Title: PSI 2019/2020 - Quantum Matter Part 1 - Lecture 1

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Collection: PSI 2019/2020 - Quantum Matter Part 1

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QUANTUM MATTER

- Emergence
  - Symmetry Breaking
  - Order
  - Emergent Symmetries
  - Emergent Laws : 2nd Law of Th.
- Quantum Correlations

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- Emergence
  - Symmetry Breaking
  - Order
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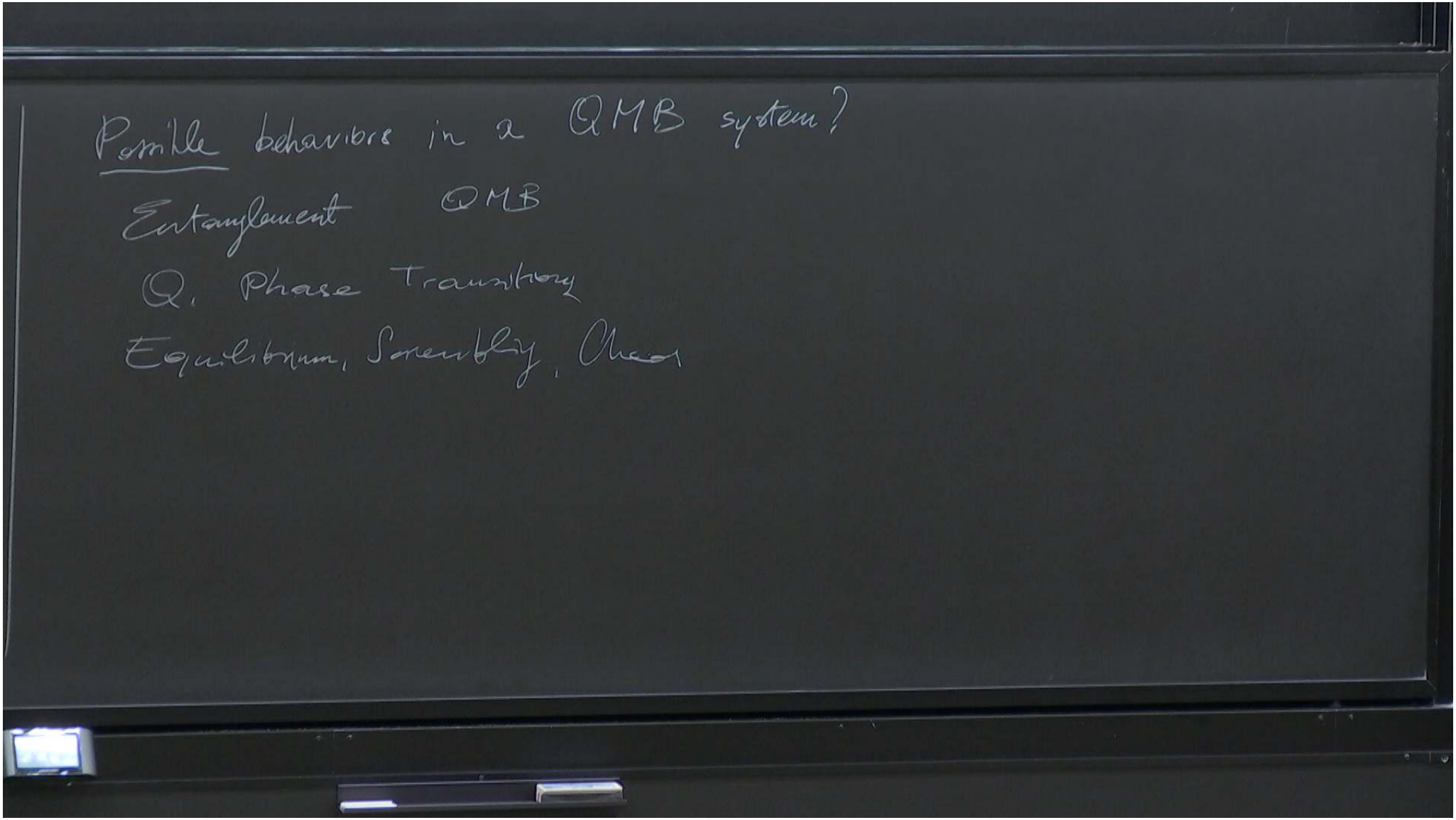
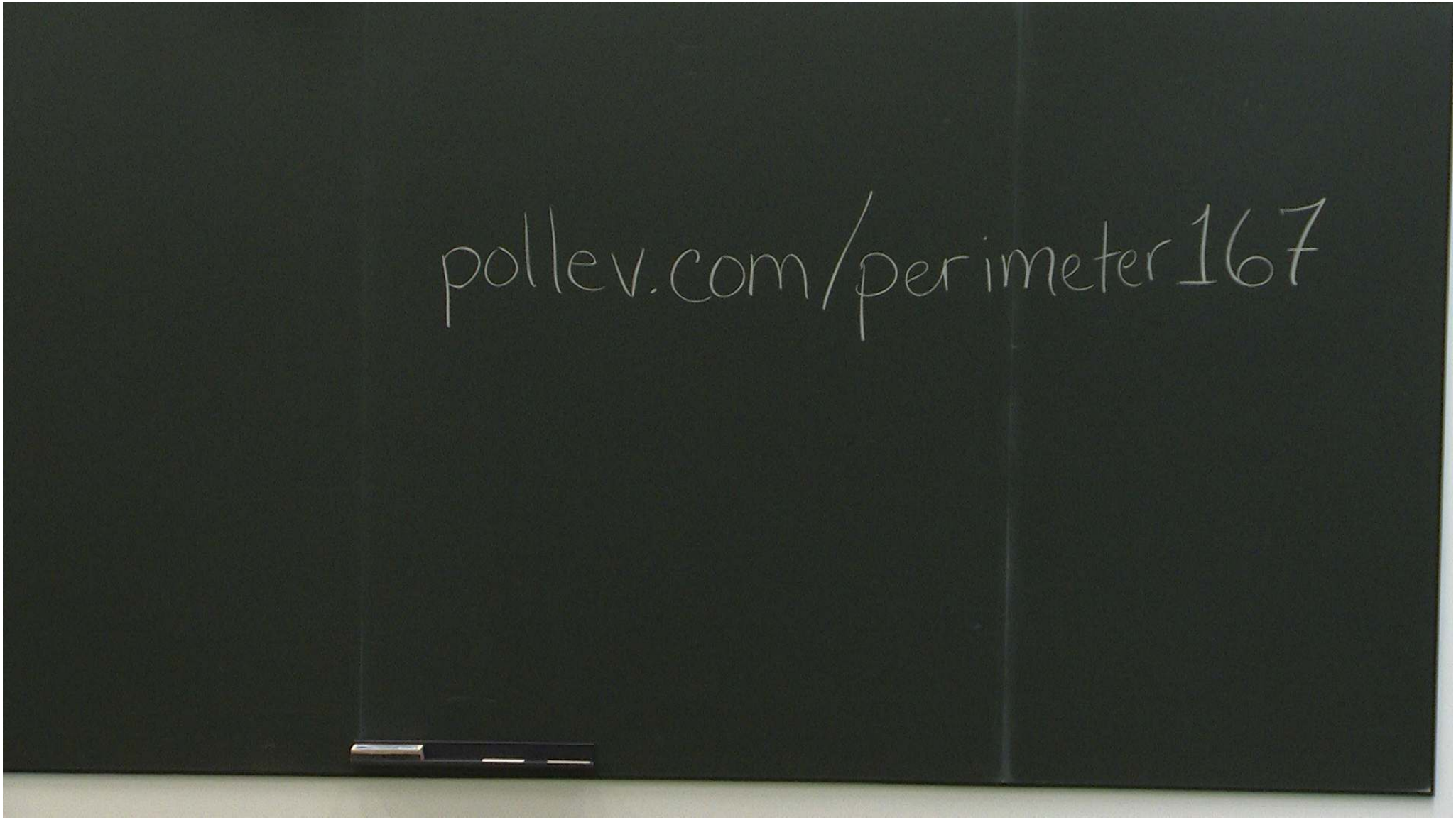
$N \to \infty$

A

# of bodies

- Quantum Correlations
  - Coherence / Decoherence
  - Entanglement
  - Quantum Chaos / Scrambling
  - Quantum Spin Liquids


Possible behaviors in a QMB system?
Entanglement QMB
Q. Phase Transitions
Equilibrium, Ensembles, Chaos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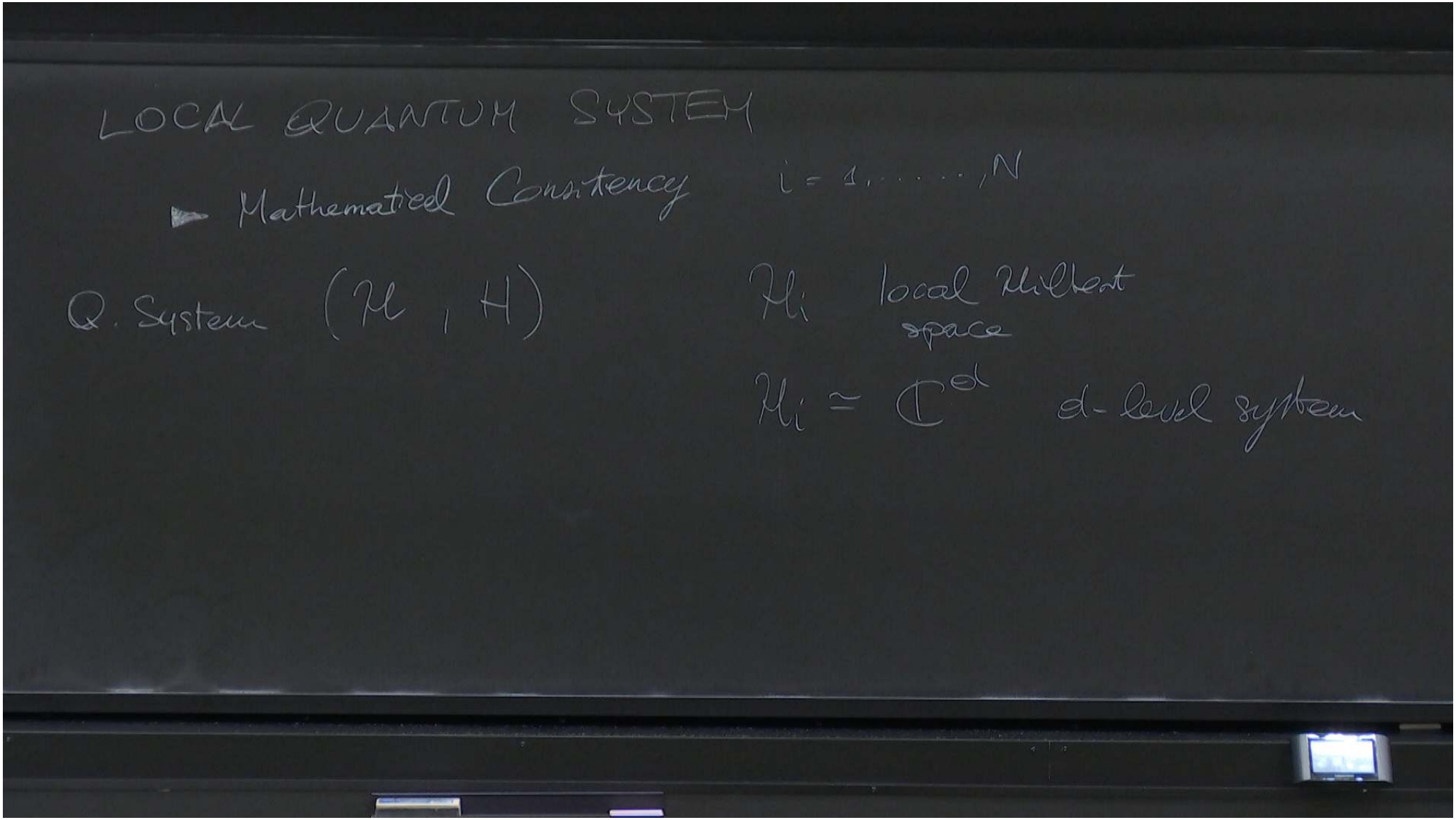

LOCAL QUANTUM SYSTEM
Mathematical Consitency
$i = 1, \ldots, N$
Q. System $(\mathcal{H}, H)$
$\mathcal{H}_i$ local Hilbert space
$\mathcal{H}_i \simeq \mathbb{C}^d$ d-level system

$\mathbb{C}^d$ d prime number

$\ldots, N$

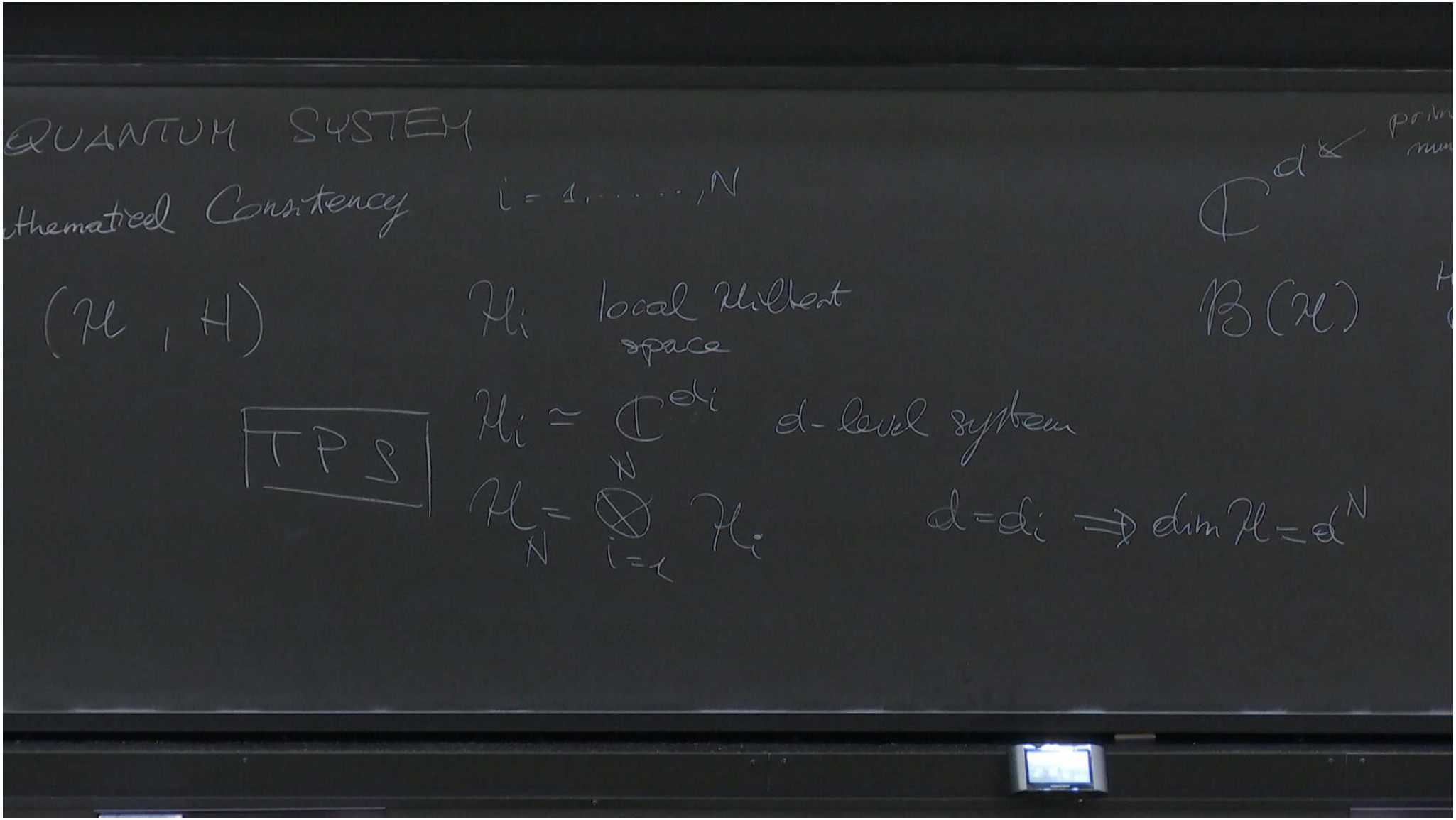
el Hilbert
pace

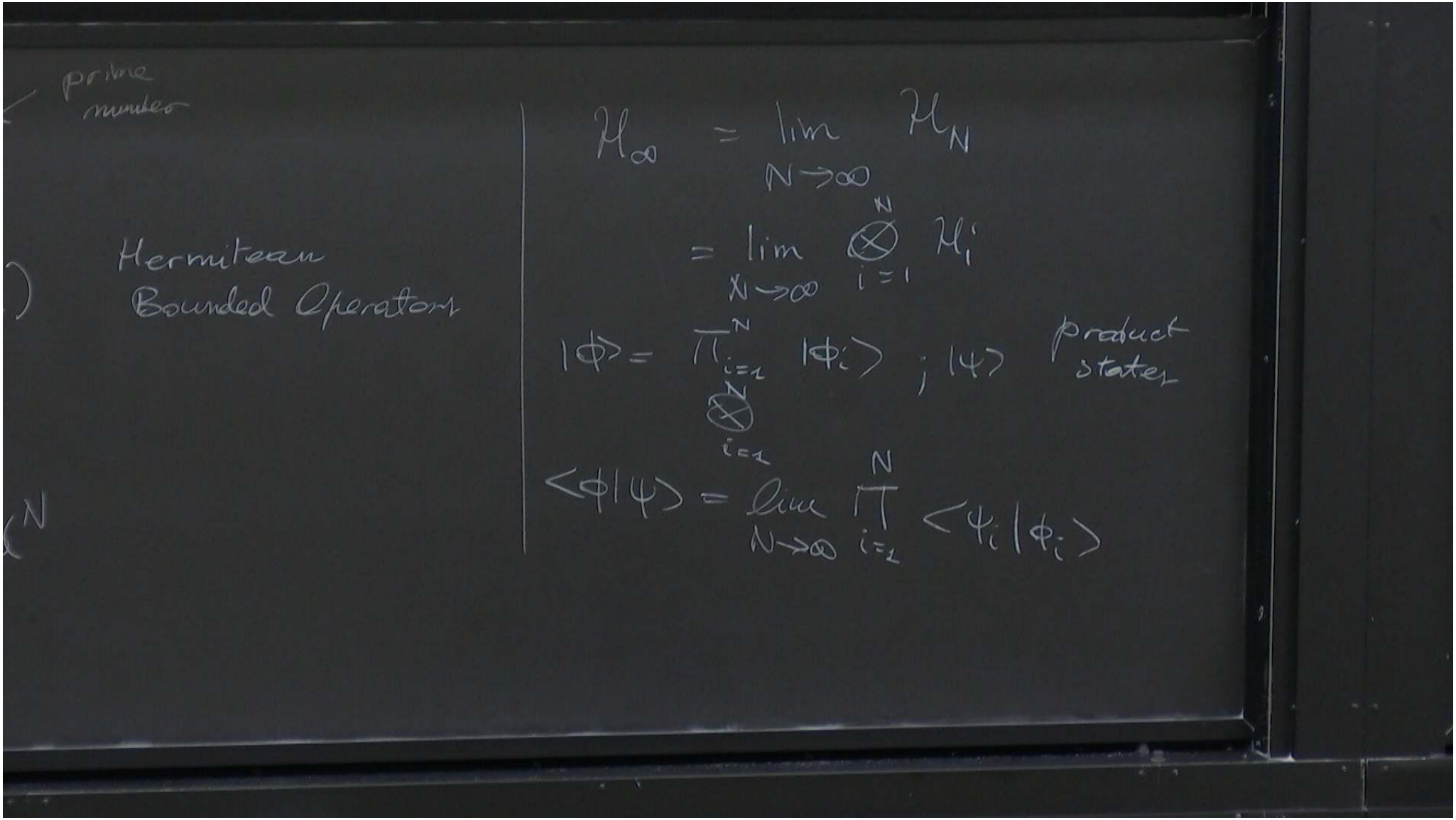
d_i

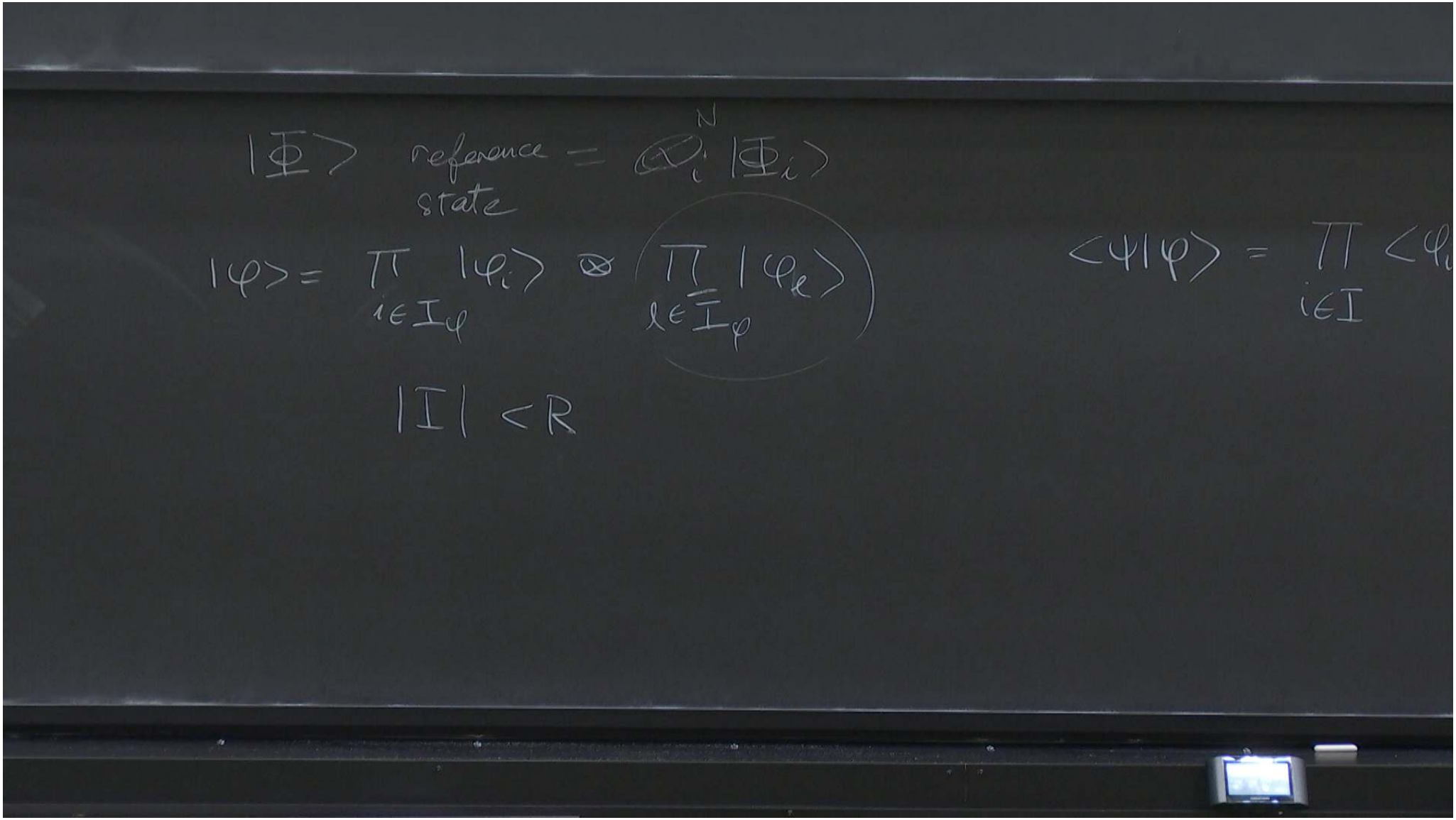
d-level system

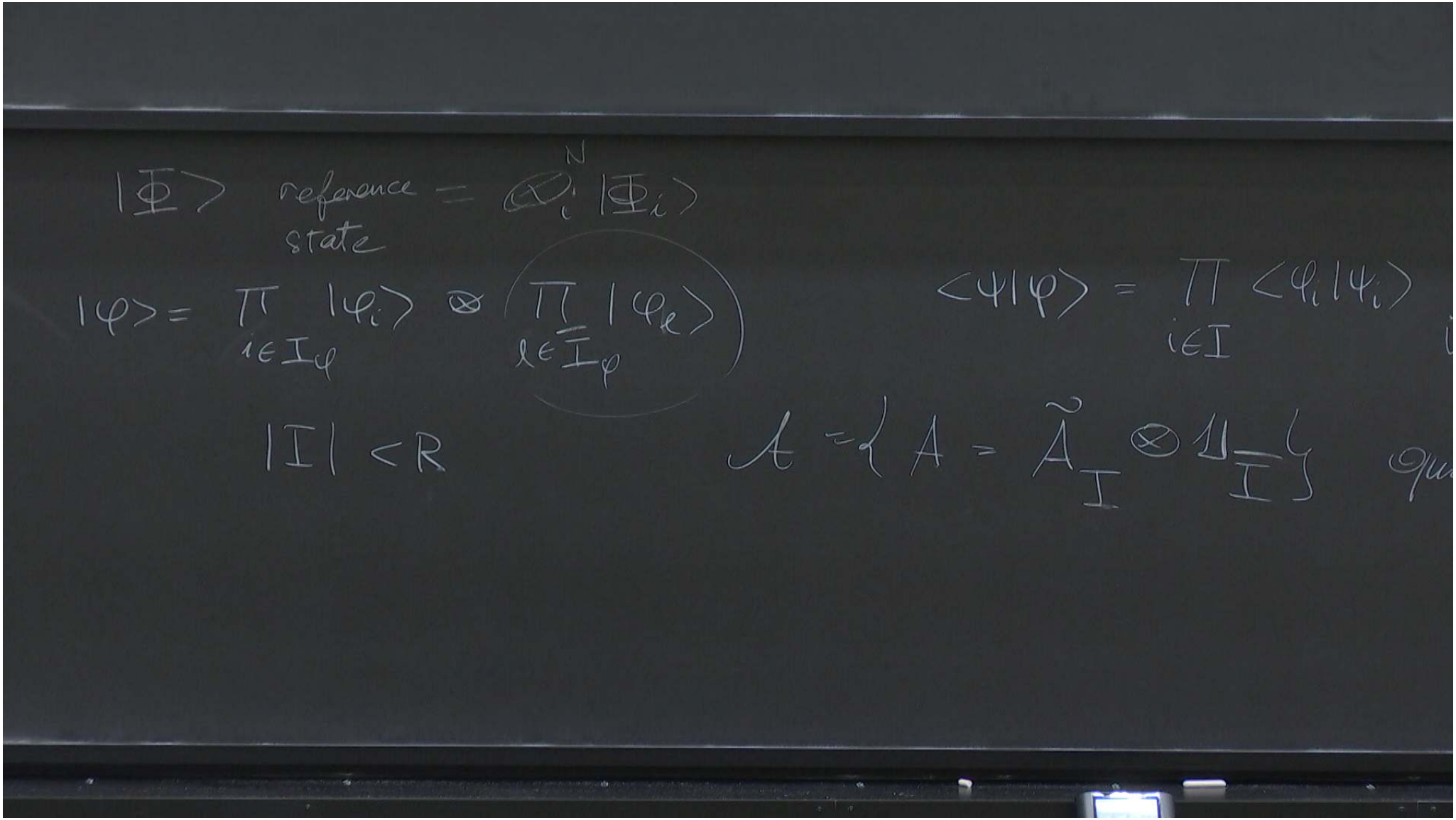
$\mathcal{H}_i$

$d = d_i \Rightarrow \dim \mathcal{H} = d^N$


QUANTUM SYSTEM
thematical Consistency
$i = 1, \ldots, N$
$\mathbb{C}^d$
$(\mathcal{H}, H)$
$\mathcal{H}_i$ local Hilbert space
$B(\mathcal{H})$
TPS
$\mathcal{H}_i \simeq \mathbb{C}^{d_i}$ d-level system
$\mathcal{H}_N = \bigotimes_{i=1}^{N} \mathcal{H}_i$
$d = d_i \Rightarrow \dim \mathcal{H} = d^N$


prime
number
Hermitean
Bounded Operators
$\mathcal{H}_\infty = \lim_{N\to\infty} \mathcal{H}_N$
$= \lim_{N\to\infty} \bigotimes_{i=1}^{N} \mathcal{H}_i$
$|\Phi\rangle = \prod_{i=1}^{N} |\phi_i\rangle$; $|\psi\rangle$ product states
$\bigotimes_{i=1}^{N}$
$\langle\Phi|\Psi\rangle = \lim_{N\to\infty} \prod_{i=1}^{N} \langle\psi_i|\phi_i\rangle$


$|\Phi\rangle$ reference state $= \bigotimes_i^N |\Phi_i\rangle$
$|\varphi\rangle = \prod_{i \in I_\varphi} |\varphi_i\rangle \otimes \prod_{\ell \in \bar{I}_\varphi} |\varphi_\ell\rangle$
$\langle\psi|\varphi\rangle = \prod_{i \in I}$
$|I| < R$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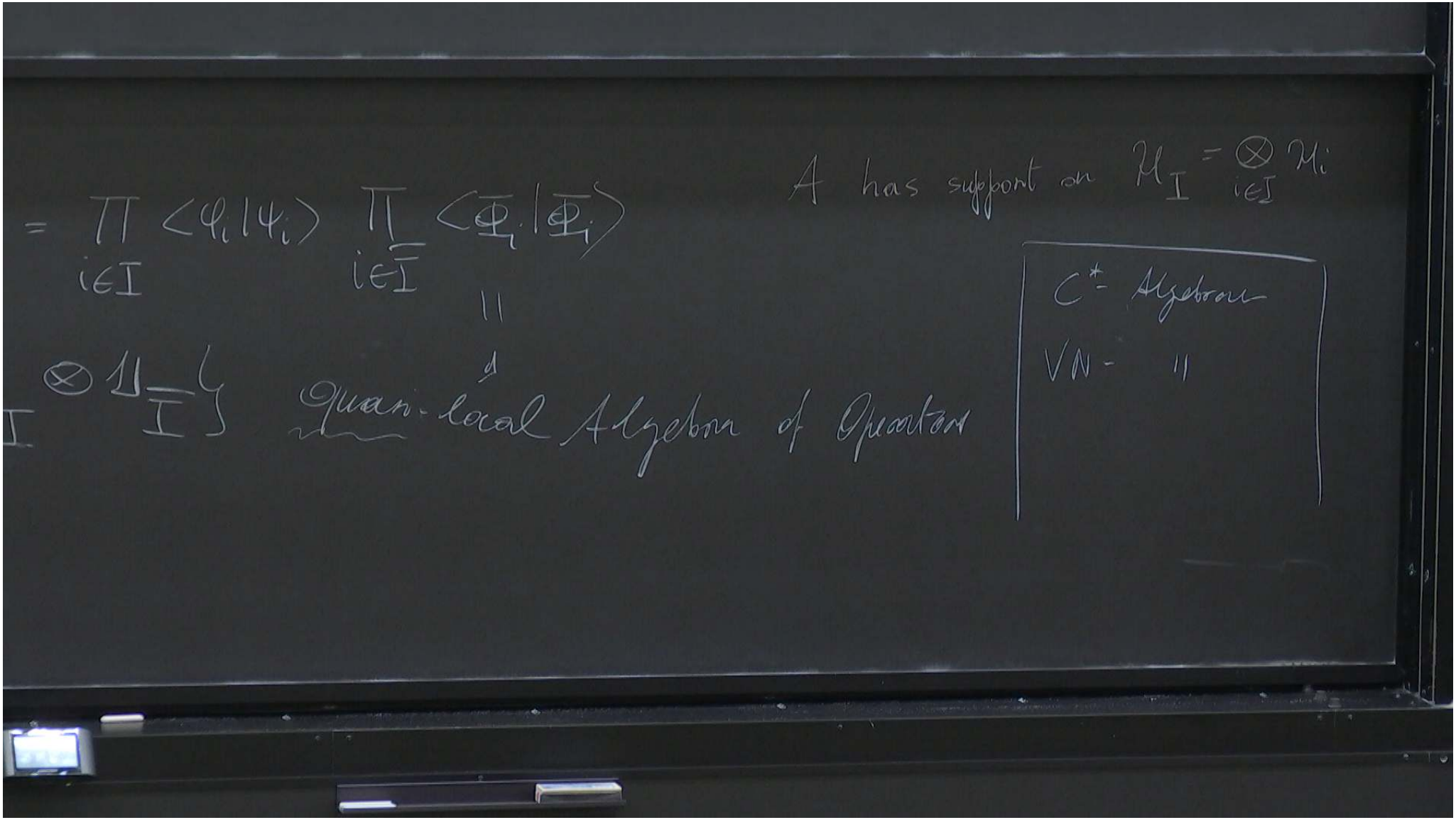
$|\varphi\rangle = \prod_{i \in I_\varphi} |\varphi_i\rangle \otimes \prod_{i \in \bar{I}_\varphi} |\varphi_i\rangle$
$\langle\psi|\varphi\rangle = \prod_{i \in I} \langle\varphi_i|\psi_i\rangle$
$|I| < R$
$\mathcal{A} = \{ A = \tilde{A}_I \otimes \mathbb{1}_{\bar{I}} \}$


$= \prod_{i \in I} \langle \varphi_i | \psi_i \rangle \prod_{i \in \bar{I}} \langle \Phi_i | \Phi_i \rangle$
$\mathcal{A}$ has support on $\mathcal{H}_I = \bigotimes_{i \in I} \mathcal{H}_i$
$\otimes \mathbb{1}_{\bar{I}}$
Quasi-local Algebra of Operators
C^*- Algebras
VN- "

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i,j} k_0 \frac{e_i e_j}{|x_i - x_j|}$$

Euclidean distance $d(i,j)$

V_N volume for N-particles $\sim O(N)$

$E_N \sim O(N^2)$

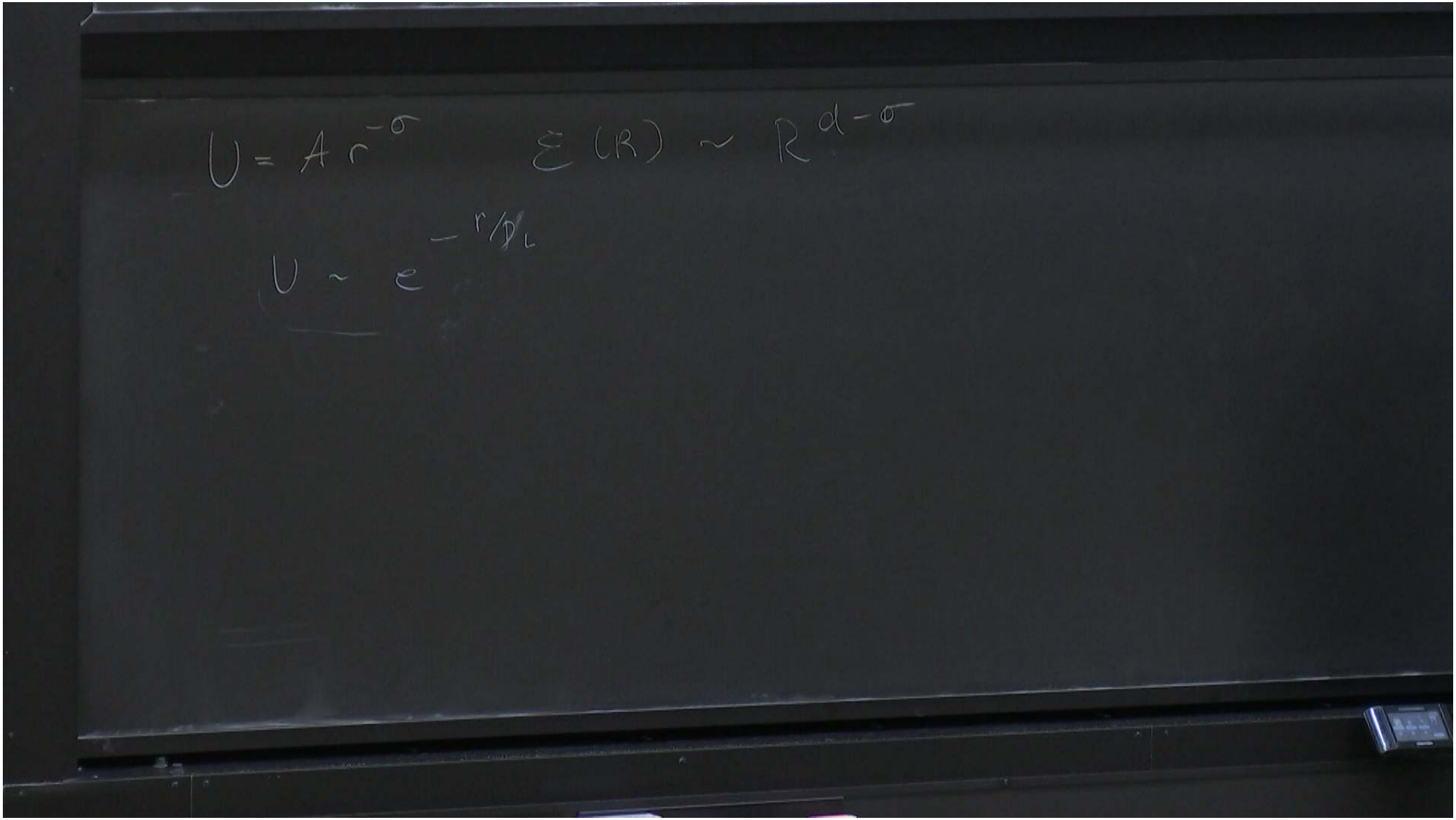
$$\mathcal{E}_N = \frac{E_N}{V_N} \xrightarrow{N \to \infty} \infty$$

Euclidean distance
$d(i,j)$

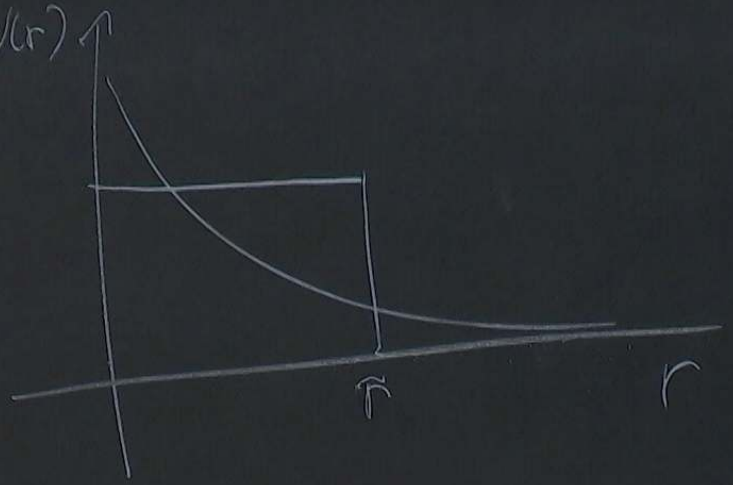
$\rho(\vec{r}) = \rho \qquad U(\vec{r}) = \frac{A}{r}$

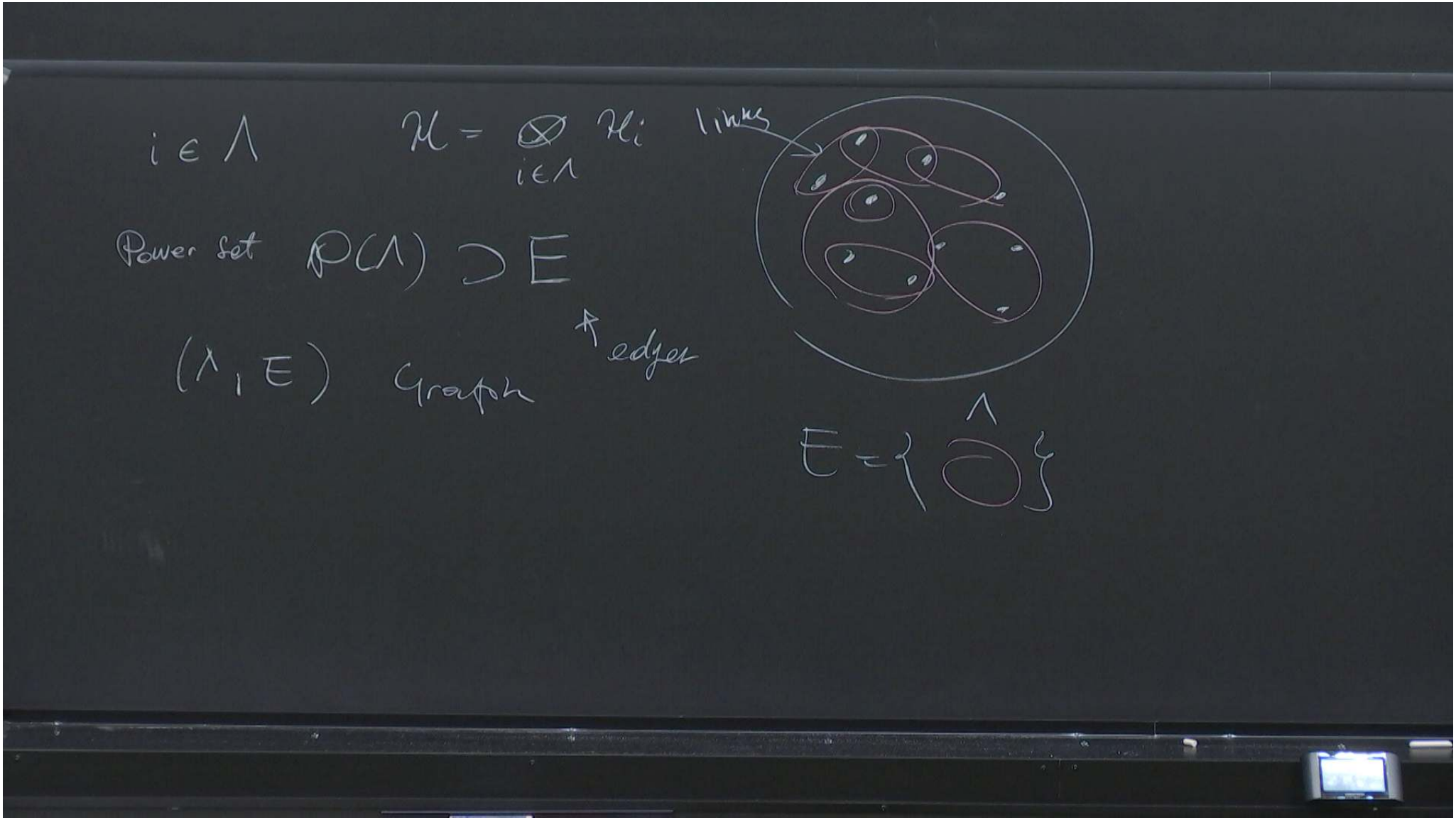
$$E_R = \int_0^R \frac{4}{3}\pi r^3 \rho \, \frac{A}{r} \, 4\pi r^2 \rho \, dr = \frac{A}{15}(4\pi)^2 \rho^2 R^5$$

$$V_R = \frac{4}{3}\pi R^3$$


$U = A r^{-\sigma}$
$\mathcal{E}(R) \sim R^{d-\sigma}$
$U \sim e^{-r/\xi}$

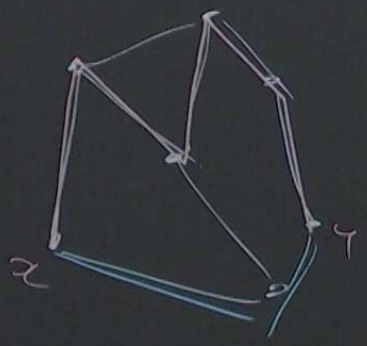
$R) \sim R^{d-\sigma}$

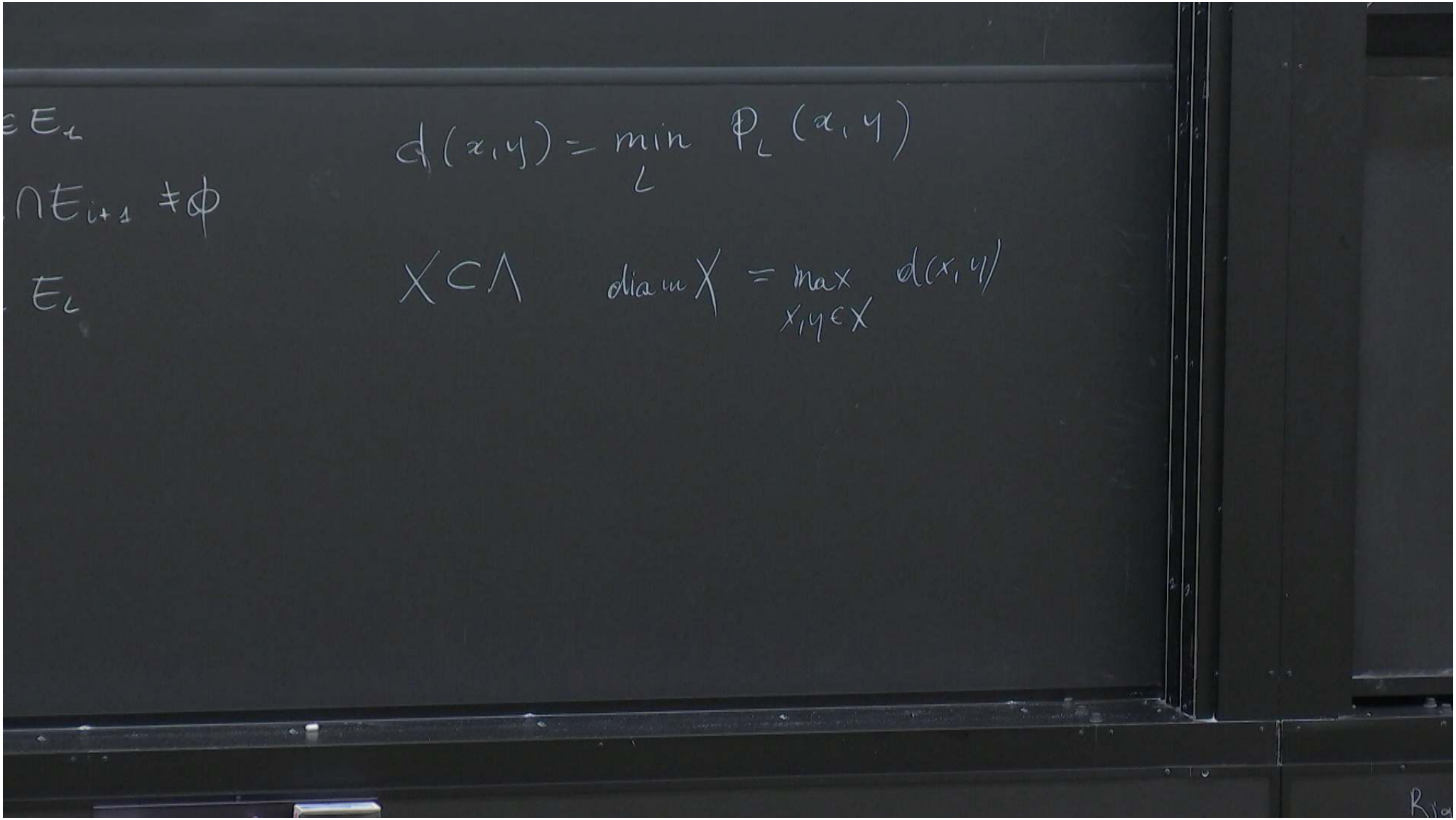


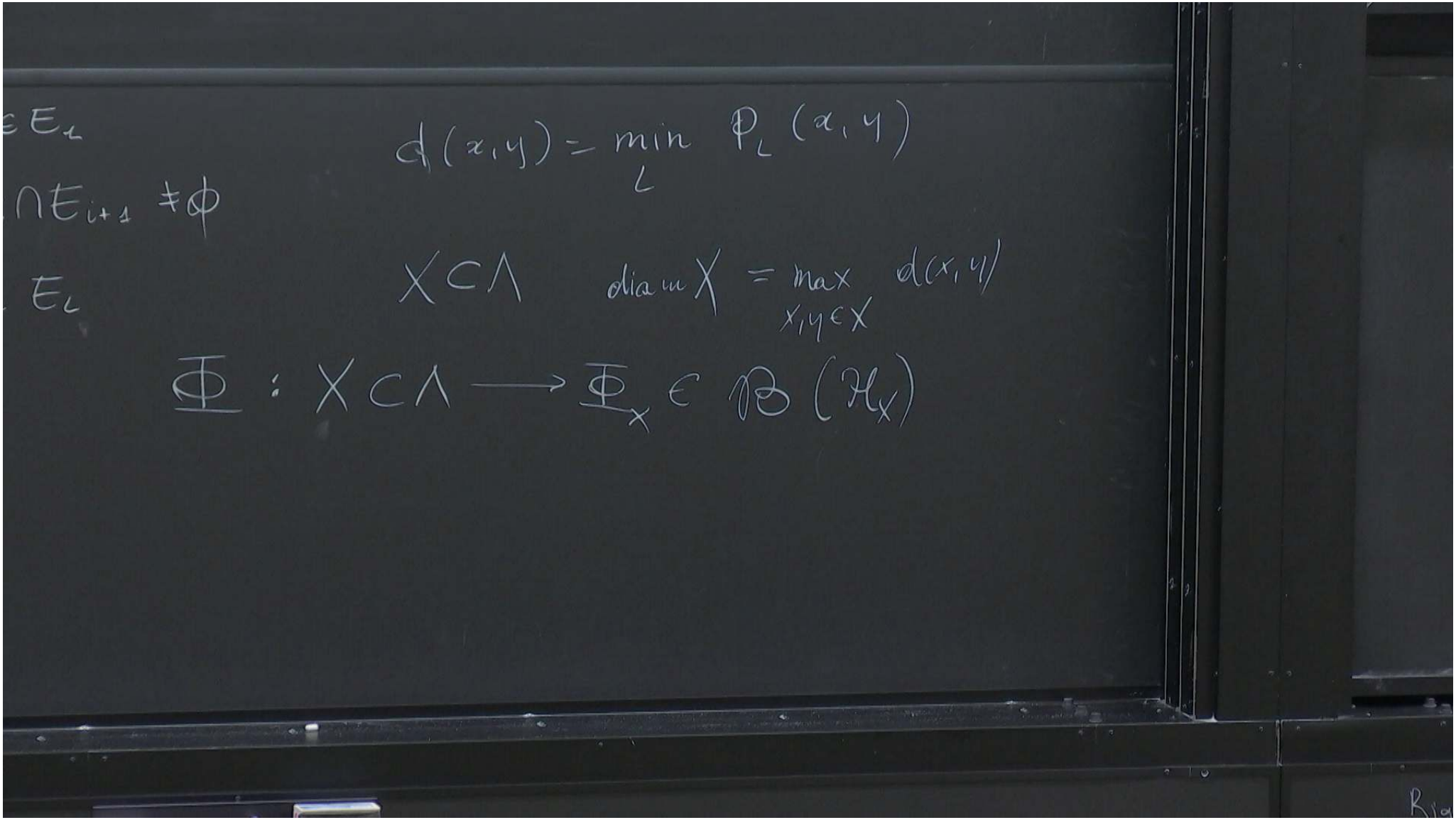

$i \in \Lambda$
$\mathcal{H} = \bigotimes_{i \in \Lambda} \mathcal{H}_i$
links
Power set $\mathcal{P}(\Lambda) \supset E$
edges
(Λ, E) Graph
$E = \{\}$

$$\text{Path}(x,y) = \begin{cases} x \in E_1 \\ E_i \cap E_{i+1} \neq \phi \\ y \in E_L \end{cases}$$

$$d(x,y) = \min_{L} P_L(x,y)$$




$\in E_L$
$\cap E_{i+1} \neq \phi$
E_L
$d(x,y) = \min_L P_L(x,y)$
$X \subset \Lambda$
$\operatorname{diam} X = \max_{x,y \in X} d(x,y)$


$\in E_L$
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E_L
$d(x,y) = \min_L P_L(x,y)$
$X \subset \Lambda$
$\text{diam } X = \max_{x,y \in X} d(x,y)$
$\Phi : X \subset \Lambda \longrightarrow \Phi_X \in \mathcal{B}(\mathcal{H}_X)$

$$P_L(x,y) = \begin{cases} x \in E_L \\ E_i \cap E_{i+1} \neq \emptyset \\ y \in E_L \end{cases}$$

$$d(x,y) = \min_L P_L(x,y)$$

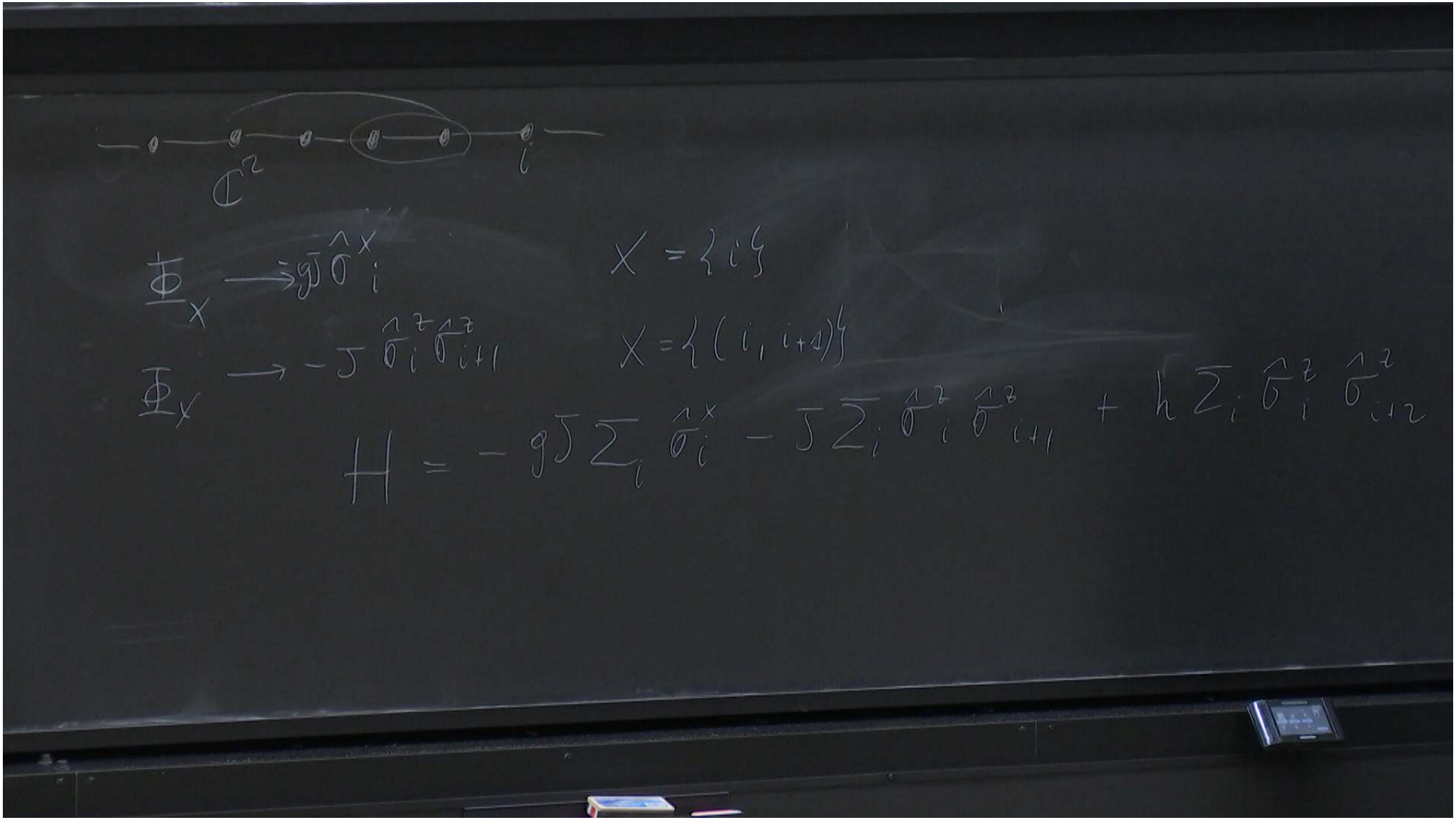
$X \subset \Lambda$ $\quad \operatorname{diam} X = \max_{x,y \in X} d(x,y)$

$$\Phi : X \subset \Lambda \longrightarrow \Phi_X \in \mathcal{B}(\mathcal{H}_X)$$

$\neq 0$ only $X : \operatorname{diam} X < \tilde{r}$

$|X| < R$

$$H = \sum_X \Phi_X$$


$\mathbb{C}^2$
i
$\Phi_x \longrightarrow g J \hat{\sigma}^x_i \qquad X = \{i\}$
$\Phi_x \longrightarrow -J \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \qquad X = \{(i, i+1)\}$
$H = -gJ \sum_i \hat{\sigma}^x_i - J \sum_i \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} + h \sum_i \hat{\sigma}^z_i \hat{\sigma}^z_{i+2}$