Title: Quantum Field Theory for Cosmology - Lecture 8

Speakers: Achim Kempf

Collection: Quantum Field Theory for Cosmology (Kempf)

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QFT for Cosmology, Achim Kempf, Lecture 8

The Unruh effect (W. G. Unruh, 1976)

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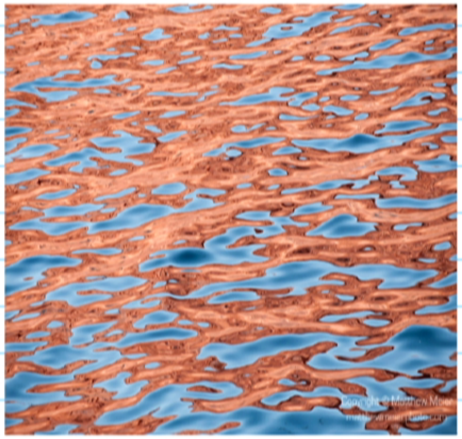
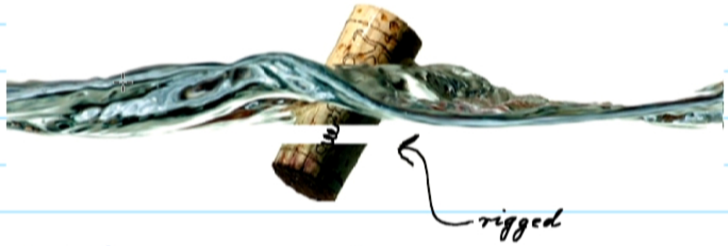
An accelerated ice cube will melt, even in vacuum.

The Unruh effect is the observation, by accelerated observers, of particles, even when the field is in the vacuum state in Minkowski space, i.e. even if inertial observers don't see particles

Intuition:

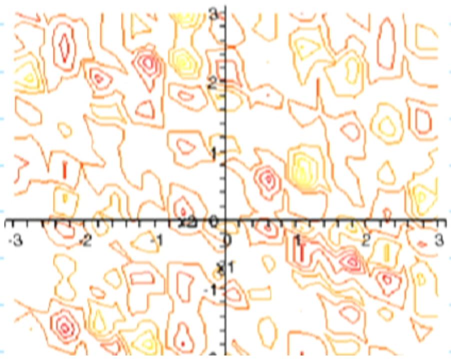


A rigged cork can act as sender and detector.



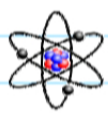
When accelerated, it gets excited - as if detecting.

Similarly:



If simplified to a 2-level system, we say that we have an Unruh DeWitt detector system.

An atom or molecule can also both emit and detect particles. This can serve as the definition of particles.



Unruh effect

We'll consider detectors at rest and in motion:

* A detector at rest has: $x^\mu(\tau) = (\tau, 0, 0, 0)$

* Case of constant velocity:

$$x^\mu(\tau) = (a\tau, \vec{b}\tau)$$

with $a^2 - \vec{b}^2 = 1$. Exercise: verify

* Case of constant acceleration in the x-direction:

$$x^0(\tau) = \alpha \sinh(\tau/\alpha)$$
$$x^1(\tau) = \alpha \left(1 + \sinh^2(\tau/\alpha)\right)^{1/2}$$
$$x^2(\tau) = x^3(\tau) = 0$$

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Exercise: ☐ verify that $\ddot{x}^\mu \ddot{x}_\mu = \text{const}$

☐ show that for $\tau \ll 1$: (i.e. for still small velocities) $x(\tau) \approx (\tau, a + b\tau^2)$

The quantum field

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The quantum field

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and $v_k(x^0) = \frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0}$ with $\omega_k = \sqrt{\vec{k}^2 + m^2}$.

- Thus: $$\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{\omega_k}} e^{i\omega_k x^0 + i\vec{k}\vec{x}}\, a_k + \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}}\, a^+_{-k} \right) d^3k$$

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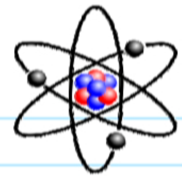
The detector system



Inertial observer's cartesian coordinates.

- Small, localized system with path $x^\mu(\tau)$

detector's eigentime

E.g.: * An atom

* An oscillator, such as the diatomic molecule H_2.

- First quantized.

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- Assume $spec(H^{detector}) = \{E_0, E_1, E_2, \dots\}$ is discrete.

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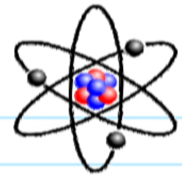

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$\Rightarrow$ The total quantum system thus consists of two subsystems, with:

- First quantized.
- Hamiltonian $\hat{H}^{detector}$ acts on Hilbert space $\mathcal{H}^{detector}$.
- Assume $spec(H^{detector}) = \{E_0, E_1, E_2, \dots\}$ is discrete.

⇒ The total quantum system thus consists of two subsystems, with:

$$\boxed{\hat{H}^{total} = \hat{H}_0^{detector} \otimes 1 + 1 \otimes \hat{H}_0^{field} + \hat{H}^{interaction}}$$

- On the total Hilbert space:

$$\mathcal{H}^{total} = \mathcal{H}^{detector} \otimes \mathcal{H}^{field}$$

- The interaction Hamiltonian $\hat{H}^{interaction}$ consists of operators of both subsystems, usually:

$$\hat{H}^{interaction}(\tau) = \underbrace{\varepsilon(\tau)}_{\substack{\text{Detector efficiency} \\ \text{(can also be used} \\ \text{as on/off switch)}}} \underbrace{\hat{Q}(\tau)}_{\substack{\text{An observable} \\ \text{of the detector's} \\ \text{quantum system}}} \underbrace{\hat{\phi}(x^0(\tau), \vec{x}(\tau))}_{\substack{\text{The field } \hat{\phi} \\ \text{at the current} \\ \text{detector location}}}$$

- Examples: $H^{int}(\tau) = \hat{S}_3(\tau) \otimes \hat{B}_3(x(\tau))$

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Detector efficiency (can also be used as on/off switch)

An observable of the detector's quantum system

The field $\hat{\phi}$ at the current detector location

- Examples: $H^{int}(\tau) = \hat{S}_3(\tau) \otimes \hat{B}_3(x(\tau))$

field is magnetic field

▯ The interaction Hamiltonian $H^{\text{interaction}}$ consists of operators of both subsystems, usually:

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▯ Examples: $H^{int}(\tau) = \hat{S}_3(\tau) \otimes \hat{B}_3(x(\tau))$

detector is a spin. field is magnetic field.

or: $H^{int}(\tau) = -\frac{e}{mc}\, \hat{p}_i \otimes \hat{A}^i(x(\tau))$ (use Axial gauge: $\partial_i A^i = 0$)

Time evolution

- If we (realistically) assume that the detector efficiency $\varepsilon(\tau)$ is small, we can use perturbation theory.

- In this case, the Dirac picture of time evolution is convenient:

  * Operators evolve according to

$$\hat{H}^{free} = \hat{H}^{detector} \otimes 1 + 1 \otimes \hat{H}^{field} \qquad (*)$$

For example:

$$\hat{Q}(\tau) = e^{i\hat{H}^{free}\tau} (\hat{Q}_0 \otimes 1)\, e^{-i\hat{H}^{free}\tau}$$

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For example:

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$$= e^{i\hat{H}^{detector}\tau}\, \hat{Q}_0\, e^{-i\hat{H}^{detector}\tau} \otimes 1$$

* States evolve according to $\hat{H}^{int}(\tau)$, i.e., according to the time evolution operator:

$$\hat{U}(\tau) = T\exp\left(i\int_{\tau_i}^{\tau_f} \hat{H}^{interaction}(\tau')\, d\tau'\right)$$

time-ordering symbol

In $\hat{H}^{interaction}$ the operators are time dependent, evolving according to (*)

Perturbative ansatz

- For small detector efficiency $\varepsilon(\tau)$ we can expand:

$$\hat{U}(\tau) = 1 + i\int_{-\infty}^{\tau} \varepsilon(\tau')\,\hat{Q}(\tau')\,\hat{\phi}(x^0(\tau'),\vec{x}(\tau'))\, d\tau' + O(\varepsilon^2)$$

$$\mathcal{U}(\tau) = T\exp\left(i\int_{\tau_i} \hat{H}^{\text{interaction}}(\tau')\,d\tau'\right)$$

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- **Note:** We can set $\tau_i = -\infty$ since we can always switch $\varepsilon(\tau)$ on or off.

Initial conditions

- We assume that the quantum field $\hat{\phi}$ starts out in a state $|\alpha\rangle$ with $|\alpha\rangle$ = Minkowski vacuum, $|\alpha\rangle = |0\rangle$, or a 1-particle state $|\alpha\rangle = |1_k\rangle$.

- We assume that the detector starts out in its ground state $|E_0\rangle$.

- Thus, the combined system starts out in the state:

$$|\psi_{in}\rangle = |E_0\rangle \otimes |\alpha\rangle$$

- Time evolution:

At time τ the total system is in the state

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$$|\psi(\tau)\rangle = \hat{U}(\tau)\,|\psi_{in}\rangle$$

Particle creation

- The problem:

What is the probability amplitude that, if we

Total detection probability:

- The probability for detection eventually is:

(we may choose $\varepsilon(\tau)$ so as to make it finite)

$$p(\infty) \simeq \langle E_n| \otimes \langle \Omega| \left(1 + i \int_{-\infty}^{+\infty} \varepsilon(\tau') \, \hat{Q}(\tau') \otimes \hat{\phi}(x(\tau')) \, d\tau' \right) |E_0\rangle \otimes |\alpha\rangle$$

Note: $\langle E_n | E_0 \rangle = 0 \Rightarrow 1^{st}$ term vanishes $\Rightarrow$

$$= i \int_{-\infty}^{+\infty} \varepsilon(\tau) \langle E_n | \hat{Q}(\tau) | E_0 \rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

Recall: $\hat{Q}(\tau) = e^{i H_0^{detector} \tau} \hat{Q}_0 \, e^{-i H_0^{detector} \tau}$

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Recall: $\hat{\phi}(x) = \frac{1}{(2\pi)^{3/2}} \int \left(\frac{1}{\sqrt{2\omega_k}} e^{i\omega_k x^0 - i\vec{k}\vec{x}} a^+_{\vec{k}} + \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k x^0 + i\vec{k}\vec{x}} a_{\vec{k}} \right) d^3k$

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Note: We can now calculate all absorption and emission processes.

Here: Let's focus on particle detection in the vacuum, $|\alpha\rangle := |0\rangle$:

* In $\hat{\phi}(x)$, only the terms $\sim a^+_k$ can contribute, because $a_k |0\rangle = 0$

* Thus, in $|\Omega\rangle$ only the one-particle components contribute:

$$p(\infty) = i \int_{-\infty}^{+\infty} \varepsilon(\tau)\, e^{i(E_n - E_0)\tau} \langle E_n | \hat{Q}_0 | E_0 \rangle \langle \Omega | \hat{\phi}(x(\tau)) | \alpha \rangle \, d\tau$$

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$$|\Omega\rangle = \cancel{\Omega_0|0\rangle} + \int \Omega_k\, a_k^+ |0\rangle\, d^3k + \cancel{\iint \Omega_{k,k'}\, a_k^+ a_{k'}^+ |0\rangle\, d^3k\, d^3k'} + \cancel{\ldots}$$

* Thus, let us consider a $|\Omega\rangle = a_{\underset{\sim}{k}}^+ |0\rangle$:

$$\Rightarrow \quad p(\infty) = i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}\varepsilon(\tau)\,e^{i(E_n-E_0)\tau}\langle 0|a_{\tilde{k}}\int\frac{e^{i\omega_k x_0(\tau)-i\vec{k}\vec{x}(\tau)}}{\sqrt{2\omega_k}}\,a_k^+\,d^3k|0\rangle\,d\tau$$

leads to:

$$\langle 0|\,a_{\tilde{k}}\,a_k^+|0\rangle = \langle 0|\,a_k^+\,a_{\tilde{k}} + \delta^3(\tilde{k}-k)|0\rangle = \delta^3(\tilde{k}-k)$$

$$\Rightarrow \quad \boxed{p(\infty) = i\underbrace{\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}}_{\text{some constant}}\int_{-\infty}^{+\infty}\frac{e^{i(E_n-E_0)\tau}\;e^{i(\omega_{\tilde{k}}x^0(\tau)-\tilde{k}\vec{x}(\tau))}}{\sqrt{2\omega_{\tilde{k}}}}\,\varepsilon(\tau)\,d\tau}$$

Special case: $|\alpha\rangle = |0\rangle$ and detector inertial:

* Choose the detector's rest frame: $x^\mu(\tau) = (\tau, 0, 0, 0)$

$\Rightarrow$

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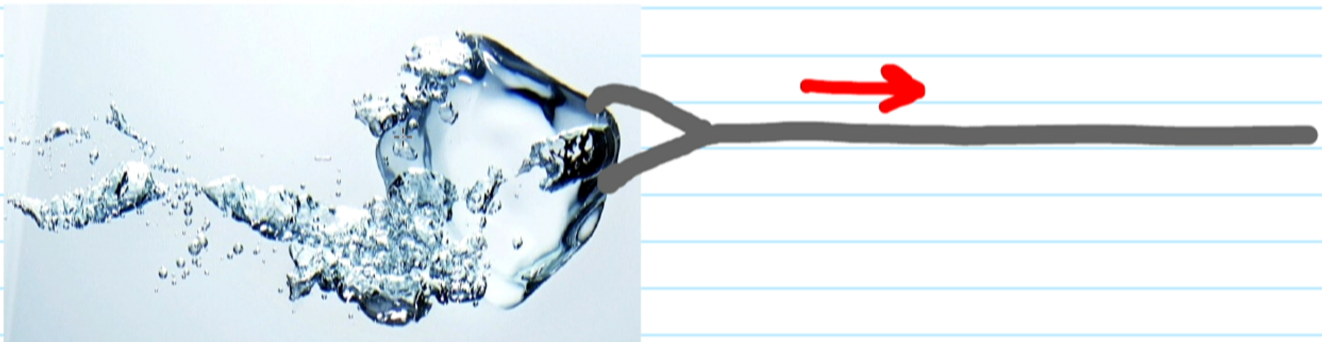
assume $\varepsilon(\tau) = 1$, i.e., "always on".

$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty} \frac{e^{i(E_n-E_0)\tau}\, e^{i\,\omega_{\tilde{k}}\tau}}{\sqrt{2\omega_{\tilde{k}}}}\, d\tau$$

$$= i\frac{\langle E_n|\hat{Q}_0|E_0\rangle}{(2\pi)^{3/2}}(2\pi)^{1/2}\,\delta\Big(\underbrace{\overbrace{E_n - E_0}^{>0} + \overbrace{\omega_{\tilde{k}}}^{\sqrt{k^2+m^2}\,>\,0}}_{\text{this cannot be } 0}\Big)\frac{1}{\sqrt{2\omega_{\tilde{k}}}}$$

$$= 0$$

Special case: $|\alpha\rangle = |0\rangle$ and detector non-inertial:



- The probability amplitude for the detector to become excited will depend on the excitation energy $E_{ex} := E_n - E_0$:

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i.e. τ and E_{ex} are a Fourier pair (if neglecting the "constant") possesses a Fourier transform

$$F(E_x) = \int_{-\infty}^{+\infty} e^{iE_x\tau} f(\tau)\, d\tau , \qquad E_x = E_n - E_0$$

which is generally nonzero for positive E_x.

$\Rightarrow$ $p(\infty) \sim F(E_x) \neq 0$ $\Rightarrow$ detector does get excited.

"proportional to" (European notation)

(while also the field gets excited)

$\rightsquigarrow$ <u>Unruh effect</u>