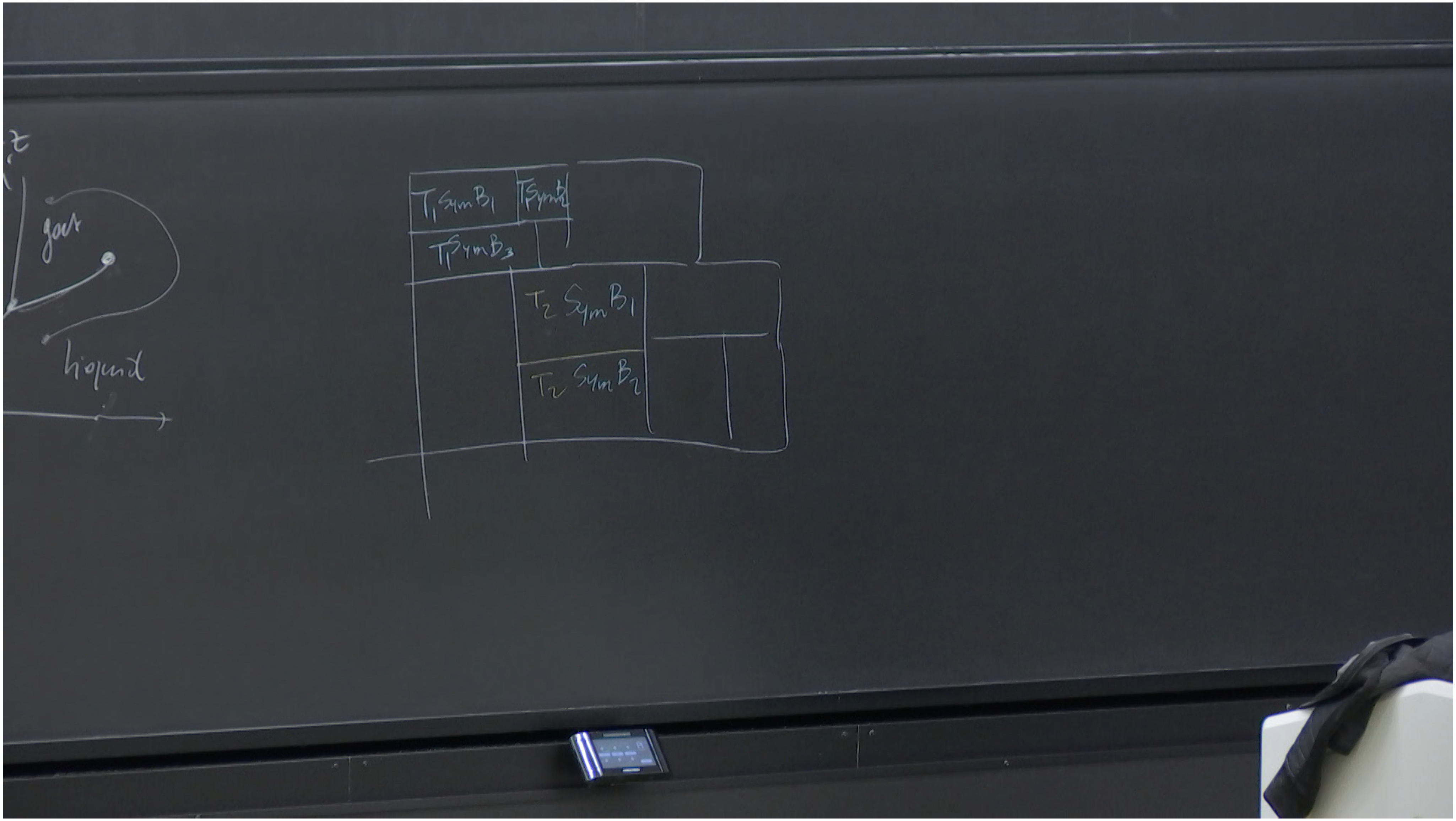
Quantum XY model

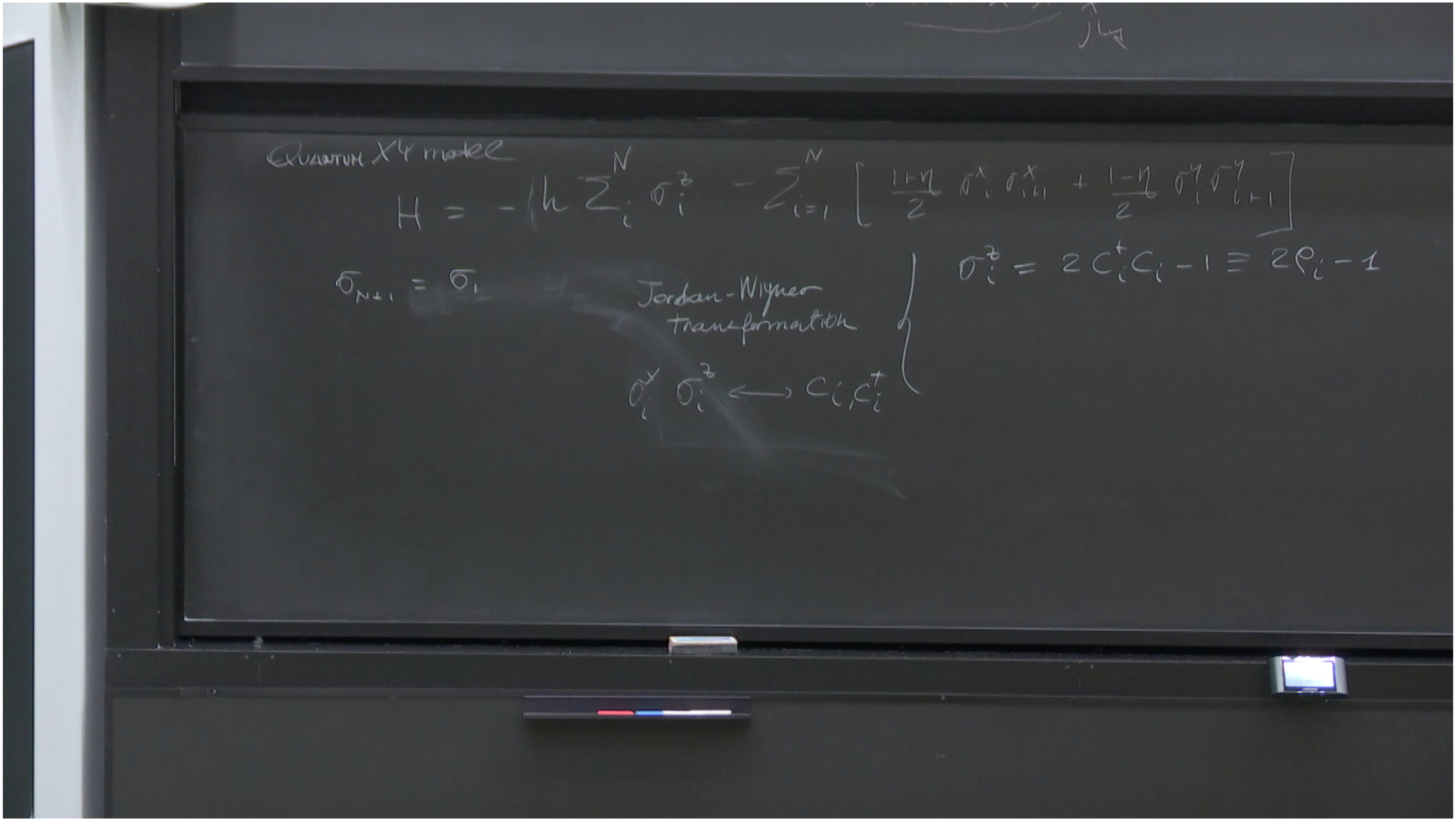
$$H = -h\sum_{i}^{N} \sigma_i^z - \sum_{i=1}^{N} \left[\frac{1+\eta}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\eta}{2} \sigma_i^y \sigma_{i+1}^y \right]$$

$\eta = 1$ $\quad H_{Ising}$ $\quad \eta$




$H = -h\sum \sigma_i^x - \sum \sigma_i^z \sigma_{i+1}^z - \lambda \sum_i \sigma_i^z$
λ
h
FERRO
h=1
PARA
gas
liquid


gas
liquid
T1 Sym B1
T1 Sym B3
T2 Sym B1
T2 Sym B2


Quantum XY model
$H = -h \sum_i^N \sigma_i^z - \sum_{i=1}^N \left[\frac{1+\eta}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\eta}{2} \sigma_i^y \sigma_{i+1}^y \right]$
$\sigma_{N+1} = \sigma_1$
Jordan-Wigner transformation
$\sigma_i^x \; \sigma_i^z \longleftrightarrow c_i, c_i^\dagger$
$\sigma_i^z = 2 c_i^\dagger c_i - 1 \equiv 2\rho_i - 1$

$$\ldots{}^x_i\,\sigma^x_{i+1} + \frac{1-\eta}{2}\,\sigma^y_i\sigma^y_{i+1}\Big]$$

$$N_f = \sum_i c^\dagger_i c_i$$

$$\sigma^z_i = 2c^\dagger_i c_i - 1 \equiv 2\rho_i - 1$$

$$\sigma^-_i = \frac{1}{2}\left(\sigma^x_i - i\sigma^y_i\right) = c_i\, e^{i\pi\sum_{j=1}^{i-1} c^\dagger_j c_j}$$

$$\{c_i, c^\dagger_j\} = \delta_{ij}$$

$$\{c_i, c_j\} = \{c^\dagger_i, c^\dagger_j\} = 0$$

$H = -gJ\sum_i \sigma^x_i - J\sum_i \sigma^z_i \sigma^z_{i+1}$

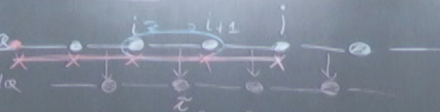
Duality

$j \equiv (i, i+1)$

$g_c : \Delta(g_c) \xrightarrow{N\to\infty} 0$

Original lattice

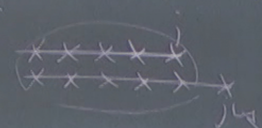
Dual lattice



$\tau^1_j = \sigma^z_j \sigma^z_{j+1}$

$\tau^3_j = \prod_{k\le j} \sigma^x_j$

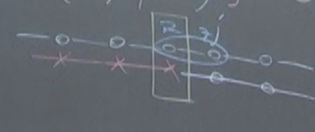
$\tau^3_j \tau^3_{j+1}$



$(\tau^1_i)^2 = 1\!\!1$

$(\tau^3_j)^2 = 1\!\!1$

$\{\tau^1_j, \tau^3_j\} = 0$



$H_\sigma(g) \mapsto H_\tau(g) = -J\sum_j \tau^1_j - gJ\sum_j \tau^3_j \tau^3_{j+1}$

$= g\left[-\frac{J}{g}\sum_j \tau^1_j - J\sum_j \tau^3_j \tau^3_{j+1}\right]$

$H_\tau(g) = g H_\sigma\left(\frac{1}{g}\right)$ Self-duality

$E(g) = gE\left(\frac{1}{g}\right)$

$\Delta E(g) = g\,\Delta E(g^{-1})$

$\Delta E = 0$

$\to g = g^{-1}$

$\to g = 1$

Quantum XY model

$H = -h\sum_i^N \sigma^z_i - \sum_{i=1}^N \left[\frac{1+\eta}{2}\sigma^x_i \sigma^x_{i+1} + \frac{1-\eta}{2}\sigma^y_i \sigma^y_{i+1}\right]$

$N_f = \sum_i c^\dagger_i c_i$

$\sigma_{N+1} = \sigma_1$

Jordan-Wigner transformation

$\sigma^z_i = 2c^\dagger_i c_i - 1 \equiv 2\rho_i - 1$

$\sigma^-_i = \frac{1}{2}(\sigma^x_i - i\sigma^y_i) = c_i\, e^{i\pi\sum_{j=1}^{i-1} c^\dagger_j c_j}$

$\sigma^y_i\ \sigma^z_i \longleftrightarrow c_i c^\dagger_i$

$\{c_i, c^\dagger_j\} = \delta_{ij}$

$\{c_i, c_j\} = \{c^\dagger_i, c^\dagger_j\} = 0$

$$H = \sum_{i=1}^{N-1}\left[-(c^\dagger_i c_{i+1} + c^\dagger_{i+1} c_i) + \eta(c^\dagger_{i+1} c^\dagger_i + c_i c_{i+1})\right] - (-1)^{N_f}\left[-(c^\dagger_N c_1 + c^\dagger_1 c_N) + \eta(c^\dagger_1 c^\dagger_N + c_N c_1)\right] - h\sum_{i=1}^N (2c^\dagger_i c_i - 1)$$

$[H, N_f] = 0$

transformation $\quad \sigma_i^- = \frac{1}{2}(\sigma_i^x - i\sigma_i^y) = c_i e$

$\sigma_i^x \; \sigma_i^z \longleftrightarrow c_i, c_i^\dagger$

$$H = \sum_{i=1}^{N-1} \left[-(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + \eta (c_{i+1}^\dagger c_i^\dagger + c_i c_{i+1}) \right] - (-1)^{N_f} \left[-(c_N^\dagger c_1 + c_1^\dagger c_N) + \eta (c_1^\dagger c_N^\dagger + c_N c_1) \right] - h \sum$$

$$[H, N_f] = 0$$

Fourier Transform $\quad c_k = \frac{1}{\sqrt{N}} \sum_{\ell=1}^{N} c_\ell e^{-ik\ell a}$ $\qquad L = Na \qquad |\phi\rangle$

$k = \left(-\frac{\pi}{a}, \frac{\pi}{a}\right) \qquad k_n = \frac{2\pi n}{Na}$

$$H = \sum_k (c_k^\dagger \; c_{-k}^\dagger) \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \begin{pmatrix} c_k \\ c_{-k} \end{pmatrix}$$

$$d_k = A_k c_k + B_k c_k^\dagger \qquad H = \sum_k \omega_k d_k^\dagger d_k - 1$$