Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 2

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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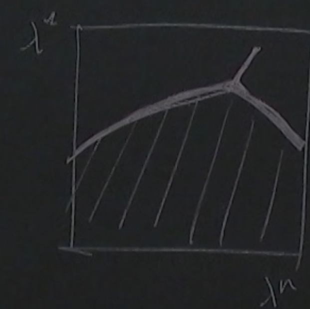
QUANTUM PHASES

$$H = \sum_{X \subset \Lambda} \Phi_X$$

Phase as equivalence Class.

$$F = E - TS = -k_B T \log Z$$

$$\frac{F}{V} = f(\lambda^1, \dots, \lambda^n)$$



Phase as equivalence Class.

$$F = E - TS = -k_B T \log Z$$

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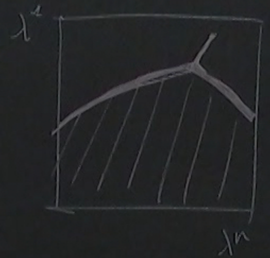


$$Z = \sum_{\{\alpha\}} e^{-\beta E(\{\alpha\})}$$

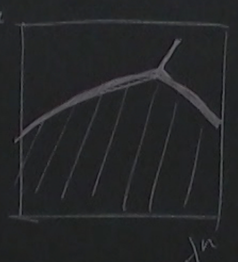
lattice $\sigma_i = \pm 1$ $\{\sigma_i\}$

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\frac{\partial f}{\partial h} = \frac{1}{N} \cancel{(-k_B T)} \frac{1}{Z} \sum_{\{\sigma\}} \cancel{(-\beta)} \left(-\sum_i \sigma_i\right) e^{-\beta H}$$

...ce Class.

$-k_B T \log Z$



$$Z = \sum_{\{\alpha\}} e^{-\beta E(\{\alpha\})}$$

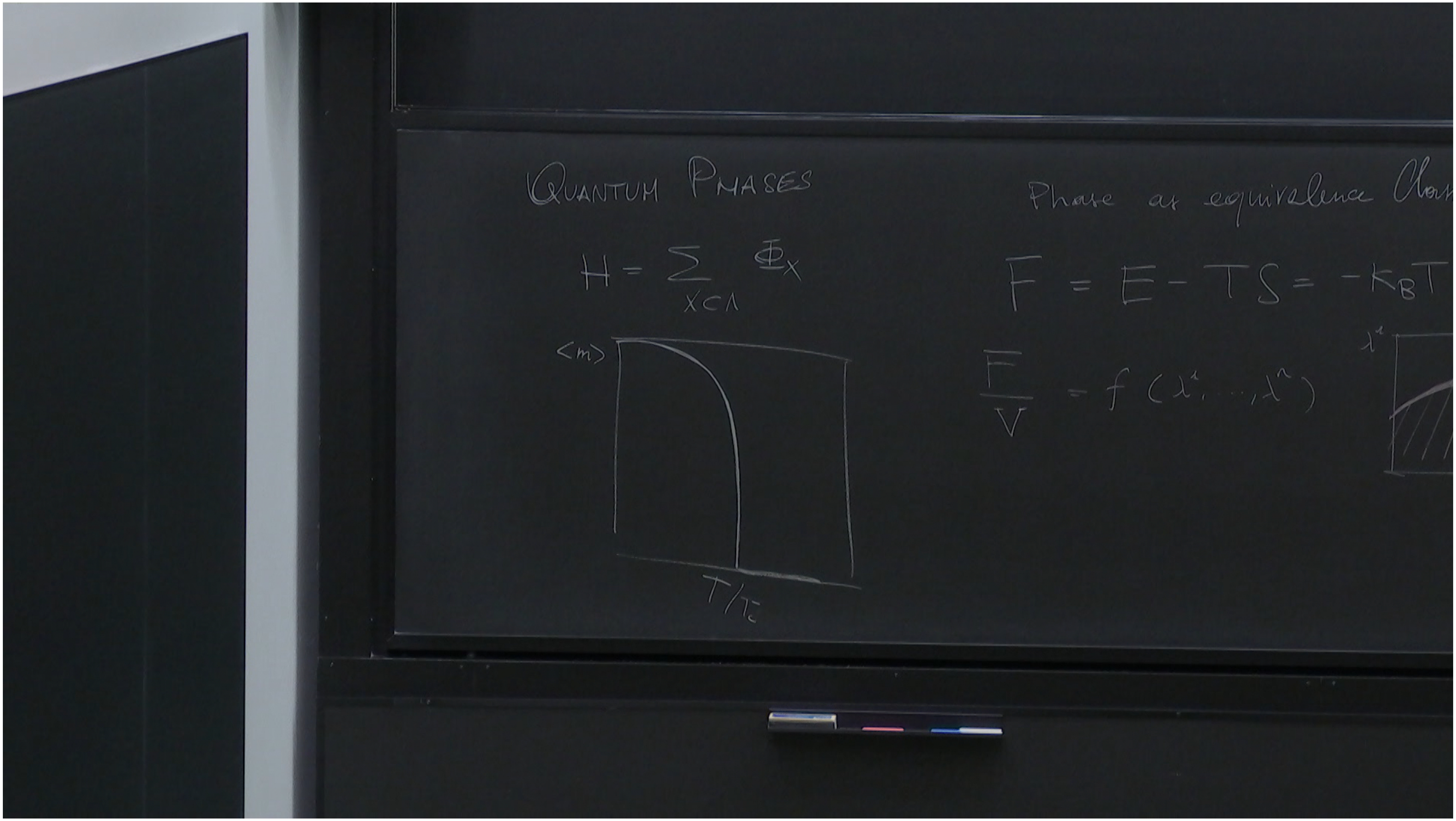
$$\boxed{\begin{gathered}\sum_i \sigma_i = M \\ m = \frac{M}{N}\end{gathered}}$$

$$-\frac{\partial f}{\partial h} = \langle m \rangle$$

lattice $\sigma_i = \pm 1$ $\{\sigma_i\}$

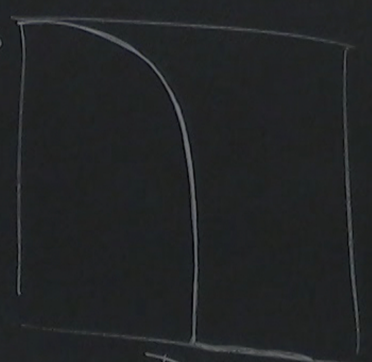
$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\frac{\partial f}{\partial h} = \frac{1}{N}\cancel{(-k_B T)}\frac{1}{Z}\sum_{\{\sigma\}}\cancel{(-\beta)}\left(-\sum_i \sigma_i\right) e^{-\beta H} = -\frac{1}{N}\left\langle \sum_i \sigma_i \right\rangle$$


QUANTUM PHASES
H = Σ Φ_X
X⊂Λ
<m>
T/T_c
Phase as equivalence Clas
F = E − TS = −k_B T
F/V = f(λ^1, …, λ^n)
λ^1

Quantum Phases

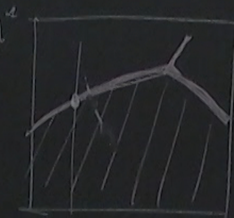
$$H = \sum_{X \subset \Lambda} \Phi_X$$



Phase as equivalence class.

$$F = E - TS = -k_B T \log Z$$

$$\frac{F}{V} = f(\lambda^1, \dots, \lambda^n)$$

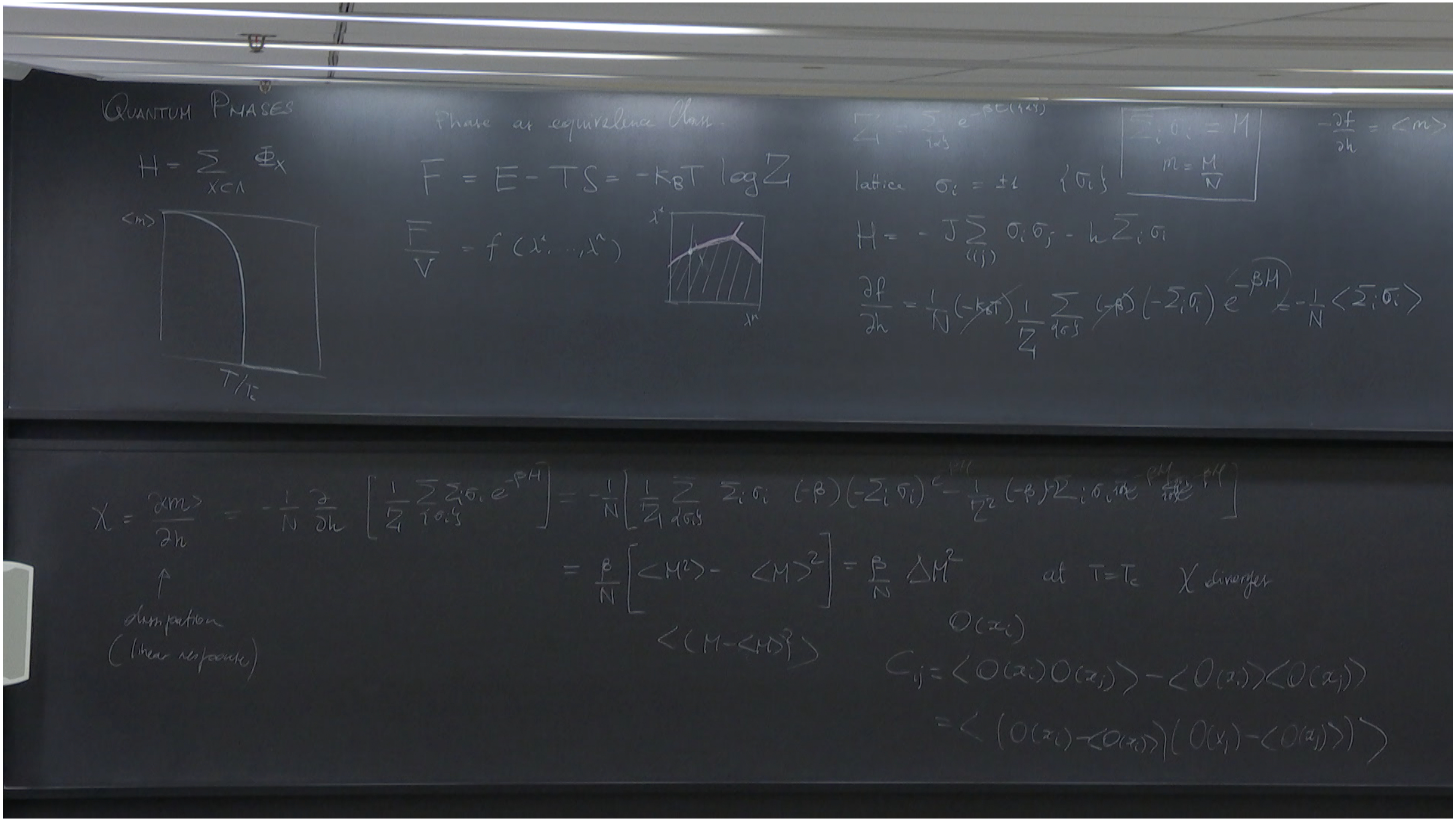


$$Z = \sum_{\{\alpha\}} e^{-\beta E(\{\alpha\})}$$

lattice $\sigma_i = \pm 1$

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

$$\frac{\partial f}{\partial h} = \frac{1}{N} \cancel{(-k_B T)} \frac{1}{Z}$$


QUANTUM PHASES
Phase as equivalence Class.
$H = \sum_{X \subset \Lambda} \Phi_X$
$F = E - TS = -k_B T \log Z$
$\frac{F}{V} = f(\lambda^1, \dots, \lambda^n)$
$\langle m \rangle$
T/T_c
$Z = \sum_{\{\sigma_i\}} e^{-\beta E(\{\sigma\})}$
$\sum_i \sigma_i = M$
$m = \frac{M}{N}$
$-\frac{\partial f}{\partial h} = \langle m \rangle$
lattice $\sigma_i = \pm 1$ $\{\sigma_i\}$
$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i$
$\frac{\partial f}{\partial h} = \frac{1}{N}(-k_B T) \frac{1}{Z} \sum_{\{\sigma\}} (-\beta)(-\sum_i \sigma_i) e^{-\beta H} = -\frac{1}{N} \langle \sum_i \sigma_i \rangle$
$\chi = \frac{\partial m}{\partial h} = -\frac{1}{N} \frac{\partial}{\partial h} \left[\frac{1}{Z} \sum_{\{\sigma\}} \sum_i \sigma_i e^{-\beta H} \right]$
dissipation
(linear response)
$= \frac{\beta}{N} \left[\langle M^2 \rangle - \langle M \rangle^2 \right] = \frac{\beta}{N} \Delta M^2$
at $T = T_c$ χ diverges
$\langle (M - \langle M \rangle)^2 \rangle$
$O(x_i)$
$C_{ij} = \langle O(x_i) O(x_j) \rangle - \langle O(x_i) \rangle \langle O(x_j) \rangle$
$= \langle (O(x_i) - \langle O(x_i) \rangle)(O(x_j) - \langle O(x_j) \rangle) \rangle$

$$C_{ij} = \langle O(x_i)\, O(x_j) \rangle$$

$$= \langle (O(x_i) - \langle O(x_i$$

$$\chi = \langle (M - \langle M \rangle)^2 \rangle = \frac{\beta}{N}\Big[\sum_{ij} \langle \sigma_i \sigma_j \rangle - \sum_{ij} \langle \sigma_i \rangle \langle \sigma_j \rangle \Big] = \frac{\beta}{N} \sum_{ij} C_{ij} = \beta \sum_i C_{i0}$$

dissipation   fluctuations

$$T > T_c \quad C_{ij} \sim e^{-|x_i - x_j|/\xi} \quad < a^2 \xi^d$$

$$\Big] = \frac{\beta}{N}\sum_{ij} C_{ij} = \beta \sum_i C_{i0}$$

$$|x_i - x_j| / \xi \quad < a^2 \xi^d$$

$$T \sim T_c$$

$$\xi \sim (T - T_c)^{-\nu}$$

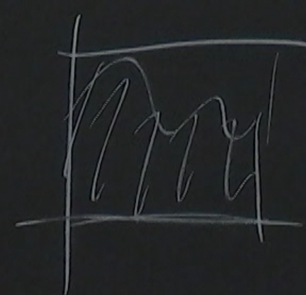
$$\chi \sim (T - T_c)^{-\gamma}$$

$$\ldots\langle\sigma_i\rangle\langle\sigma_j\rangle\Big] = \frac{\beta}{N}\sum_{ij} C_{ij} = \beta \sum_i C_{i0}$$

$$\sim e^{-|x_i - x_j|/\xi} < a^2 \xi^d$$

$$T \sim T_c$$

$$\xi \sim (T - T_c)^{-\nu}$$



$$\chi \sim (T - T_c)^{-\gamma}$$

$$\hat{O} = \sum_i O_i |i\rangle\langle i|$$

$$\psi = \sum_\alpha q_\alpha |\alpha\rangle\langle\alpha|$$

$$\psi \geq 0$$

$$\mathrm{tr}\,\psi = 1$$

$\Pr($

$\rangle\langle i|$

$\alpha\rangle\langle\alpha|$

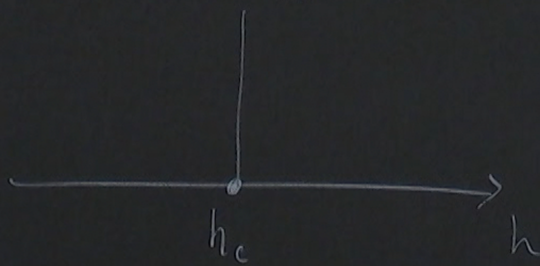
$$P_r(O_i) = \sum_\alpha q_\alpha \, |\langle\alpha|i\rangle|^2 = \mathrm{Tr}\left[\, |i\rangle\langle i| \, \rho \,\right]$$

$$\langle \hat{O} \rangle = \mathrm{Tr}(O\rho)$$

$$|\alpha\rangle\langle\alpha| \rightarrow P_i = |\langle\alpha|i\rangle|^2$$

$$\frac{e^{-\beta H}}{Z} \xrightarrow{\beta\to\infty} \text{pure state}$$

$$H(\lambda) = \sum_x \Phi_x(\lambda)$$



$$F = E - TS$$

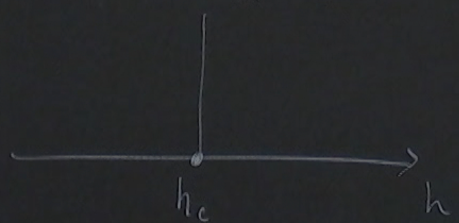
$$f = \mathcal{E} - TS/N \xrightarrow{T=0} \mathcal{E}$$

$$H(\lambda) = \sum_x \Phi_x(\lambda)$$ smooth in λ

$$F = E - TS$$

$$f = \mathcal{E} - TS/N \xrightarrow{T=0} \mathcal{E}$$

QCP are λ s.t. $\mathcal{E}(\lambda)$ is non-analytic



$$H(\lambda) \to E_0(\lambda)$$