Title: Quantum Field Theory for Cosmology - Lecture 3

Speakers: Achim Kempf

Collection: Quantum Field Theory for Cosmology (Kempf)

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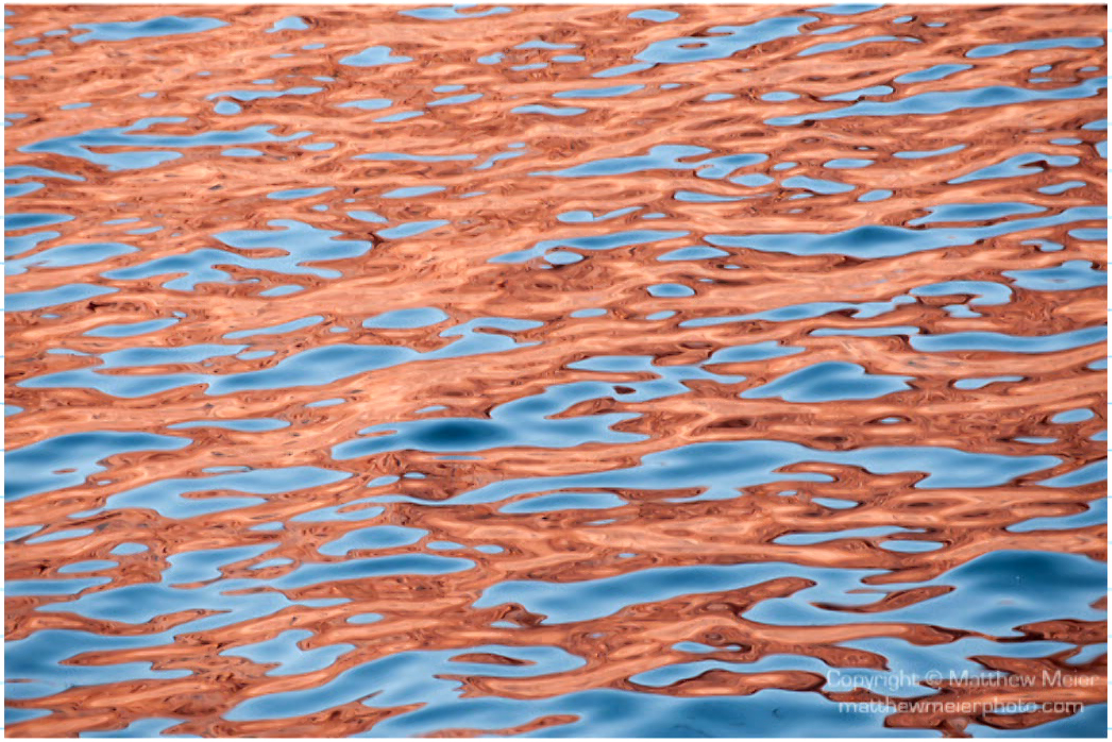
QFT for Cosmology, Achim Kempf, Lecture 3

Note Title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ONLY

☐ Quantization conditions:

analogous to:

$$[\hat\phi(x,t), \hat\pi(x',t)] = i\hbar\, \delta^3(x-x') \qquad [\hat q_a(t), \hat p_{a'}(t)] = i\hbar\, \delta_{aa'}$$

$$[\hat\phi(x,t), \hat\phi(x',t)] = 0 \qquad [\hat q_a(t), \hat q_{a'}(t)] = 0$$

$$[\hat\pi(x,t), \hat\pi(x',t)] = 0 \qquad [\hat p_a(t), \hat p_{a'}(t)] = 0$$

☐ We keep the equations of motion:

$$\dot{\hat\phi}(x,t) = \hat\pi(x,t) \qquad \text{(E1)} \qquad \dot{\hat q}_a(t) = \hat p_a(t)$$

$$\dot{\hat\pi}(x,t) = -(-\Delta + m^2)\,\hat\phi(x,t) \qquad \text{(E2)} \qquad \dot{\hat p}_a(t) = -k_a \hat q_a(t)$$

☐ Note: $\phi^*(x,t) = \phi(x,t)$ now implies hermiticity: $\hat\phi^\dagger(x,t) = \hat\phi(x,t)$

☐ Proposition:

$E1, E_2$ follow from the Heisenberg equs

analogous to:

$$i\hbar\,\dot{\hat{\phi}}(x,t) = [\hat{\phi}(x,t), \hat{H}]$$

$$i\hbar\,\dot{\hat{\pi}}(x,t) = [\hat{\pi}(x,t), \hat{H}]$$

$$i\hbar\,\dot{\hat{q}}_a(t) = [\hat{q}_a(t), \hat{H}]$$

$$i\hbar\,\dot{\hat{p}}_a(t) = [\hat{p}_a(t), \hat{H}]$$

with this QFT Hamiltonian:

$$\hat{H} = \int_{\mathbb{R}^3} \frac{1}{2}\hat{\pi}^2(x',t) + \frac{1}{2}\hat{\phi}(x',t)\,(m^2-\Delta)\,\hat{\phi}(x',t)\, d^3x'$$

$$\hat{H} = \sum_a{}' \frac{\hat{p}_a^2}{2} + \frac{\omega_a^2}{2}\hat{q}_a^2$$

Plan:

1. Recall harmonic oscillators ✓
2. Relativistic fields ✓
3. 2nd quantization ✓
4. Harmonic oscillators in fields ⇒ vacuum fluctuations

4. Harmonic oscillators in quantum fields

- From the above, we need to solve 2 equations:

a.) The K.G. eqn: $$\left(\frac{\partial^2}{\partial t^2} - \Delta + m^2\right)\hat{\phi}(\overset{x=(x_1,x_2,x_3)}{x},t) = 0$$

3. 2nd quantization ✓

4. Harmonic oscillators in fields ⇒ vacuum fluctuations

4. Harmonic oscillators in quantum fields

- From the above, we need to solve 2 equations:

$x = (x_1, x_2, x_3)$

a.) The K.G. eqn: $\left(\frac{\partial^2}{\partial t^2} - \Delta + m^2\right)\hat{\phi}(x,t) = 0$

b.) The commutation rels: $\left[\hat{\phi}(x,t), \dot{\hat{\phi}}(x',t)\right] = i\hbar\,\delta(x-x')$

- Q: How to solve these eqns?

  A: Use similarity to harmonic oscillator problem after overcoming a few technical difficulties:

1st Difficulty: (in reducing the QFT problem to harmonic oscillators)

- In the K.G. equation,

$$\dot{\hat{\pi}}(x,t) = -\left(-\Delta + m^2\right)\hat{\phi}(x,t) \xleftrightarrow{\text{Analogy}} \dot{\hat{p}}_a(t) = -K_a\,\hat{q}_a(t)$$

  we notice that $(-\Delta + m^2)$, unlike K_a, is not a number!

- Q: Can we "transform" $(-\Delta + m^2)$ into a number?

after overcoming a few technical difficulties:

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we notice that $(-\Delta + m^2)$, unlike K_a, is not a number!

- **Q**: Can we "transform" $(-\Delta + m^2)$ into a number?

A: Yes: Fourier transform turns derivatives into numbers!

The local field oscillators are coupled.

$\Rightarrow$ Excitations spread.


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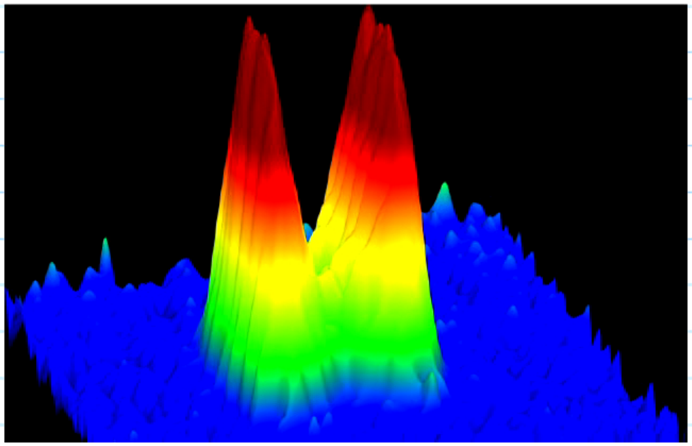
The oscillators that are local in momentum space are uncoupled.

$\Rightarrow$ Excitations



Fourier transform of the spatial variables x_i:

☐ Definition:

$xk = \sum_{i=1}^{3} x_i k_i \quad ; \quad k = (k_1, k_2, k_3)$

$$\hat{\phi}(k,t) := (2\pi)^{-3/2} \int_{\mathbb{R}^3} e^{-ixk}\, \hat{\phi}(x,t)\, d^3x$$

Traditional notation: $\hat{\phi}_k(t) := \hat{\phi}(k,t)$

Traditional terminology: $\hat{\phi}_k(t)$ is called the field's k-mode.

☐ Inverse Fourier transform:

$$\hat{\phi}(x,t) = (2\pi)^{-3/2} \int e^{ixk}\, \hat{\phi}(t)\, d^3k$$

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☐ <u>Inverse Fourier transform:</u>

$$\hat{\phi}(x,t) = (2\pi)^{-3/2} \int_{\mathbb{R}^3} e^{ixk}\, \hat{\phi}_k(t)\, d^3k$$

☐ Proposition: (Exercise: show this)

Analogous to:

$k^2 = \sum_{i=1}^{3} k_i^2$

a) $$\hat{H} = \int_{\mathbb{R}^3} \frac{1}{2}\hat{\pi}_k^{+}(t)\,\hat{\pi}_k(t) + \frac{1}{2}\hat{\phi}_k^{+}(t)\left(k^2+m^2\right)\hat{\phi}_k(t)\, d^3k$$

$$\hat{H} = \sum_a{}' \frac{1}{2}\hat{p}_a\hat{p}_a + \frac{1}{2}\omega_a\hat{q}_a\hat{q}_a$$

b) $$\left[\hat{\phi}_k(t), \hat{\pi}_{k'}(t)\right] = i\hbar\,\delta^3(k+k')$$

$$\left[\hat{q}_a, \hat{p}_{a'}\right] = i\hbar\,\delta_{a,a'}$$

$$\left[\hat{\phi}_k(t), \hat{\phi}_{k'}(t)\right] = 0$$

$$\left[\hat{q}_a(t), \hat{q}_{a'}(t)\right] = 0$$

$$\left[\hat{\pi}_k(t), \hat{\pi}_k(t)\right] = 0$$

$$\left[\hat{p}_a(t), \hat{p}_{a'}(t)\right] = 0$$

c) $$\dot{\hat{\phi}}_k(t) = \hat{\pi}_k(t)$$

$$\dot{\hat{q}}_a(t) = \hat{p}_a(t)$$

$$\dot{\hat{\pi}}_k(t) = -\left(k^2+m^2\right)\hat{\phi}_k(t)$$

$$\dot{\hat{p}}_a(t) = -\omega_a^2\,\hat{\phi}_a(t)$$

a.) $H = \int_{\mathbb{R}^3} \frac{1}{2}\hat{\pi}_k^+(t)\hat{\pi}_k(t) + \frac{1}{2}\hat{\phi}_k^+(t)\left(k^2+m^2\right)\hat{\phi}_k(t)\, d^3k$	$\hat{H} = \sum_a{}' \frac{1}{2}\hat{p}_a\hat{p}_a + \frac{1}{2}\omega_a\hat{q}_a\hat{q}_a$
b.) $[\hat{\phi}_k(t), \hat{\pi}_{k'}(t)] = i\hbar\,\delta^3(k+k')$	$[\hat{q}_a, \hat{p}_{a'}] = i\hbar\,\delta_{a,a'}$
$[\hat{\phi}_k(t), \hat{\phi}_{k'}(t)] = 0$	$[\hat{q}_a(t), \hat{q}_{a'}(t)] = 0$
$[\hat{\pi}_k(t), \hat{\pi}_k(t)] = 0$	$[\hat{p}_a(t), \hat{p}_{a'}(t)] = 0$
c.) $\dot{\hat{\phi}}_k(t) = \hat{\pi}_k(t)$	$\dot{\hat{q}}_a(t) = \hat{p}_a(t)$
$\dot{\hat{\pi}}_k(t) = -(k^2+m^2)\hat{\phi}_k(t)$	$\dot{\hat{p}}_a(t) = -\omega_a^2\hat{\phi}_a(t)$

$\Rightarrow$ For each mode $\vec{k}$ we seem to have a harmonic oscillator with $\omega_k = \sqrt{k^2+m^2}$.

Exercise:

Show that a) + b) + Heisenberg eqn $\dot{\hat{f}}(t) = \frac{1}{i\hbar}[\hat{f}(t), \hat{H}[$ yields c.)

($\hat{f}$ is arbitrary. E.g. $\hat{f} = \hat{\phi}_k$ or $\hat{f} = \hat{\pi}_k$)

2^{nd} Difficulty:

(in reducing the QFT problem to harmonic oscillators)

We notice that the commutation relations

$$[\hat{\phi}_k(t), \hat{\pi}_{k'}(t)] = i\hbar\,\delta^3(k+k') \quad \text{and} \quad [\hat{q}_a, \hat{p}_{a'}] = i\hbar\,\delta_{a,a'}$$

do not match, because the Kronecker δ is only

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do not match, because the Kronecker δ is only either 0 or 1, unlike the Dirac δ!

- Idea: If we use Fourier series instead, should have discrete values of k, thus Kronecker δ for CCR!

- Strategy:

  1. Put system into a large box $[-L/2, L/2]^{\times 3}$
  2. Assume (for example) periodic boundary conditions.
     (If box large enough it should not matter here what happens at the boundary of the box)
  3. Instead of Fourier transform, we can now use Fourier series.

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Terminology: Putting a system in a box is called

☐ Strategy:

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2. Assume (for example) periodic boundary conditions.
   (If box large enough it should not matter here what happens at the boundary of the box)
3. Instead of Fourier transform, we can now use Fourier series.

Terminology: Putting a system in a box is called "Infrared regularization".

↑ because "long" wavelengths are removed.

☐ Infrared regularization:

- ✱ $(k_1, k_2, k_3) = \frac{2\pi}{L}(n_1, n_2, n_3)$ with $n_1, n_2, n_3 \in \mathbb{Z}$

- ✱ $V = L^3$ (Volume of box)

- ✱ Fourier series expansion coefficients:

$$\hat{\phi}_k(t) = V^{-1/2} \int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \hat{\phi}(x,t)\, e^{-ixk}\, d^3x$$

- ✱ The inverse is the Fourier series:

$$\hat{\phi}(x,t) = V^{-1/2} \sum \hat{\phi}(t)\, e^{ixk}$$

The QFT problem in the box:

analogous to

a) $$\hat{H} = \sum_k \frac{1}{2} \hat{\pi}_k^{+} \hat{\pi}_k + \frac{1}{2} \omega_k^2 \hat{\phi}_k^{+} \hat{\phi}_k$$

$$\hat{H} = \sum_a \frac{1}{2} \hat{p}_a \hat{p}_a + \frac{1}{2} \omega_a^2 \hat{q}_a \hat{q}_a$$

$$\omega_k^2 = k^2 + m^2$$

Kronecker δ

b) $$[\hat{\phi}_k(t), \hat{\pi}_{k'}(t)] = i\hbar\, \delta_{k,-k'}$$

$$[\hat{q}_a, \hat{p}_{a'}] = i\hbar\, \delta_{a,a'}$$

$$[\hat{\phi}_k(t), \hat{\phi}_{k'}(t)] = 0$$

$$[\hat{q}_a(t), \hat{q}_{a'}(t)] = 0$$

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c) $$\dot{\hat{\phi}}_k(t) = \hat{\pi}_k(t)$$

$$\dot{\hat{q}}_a(t) = \hat{p}_a(t)$$

$$\dot{\hat{\pi}}_k(t) = -(k^2 + m^2)\, \hat{\phi}_k(t)$$

$$\dot{\hat{p}}_a(t) = -\omega_a^2\, \hat{\phi}_a(t)$$

$\uparrow \omega_k^2 = k^2 + m^2$

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b) $[\hat{\phi}_k(t), \hat{\pi}_{k'}(t)] = i\hbar\, \delta_{k,-k'}$	$[\hat{q}_a, \hat{p}_{a'}] = i\hbar\, \delta_{a,a'}$
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$\dot{\hat{\pi}}_k(t) = -(k^2+m^2)\,\hat{\phi}_k(t)$	$\dot{\hat{p}}_a(t) = -\omega_a^2\, \hat{\phi}_a(t)$

3rd Difficulty: (in reducing the QFT problem to harmonic oscillators)

- **Hermiticity:**

We notice that $\hat{\phi}^+(x,t) = \hat{\phi}(x,t)$, $\hat{\pi}^+(x,t) = \hat{\pi}(x,t)$ implies

$$\hat{\phi}_k^+(t) = \hat{\phi}_{-k}(t), \quad \hat{\pi}_k^+(t) = \hat{\pi}_{-k}(t) \qquad (H)$$

(Indeed:

$$\hat{\phi}_k^+(t) = (2\pi)^{-3/2}\int_{\mathbb{R}^3} e^{ixk}\hat{\phi}^+(x,t)\,d^3x = (2\pi)^{-3/2}\int_{\mathbb{R}^3} e^{ixk}\hat{\phi}(x,t)\,d^3x = \hat{\phi}_{-k}(t)$$
)

But eqns (H) do not match:

$$\hat{q}_a^+(t) = \hat{q}_a(t) \qquad \hat{p}_a^+(t) = \hat{p}_a(t)$$

☐ **Correspondingly:**

The analogy between

$[\hat{\phi}_k(t), \hat{\pi}_{k'}(t)] = i\hbar\,\delta_{k,-k'}$ and $[\hat{q}_a, \hat{p}_{a'}] = i\hbar\,\delta_{a,a'}$

suffers from $\delta_{k,-k'}$ instead of $\delta_{k,k'}$. (we do have $[\hat{\phi}_k(t), \hat{\pi}^+_{k'}(t)] = i\hbar\,\delta_{k,k'}$)

☐ **Mukhanov:**

Neglects hermiticity issue and treats the field's oscillators just like ordinary quantum oscillators but with complex, i.e. non-hermitean amplitudes

Proper treatment:

☐ Define new variables $\hat{q}_k, \hat{p}_k$, which are proper oscillators:

Eqns of motion: $\hat{p}_k = \dot{\hat{q}}_k$, $\dot{\hat{p}}_k = -\omega_k^2 q_k$

Canon. com. rels: $[\hat{q}_k, \hat{p}_{k'}] = i\,\delta_{k,k'}$

Hermiticity: $\hat{q}_k^+ = \hat{q}_k$ $\hat{p}_k^+ = \hat{p}_k$

☐ Then, try ansatz:

$$\hat{\phi}_k = \frac{1}{2}\left(\hat{q}_k + \hat{q}_{-k}\right) + \frac{i}{2\omega_k}\left(\hat{p}_k - \hat{p}_{-k}\right) \qquad (A)$$

Remark: In practice, it'll be more convenient to work with a_k, a_k^+:

↙ Exercise!

☐ Now, show that ansatz (A) succeeds, i.e., that indeed:

Hamiltonian $$\hat{H} = \sum_k \frac{1}{2}\hat{p}_k^2 + \frac{1}{2}\omega_k^2 \hat{q}_k^2 \qquad (H)$$

Eqns of motion: $$\hat{\pi}_k = \dot{\hat{\phi}}_k\,, \quad \dot{\hat{\pi}}_k = -\omega_k^2 \hat{\phi}_k$$

Canon. com. rels: $$[\hat{\phi}_k, \hat{\pi}_{k'}] = i\,\delta_{k,-k'}$$

Hermiticity cond.: $$\hat{\phi}_k^{+} = \hat{\phi}_{-k}\,, \quad \hat{\pi}_k^{+} = \hat{\pi}_{-k}$$

☐ Finally, via inverse Fourier series, show that:

$$\hat{\phi}(x) = \sqrt{\frac{2}{V}} \sum_k \left\{ \cos(xk)\,\hat{q}_k - \frac{1}{\omega_k}\sin(xk)\,\hat{p}_k \right\} \qquad (B)$$

Canon. com. rels: $[\hat{q}_k, \hat{p}_{k'}] = i\,\delta_{k,k'}$

Hermiticity: $\hat{q}_k^{+} = \hat{q}_k \qquad \hat{p}_k^{+} = \hat{p}_k$

☐ Then, try ansatz:

$$\hat{\phi}_k = \frac{1}{2}\left(\hat{q}_k + \hat{q}_{-k}\right) + \frac{i}{2\omega_k}\left(\hat{p}_k - \hat{p}_{-k}\right) \qquad (A)$$

Remark: In practice, it'll be more convenient to work with a_k, a_k^{+}:

With $a_k := \sqrt{\omega_k}\,\hat{x}_k + \frac{i}{\sqrt{\omega_k}}\hat{p}_k$ the ansatz reads: $\hat{\phi}_k = \frac{1}{\sqrt{2\omega_k}}\left(a_k + a_{-k}^{+}\right)$

$$\hat{\phi}_k = \frac{1}{2}\left(\hat{q}_k + \hat{q}_{-k}\right) + \frac{i}{2\omega_k}\left(\hat{p}_k - \hat{p}_{-k}\right) \qquad (A)$$

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↙ Exercise!

☐ Now, show that ansatz (A) succeeds, i.e., that indeed:

Hamiltonian $\quad \hat{H} = \sum_k \frac{1}{2}\hat{p}_k^2 + \frac{1}{2}\omega_k^2\hat{q}_k^2 \qquad (H)$

"No particles state"

Significance of non-uniqueness?

- * Ground state of q_k, p_k oscillators ⇝ Vacuum
- * This need not be lowest energy state of the QFT Hamiltonian

→ Problem of vacuum identification on curved space.

→ See later.

For now: We solved, using (A), the QFT eqns of the K.G. field.

Namely, we have now solved:

$$\text{Eqns of motion:}\begin{cases}\dot{\hat{\phi}}(x,t) = \hat{\pi}(x,t)\\ \dot{\hat{\pi}}(x,t) = -(-\Delta + m^2)\,\hat{\phi}(x,t)\end{cases}$$

Hermiticity: $\hat{\phi}^+(x,t) = \hat{\phi}(x,t)$, $\hat{\pi}^+(x,t) = \hat{\pi}(x,t)$

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Hermiticity: $\hat{\phi}^{+}(x,t) = \hat{\phi}(x,t)$, $\hat{\pi}^{+}(x,t) = \hat{\pi}(x,t)$

Can. com. rels: $\left[\hat{\phi}(x,t), \hat{\pi}(x',t)\right] = i\hbar\,\delta^3(x-x')$

Example: How to calculate quant. fluct. of K.G. field?

1. Solve the system of ∞ many quantum harmonic oscillator degrees of freedom

$$\hat{q}_k, \hat{p}_k$$

with $\hat{H} = \sum_k \frac{1}{2} \hat{p}_k^2 + \frac{\omega_k^2}{2} \hat{q}_k^2, \qquad \omega_k = \sqrt{k^2 + m^2}$

for all $k = (k_1, k_2, k_3) = \frac{2\pi}{L}(n_1, n_2, n_3)$ where $n_1, n_2, n_3 \in \mathbb{Z}$

2. Choose a state $|\psi\rangle$ of that quantum system.

Example: The oscillators could all be in harmonic oscillator degrees of freedom

$$\hat{q}_k \,, \; \hat{p}_k$$

with $\hat{H} = \sum_k \frac{1}{2}\hat{p}_k^2 + \frac{\omega_k^2}{2}\hat{q}_k^2 ,$ $\quad \omega_k = \sqrt{k^2+m^2}$

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2. Choose a state $|\psi\rangle$ of that quantum system.

Example: The oscillators could all be in their lowest energy state.

- We preliminarily call this $|\psi\rangle$ the vacuum state $|\psi_0\rangle$.

3. Given a state $|\psi\rangle$, we can calculate the probability (amplitude density) for finding arbitrary values $q_k(t)$, $p_k(t)$.

Example:

- In vacuum state, we know that the probability distribution of the $\hat{q}_k$ (and $\hat{p}_k$ as well) is gaussian:
- 

$$\text{prob}(q_k) \sim e^{-\omega_k q_k^2}$$

the vacuum state $|\psi_0\rangle$.

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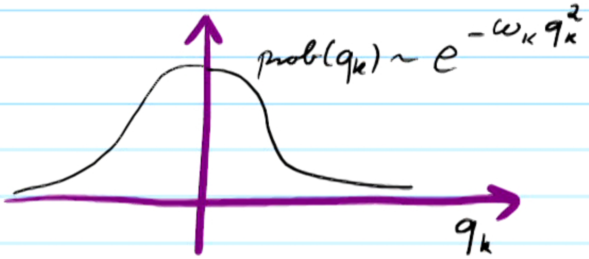
- 

4. Given $|\Psi\rangle$, calculate the probability distribution of the Fourier coefficients:

$$\hat{\phi}_k, \hat{\pi}_k$$

Can do because they are simply linear combinations of the harmonic oscillator variables $\hat{q}_k, \hat{p}_k$. (Exercise: calculate)

Example:

For $|\Psi_0\rangle$, since q_k, p_k are gaussian distributed, also the ϕ_k, π_k are gaussian distributed:

$$\text{prob}(\phi_k) \sim e^{-\omega_k \phi_k^* \phi_k}$$ (straightforward but tedious to show)

of the Fourier coefficients:

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Example:

- For $|\psi_0\rangle$, since q_k, p_k are gaussian distributed, also the ϕ_k, π_k are gaussian distributed:

$$\text{prob}(\phi_k) \sim e^{-\omega_k \phi_k^* \phi_k}$$

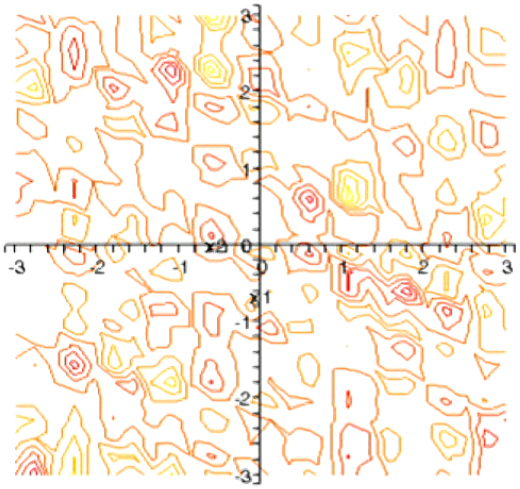
(straightforward but tedious to show)

5. Given the prob. distribution of the ϕ_k, use Fourier to obtain prob. distribution of $\phi(x)$!

Example:

- Consider $|\psi_0\rangle$.
- Draw a field $\phi(x)$ from the above calculated probability distribution for fields $\phi(x)$.

Contour lines of a typical $\phi(x,t)$ drawn from the vacuum's probability distribution for ϕ's.



Actual draw from that distribution.

The fluctuations trace back to the Fourier coefficients and to the $\hat{q}_k, \hat{p}_k$ which

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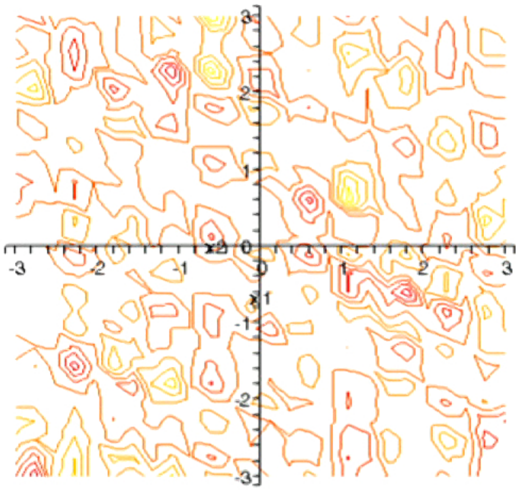

← Actual draw from that distribution.

The fluctuations trace back to the Fourier coefficients and to the $\hat{q}_k, \hat{p}_k$ which fluctuate even in lowest energy state.