Title: 10 years of the quantum SWITCH: state of the art and new perspectives

Speakers: Giulio Chiribella

Collection: Indefinite Causal Structure

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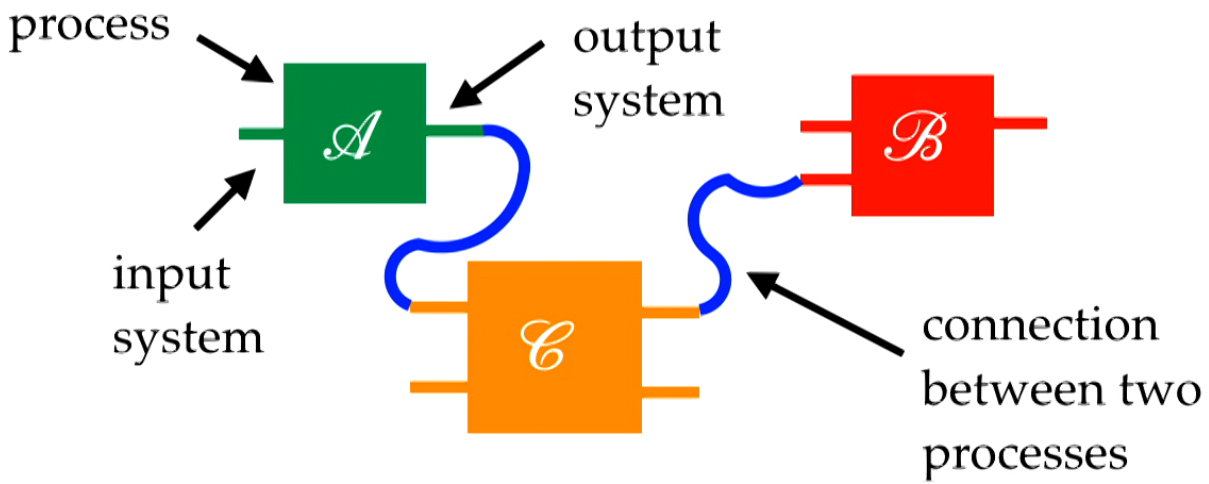
Abstract: The quantum SWITCH is the simplest example of indefinite causal structure. Technically, it is a higher-order transformation that takes two physical processes A and B in input and combines them in a coherent superposition of two alternative orders, AB and BA. In the past decade, the quantum SWITCH has been the object of active research, both theoretically and experimentally. In this talk, I will review the state of the art, and outline two new applications to quantum Shannon theory and quantum metrology.

OUTLINE

- The quantum SWITCH: definition and basic facts
- Realization with known physics
- Information-processing advantages
- Quantum SWITCH in Shannon theory
- Quantum SWITCH in quantum metrology.

THE QUANTUM SWITCH: DEFINITION

NETWORKS OF QUANTUM PROCESSES



- Quantum processes, a.k.a. quantum channels
  = ***maps*** transforming density matrices into density matrices
- Connections = ***higher-order maps***, transforming channels into channels. Can be used to represent causal structure.

cf. Hardy's causaloid (JPA, 2005),
Chiribella-D'Ariano-Perinotti supermaps (PRL 2008,EPL 2008,PRA 2009)

THE (SIMPLIFIED) QUANTUM SWITCH

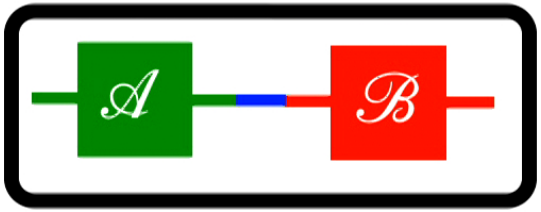
The *(simplified)* quantum SWITCH is the supermap that

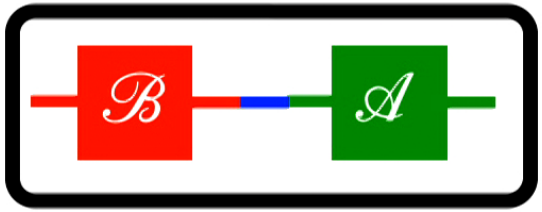
takes as input the two processes

and

with equal inputs/outputs

and *connects* them in a coherent superposition of the two configurations



and 

Chiribella, D'Ariano, Perinotti, Valiron, arXiv:0912.0195;
Phys. Rev. A 88, 022318 (2013),

DEFINITION OF SIMPLIFIED QUANTUM SWITCH

- Input channels:

$$\mathscr{A}(\rho) = \sum_i A_i \rho A_i^\dagger \qquad \text{and} \qquad \mathscr{B}(\rho) = \sum_j B_j \rho B_j^\dagger$$

- Output of simplified quantum SWITCH:

$$\mathscr{S}(\mathscr{A}, \mathscr{B})(\rho) = \sum_{i,j} S_{ij} \rho S_{ij}^\dagger$$

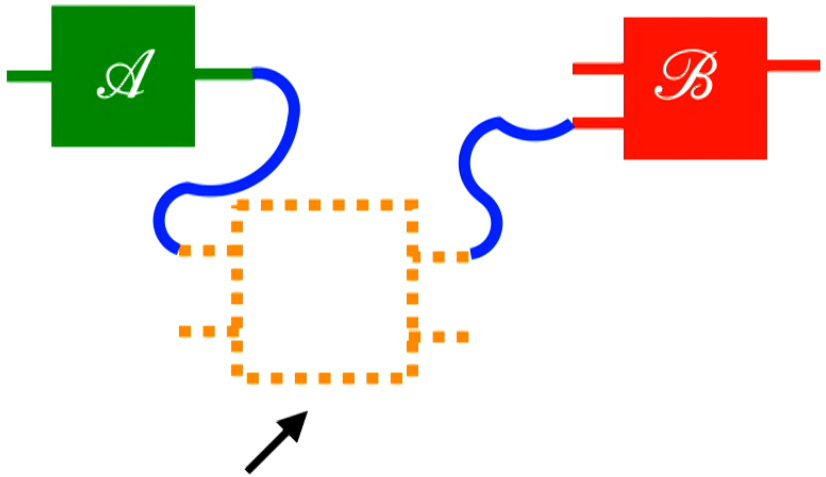
$$S_{ij} := A_i B_j \otimes |0\rangle\langle 0| + B_j A_i \otimes |1\rangle\langle 1|$$

states of an "order qubit"

- Simplified Quantum SWITCH:
  supermap $\mathscr{S} : (\mathscr{A}, \mathscr{B}) \mapsto \mathscr{S}(\mathscr{A}, \mathscr{B})$

MORE GENERALLY

It is useful to consider also *neworks with open slots*



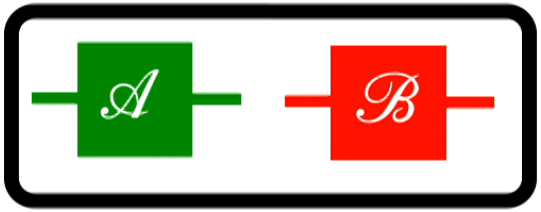
open slot,
where variable process can be inserted

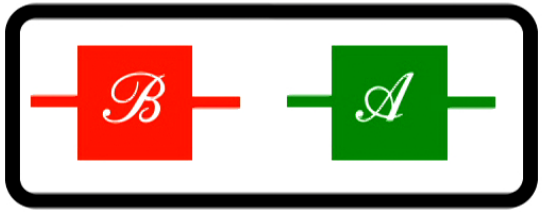
THE FULL QUANTUM SWITCH

The quantum SWITCH is the higher-order transformation that takes as input the two processes

and

and places them in a coherent superposition of the two configurations



and 

Chiribella, D'Ariano, Perinotti, Valiron, arXiv:0912.0195;
Phys. Rev. A 88, 022318 (2013),

DEFINITION OF FULL QUANTUM SWITCH

- Output channel of simplified quantum SWITCH:

$$\mathcal{S}(\mathcal{A},\mathcal{B})(\rho) = \sum_{i,j} S_{ij}\rho S_{ij}^{\dagger}$$

$$S_{ij} := A_i \otimes B_j \otimes |0\rangle\langle 0| + B_j \otimes A_i \otimes |1\rangle\langle 1|$$

acting on systems in the first time slot

acting on systems in the second time slot

- Quantum SWITCH:
  supermap $\mathcal{S} : (\mathcal{A},\mathcal{B}) \mapsto \mathcal{S}(\mathcal{A},\mathcal{B})$

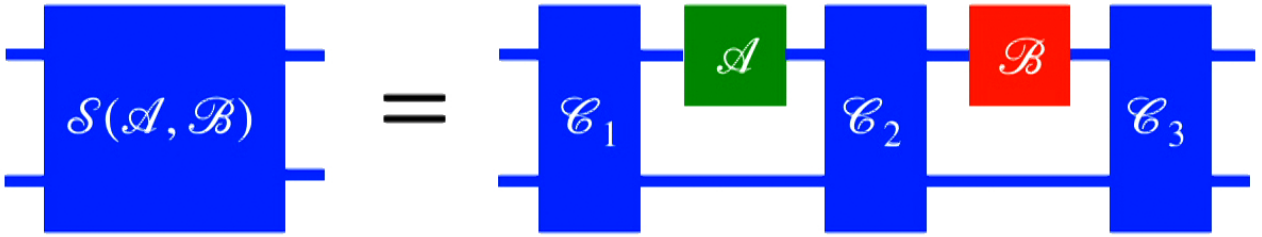
Full quantum SWITCH is important because it generates all $N!$ permutations of N channels.

BASIC FACTS

INCOMPATIBILITY WITH FIXED CAUSAL ORDER

Theorem

It is impossible to find quantum channels
and quantum channels $\mathscr{C}_1$, $\mathscr{C}_2$, and $\mathscr{C}_3$ such that



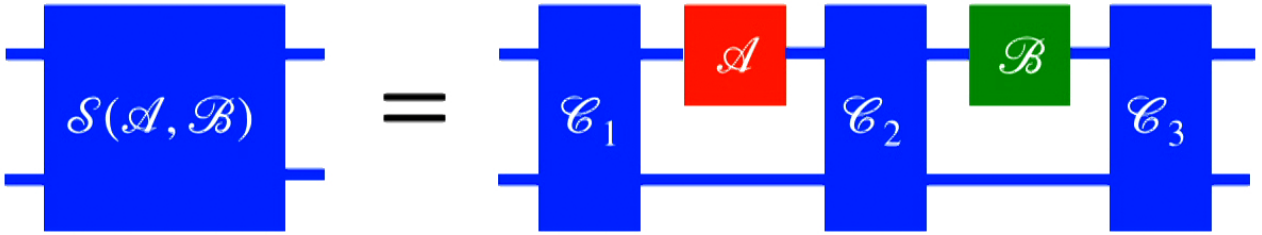
for all unitary and

(same holds with $\mathcal{A}$ and $\mathcal{B}$ in the opposite order,
and for classical mixtures of the two orders)

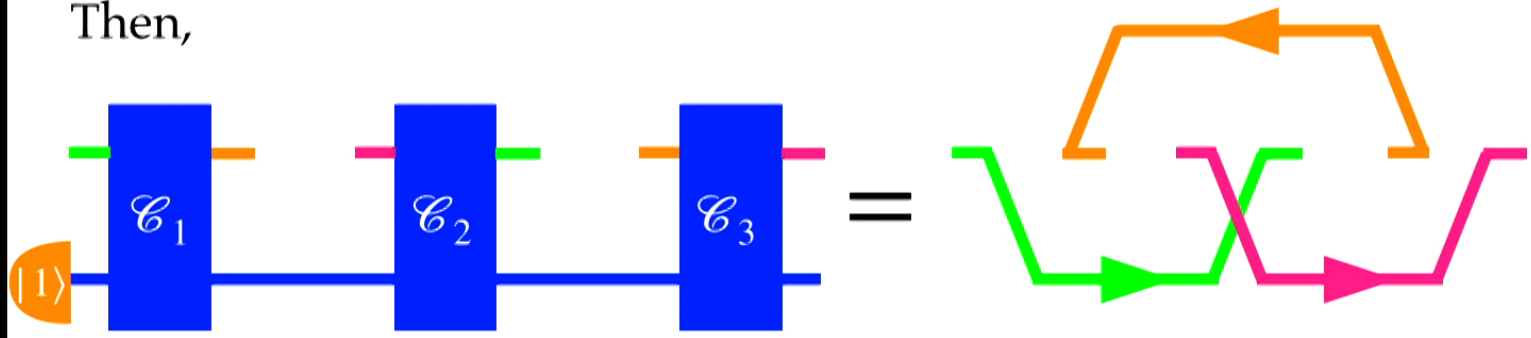
The quantum SWITCH has *indefinite causal structure.*

IDEA OF PROOF

By contradiction: suppose that

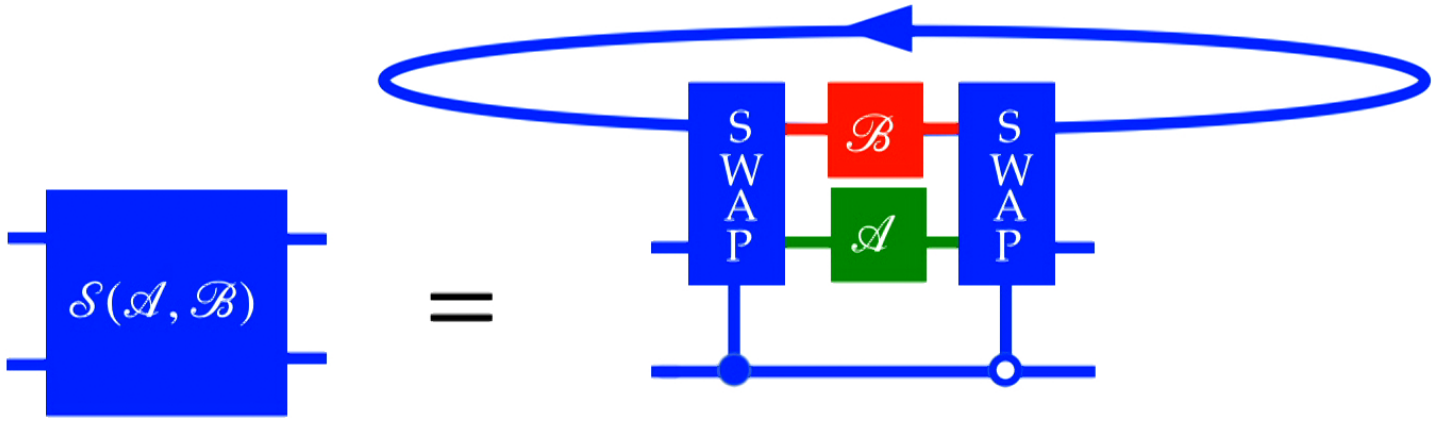


Then,



The r.h.s. cannot be a sequence of quantum channels:
the orange line is a time loop! (output appears before input)

REALIZATION OF THE SWITCH IN A CIRCUIT WITH LOOP



Chiribella, D'Ariano, Perinotti, Valiron, arXiv:0912.0195;
Phys. Rev. A 88, 022318 (2013),

- **True time loop:** maybe in exotic quantum gravity scenarios
- **Simulated time loop:** with conclusive teleportation

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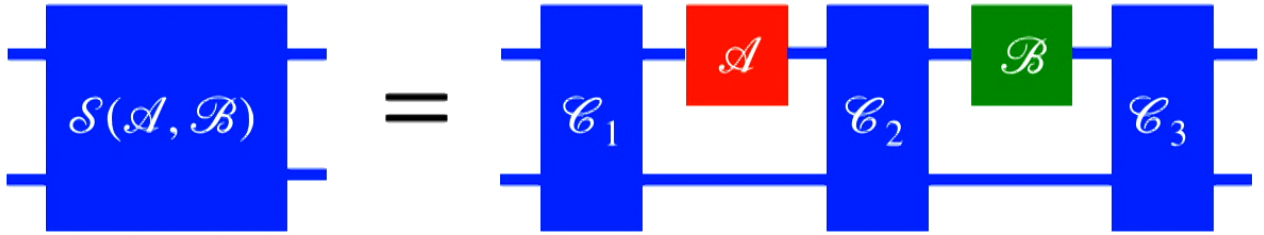

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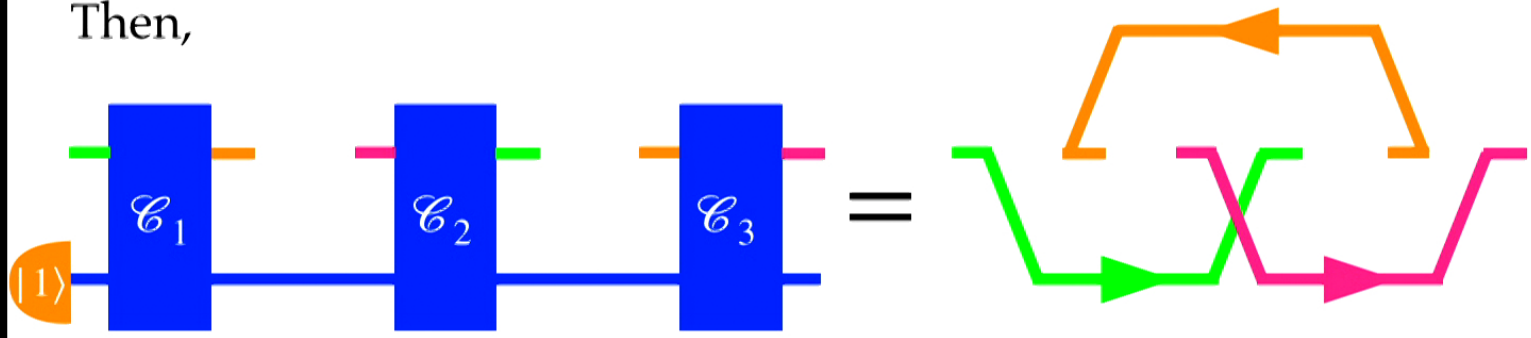

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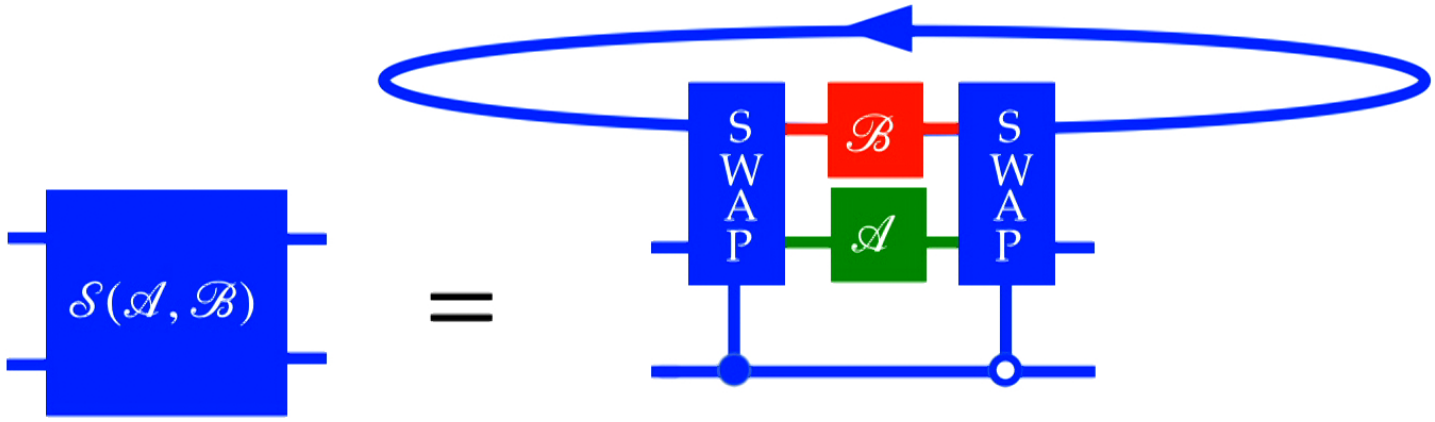

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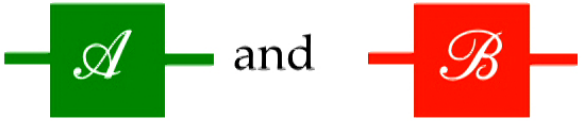
INFORMATION-PROCESSING ADVANTAGES OF THE QUANTUM SWITCH

REDUCING QUERY COMPLEXITY

- Testing properties of processes 

e.g. discovering if Kraus operators commute or anti-commute

probability of correct answer = 1 with the quantum SWITCH

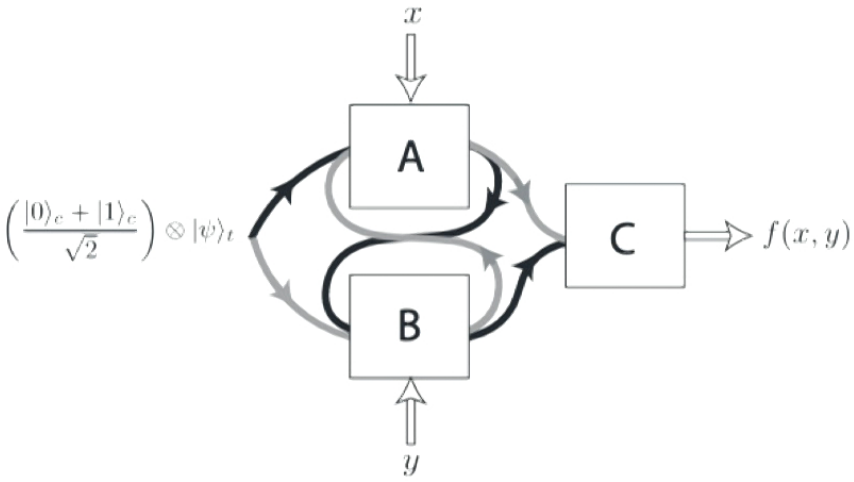
< 1 for every testing strategy where $\mathscr{A}$ and $\mathscr{B}$ are connected in a definite order.

Chiribella, PRA 86, 040301(R) (2012)

Extension to N channels:
Araujo, Costa, and Brukner, PRL 113, 250402 (2014)

REDUCING COMMUNICATION COMPLEXITY

The ability to distinguish between commuting and anticommuting channels is a primitive that can be used to reduce the amount of communication needed by 3 distant parties to compute a desired function.



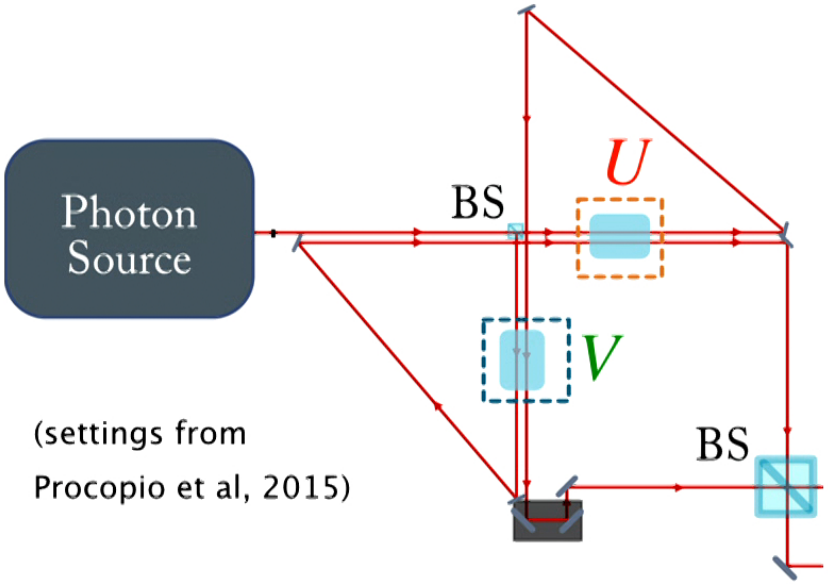
- Causally ordered: $O(N)$ bits
- SWITCH: $O(\log N)$ bits

N = size of the input strings

Guerin, Feix, Araujo, and Brukner, PRL 117, 100502 (2016)

REALIZATION
OF
(THE OUTPUT OF)
THE QUANTUM SWITCH
WITH
KNOWN PHYSICS

PHOTONIC REALIZATIONS



(settings from Procopio et al, 2015)

Procopio et al, Nat. Commun. 6:7913 (2015)

Rubino et al, Sci. Adv. 3 1602589 (2017)

Goswami et al, PRL 121 090503 (2018)

Goswami et al arXiv:1807.07383

Guo et al arXiv:1811.07526

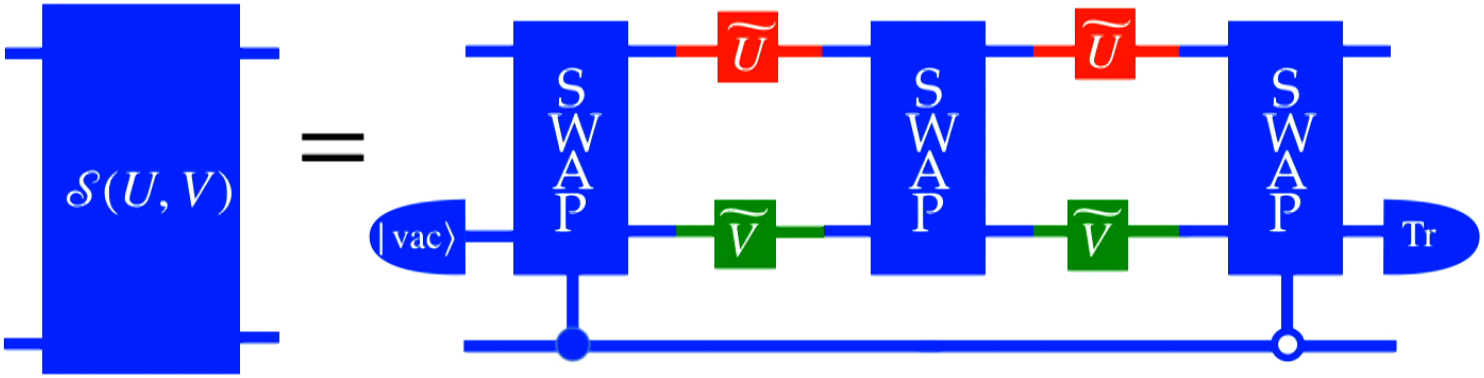
Wei et al, PRL 122, 120504 (2019)

Experiments realize the *output of the quantum SWITCH*

$$S(U,V) := UV \otimes |0\rangle\langle 0| + VU \otimes |1\rangle\langle 1|$$

Photon experiences the processes U and V
in a superposition of two causal orders.

CIRCUIT REPRESENTATION OF THE EXPERIMENT



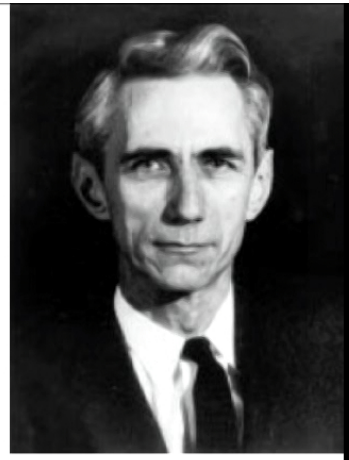
Differences with the original quantum SWITCH:

- requires access to *extended* unitaries $\widetilde{U}$ and $\widetilde{V}$, acting on single-photon subspace and on vacuum
- requires *coherence with the vacuum*
- requires *coherence in time* (action of the devices should not change between one time step and the next)

QUANTUM SHANNON THEORY ON INDEFINITE CAUSAL STRUCTURE

(CLASSICAL) SHANNON THEORY:

The carriers of information are classical:
classical states, classical channels,
classical configuration of the channels



Claude E. Shannon

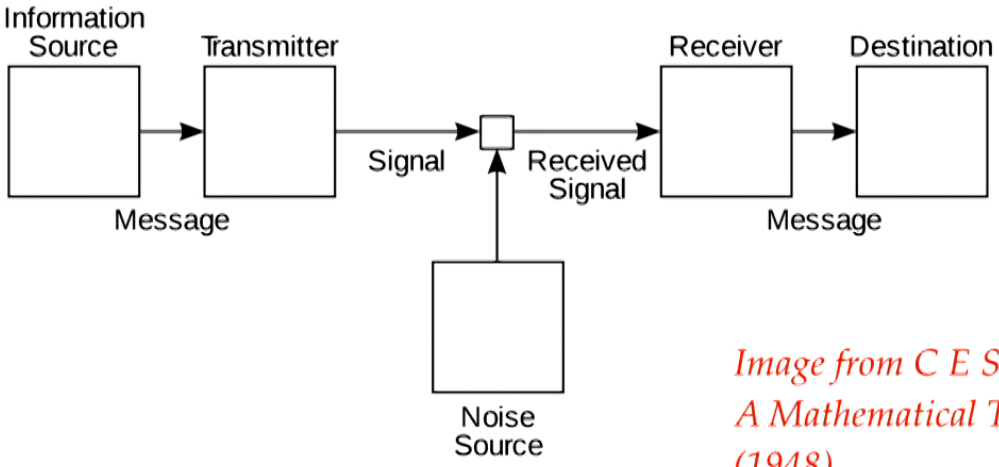


Image from C E Shannon,
A Mathematical Theory of Communication
(1948)

QUANTUM SHANNON THEORY

Allows the state of the information carriers and the channels be quantum.

Messages can be quantum:
not just strings of bits, like 0010110111,
but also quantum superpositions, like

$$|\Psi\rangle = \frac{|0010110111\rangle + |1010100011\rangle}{\sqrt{2}}$$

Still, the configuration of the communication channels is fixed.

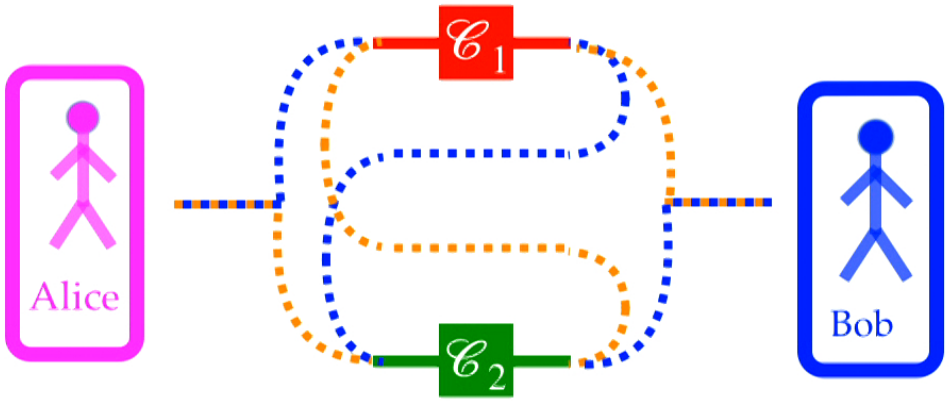
Alexander Holevo

Benjamin Schumacher

Charles Bennett Gilles Brassard

QUANTUM SHANNON THEORY ASSISTED BY SUPERPOSITION OF CAUSAL STRUCTURES

Suppose that Alice and Bob are embedded in an indefinite causal structure, so that the communication devices between them are placed in an indefinite order.



How is the exchange of information affected?

PLACEMENT OF THE CHANNELS

Suppose that the communication between a sender (Alice) and a receiver (Bob) uses n communication devices, corresponding to channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$

The causal structure determines
how the devices are placed between Alice and Bob.

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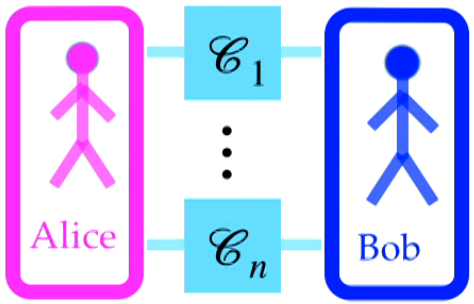
e.g. parallel placement



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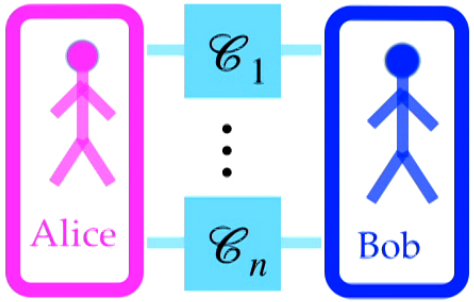
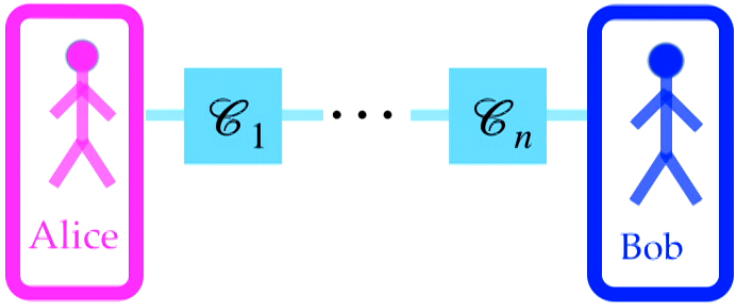


vs sequential placement



PLACEMENT SUPERMAPS

A *placement* is a supermap transforming a list of channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$ into a communication channel between Alice and Bob

e.g. parallel placement:

$$(\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n) \mapsto \mathscr{C}_1 \otimes \mathscr{C}_2 \otimes \cdots \otimes \mathscr{C}_n$$

e.g. sequential placement with repeaters:

$$(\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n) \mapsto \mathscr{C}_n \circ \mathscr{R}_{n-1} \circ \mathscr{C}_{n-1} \circ \cdots \mathscr{C}_2 \circ \mathscr{R}_1 \circ \mathscr{C}_1$$

Idea: adding superposition of orders to the allowed placements.

SHANNON THEORETIC ADVANTAGES

- Model 1: parallel + sequential placement only
- Model 2: parallel, sequential, and superposition of orders

Model 2 outperforms Model 1 in several situations:

-classical data transmission:

Model 2 enables transmission of bits with completely depolarizing channels, which would not permit any transmission in Model 1

Ebler, Salek, Chiribella, Phys. Rev. Lett. 120, 120502 (2018)

Goswami, Romero, White Goswami arXiv:1807.07383.

-quantum data transmission:

Model 2 enables transmission of qubits channels that would not permit any transmission in Model 1

Salek, Ebler, Chiribella, arXiv:1809.06655

Chiribella et al, arXiv:1810.10457

NEW RESOURCE THEORIES OF QUANTUM COMMUNICATION

The extension of quantum Shannon theory with superposition of orders has revived interest in other forms of superpositions:

-superpositions of communication channels
Gisin, Linden, Massar, Popescu, PRA 72, 012338 (2005).
Abbott, Wechs, Horsman, Mhalla, Branciard, arXiv:1810.09826
Chiribella and Kristjánsson, Proc. Royal Soc. A 475, 20180903 (2019)

-superpositions of encoding/decoding operations
Guerin, Rubino, Brukner PRA 99, 062317 (2019)

General framework for resource theories of quantum communication,
Kristjánsson, Salek, Ebler, Chiribella, arXiv:1910.08197

QUANTUM METROLOGY WITH INDEFINITE CAUSAL ORDER

Zhao, Yang, Chiribella 1912.02449

BACK TO THE HARMONIC OSCILLATOR!

Consider harmonic oscillator with position and momentum operators X and P, respectively, and Hamiltonian $H = (X^2 + P^2)/2$

Canonical commutation relation $[X, P] = i\,I$
or equivalently, $D_x D_p = e^{ixp}\, D_p D_x$
with $D_x = \exp[-ixP]$ and $D_p = \exp[ipX]$

Phase space area: $A = x\,p$

MEASURING THE PRODUCT OF TWO AVERAGE DISPLACEMENTS

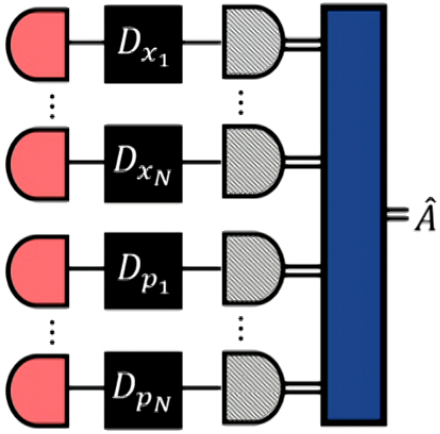
A harmonic oscillator is subject to a sequence of displacements, either of its position or of its momentum

$$D_{x_j} = \exp[-ix_j P] \qquad j \in \{1,\ldots,N\}$$
$$D_{p_k} = \exp[ip_k X] \qquad k \in \{1,\ldots,N\}$$

Task: estimate the product $A = \bar{x}\,\bar{p}$ of the average displacements $\bar{x} = \frac{1}{N}\sum_j x_j$ and $\bar{p} = \frac{1}{N}\sum_k p_k$

CAUSALLY ORDERED STRATEGIES (1)

Strategy 1: estimate each displacement independently



For a single displacement z,
the *Root Mean Square Error (RMSE)* is

$$\Delta z = \frac{1}{\sqrt{8\nu E}} \quad \text{with}$$

ν = number of repetitions of the experiment

$E = \langle X^2 + P^2 \rangle / 2$ = average energy of the probe

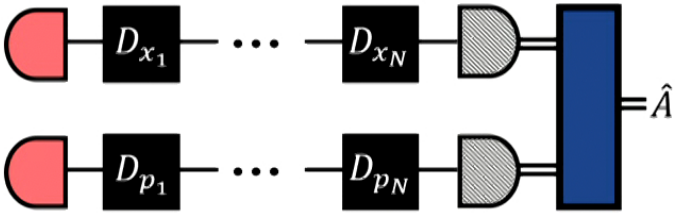
The averages and their product are computed classically.

RMSE for the product: $\Delta A = O\left(\frac{1}{\sqrt{\nu N}}\right)$

(standard quantum limit w.r.t. N)

CAUSALLY ORDERED STRATEGIES (2)

Strategy 2: directly estimate the two average displacements



For an average displacement $\bar{z}$, the *RMSE* is $\Delta\bar{z} = \dfrac{1}{\sqrt{8\nu E}\,N}$

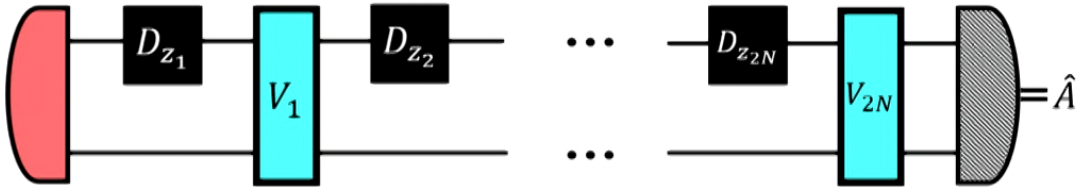
The product of $\bar{x}$ and $\bar{p}$ is computed classically.

RMSE for the product: $\Delta A = O\left(\dfrac{1}{\sqrt{\nu}N}\right)$

(Heisenberg limit w.r.t. N)

CAUSALLY ORDERED STRATEGIES (3)

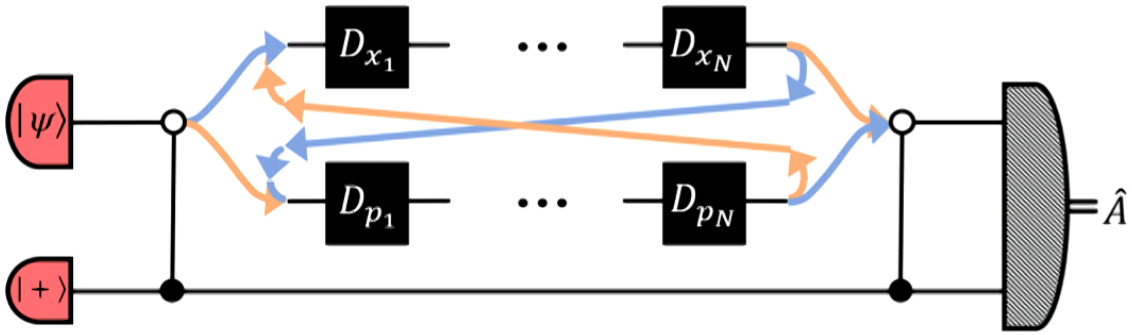
Most general causally ordered strategy



General bound on RMSE: $$\Delta A_{\text{fixed}} = \Omega\left(\frac{1}{\sqrt{\nu N}}\right)$$

No causally ordered scheme can beat Heisenberg limit w.r.t. N

SWITCH-ENHANCED ESTIMATION



Final state of the probe and control qubit:

$$\frac{1}{\sqrt{2}} D_{N\bar{p}} D_{N\bar{x}} |\psi\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} D_{N\bar{x}} D_{N\bar{p}} |\psi\rangle \otimes |1\rangle$$

$$= D_{N\bar{p}} D_{N\bar{x}} |\psi\rangle \otimes \frac{|0\rangle + e^{iN^2 A} |1\rangle}{\sqrt{2}}$$

ADVANTAGE OF THE QUANTUM SWITCH

For small A, the quantum SWITCH yields

$$\Delta A_{\text{switch}} = \frac{1}{\sqrt{\nu} N^2}$$

to be contrasted with the bound $\Delta A_{\text{fixed}} = \Omega\left(\frac{1}{\sqrt{\nu} N}\right)$ for general strategies with definite causal order.

In general, the quantum SWITCH enables estimation of the phase $\phi = \sum_{i,j} x_i p_j \mod 2\pi$ with error $\Delta\phi = \frac{1}{\sqrt{\nu}}$

whereas causally-ordered strategies have error $\Delta\phi = \Omega\left(\frac{N}{\sqrt{\nu}}\right)$

TOWARDS IMPLEMENTATIONS

- Applications to tests of canonical commutation relations, measurements of fundamental constants.

- Experimental challenge: to implement *quantum SWITCH on a harmonic oscillator.*
  e.g.
  -vibrational modes of molecule, with path as control
  -axial modes of ion traps, with spin as control.

TAKE HOME MESSAGES

(1) Quantum SWITCH VS simplified quantum SWITCH

(2) Relation with time loops

(3) Information-processing advantages:
query complexity + communication complexity

(4) Realizations with known physics:
reproducing the output of the quantum SWITCH

(5) Quantum Shannon theory with assistance of indefinite causal order: placement of channels as a resource

(6) Quantum metrology with indefinite causal order:
super-Heisenberg limit