Title: Observables and non-locality in perturbative algebraic QFT

Speakers: Katarzyna Rejzner

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Abstract: I will start with an introduction into the framework of perturbative algebraic quantum field theory (pAQFT), which is a mathematically rigorous approach to perturbative QFT. In its original formulation, it is based on the Haag-Kastler axiomatic framework, where locality is a fundamental principle. In my talk I will discuss how it can be extended to treat also non-local observables, with potential applications to effective quantum gravity

Observables and non-locality in perturbative algebraic QFT

Kasia Rejzner

University of York

PI, 08.01.2020

Outline of the talk

1. Algebraic QFT and its generalizations
2. pAQFT
   - Outline of the framework
   - Non-local observables

1 Algebraic QFT and its generalizations

Outline of the framework
Non-local observables

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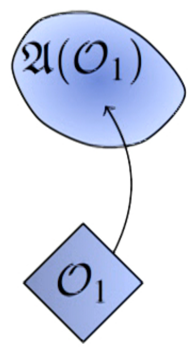

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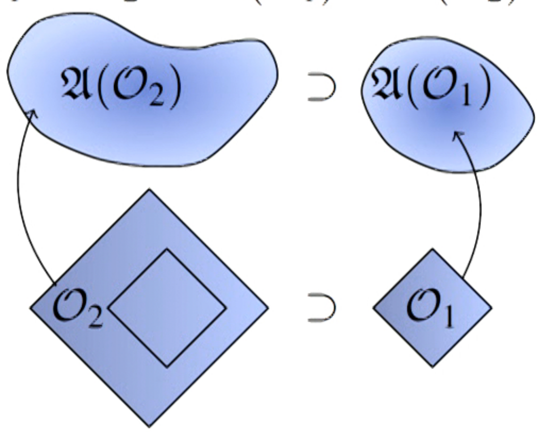
- The physical notion of subsystems is realized by the condition of isotony, i.e.: $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$. We obtain a net of C^*-algebras.



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- Key idea: algebras of observbles constructed independently of the choice of state ("vacuum"), so allows for degenerate vacuua. This idea can be applied more generally, as we will see later.

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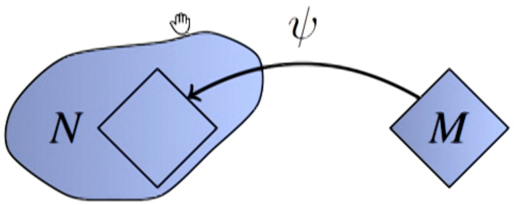

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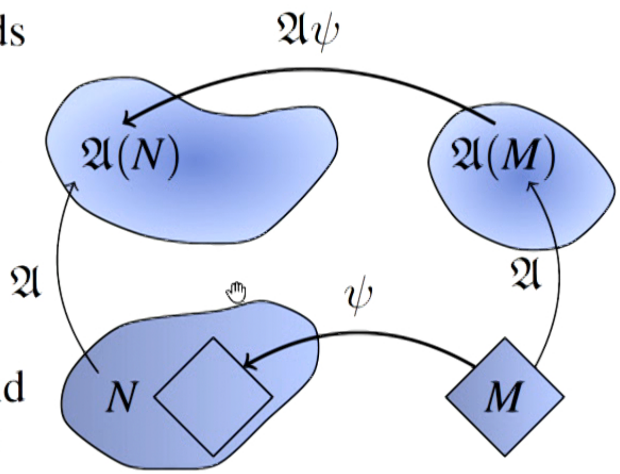
- A model in LCQFT is defined by assigning observable algebras $\mathfrak{A}(M)$ to spacetimes and algebra morphisms $\mathfrak{A}\psi$ to embeddings.



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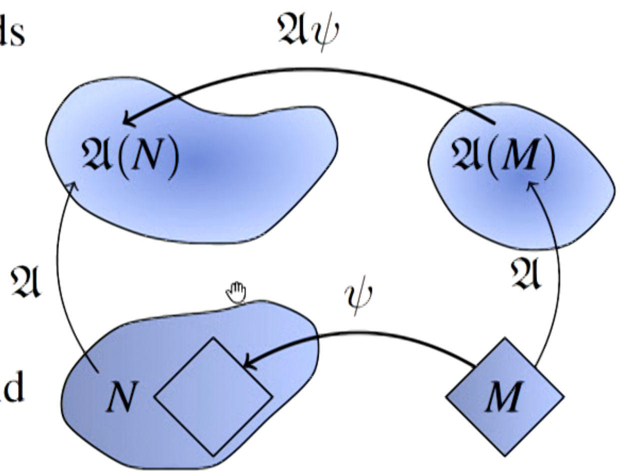
- Covariance requirement: $\mathfrak{A}$ is a functor.



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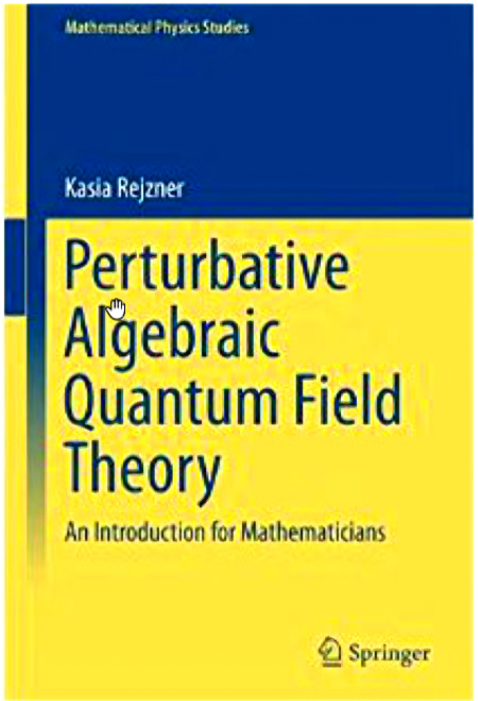
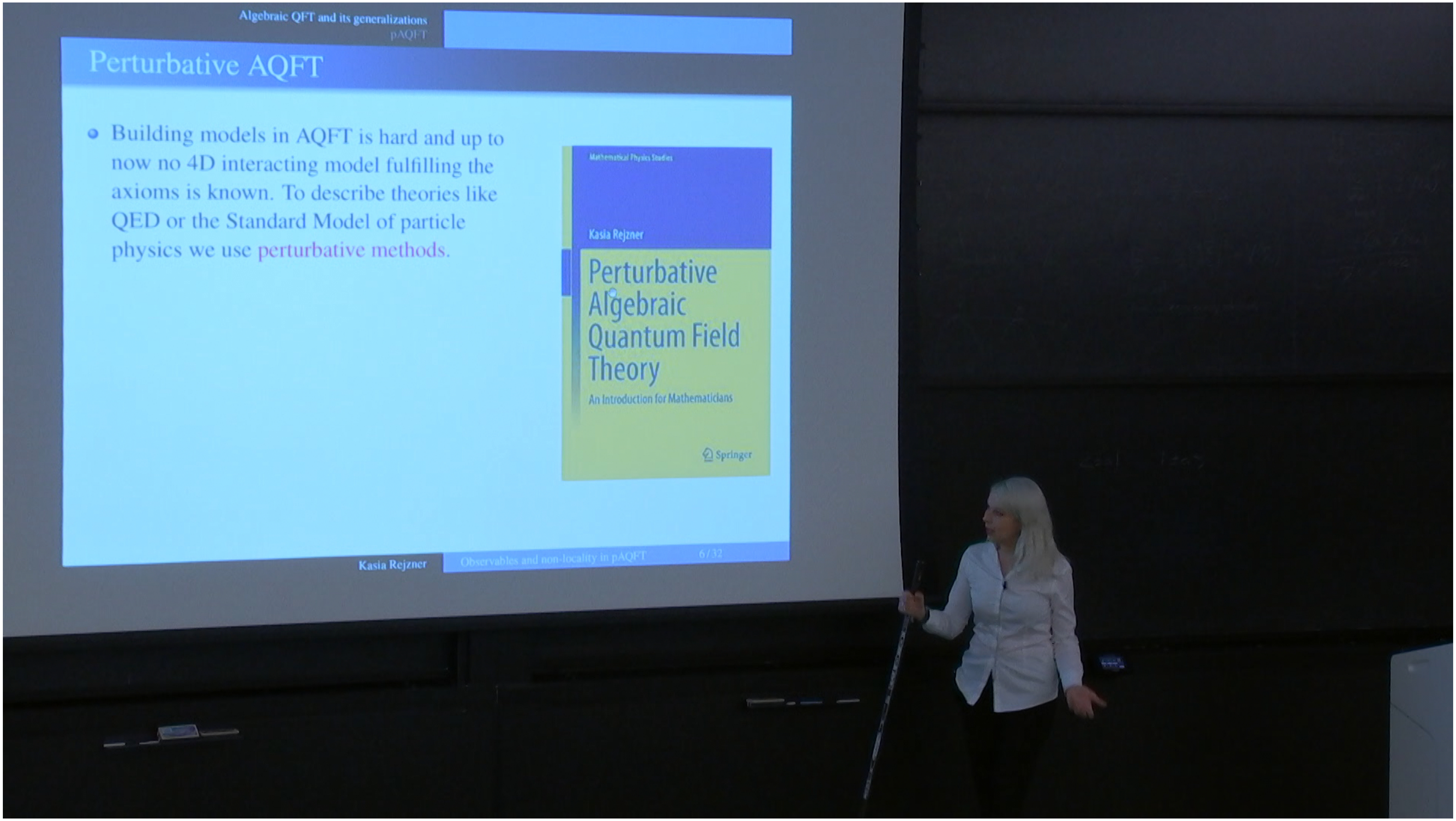
- Building models in AQFT is hard and up to now no 4D interacting model fulfilling the axioms is known. To describe theories like QED or the Standard Model of particle physics we use perturbative methods.

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Mathematical Physics Studies
Kasia Rejzner
Perturbative Algebraic Quantum Field Theory
An Introduction for Mathematicians
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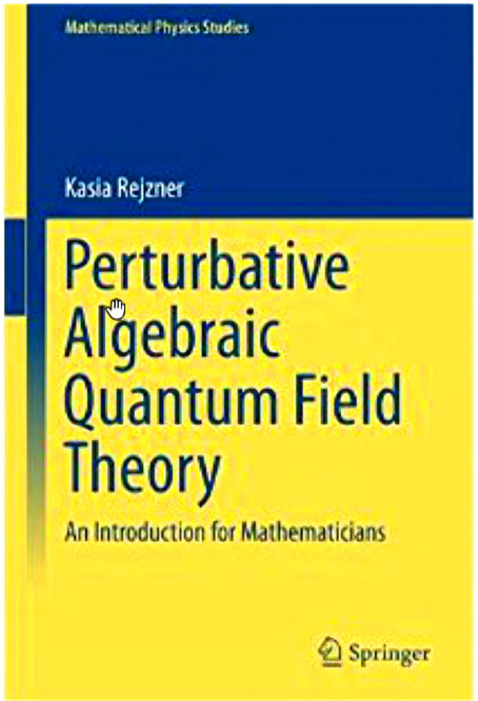

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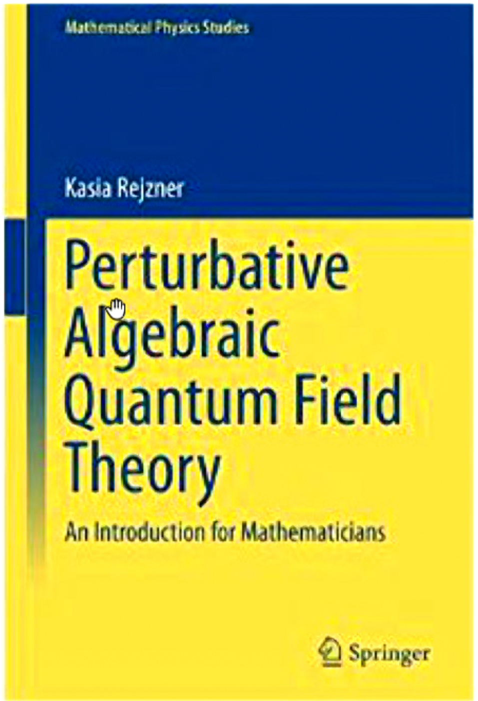

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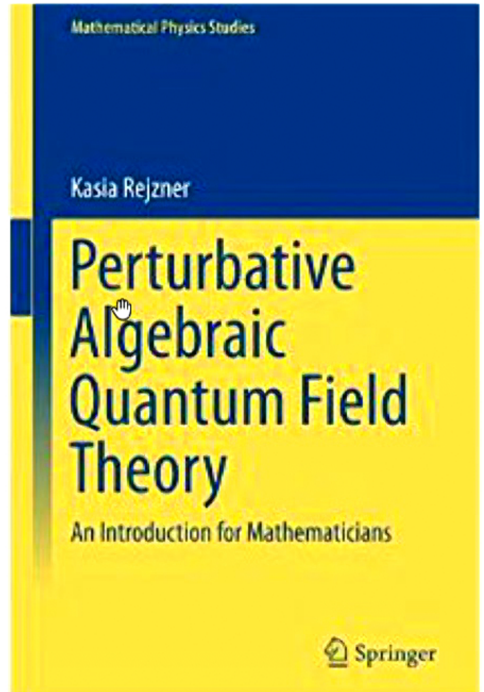
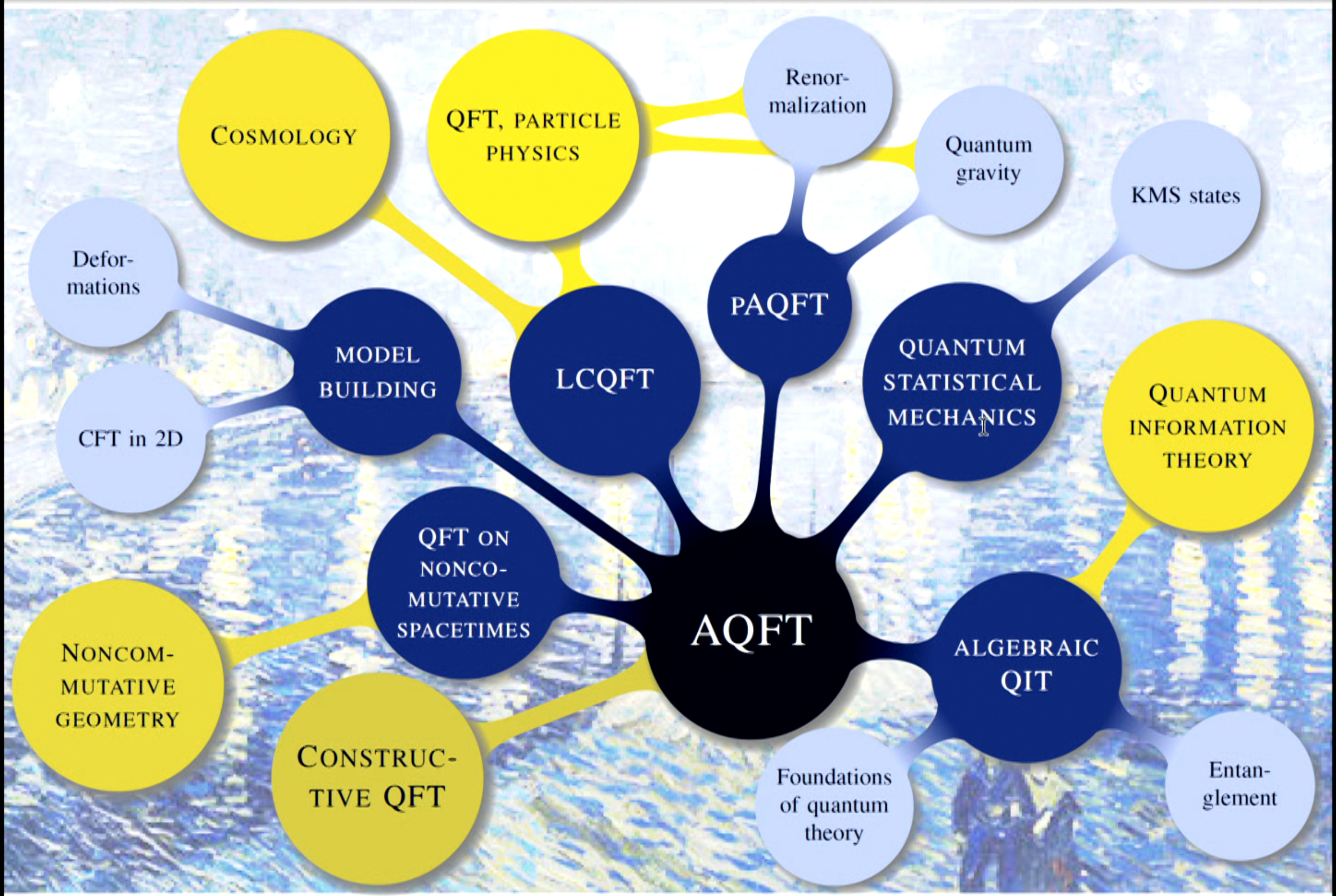
- Mathematical foundations of pAQFT have been reviewed in: *pAQFT. An Introduction for Mathematicians*, KR, **Springer 2016**.



Different aspects of AQFT and relations to physics



2 pAQFT

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- Dynamics: we use a modification of the Lagrangian formalism (fully covariant).

Classical observables

- Classical observables are modeled as smooth functionals on $\mathcal{E}(M)$, i.e. elements of $\mathcal{C}^\infty(\mathcal{E}(M),\mathbb{C})$. For simplicity of notation (and because of functoriality), we drop M, if no confusion arises, i.e. write $\mathcal{E}$, $\mathcal{C}^\infty(\mathcal{E},\mathbb{C})$, etc.

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- Localization of functionals governed by their spacetime support:

$$\begin{aligned}\operatorname{supp} F = \{x \in M | \forall \text{ neighbourhoods } U \text{ of } x\ \exists \varphi,\psi \in \mathcal{E}, \\ \operatorname{supp}\psi \subset U \text{ such that } F(\varphi+\psi) \neq F(\varphi)\}\ .\end{aligned}$$

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The main message of this talk:

pAQFT is a machinery to turn functionals of classical field configurations (classical observables) into quantum observables. This is done without referring to a Hilbert space representation and works for a large class of (potentially non-local) functionals.

Local functionals

- We define $\mathcal{F}_{\mathrm{loc}}$, local functionals on $\mathcal{E}$, as functionals that satisfy:

$$F(\varphi_1 + \varphi_2 + \varphi_3) = F(\varphi_1 + \varphi_2) + F(\varphi_2 + \varphi_3) - F(\varphi_2)\,,$$

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- We have shown (*Functionals and their derivatives in quantum field theory*, C. Brouder, N.V. Dang, C. Laurent-Gengoux, KR, **JMP 2017**) that this is equivalent to saying that F is of the form

$$F(\varphi) = \int f(j^k_x(\varphi))d\mu_g\,,$$

  for a smooth, compactly supported, function f on the jet bundle.

Regular functionals

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- Note that regular, polynomial functionals of degree 2 and higher are not local. Take for example

$$F(\varphi) = \int f(x,y)\varphi(x)\varphi(y)d\mu(x)d\mu(y)\,, \qquad f \in \mathcal{D}(M^2)\,.$$

- Now take $f \in \mathcal{D}(M)$ and consider

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- To avoid technical analytic issues, I will formulate the rest of this introduction for $\mathcal{F}$. However, all of this generalizes to $\mathcal{F}_{\mathrm{loc}}$.

Dynamics

- Dynamics is introduced by a generalized Lagrangian S, a localization preserving map $S : \mathcal{D} \to \mathcal{F}_{\text{loc}}$, where $\mathcal{D}(M) = \mathcal{C}_0^\infty(M, \mathbb{R})$. Examples:
  - $S(f)[\varphi] = \int_M \left(\frac{1}{2}\varphi^2 + \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi \right) f d\mu,$

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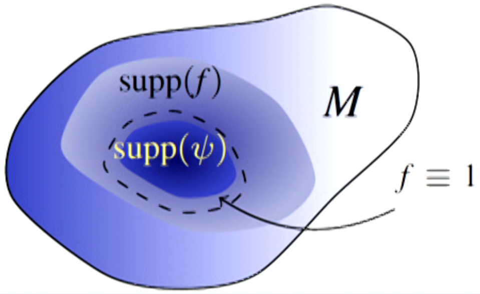
- The Euler-Lagrange derivative of S is denoted by dS and defined by $\langle dS(\varphi), \psi\rangle = \left\langle S(f)^{(1)}[\varphi], \psi\right\rangle$, where $f \equiv 1$ on suppψ, $\psi \in \mathcal{D}(M)$.



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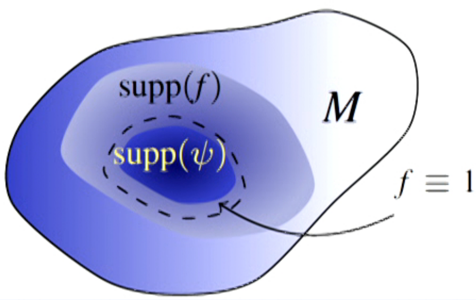
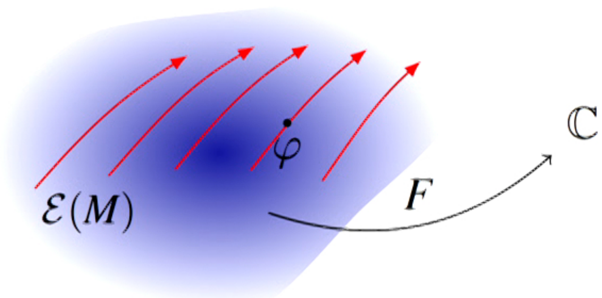
- The field equation is: $dS(\varphi)=0$, so geometrically, the solution space is the zero locus of dS (seen as a 1-form on $\mathcal{E}$).



Symmetries

- We use the BV framework, where symmetries are identified with vector fields (directions) on $\mathcal{E}$.



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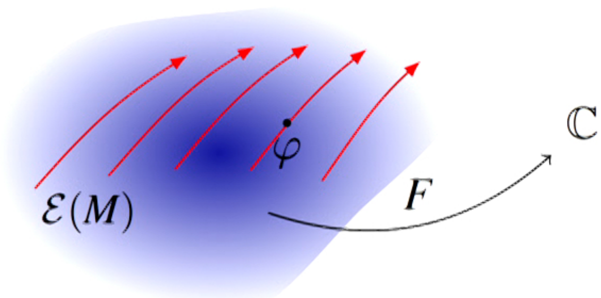

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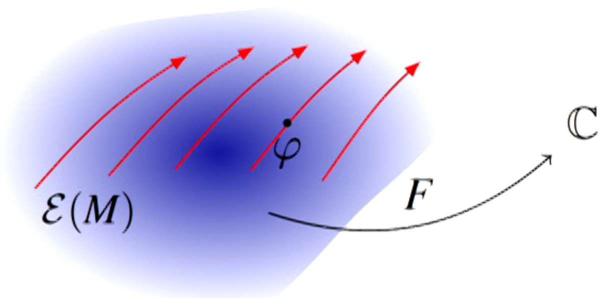

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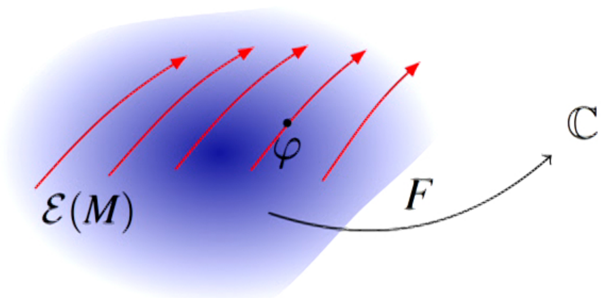
- A symmetry of S is a direction in $\mathcal{E}$ in which the action is constant, i.e. it is a vector field $X \in \mathcal{V}$ such that $\forall\varphi \in \mathcal{E}$: $0 = \langle dS(\varphi), X(\varphi)\rangle =: \delta_S(X)(\varphi)$.



Free scalar field (classical)

- $\mathcal{E} = \mathcal{C}^\infty(M, \mathbb{R})$ and the equation of motion is $dS(\varphi) = P\varphi = 0$, where $P = -(\Box + m^2)$.
- Space of solutions: $\mathcal{E}_S \subset \mathcal{E}$. Denote functionals that vanish on $\mathcal{E}_S$ by $\mathcal{F}_0$. Assume that they are of the form: $\delta_S(X)$ for some $X \in \mathcal{V}$.

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- The space of on-shell observables (i.e. functionals on $\mathcal{E}_S$) $\mathcal{F}_S$ is the quotient $\mathcal{F}_S = \mathcal{F}/\mathcal{F}_0$.
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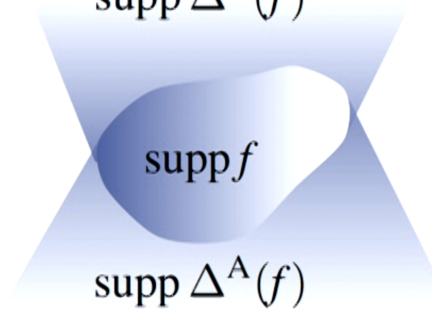
- We obtain a sequence: $0 \to \text{Sym} \hookrightarrow \mathcal{V} \xrightarrow{\delta_S} \mathcal{F} \to 0$.
- For the beginning we consider the case where there are no non-trivial (not vanishing on $\mathcal{E}_S$) local symmetries,
- In this case: $\mathcal{BV} \doteq (\Lambda\mathcal{V}, \delta_S)$. Then the space of classical on-shell observables is given by $\mathcal{F}_S = H_0(\mathcal{BV})$ and higher cohomology groups vanish.

Peierls bracket

- For M globally hyperbolic, P possesses unique retarded and advanced Green's functions Δ^{R}, Δ^{A}.
- Their difference is the Pauli-Jordan function
$$\Delta \doteq \Delta^{\mathrm{R}} - \Delta^{\mathrm{A}}.$$
- The Poisson bracket (Peierls bracket) of the free theory is
$$\lfloor F, G \rfloor \doteq \left\langle F^{(1)}, \Delta G^{(1)} \right\rangle ,$$
for F, G local functions on $\mathcal{E}(M)$.



- This structure extends to $\mathcal{BV}$ and we obtain $(\mathcal{BV}(M), \lfloor ., . \rfloor)$ as the dg classical filed theory model on M. The on-shell classical theory is obtained as $(H_0(\mathcal{BV}(M)), \lfloor ., . \rfloor, \cdot)$, where $\cdot$ is the pointwise product of functionals.

Free scalar field quantization

- We define a $\star$-product (deformation quantization of the classical Poisson algebra):

$$(F \star G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), (\Delta_+)^{\otimes n} G^{(n)}(\varphi) \right\rangle \ ,$$

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Interacting regular observables

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- We define the interacting star product as:

$$F \star_{int} G \doteq R_V^{-1}\left(R_V(F) \star R_V(G)\right)\,,$$

Recent results on convergence

Theorem (Hawkins, KR 2016)

$$F \star_{int} G = \sum_{\gamma} \frac{(-i)^{v(\gamma)+d(\gamma)} \hbar^{e(\gamma)-v(\gamma)}}{|\operatorname{Aut}\gamma|} \vec{\gamma}(F,G)\,,$$

the sum runs over certain class of graphs. Here $d(\gamma)$ denotes the number of directed edges and $\vec{\gamma}$ defines an n-ary multidifferential operator. Importantly, this is a finite sum at each order in $\hbar$.

Relation to the Costello-Gwilliam approach (free theory)

- *Comparing nets and factorization algebras of observables: the free scalar field*, O. Gwilliam, KR, **CMP 2020**.

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- *Comparing nets and factorization algebras of observables: the free scalar field*, O. Gwilliam, KR, **CMP 2020**.
- In the free theory we have $(\Lambda\mathcal{V}[[\hbar]], \star, \cdot_{\mathcal{T}}, \delta_S)$ and $H_0(\Lambda\mathcal{V}[[\hbar]], \delta_S)$ gives the classical observables.
- Using $\mathcal{T}^{-1}$ we can map $(\Lambda\mathcal{V}[[\hbar]], \cdot_{\mathcal{T}}, \delta_S) \xrightarrow{\mathcal{T}^{-1}} (\Lambda\mathcal{V}[[\hbar]], \cdot, \hat{s}_0)$, where $\hat{s}_0 \doteq \mathcal{T}^{-1} \circ \delta_S \circ \mathcal{T}$ is the quantum BV operator, which can also be written as

$$\hat{s}_0 = \{., S\} - i\hbar\triangle\,,$$

where $\triangle$ is the BV Laplacian (divergence on $\mathcal{V}$, extended to $\Lambda\mathcal{V}$ with appropriate signs) and $\{.,.\}$ is the Schouten bracket (shifted Poisson bracket on $\Lambda\mathcal{V}$ generalizing the commutator on $\mathcal{V}$).

Relation to the Costello-Gwilliam approach (summary)

Bottom line:

In pAQFT we deform the product, while in CG approach one deforms the differential. Both viewpoints are shown to be equivalent, using the maps:

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- $\mathcal{T}$ in the free case.

Gauge theories and gravity

- Gauge theories: the action is invariant under some infinite dimensional Lie group $\mathcal{G}$ and the theory possesses local symmetries. In such case $\mathcal{E}$ has to be replaced by the space of orbits of $\mathcal{G}$.

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- Since the global structure of this space could be very complicated, we work with the derived version of this space and consider the Chevalley-Eilenberg complex associated with the Lie algebra action of $\mathfrak{g} = Lie(\mathcal{G})$.
- Effectively, one replaces $\mathcal{E}$ with a graded infinite dimensional manifold $\overline{\mathcal{E}} = \mathcal{E} \oplus \mathfrak{g}[1]$.

Some literature

- Treatment of gauge theories using the BV formalism: *Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory*, K. Fredenhagen, KR, **CMP 2013**.
- Application of the pAQFT framework to perturbative quantum gravity: *Quantum gravity from the point of view of locally covariant quantum field theory*, R. Brunetti, K. Fredenhagen, KR, **CMP 2016**.
- Application to quantum cosmology has been outlined in *Cosmological perturbation theory and quantum gravity*, R. Brunetti, K. Fredenhagen, T.-P. Hack, B. Pinamonti, KR, **JHEP 2016**.

Diffeomorphism invariant observables

- In classical theory we have the metric g on a manifold M and observables are (smooth) functionals of the metric.
- Locality requirement for functionals $F(g)$ is in conflict with diffeomorphism invariance (at least for non-compact M). Main proposals for non-local diff invariant observables:

Relational observables I

- Consider four scalars X_g^μ, $\mu = 0, \dots, 3$ which will parametrize points of spacetime. The fields X_g^μ should transform under diffeomorphisms χ as

$$X^\mu_{\chi^* g} = X^\mu_g \circ \chi \,,$$

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- One can think of the choice of X^{μ} as the choice of observer (cf. Freidel).
- Fix a background g_0 such that the map

$$X_{g_0} : x \mapsto (X^{0}_{g_0}, \ldots, X^{3}_{g_0})$$

is injective.

Relational observables II

- Take $g = g_0 + h$ sufficiently near to g_0 and set

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- α_g transforms under formal diffeomorphisms as

$$\alpha_{\chi^* g} = \chi^{-1} \circ \alpha_g \,.$$

- Take another local field $A_g(x)$ (e.g. a metric scalar). Then

$$\mathcal{A}_g := A_g \circ \alpha_g$$

is invariant under diffeos.

Relational observables III

Physical interpretation

Fields X_g^μ are configuration-dependent coordinates such that $[A_g \circ X_g^{-1}](Y)$ corresponds to the value of the quantity A_g provided that the quantity X_g has the value $X_g = Y$.

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$$F_{\mathcal{A}}(g) = \int \mathcal{A}_g(x) f(x) = \int A_g(X_g^{-1}(Y)) f(X_{g_0}^{-1}(Y)),$$

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  for a test density f. This functional depends on the choice of observable A and "observer" X.
- If X_g^μ and A_g are all local fields themselves, then $F_{\mathcal{A}}$ is non-local with local derivatives.

Theories with boundary and BFV formalism

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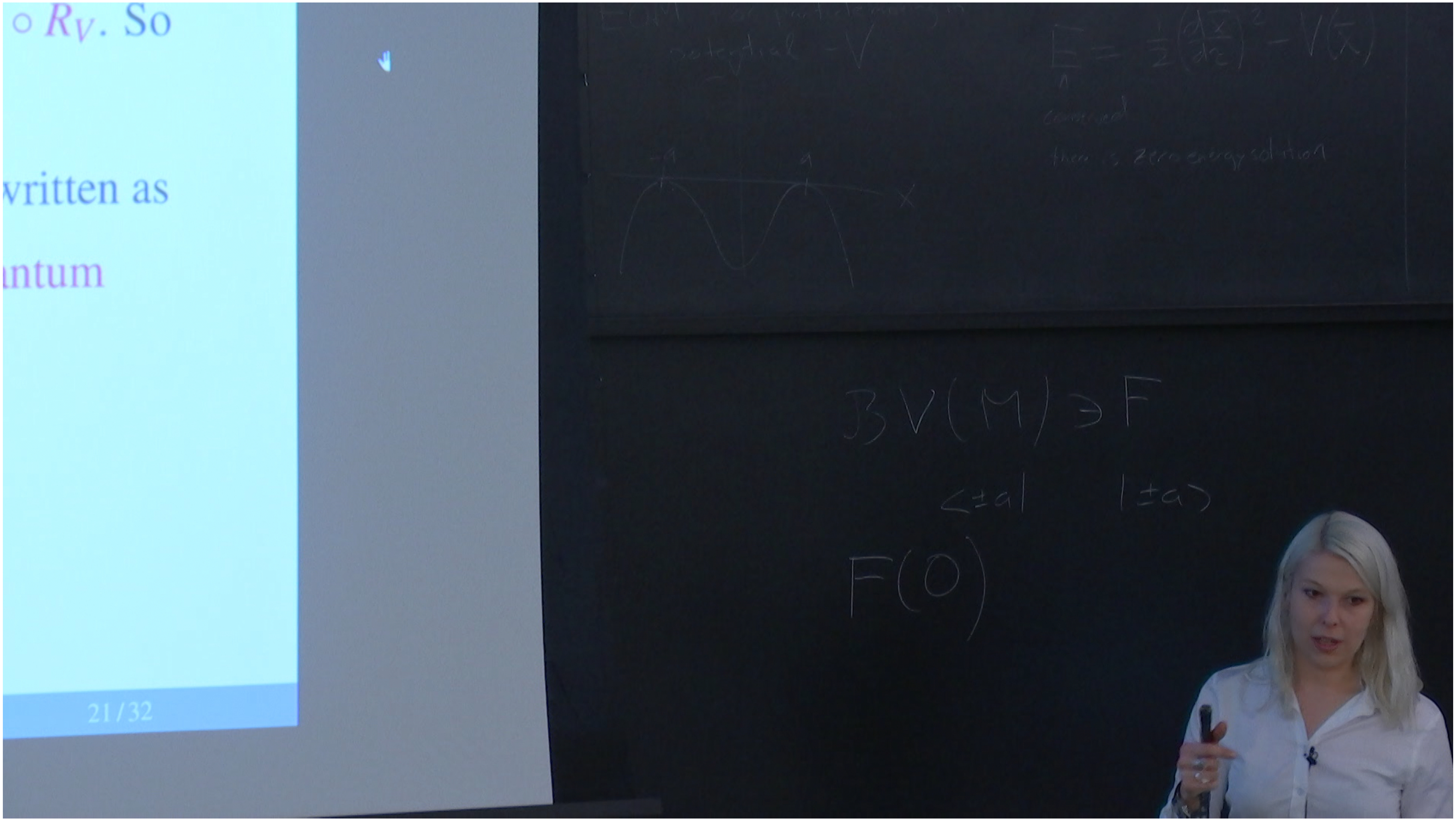
- Study the interacting star product from the point of view of geometric quantization and apply this to discrete spacetime models (with E. Hawkins and C. Minz).
- Use Borel summability and resurgence techniques to push the convergence results further an go “beyond perturbation theory” (with P. Clavier).

Thank you very much for your attention!

Relation to the Costello-Gwilliam approach (interacting)

- In the interacting theory, with interaction V, we have $(\Lambda\mathcal{V}[[\hbar]], \star_{int})$ as a further deformation of $(\Lambda\mathcal{V}[[\hbar]], \star)$ by means of R_V.
- Define the interacting BV differential by $\hat{s}_V \doteq R_V^{-1} \circ \delta_S \circ R_V$. So we obtain: $(\Lambda\mathcal{V}[[\hbar]], \star, \delta_S) \xrightarrow{R_V^{-1}} (\Lambda\mathcal{V}[[\hbar]], \star_{int}, \hat{s}_V)$
- Assume the following: $\delta(\mathcal{S}(V)) = 0$. This can be also written as $\frac{1}{2}\{S+V, S+V\} - i\hbar \bigtriangleup (S+V) = 0$ and is called quantum master equation (QME).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21 / 32
BV(M) ∋ F
<±a| |±a>
F(0)


Interaction
product
Interacting

Theories with boundary and BFV formalism

- Another way to introduce non-locality is to consider theories with boundary, using a modification of the BV formalism, called BFV formalism.
- In this framework, one associates observables to the bulk, to the boundary and possibly to corners (depending on the dimension).
- We can also consider a situation, where the boundary is added *at infinity*, so we have the bulk observables and the asymptotic observables.
- Quantizing asymptotic observables in quantum gravity and QED goes under the name of asymptotic quantization (going back to Ashtekar) and has been used by Herdegen (*Asymptotic algebra for charged particles and radiation*, **JMP 96**) to address the infrared problem in QED.