Title: Quantum Field Theory for Cosmology - Lecture 1

Speakers: Achim Kempf

Collection: Quantum Field Theory for Cosmology (Kempf)

Date: January 07, 2020 - 4:00 PM

URL: http://pirsa.org/20010000

Quantum Field Theory for Cosm

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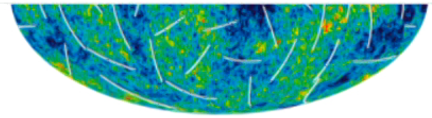
consent of instructor. Some knowledge of general relativity.



- **Time:** Tuesdays and Fridays, 4-5:20pm
- **First lecture:** 4pm, Tuesday, January 7th, 2020, Alice Room, PI
- **Venue:** Alice room, Perimeter Institute
- **Office hours:** by appointment

Content

This course introduces quantum field theory from scratch and then develops the theory of the quantum fluctuations of fields and particles. We will focus, in particular, on how quantum fields are affected by curvature and by spacetime horizons. This will lead us to the Unruh effect, Hawking radiation and to inflationary cosmology. Inflationary cosmology, which we will study in detail, is part of the current standard model of cosmology which holds that all structure in the universe - such as the distribution of galaxies - originated in tiny quantum fluctuations of a scalar field and of space-time itself. For intuition, consider that quantum field fluctuations of significant amplitude normally occur only at very small length scales. Close to the big bang, during a brief initial period of nearly exponentially fast expansion (inflation), such small-wavelength but large-amplitude quantum fluctuations were stretched out to cosmological wavelengths. In this way, quantum fluctuations are thought to have seeded the observed inhomogeneities in the cosmic microwave background - which in turn seeded the condensation of hydrogen into galaxies and stars, all closely matching the increasingly accurate astronomical observations over recent years. The prerequisites for this course are a solid understanding of quantum theory and some basic knowledge of general relativity, such as FRW spacetimes.

Project

- The grades will be based on a project. The topic is quantum field fluctuations in flat and curved space-time.
- Deadline (to email in a max of 20 pages pdf): **Sunday, 19 April 2020, 6:00 pm**

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Note Title

QFT for Cosmology, Achim Kempf, Lecture 1

Historical background:

□ ≈ 1900:

Classical mechanics became experimentally untenable:

- Black body radiation ("Ultraviolet catastrophe")
- Photoelectric effect (Ionization depends on color, not intensity)

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- Photoelectric effect (Ionization depends on color, not intensity)

- Stability of matter ($\Delta x \Delta p \geq \frac{\hbar}{2}$ implies that e^- do not spiral into the nuclei)

Finally, in 1925:

Heisenberg discovers nonrelativistic quantum mechanics (QM)

In essence:

- Equations of motion stay the same, e.g.:

$$m\ddot{\hat{x}} = -k\hat{x} \qquad \text{(harm. oscillator)}$$

- but we have noncommutativity:

$$[\hat{x}, \underbrace{m\dot{\hat{x}}}] = i\hbar \qquad \text{"canonical commutation relation"}$$

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- A few months later:

Schrödinger discovered his equation

$$i\hbar \frac{d}{dt} \psi(x,t) = -\frac{\hbar^2}{2m} \Delta \psi(x,t) + V(x,t)\,\psi(x,t)$$

- A few more months later:

Dirac showed equivalence to Heisenberg's.

Quantization implied fundamental changes:

Math: $[\hat{x}(t), \hat{p}(t)] = i\hbar 1 \neq 0 \Rightarrow \hat{x}(t), \hat{p}(t)$ not number-valued.

Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

A: No: If $\hat{x}(t), \hat{p}(t)$ were $N \times N$ matrices, then:

$$\boxed{Tr([\hat{x}, \hat{p}]) = Tr(i\hbar 1) \Rightarrow 0 = i\hbar N} \quad ↯$$

$\Rightarrow \hat{x}(t), \hat{p}(t)$ must not have well-defined trace, i.e., must act on ∞ dim. Hilbert space, i.e., must be operator-valued.

Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2} \delta_{ij}$

$\Rightarrow$ Uncertainty, i.e. "quantum fluctuations"

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Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2} \delta_{ij}$

$\rightsquigarrow$ Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

- **But:** Nonrelativistic quantum mechanics, i.e.,

$$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij} \quad \text{and} \quad i\hbar \frac{d}{dt} \hat{f}(\hat{x},\hat{p}) = [\hat{f}(\hat{x},\hat{p}), \hat{H}]$$

soon became unsatisfactory.

- **Why?** QM is not consistent with special relativity:

E.g. typical momentum of e^- in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

$\Rightarrow$ The effects of special relativity were soon spectroscopically measurable.

$[x_i, p_j] = i\hbar\delta_{ij}$ and $i\hbar \frac{d}{dt} f(x,p) = [f(x,p), H]$

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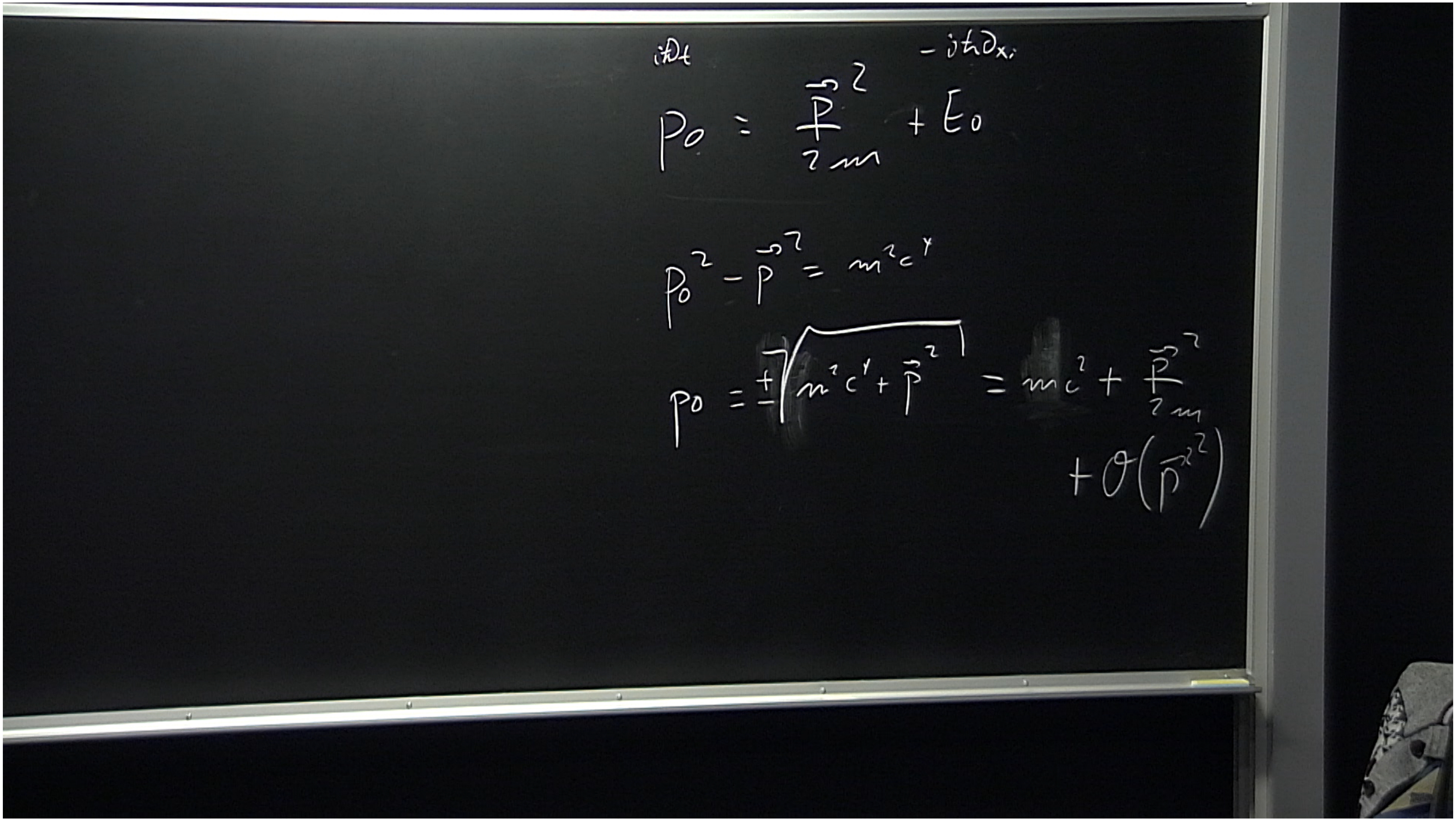
$\Rightarrow$ The effects of special relativity were soon spectroscopically measurable.

$\Rightarrow$ measurable contradiction to QM!

- Attempts to find a covariant generalization of the Schrödinger equation led to:
  - "Dirac Equation"
  - "Klein Gordon Equation" (see later)

- They had some success, but suffer serious problems too:
  - Energy not bounded from below ⇒ "instability"
  - Unitarity of time evolution unclear
  - Also: It remained unclear how particle creation and annihilation processes could be calculated.


$i\hbar\partial_t$ $-i\hbar\partial_{x_i}$
$p_0 = \frac{\vec{p}^2}{2m} + E_0$
$p_0^2 - \vec{p}^2 = m^2c^4$
$p_0 = \pm\sqrt{m^2c^4 + \vec{p}^2} = mc^2 + \frac{\vec{p}^2}{2m}$
$+ \mathcal{O}(\vec{p}^{2^2})$

- Dirac Equation"
- "Klein Gordon Equation" (see later)

- They had some success, but suffer serious problems too:
  - Energy not bounded from below ⇒ "instability"
  - Unitarity of time evolution unclear
  - Also: It remained unclear how particle creation and annihilation processes could be calculated.
- Thus, a new idea was needed!

The idea of 2nd quantization: (Heisenberg and others, 1930s)

- **Observation:**

In QM, all is subject to quantum fluctuations and therefore to uncertainty – except for the wave function $\psi(x,t)$:

Namely:

As in classical theories, if the wave function's initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or Dirac equation) determine the evolution of $\psi(x,t)$

- **Idea:**

In 2nd quantization, quantize ψ !

- **Program:**

Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in suitable units)

introduce a "momentum wave function":

$$\hat{\pi}(x,t) = \dot{\hat{\psi}}(x,t)$$

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, require:

$$[\hat{\psi}(x,t), \hat{\pi}(x',t)] = i\hbar\,\delta(x-x')$$

- Success!

- **Program:**

Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in suitable units)

introduce a "momentum wave function":

$$\hat{\pi}(x,t) = \dot{\hat{\varphi}}(x,t)$$

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, require:

$$[\hat{\varphi}(x,t), \hat{\pi}(x',t)] = i\hbar\delta(x-x')$$

- **Success!**

Problems with energy positivity, unitarity etc can be solved.

Consequences:

Math:

→ $\hat{\phi}(x,t)$ and $\hat{\pi}(x,t)$ can no longer be number-valued.

→ For each x and t the "value"

$$\hat{\phi}(x,t)$$

is an operator on a Hilbert space!

Notice: (Recall: The eqns of motion stay the same also in 1st quantization)

The equations of motion (Schrödinger, Klein-Gordon or Dirac equation) stay the

Notice: (Recall: The eqns of motion stay the same also in 1st quantization)

The equations of motion (Schrödinger, Klein Gordon or Dirac equation) stay the same only now with $\hat{\psi}, \hat{\pi}$ noncommutative.

Physics:

$$\Delta\psi(x,t)\,\Delta\pi(x',t) \geq \frac{\hbar}{2}\,\delta^3(x-x')$$

(we'll need to discuss that)

$x = (x_1, x_2, x_3)$

Physics:

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⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

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1.) Regarding particles:

(E.g. norm of wave fctn i.e. particle number no

⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

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1.) Regarding particles:

Particle creation / annihilation ↙ (E.g. norm of wave fctn i.e. particle number no longer fixed)

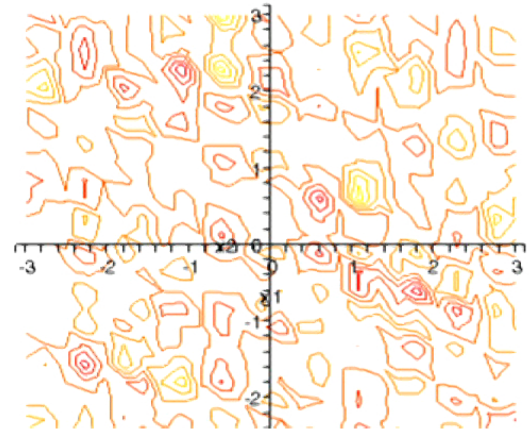
Existence of antiparticles ← (the negative energy (or mass) states can be interpreted as particles propagating backwards in time, thus to us appearing to have positive energy (or mass).)

2.) Regarding fields:

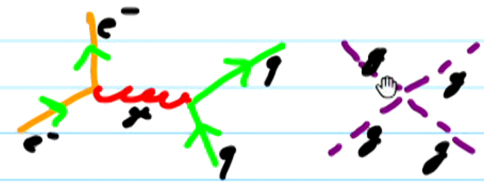
Even in the lowest energy state (i.e. no particles, i.e. in the Vacuum, the statement

$$\bar{\phi}(x,t) = \langle vacuum | \hat{\phi}(x,t) | vacuum \rangle = 0$$

allows for the values of $\hat{\phi}(x,t)$ when measured, to fluctuate:



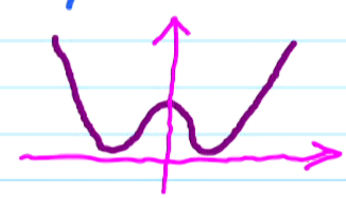
1) The Standard Model of Particle Physics



* EM, weak and strong forces

* Screening, anti-screening and renormalization

* How fundamentally massless particles can effectively acquire a mass: "Spontaneous symmetry breaking"



Namely: Groundstate has less symmetry than the action: "Higgs" particle.

* Anomalies: Quantum fluctuations reduce symmetey of the action itself.

The contrain the Standard Model of Particle Physics's structure.

2) The Standard Model of Cosmology

(the aim of this course)

Classical General Relativity + QFT
[Mostly

i.e.: Accelerations, curvature, horizons + QFT

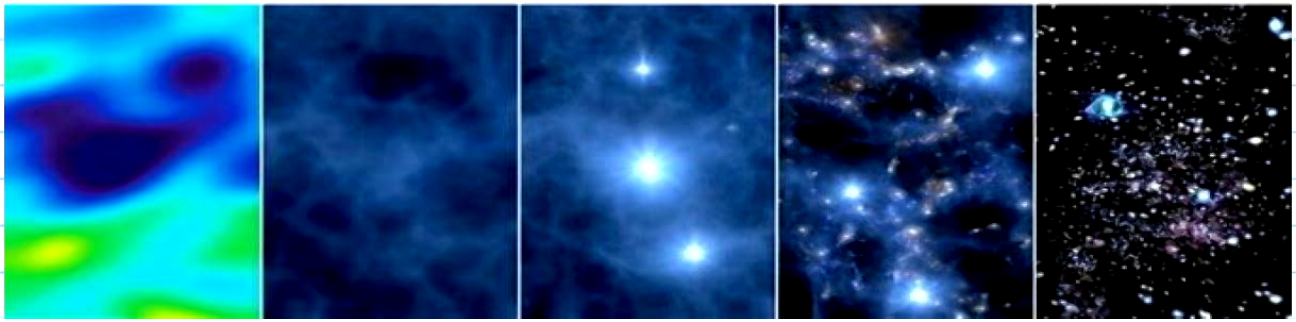
* Unruh Effect: What is a "particle"?

* Hawking Effect: Can nature destroy information?

* Cosmic Inflation: Where did it all come from?

Cosmic Inflation:

- A local quantum fluctuation of high potential $V(\phi)$ may occur.
- Acting as temporary cosm. constant, may spawn a rapidly-expanding daughter universe.
- Finally, $V(\phi) \rightarrow 0$, energy goes into particle production: plasma
- Rapid expansion amplified quantum field fluctuations.
- These fluctuations imprinted on primordial plasma, seeding galaxy formation:



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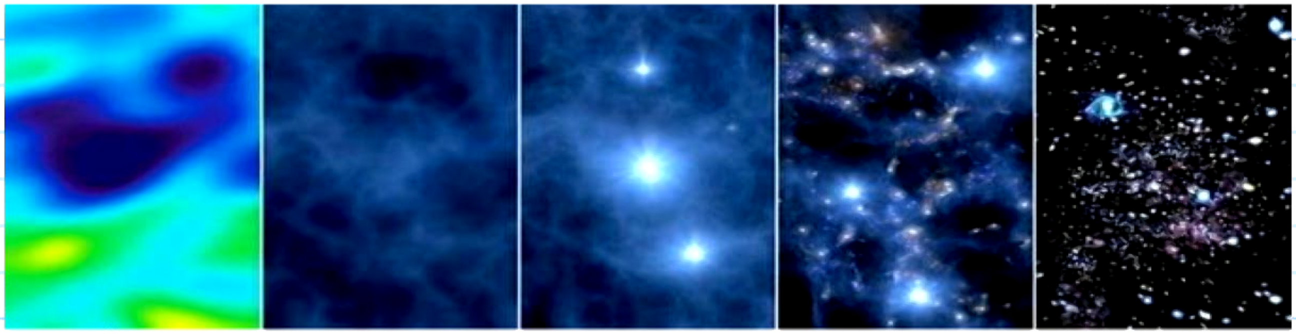

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Quantum Field Theory for Cosmology (AMATH 872/PHYS 785) in W2020

- **Term:** Winter 2020.
- **Course codes:** AMATH872 / PHYS785
- **Instructor:** Achim Kempf
- **Prerequisite:** AMATH 673 or PHYS 702 or consent of instructor. Some knowledge of general relativity.
- **Time:** Tuesdays and Fridays, 4-5:20pm
- **First lecture:** 4pm, Tuesday, January 7th, 2020, Alice Room, PI
- **Venue:** Alice room, Perimeter Institute
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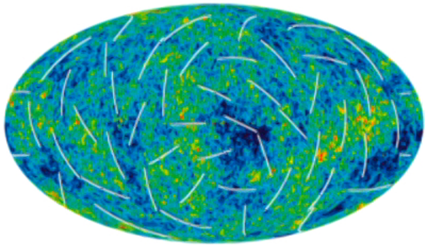

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