Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 3

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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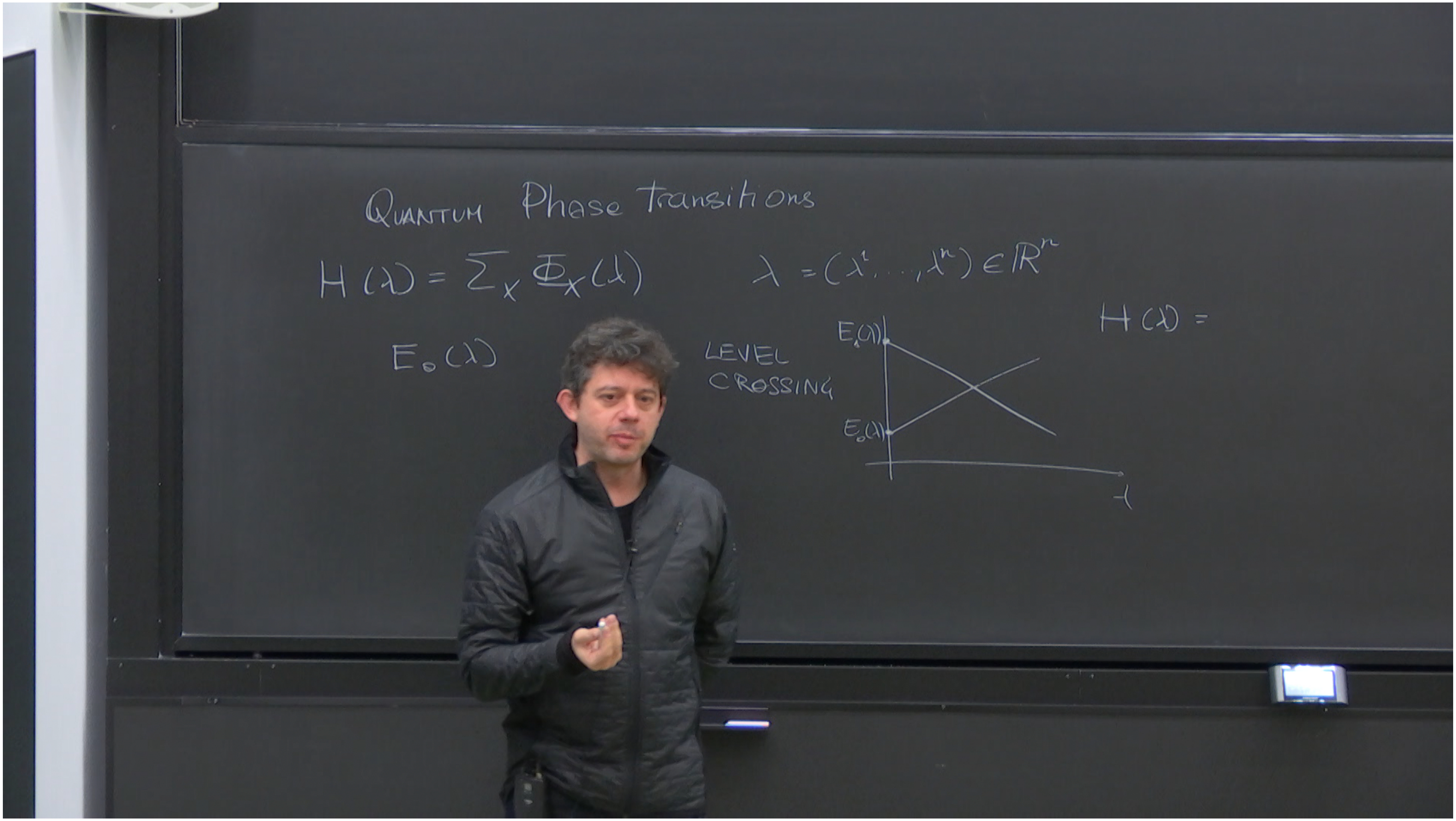
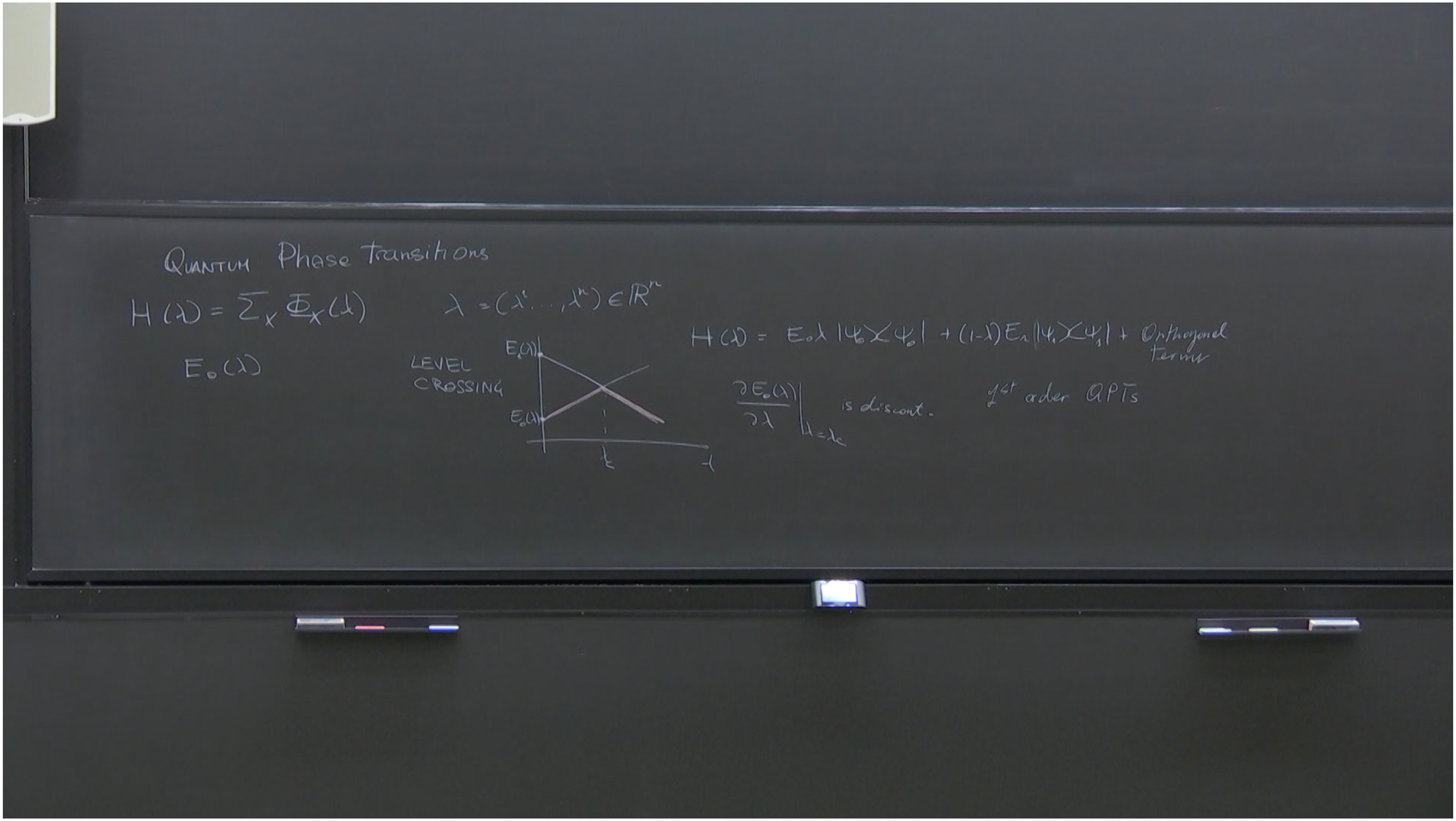
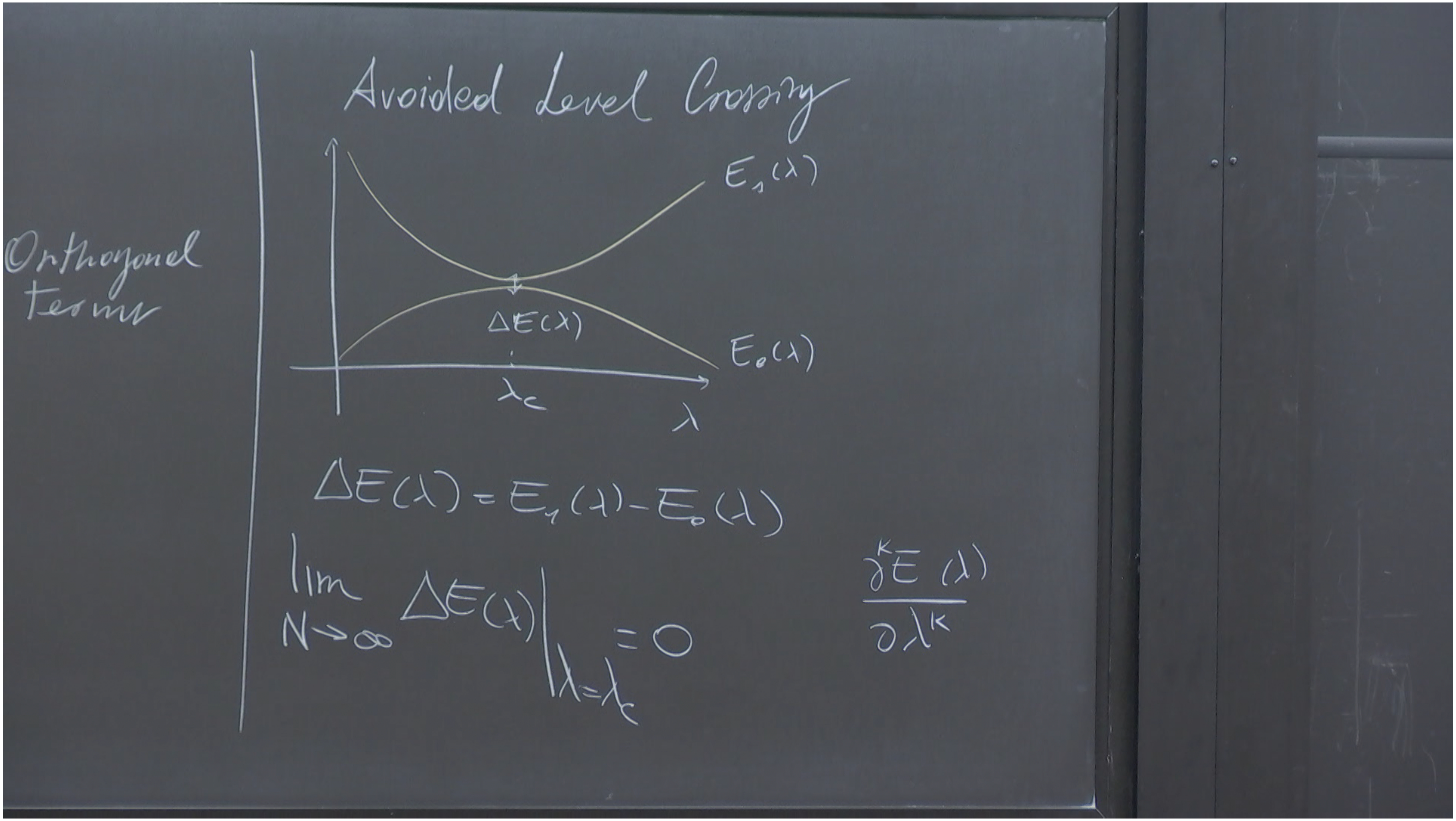

QUANTUM Phase transitions
$H(\lambda) = \sum_X \Phi_X(\lambda)$
$\lambda = (\lambda^1, \ldots, \lambda^n) \in \mathbb{R}^n$
$H(\lambda) =$
$E_0(\lambda)$
LEVEL CROSSING
$E_1(\lambda)$
$E_0(\lambda)$
λ


QUANTUM Phase transitions
$H(\lambda) = \sum_X \Phi_X(\lambda)$
$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$
$E_0(\lambda)$
LEVEL CROSSING
$H(\lambda) = E_0 \lambda |\psi_0\rangle\langle\psi_0| + (1-\lambda) E_1 |\psi_1\rangle\langle\psi_1|$ + Orthogonal Terms
$\left.\frac{\partial E_0(\lambda)}{\partial \lambda}\right|_{\lambda=\lambda_c}$ is discont.
1st order QPTs


Avoided Level Crossing
Orthogonal terms
$E_1(\lambda)$
$E_0(\lambda)$
$\Delta E(\lambda)$
λ_c
λ
$\Delta E(\lambda) = E_1(\lambda) - E_0(\lambda)$
$\lim_{N\to\infty} \Delta E(\lambda)\Big|_{\lambda=\lambda_c} = 0$
$\frac{\partial^k E(\lambda)}{\partial \lambda^k}$

$\lambda \sim \lambda_c$

$\Delta E \sim |\lambda - \lambda_c|^{z\nu}$; $G(r) \sim e^{-r/\xi}$

$\xi \sim |\lambda - \lambda_c|^{-\nu}$

$\Delta^{-1} \sim \xi^{z}$

QUANTUM ISING MODEL

Graph (Λ, E) , $\mathcal{H}_\Lambda = \bigotimes_{i\in\Lambda} \mathcal{H}_i$ $\qquad |\Lambda| = N$

$\mathcal{H}_i = \mathbb{C}^2 = \text{span}\{|0\rangle, |1\rangle\}$

$$H(g) = -gJ\sum_i \sigma_i^x \otimes \mathbb{1}_{\{\Lambda - i\}} - J\sum_{(ij)\in E} \sigma_i^z \otimes \sigma_j^z \otimes \mathbb{1}_{\{\Lambda - \{i,j\}\}}$$

$|\psi_0(g)\rangle$

1 $\quad g$

$|\psi_0(\ldots$

QUANTUM ISING MODEL

Graph (Λ, E) , $\mathcal{H}_\Lambda = \bigotimes_{i\in\Lambda} \mathcal{H}_i$

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$|\psi_0(g)\rangle$

① $g \to \infty$ Paramagnetic Phase

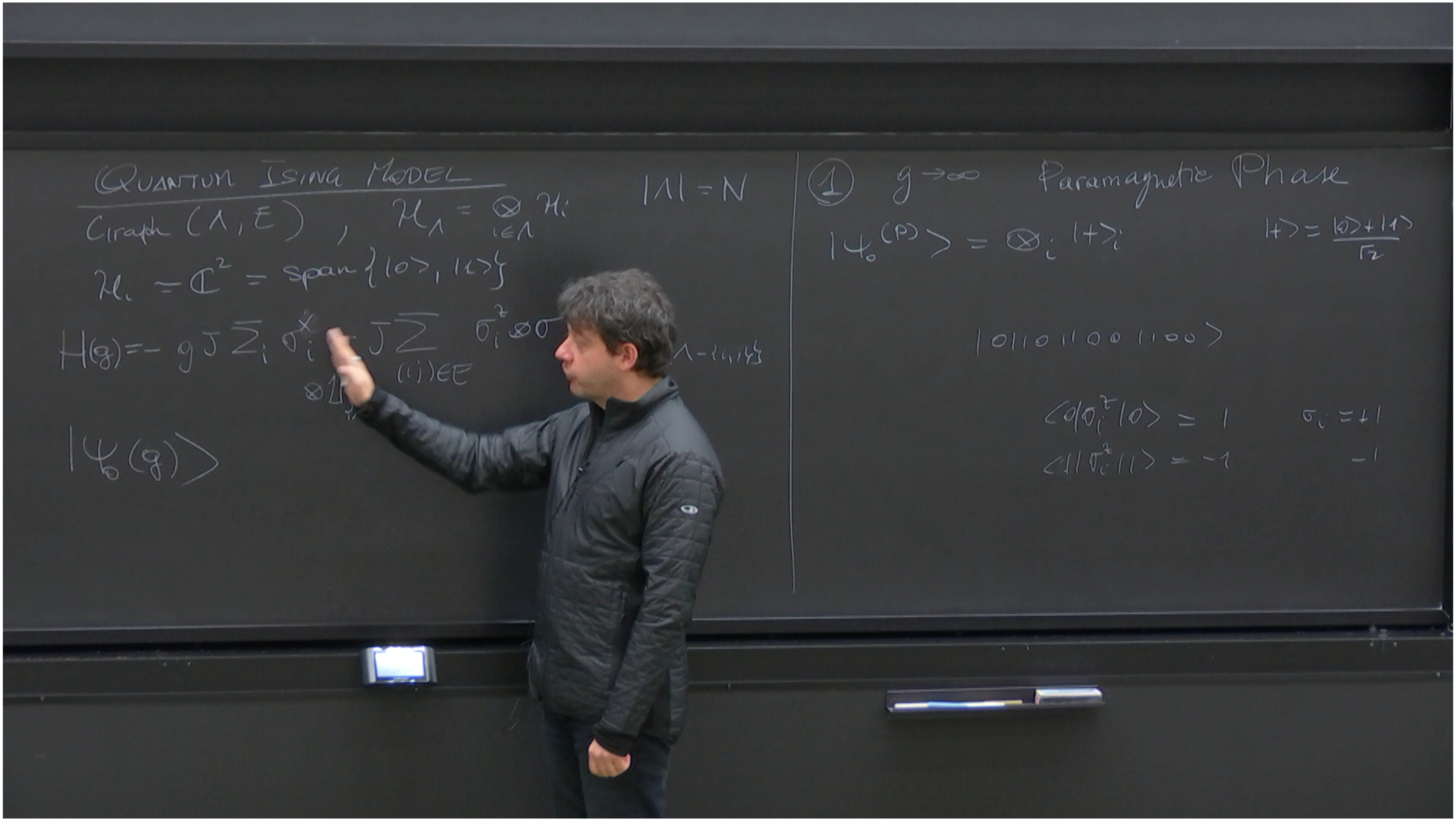
$|\psi_0^{(P)}\rangle = \bigotimes_i |+\rangle_i$

$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$|0110110011100\rangle$

$\langle 0|\sigma_i^z|0\rangle = 1$ $\sigma_i = +1$

$\langle 1|\sigma_i^z|1\rangle = -1$ -1


QUANTUM ISING MODEL
|Λ| = N
Graph (Λ, E) , H_Λ = ⊗_{i∈Λ} H_i
H_i = C^2 = span{|0>, |1>}
H(g) = − gJ Σ_i σ_i^x − J Σ_{(ij)∈E} σ_i^z ⊗ σ
|Ψ_0(g)>
① g → ∞ Paramagnetic Phase
|Ψ_0^(P)> = ⊗_i |+>_i
|+> = (|0> + |1>)/√2
|0110110011100>
<0|σ_i^z|0> = 1
<1|σ_i^z|1> = −1
σ_i = +1
−1

$|\Lambda| = N$

$\mathbb{1}_{\{\Lambda - \{i_1, i_2\}\}}$

① $g \to \infty$ Paramagnetic Phase

$$|\psi_0^{(P)}\rangle = \otimes_i |+\rangle_i \qquad |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|0110110011100\rangle$$

$$\langle 0|\sigma_i^z|0\rangle = 1 \qquad \sigma_i = +1$$

$$\langle 1|\sigma_i^z|1\rangle = -1 \qquad -1$$

$= N$

$\{i, i'\}$

① $g \to \infty$ Paramagnetic Phase

$$|\psi_0^{(P)}\rangle = \otimes_i |+\rangle_i \qquad |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$m = \frac{1}{N} \sum_i \langle \psi_0 | \sigma_i^z | \psi_0 \rangle = 0$$

order parameter

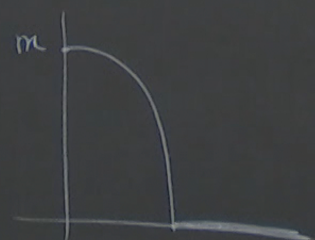
disordered phase

$\lambda \sim \lambda_c$

$$\Delta E \sim |\lambda - \lambda_c|^{z\nu} \; ; \quad G(r) \sim e^{-r/\xi}$$

$$\xi \sim |\lambda - \lambda_c|^{-\nu}$$

$$\Delta^{-1} \sim \xi^{z}$$



QUANTUM ISING MODEL

$|\Lambda| = N$

$$\text{Graph } (\Lambda, E) \; , \quad \mathcal{H}_\Lambda = \bigotimes_{i\in\Lambda} \mathcal{H}_i$$

$$\mathcal{H}_i = \mathbb{C}^2 = \text{span}\{|0\rangle, |1\rangle\}$$

$$H(g) = -gJ \sum_i \sigma_i^x \otimes \mathbb{1}_{\{\Lambda - i\}} - J \sum_{(ij)\in E} \sigma_i^z \otimes \sigma_j^z \otimes \mathbb{1}_{\{\Lambda - \{i,j\}\}}$$

$$|\psi_0(g)\rangle$$

$\mathcal{H}_i$

$|\Lambda| = N$

$\sigma^z_j \otimes \mathbb{1}_{\{\Lambda - \{i,j\}\}}$

① $g \to \infty$ Paramagnetic Phase

$$|\psi_0^{(P)}\rangle = \otimes_i |+\rangle_i \qquad |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$m = \frac{1}{N} \sum_i \langle \psi_0 | \sigma^z_i | \psi_0 \rangle = 0$$

order parameter

disordered phase

$$\langle \psi_0 | \sigma^z_i \sigma^z_j | \psi_0 \rangle = 0$$

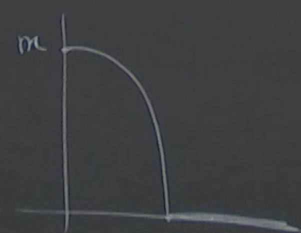
$$\boxed{g \gg 1}$$

$\lambda \sim \lambda_c$

$\Delta E \sim |\lambda - \lambda_c|^{z\nu}$; $G(r) \sim e^{-r/\xi}$

$\xi \sim |\lambda - \lambda_c|^{-\nu}$

$\Delta^{-1} \sim \xi^{z}$



QUANTUM ISING MODEL

$|\Lambda| = N$

Graph (Λ, E) , $\mathcal{H}_\Lambda = \bigotimes_{i\in\Lambda} \mathcal{H}_i$

$\mathcal{H}_i \simeq \mathbb{C}^2 = \mathrm{span}\{|0\rangle, |1\rangle\}$

$$H(g) = -gJ\sum_i \sigma_i^x \otimes 1\!\!1_{\Lambda - i} - J \sum_{(ij)\in E} \sigma_i^z \otimes \sigma_j^z \otimes 1\!\!1_{\Lambda - \{i,j\}}$$

$|\psi_0(g)\rangle$

$\boxed{\text{Remark}}$ $\quad [H(g), T] = 0$

T global symmetry $\qquad T = \bigotimes_{i=1}^{N} \sigma_i^x \otimes \cdots \otimes \sigma_N^x$

$$\psi^{(P)} = |+\rangle\langle+| \qquad T \bigotimes_i |+\rangle_i = \bigotimes_i |+\rangle_i$$

$$[\psi^{(P)}, T] = 0$$

metry is not Broken

$= \otimes_i |+\rangle_i$

$\mathbb{Z}_2$

$\boxed{g = 0}$

$|\psi_0^{\uparrow}\rangle = |0 \cdots 0\rangle$

$|\psi_0^{\downarrow}\rangle = |1 \cdots 1\rangle$

$\mathcal{L}_{GS} = \text{span}\{|\psi_0^{\uparrow}\rangle, |\psi_0^{\downarrow}\rangle\}$

$|\psi_0^{\uparrow}\rangle$

$$\underbrace{\frac{|\psi^{\uparrow}\rangle \pm |\psi^{\downarrow}\rangle}{\sqrt{2}}}_{\text{do not break the symmetry}}$$

$T|\psi_0^{\uparrow}\rangle = |\psi_0^{\downarrow}\rangle$

$[\psi_0^{\uparrow\downarrow}, T] \neq 0$

Symmetry is Broken

...metry is not Broken

$= \otimes_i |+\rangle_i$

$\mathbb{Z}_2$

$\boxed{g = 0}$

$|\psi_0^{\uparrow}\rangle = |0 \cdots 0\rangle$

$|\psi_0^{\downarrow}\rangle = |1 \cdots 1\rangle$

$\mathcal{L}_{GS} = \mathrm{span}\{|\psi_0^{\uparrow}\rangle, |\psi_0^{\downarrow}\rangle\}$

$|\psi_0^{\uparrow\downarrow}\rangle$

$$\underbrace{\frac{|\psi^{\uparrow}\rangle \pm |\psi^{\downarrow}\rangle}{\sqrt{2}}}_{\text{do not break the symmetry}}$$

$T|\psi_0^{\uparrow}\rangle = |\psi_0^{\downarrow}\rangle$

$[\psi_0^{\uparrow\downarrow}, T] \neq 0$

Symmetry is Broken

$$\max_{\psi_0 \in \mathcal{L}_{GS}} \|[\psi_0, T]\|$$

$\psi_0 = \frac{1}{2}(\psi_0^{\uparrow} + \psi_0^{\downarrow})$

$$m = \frac{1}{N}\sum \langle \psi_0^{\uparrow} | \sigma_i^z | \psi_0^{\uparrow} \rangle = n_0(g)$$

$$\langle \sigma_i \sigma_j \rangle = 1$$

$$n_0^2(g)$$

long-range order