Title: Spring-loading electrons and other shenanigans of superoscillatory wave functions

Speakers: Achim Kempf

Series: Quantum Foundations

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Abstract: A superoscillatory function is a bandlimited function that, on some interval, oscillates faster than the highest frequency component shown in the function's Fourier transform. Superoscillations can be arbitrarily fast and of arbitrarily long duration but come at the expense of requiring a correspondingly large dynamic range. I will review how superoscillatory wave forms can be constructed and I will discuss the unusual behavior of wave functions that superoscillate. For example, they can describe particles that automatically strongly accelerate when passing through a slit. A postselected stream of them represents a ray that cools the slit walls, raising foundational and thermodynamic questions. Superoscillatory wave forms are already being used for practical applications such as spatial resolution beyond the diffraction limit.

Spring-loading electrons and other shenanigans of superoscillatory wave functions

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Joint work with
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PI Foundations seminar, 3 Dec. 2019

Overview

1. What are superoscillations?
2. How to make them?
3. What are they good for?
4. At what cost?
5. What can we learn from their existence?

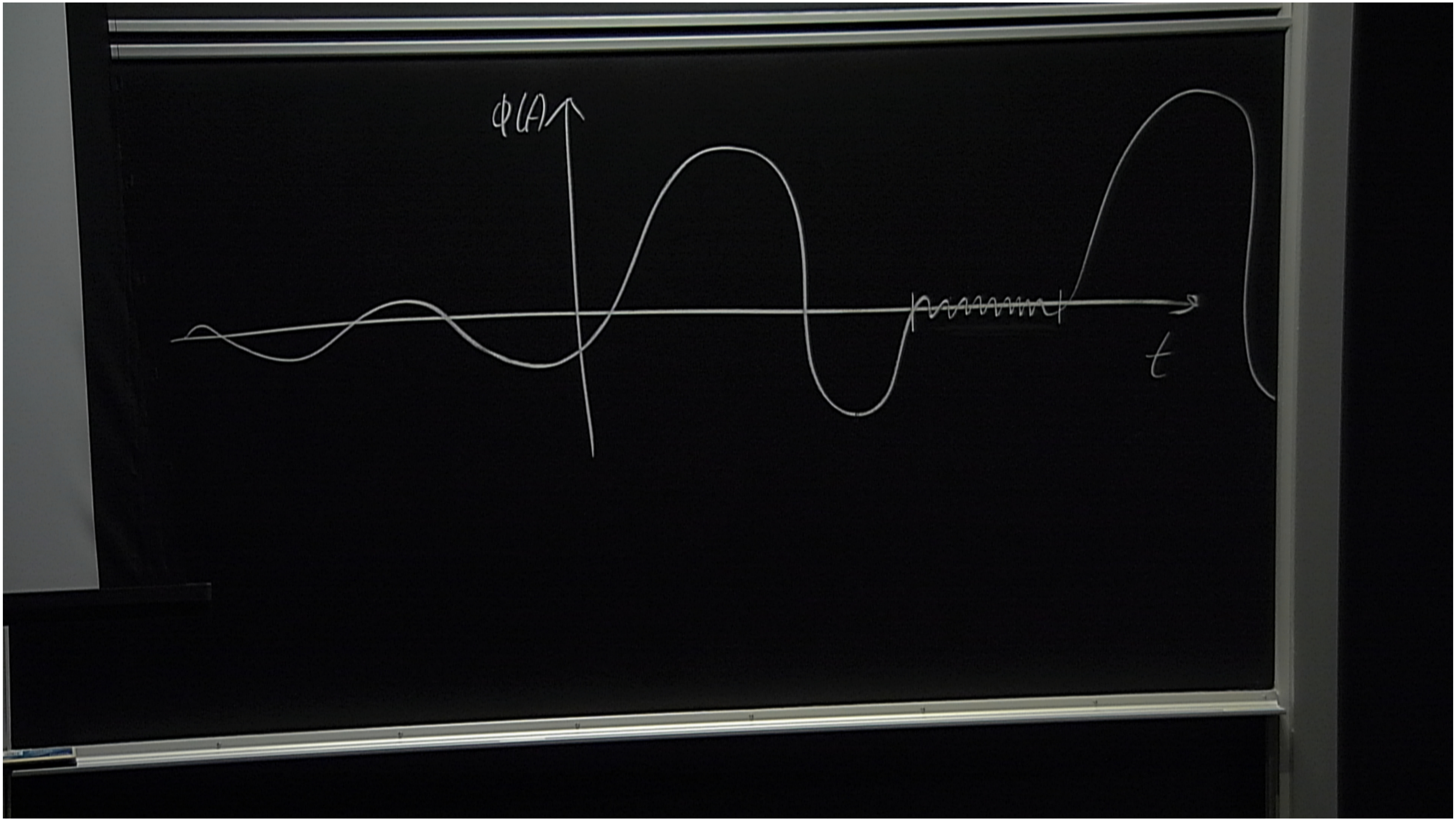
What are superoscillations?

Superoscillations are functions that locally oscillate faster than their fastest Fourier component.

Development of the theory:

- 1990s: Discovered numerically (Aharonov, Berry et al)
- Constructed analytically, unstable (AK et al)
- Several constructions, unstable (multiple groups, AK et al).
- First numerically stable method (AK, L. Chojnacki).
- Oct 2019: new stable method allows detailed design (AK, B. Soda)

In hindsight: there were early theoretical and experimental hints at the existence of superoscillations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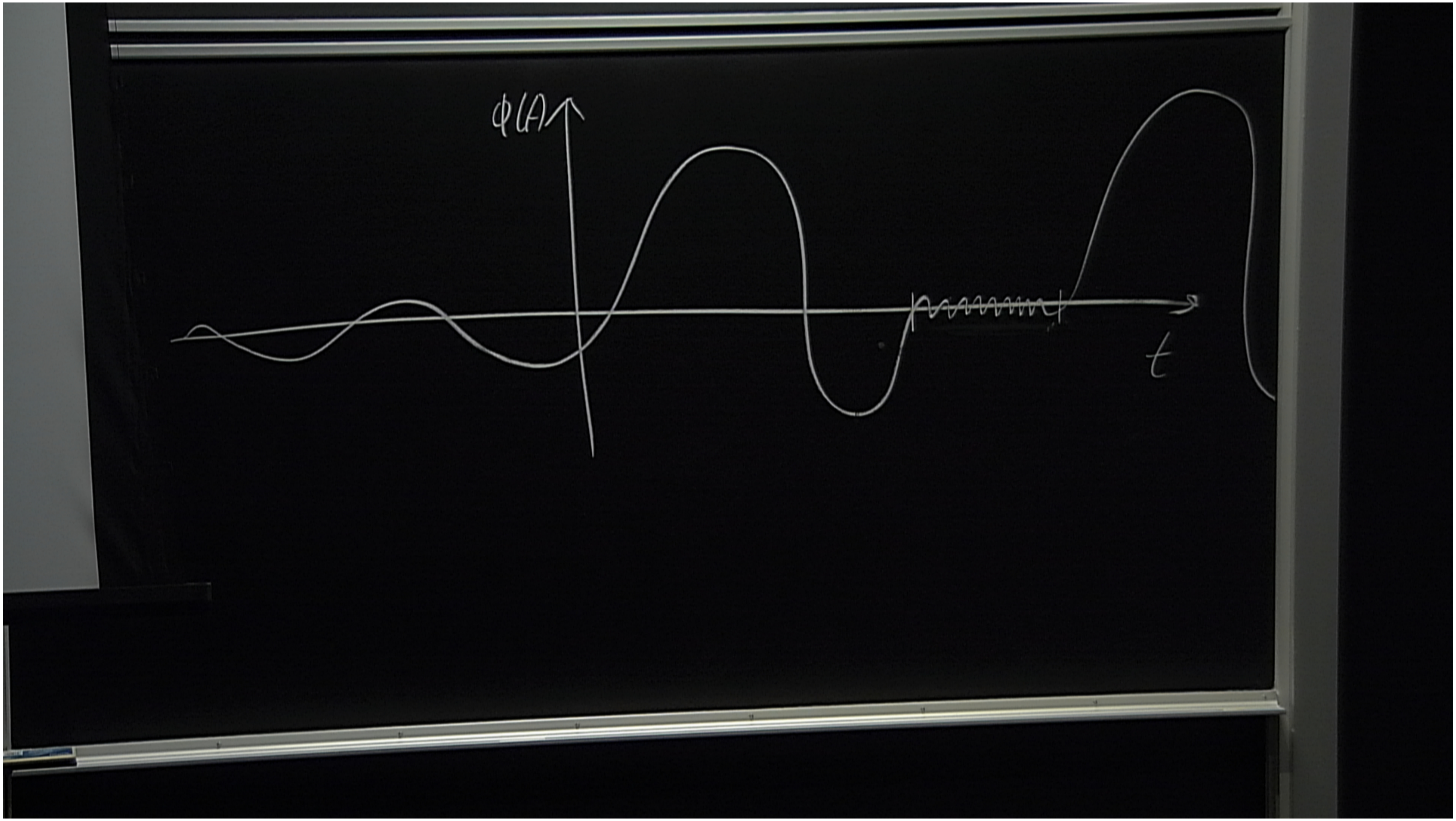
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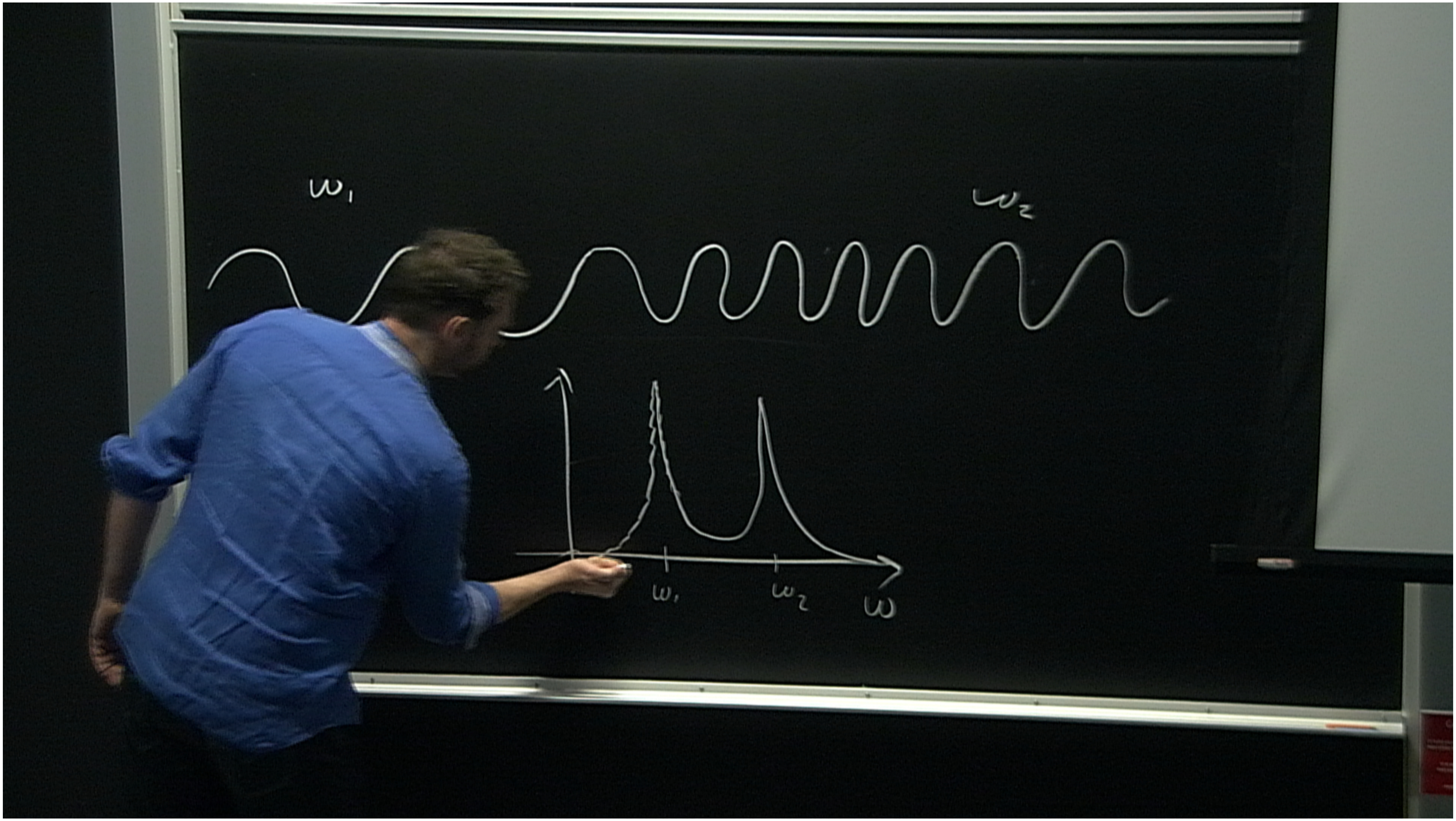

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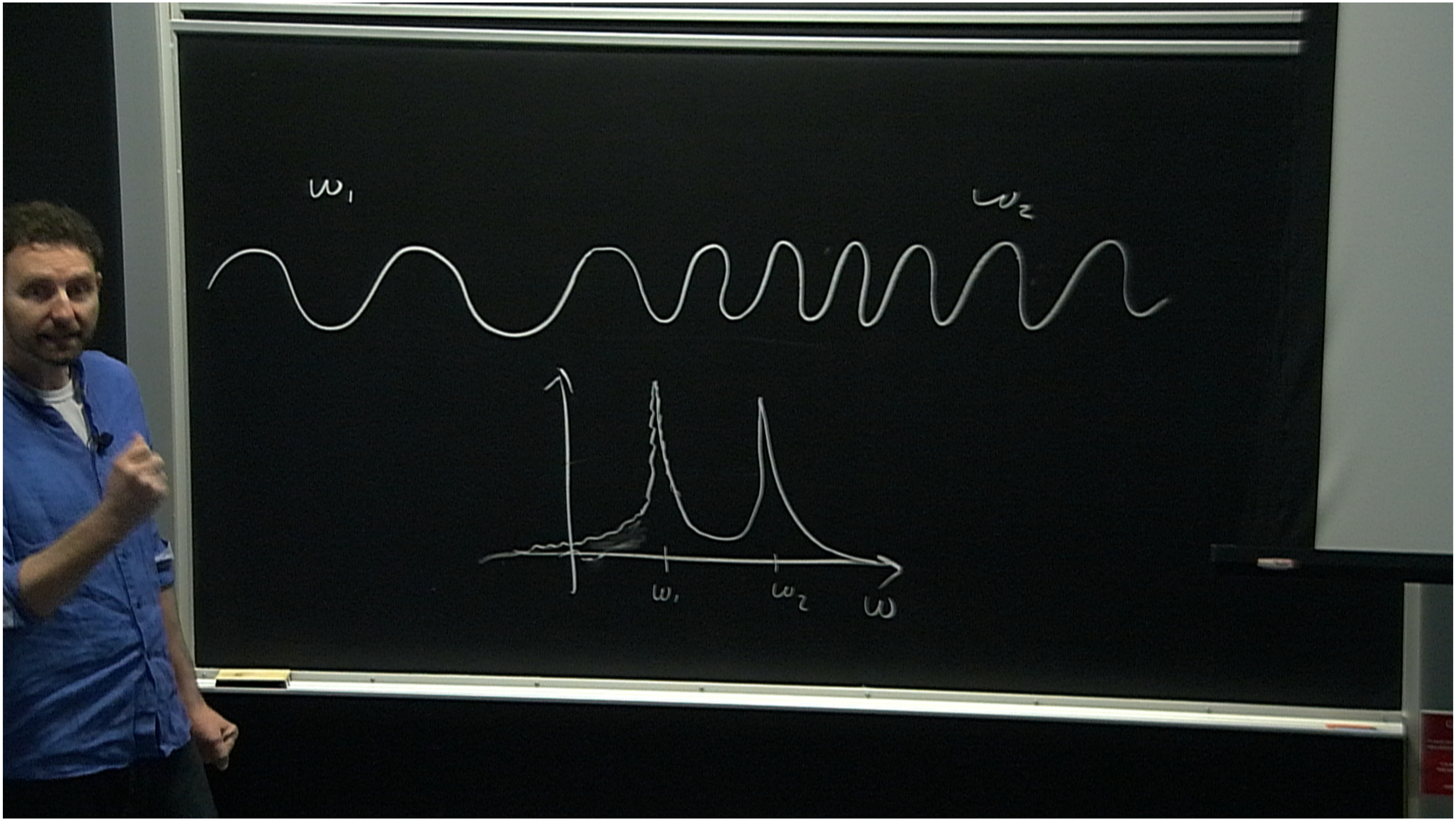
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2. How to construct superoscillations?

First numerically stable method (AK, L. Chojnacki, 2016):

Assume N functions $\{g_n\}_{n=1}^{N}$ are each Ω/N-bandlimited. Then:

- $\phi := \prod_{n=1}^{N} g_n$ is Ω-bandlimited.
- ϕ possesses the zero-crossings of all of the g_n.

Choose the functions g_n to possess close-by zero crossings (e.g., functions that are small translates of another).

$\implies$ h is superoscillatory.

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2. How to construct superoscillations?

First numerically stable method that also allows one to design the shape of the superoscillations freely. (AK, Barbara Šoda, Oct. 2019):

Basic idea: Use the fact that polynomials have zero bandwidth.

1. Choose a smooth function f with desired behavior in $[-L, L]$.
2. Calculate the Taylor series $f_N(x) := \sum_{n=0}^{N} \frac{f^{(n)}}{n!} x^n$
3. Chose a function g of bandlimit Ω obeying $g(x) \approx 1 \quad \forall x \in [-L, L]$ with some desired accuracy. Example: a flat sinc.
4. $\psi(x) := f_N(x) g(x)$ has BW Ω and $\psi(x) \approx f(x)$ in $[-L, L]$

What are superoscillations good for?

- To overcome general wavelength-limitations
  - e.g. in optogenetics
  - e.g. in bandwidth-limited (rather than S/N-limited) communication
- For super-resolution, i.e., for resolution beyond the diffraction limit.
  - e.g. in optics
  - e.g. in radar
- To probe fast dissipative processes in media

Example: Landmine detection

Landmines possess sizes of order of 10cm.

* Radar of such wavelengths is absorbed by humidity.

* Use superoscillatory radar pulses with wavelengths above 1m.

* The water molecules **should** only temporarily get excited by the 10cm wavelength superoscillations.

$\Longrightarrow$ If these radar pulses aren't absorbed, obtain superresolution!

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What is the cost for having superoscillations?

We proved two scaling laws.

The minimum dynamic range of superoscillatory functions grows:

- exponentially with the number of superoscillations,
- polynomially with the frequency of the superoscillations.

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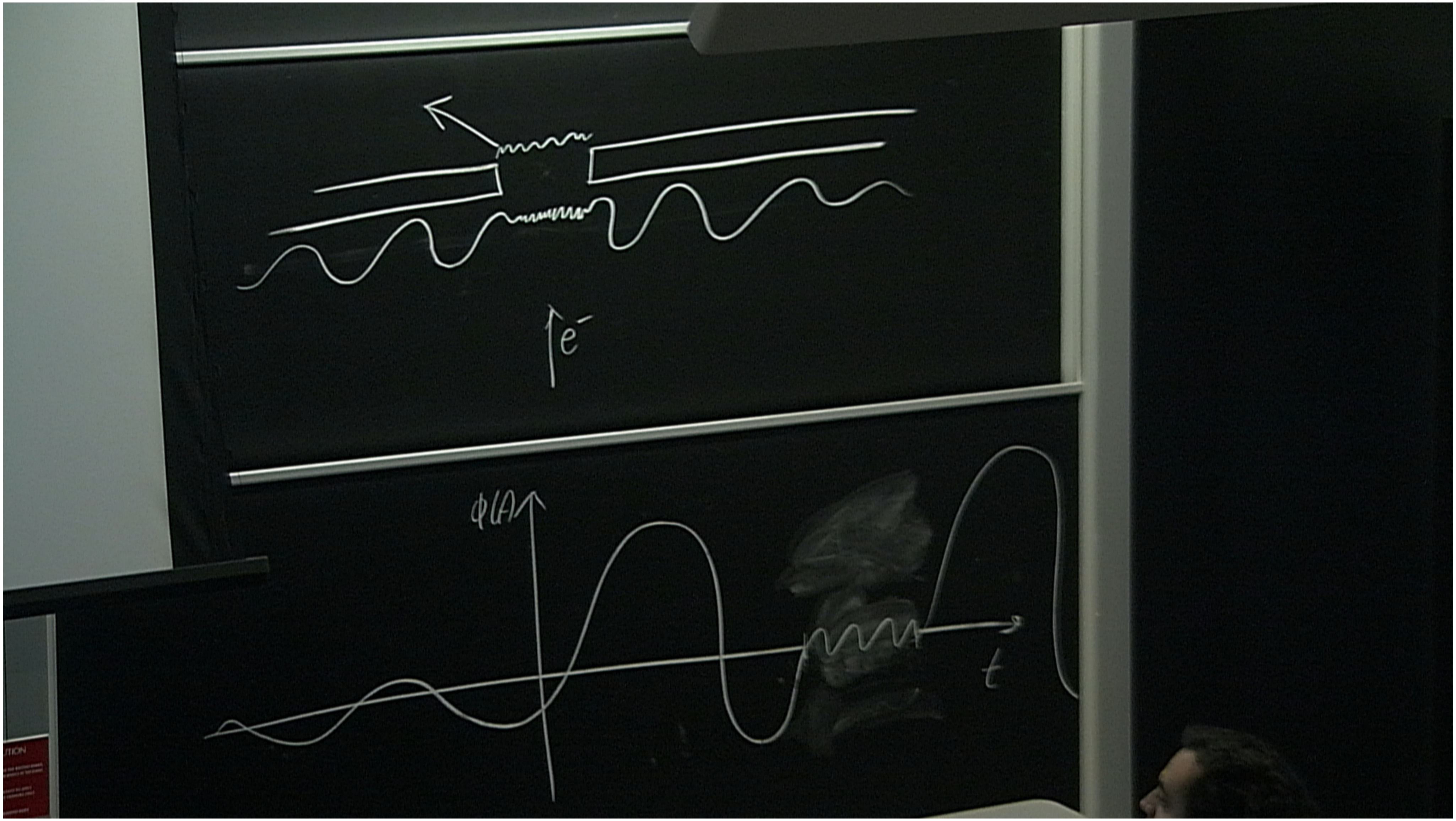
What insights may be gained from superoscillations?

(A) Shannon's noisy channel capacity theorem may be generalizable to cover all noise models at once.

(B) Quantum thermodynamics needs to account for superoscillatory **cooling rays.**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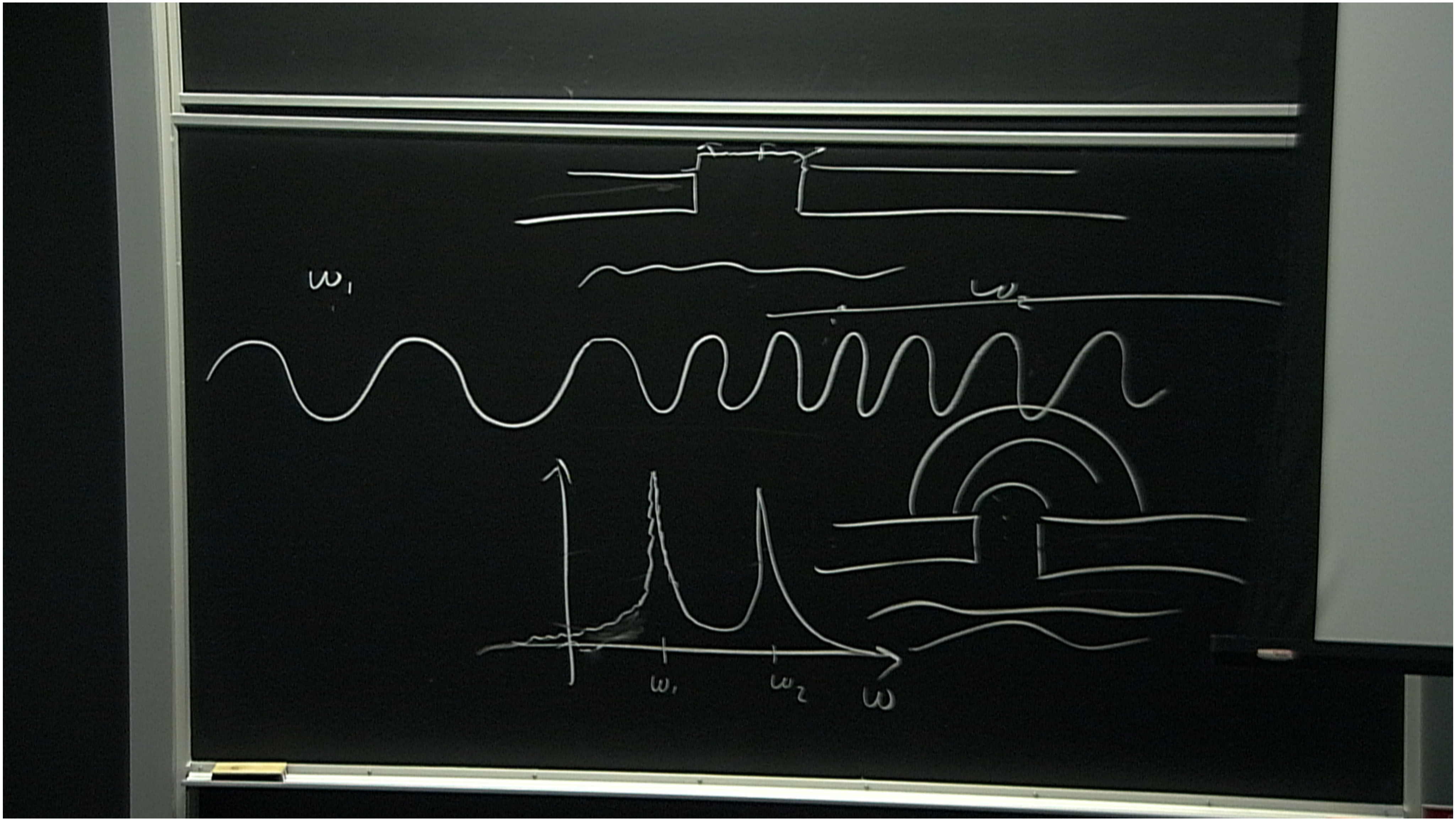

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(B) Cooling Rays

Assume a beam of electrons towards a screen with a slit, superoscillatory where the slit is.

- Electrons that pass the slit will be accelerated.
- The energy comes from the slit walls (or else slit location must be uncertain).

How does quantum thermodynamics account for this?

Experimental challenge: How to experimentally produce superoscillatory wave functions?

(A) Superoscillations and channel capacity

Recall the Shannon Hartley theorem:

Consider an Ω-bandlimited channel with *additive Gaussian noise* and average signal-to-noise power ratio S/N. Its capacity, C, is:

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

How can this theorem be compatible with the existence of superoscillations?

It's compatible because of the scaling laws of superoscillations!

Generalized channel capacity formula from SOs?

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

Observation:

- The Shannon Hartley theorem assumes Gaussian additive noise.
- But the scaling laws of SOs do not require any noise model!

Challenge: is there a noise-model independent Shannon-Hartley type theorem which, roughly, replaces S/N by the dynamic range?

Summary

- What are superoscillations? Faster than Fourier.
- How to make them? Multiplicatively.
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  Super-resolution, probing fast dissipation, optogenetics ...
- At what cost?
  Large dynamic ranges. Possible, e.g., in photon counts.
- What can we learn from their existence?
  * possibly deeper understanding of channel capacities
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